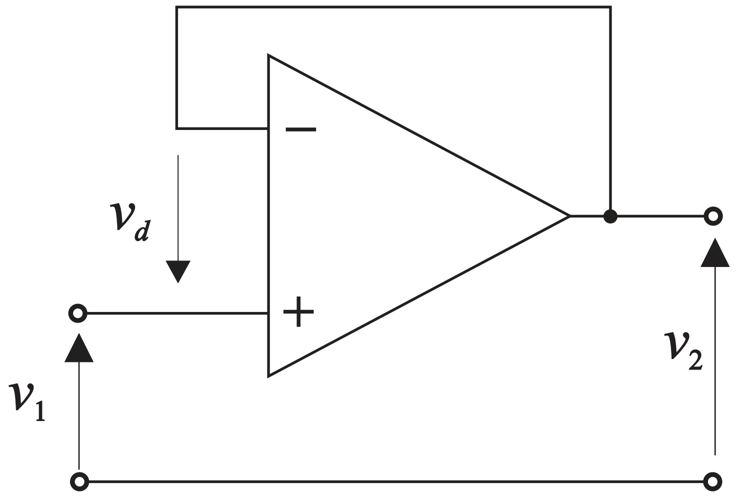
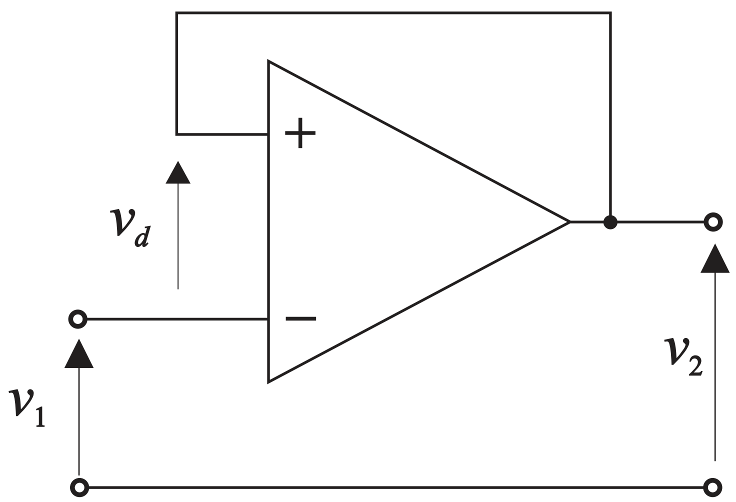
TEORIA dei CIRCUITI

– Voltage follower –



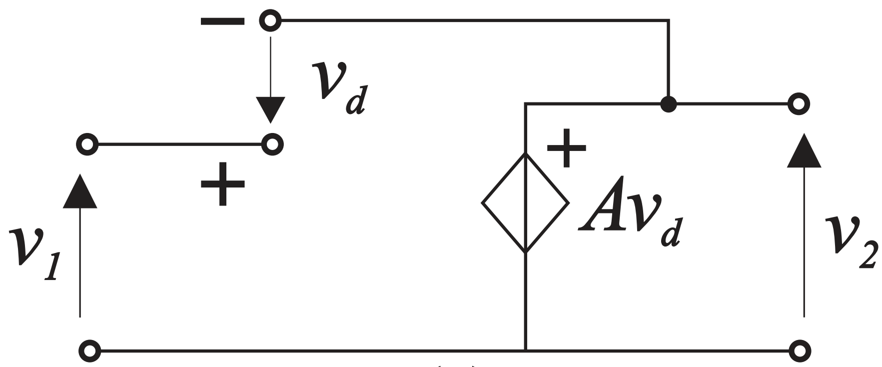
(a)



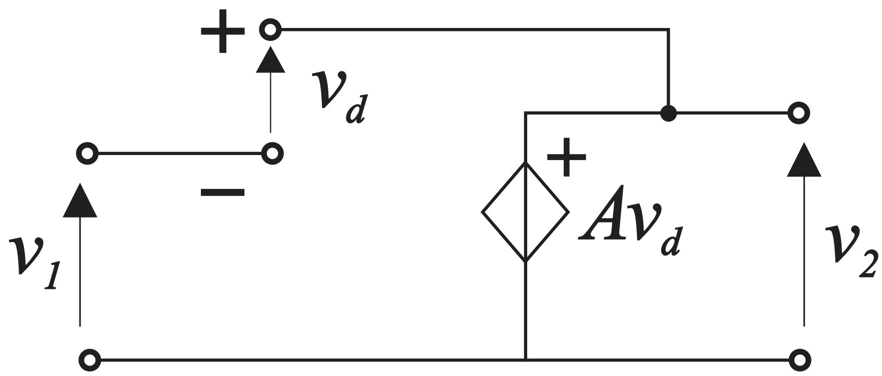
(b)

- Due “emitter follower” apparentemente identici. In realtà sono:
  a) Circuito stabile
  b) Circuito instabile

- Modelli in zona lineare:
  ($|v_2| \leq V_{sat}$, $A >> 1$):



