ES 1

$$\{p, \quad q\}$$

7. $\left\{ \frac{1}{2}\tilde{p}^{\alpha}\cos\tilde{q},\ \beta\tilde{p}^{\alpha}\operatorname{sen}\tilde{q} \right\} =$

$\{p,p\}=0$
$\{p,p\}=0$ in ordine di p, p.

$$= -\operatorname{sen}\tilde{q}\,\frac{\tilde{p}^{\alpha}}{2}\,\alpha\beta\tilde{p}^{\alpha-1}\operatorname{sen}\tilde{q} - \frac{\alpha}{2}\tilde{p}^{\alpha-1}\cos\tilde{q}\,\beta\tilde{p}^{\alpha}\cos\tilde{q} =$$

$$= -\underbrace{(\operatorname{sen}^2\tilde{q}+\cos^2\tilde{p})}_{=1}\frac{\alpha\beta}{2}\tilde{p}^{2\alpha-1}$$

$\longrightarrow$ qto deviessere cost affinchè traf canonica

$\Rightarrow \quad \alpha = 1/2$

$$\{q, p\} = \frac{\beta}{4} \quad \rightsquigarrow \quad \frac{1}{c} = \beta/4$$

(se $\beta=4$ traf. è unival.)

8. $K(\tilde{p},\tilde{q}) = c\,H(p(\tilde{p},\tilde{q}), q(\tilde{p},\tilde{p}))$

$$H = \frac{p^2}{2} + \frac{q^2}{2} \qquad p = \frac{\sqrt{\tilde{p}}}{2}\cos\tilde{q} \qquad q = \beta\sqrt{\tilde{p}}\operatorname{sen}\tilde{p}$$

$$\Rightarrow K = \frac{4}{\beta}\left(\frac{\tilde{p}}{8}\cos^2\tilde{q} + \tilde{p}\frac{\beta}{2}\operatorname{sen}^2\tilde{q} \right)$$

difficile a meno di sceglire $\beta = 1/4$

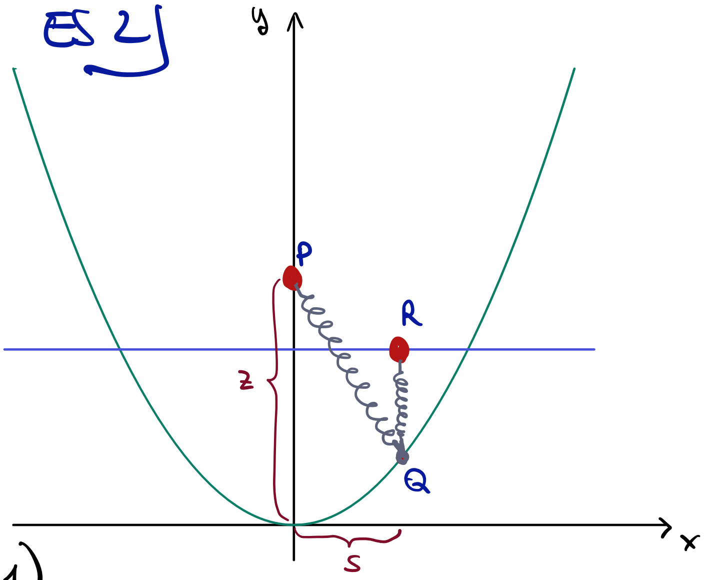
$$\Rightarrow K = \frac{\cancel{16}\,2\tilde{p}}{8} \Rightarrow \begin{cases} \dot{\tilde{p}} = 0 \\ \dot{\tilde{q}} = 2 \end{cases} \Rightarrow \begin{array}{l} \tilde{p}(t) = A^2 \\ \tilde{q}(t) = 2t + \varphi \end{array}$$

cost.

$$p(t) = \frac{A}{2}\cos(2t+\varphi) \qquad q(t) = \frac{A}{4}\operatorname{sen}(2t+\varphi)$$

(consist. con $p = \dot{q}$)

Es 2)



$x_P = 0$
$y_P = z$
$x_R = s$
$y_s = 0$

$x_Q = s$
$y_Q = \frac{s^2}{a}$

$$T = \frac{m}{2}\dot z^2 + \frac{m}{2}\dot s^2$$

1)
$$V = mg y_P + \frac{k}{2}\left(d_{PQ}^2 + d_{RQ}^2\right) =$$
$$= mgz + \frac{k}{2}\left(s^2 + \left(\frac{s^2}{a} - z\right)^2 + \left(\frac{s^2}{a} - a\right)^2 \right)$$
$$= mgz + \frac{k s^4}{a^2} + \frac{k}{2}s^2\left(\cancel{1} - \cancel{2} - 1 - 2\frac{z}{a}\right) + \frac{k}{2}z^2 + \cancel{\frac{ka^2}{2}}$$ trascuriamo perchè cost.
$$= mgz + \frac{k}{2}z^2 - \frac{k}{a}zs^2 - \frac{k}{2}s^2 + \frac{k}{a^2}s^4$$

$$L = \frac{m}{2}\dot z^2 + \frac{m}{2}\dot s^2 - mgz - \frac{k}{2}z^2 + \frac{k}{a}zs^2 + \frac{k}{2}s^2 - \frac{k}{a^2}s^4$$

2) $$\frac{d}{dt}\frac{\partial L}{\partial \dot z} - \frac{\partial L}{\partial z} : \boxed{m\ddot z + mg + kz - \frac{k}{a}s^2 = 0}$$

3) $x \to -x$ $y \to y$ $\Rightarrow$ $s \to -s$ e $z \to z$ sim. discreta, non si applica Noether.