(a)



(b)

- Equazioni del modello a)

$$\begin{cases} v_d = v_1 - v_2 \\ v_2 = Av_d \end{cases}$$

$$\Rightarrow v_2 = A(v_1 - v_2) \Rightarrow A_v = \frac{v_2}{v_1} = \frac{A}{A+1} < 1$$

$$\lim_{A\to\infty} A_v = 1 \qquad (v_d = 0)$$

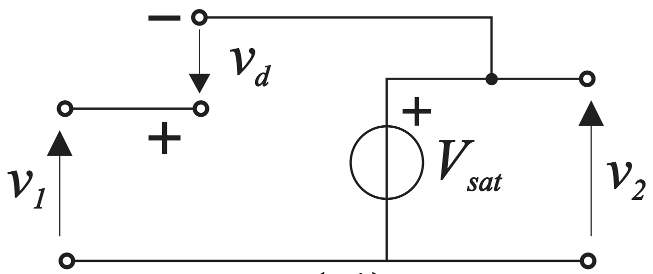
- Equazioni del modello b)

$$\begin{cases} v_d = v_2 - v_1 \\ v_2 = Av_d \end{cases}$$

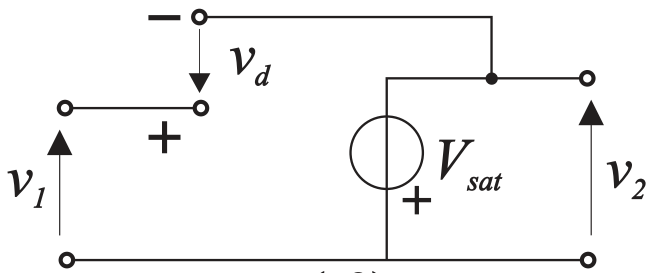
$$\Rightarrow v_2 = A(v_2 - v_1) \Rightarrow A_v = \frac{v_2}{v_1} = \frac{A}{A-1} > 1$$

$$\lim_{A\to\infty} A_v = 1 \qquad (v_d = 0)$$

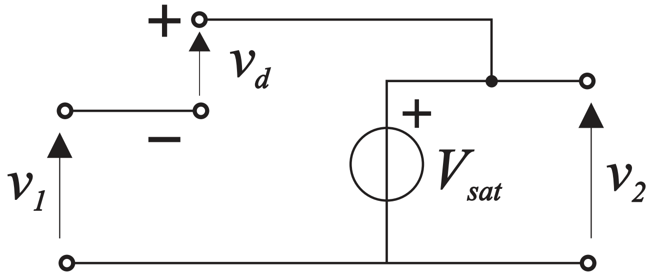
- Modelli in zona di saturazione:



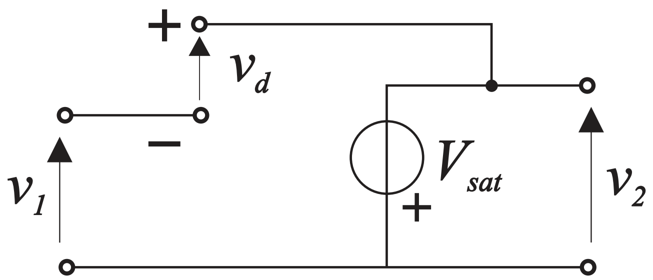
(a1)



(a2)



(b1)



(b2)

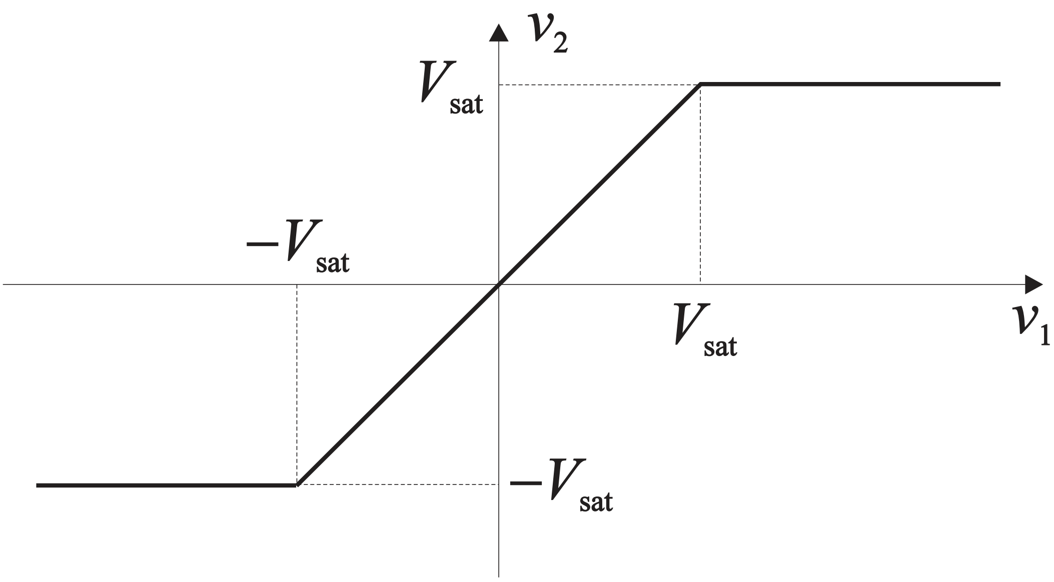
- Equazioni del modello stabile a)
- a1) $v_d = v_1 - v_2 > 0 \Rightarrow v_1 > v_2$
  - $v_1 > V_{sat}$
  - $v_2 = V_{sat}$