4) $$\partial_z V = mg + kz - \frac{k}{a}s^2 = 0 \longrightarrow z = \frac{s^2}{a} - \frac{mg}{k}$$

$$\partial_s V = -2\frac{k}{a}zs - ks + 4\frac{k}{a^2}s^3 = 0$$

$$\hookrightarrow -2\frac{k}{a}s\left(\frac{s^2}{a} - \frac{mg}{k}\right) - ks + 4\frac{k}{a^2}s^3 = 0$$

$$ks\left[-2\frac{s^2}{a^2} + 2\frac{mg}{ka} - 1 + 4\frac{s^2}{a^2}\right] = 0$$

$$2Ks\left[\frac{s^2}{a^2}-\frac{1}{2}+\frac{mg}{Ka}\right]$$

$$\rightarrow \quad s=0$$

$$s=\pm s_* \quad \text{con} \quad s_*^2=\frac{a^2}{2}\left(1-\frac{2mg}{Ka}\right) \leftarrow \exists \text{ solo quando } \frac{mg}{Ka}\leq\frac{1}{2}$$

$$s_*=a\sqrt{\frac{1}{2}-\frac{mg}{Ka}}$$

$$(z,0)=\rightarrow\left(-\frac{mg}{K},\ 0\right)$$

$$\rightarrow \left(\underbrace{\frac{s_*^2}{a}-\frac{mg}{K}}_{\equiv z_*},\ \pm s_*\right) \quad \text{con} \quad z_*=a\left(\frac{1}{2}-\frac{2mg}{Ka}\right)$$

$$\partial^2 V=\begin{pmatrix} K & -2\frac{Ks}{a} \\ -2\frac{Ks}{a} & -2\frac{K}{a}z-K+12\frac{K}{a^2}s^2 \end{pmatrix}$$

$$\partial^2 V_{\left(-\frac{mg}{K},0\right)}=\begin{pmatrix} K & 0 \\ 0 & 2\frac{mg}{a}-K \end{pmatrix}$$

STAB.

≥ 0 quando $\frac{mg}{Ka}\geq\frac{1}{2}$

(instab. altrim.)

$s_*=a\sqrt{\frac{1}{2}-\frac{mg}{Ka}}$

$$\partial^2 V_{(z_*,\pm s_*)}=\begin{pmatrix} K & \mp 2K\sqrt{\frac{1}{2}-\frac{mg}{Ka}} \\ \mp 2K\sqrt{\frac{1}{2}-\frac{mg}{Ka}} & -2K\left(\frac{1}{2}-\frac{2mg}{Ka}\right)-2K\frac{1}{2}+12K\left(\frac{1}{2}-\frac{mg}{Ka}\right) \end{pmatrix}$$

$$= \begin{pmatrix} k & \mp 2k\sqrt{\frac{1}{2} - \frac{mg}{ka}} \\ \mp 2k\sqrt{\frac{1}{2} - \frac{mg}{ka}} & \left(\frac{1}{2} - \frac{mg}{ka}\right)(\underbrace{12k - 4k}_{8k}) \end{pmatrix}$$

$$Tr = k + 8k\left(\frac{1}{2} - \frac{mg}{ka}\right)$$

$$\det = 8k^2\left(\frac{1}{2} - \frac{mg}{ka}\right) - 4k^2\left(\frac{1}{2} - \frac{mg}{ka}\right) =$$

$$= 4k^2\left(\frac{1}{2} - \frac{mg}{ka}\right) \geq 0 \quad \text{x} \quad mg \leq 1/2$$

in pto con entrambi → stab.

Quindi:

- Per $\frac{mg}{ka} > \frac{1}{2}$: c'è un solo pto di equil. $\left(-\frac{mg}{k}, 0\right)$ ed è STABILE

- Per $\frac{mg}{ka} < \frac{1}{2}$: ci sono tre pti di equil.

  $\left(-\frac{mg}{k}, 0\right)$ INSTABILE

  $\left(z_*, \pm s_*\right)$ STABILI

5) $a = \frac{3mg}{K}$ $\quad \left(\frac{mg}{Ka} = \frac{1}{3} < \frac{1}{2} \right)$

$\left(-\frac{a}{2}, -S_*\right)$ $\quad S_* = a\sqrt{\frac{1}{2} - \frac{mg}{Ka}} = a\sqrt{\frac{1}{2} - \frac{1}{3}} = \frac{a}{\sqrt{6}}$

$$A = a|_{min} = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$B = \left.\begin{pmatrix} K & -2K\sqrt{\frac{1}{2} - \frac{mg}{Ka}} \\ -2K\sqrt{\frac{1}{2} - \frac{mg}{Ka}} & 8K\left(\frac{1}{2} - \frac{mg}{Ka}\right) \end{pmatrix}\right|_{\frac{mg}{Ka} = \frac{1}{3}}$$

$$= \begin{pmatrix} K & 2K/\sqrt{6} \\ 2K/\sqrt{6} & 8K/6 \end{pmatrix} = \begin{pmatrix} K & \sqrt{\frac{2}{3}}K \\ \sqrt{\frac{2}{3}}K & \frac{4}{3}K \end{pmatrix}$$

$$\det(B - \lambda A) = \det\begin{pmatrix} K - m\lambda & \sqrt{\frac{2}{3}}K \\ \sqrt{\frac{2}{3}}K & \frac{4}{3}K - m\lambda \end{pmatrix} =$$

$$= m^2\left\{\left(\frac{K}{m} - \lambda\right)\left(\frac{4K}{3m} - \lambda\right) - \frac{2}{3}\left(\frac{K}{m}\right)^2\right\} = 0$$

$$3\lambda^2 - 7\frac{K}{m}\lambda + 2\left(\frac{K}{m}\right)^2 = 0$$

$$\Delta = 49 - 4\cdot 3\cdot 2 = 25$$

$$\lambda_{1,2} = \left(\frac{7}{6} \pm \frac{5}{6}\right)\frac{K}{m} \begin{cases} \frac{1}{3}\frac{K}{m} = \frac{g}{a} \\ 2\frac{K}{m} = 6\frac{g}{a} \end{cases}$$

$$L = \frac{1}{2}(\delta\dot z, \delta\dot s) A \begin{pmatrix} \delta\dot z \\ \delta\dot s \end{pmatrix} - \frac{1}{2}(\delta z, \delta s) B \begin{pmatrix} \delta z \\ \delta s \end{pmatrix} =$$

$$= \frac{1}{2} m \delta\dot z^2 + \frac{1}{2} m \delta\dot s^2 - \frac{1}{2}(\delta z, \delta s) \begin{pmatrix} k & \sqrt{\frac{2}{3}}k \\ \sqrt{\frac{2}{3}}k & \frac{4}{3}k \end{pmatrix} \begin{pmatrix} \delta z \\ \delta s \end{pmatrix} =$$

$$= \frac{m\delta\dot z^2}{2} + \frac{m\delta\dot s^2}{2} - \frac{k}{2}\delta z^2 - \frac{2}{3}k\delta s^2 - \sqrt{\frac{2}{3}}k\,\delta z\,\delta s$$

Dove $z = z_* + \delta z$ e $s = -s_* + \delta s$

$$B = \begin{pmatrix} k & \sqrt{\frac{2}{3}}k \\ \sqrt{\frac{2}{3}}k & \frac{4}{3}k \end{pmatrix}$$

6) $\lambda_1 = \frac{1}{3}\frac{k}{m}$

$$B - \lambda_1 A = \begin{pmatrix} \frac{2}{3}k & \sqrt{\frac{2}{3}}k \\ \sqrt{\frac{2}{3}}k & k \end{pmatrix} \rightsquigarrow \ker = \Big\langle \underbrace{\begin{pmatrix} 1 \\ -\sqrt{\frac{2}{3}} \end{pmatrix}}_{= u_1} \Big\rangle$$

$\lambda_2 = \frac{2k}{m}$

$$B - \lambda_2 A = \begin{pmatrix} -k & \sqrt{\frac{2}{3}}k \\ \sqrt{\frac{2}{3}}k & -\frac{2}{3}k \end{pmatrix} \rightsquigarrow \ker = \Big\langle \underbrace{\begin{pmatrix} \sqrt{\frac{2}{3}} \\ 1 \end{pmatrix}}_{= u_2} \Big\rangle$$

$$\begin{pmatrix} z(t) \\ s(t) \end{pmatrix} = \begin{pmatrix} z_* \\ -s_* \end{pmatrix} + A_1 u_1 \cos\left(\sqrt{\frac{k}{3m}}\,t + \varphi_1\right)$$

$$+ A_2 u_2 \cos\left(\sqrt{\frac{2k}{m}}\,t + \varphi_2\right)$$

7) $\delta V = -Fs$ $\qquad L_{new} = L_{old} + Fs$

$s^2 = a^2$

$$\partial_z V_{new} = mg + kz - \frac{k}{a}s^2 = 0 \qquad z = -\frac{mg}{k} + \frac{3mg}{k} = 2mg/k$$

$$\partial_s V_{new} = -2\frac{k}{a}zs - ks + 4\frac{k}{a^2}s^3 - F$$

$$-2\frac{k}{\cancel{a}}\frac{2mg}{k}\cancel{a} - k\frac{3mg}{k} + 4k\frac{3mg}{k} - F$$

$$\Rightarrow F = mg(-4-3+12) = 5mg$$