- a2) $v_d = v_1 - v_2 < 0 \Rightarrow v_1 < v_2$
  - $v_1 < - V_{sat}$
  - $v_2 = - V_{sat}$

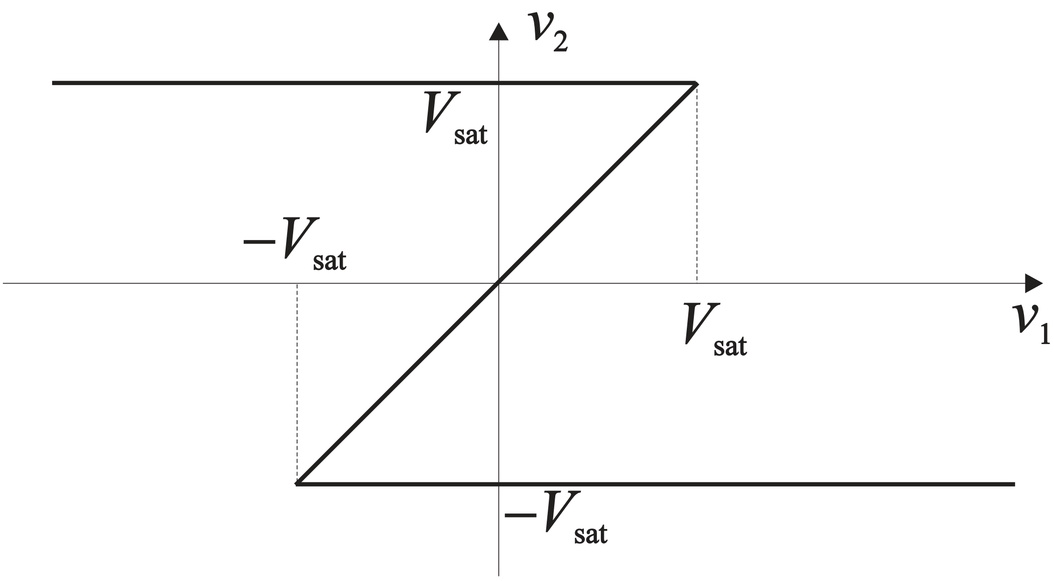
- Equazioni del modello instabile b)
- b1) $v_d = v_2 - v_1 > 0 \Rightarrow v_1 < v_2$
  - $v_1 < V_{sat}$
  - $v_2 = V_{sat}$

- b2) $v_d = v_2 - v_1 < 0 \Rightarrow v_1 > v_2$
  - $v_1 > - V_{sat}$
  - $v_2 = - V_{sat}$

- Funzione di rete resistiva dei due modelli:

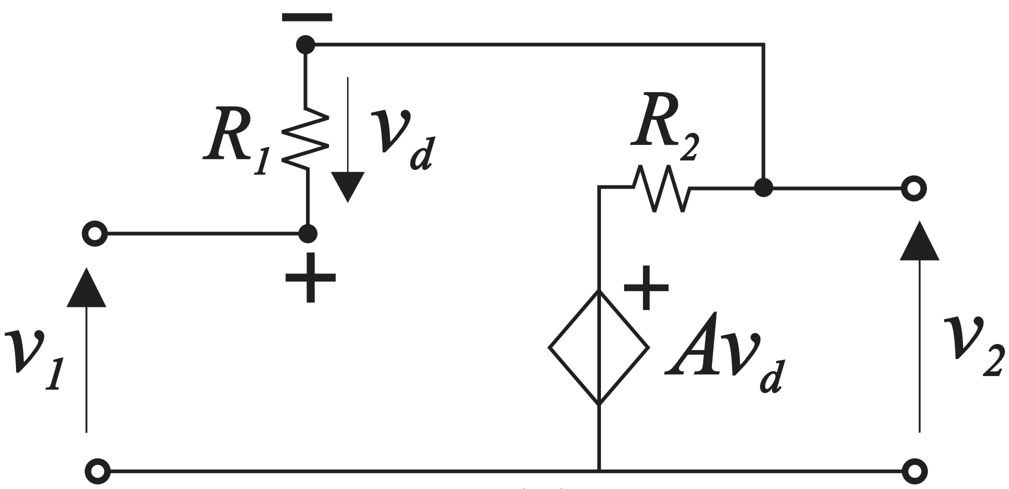


(a)

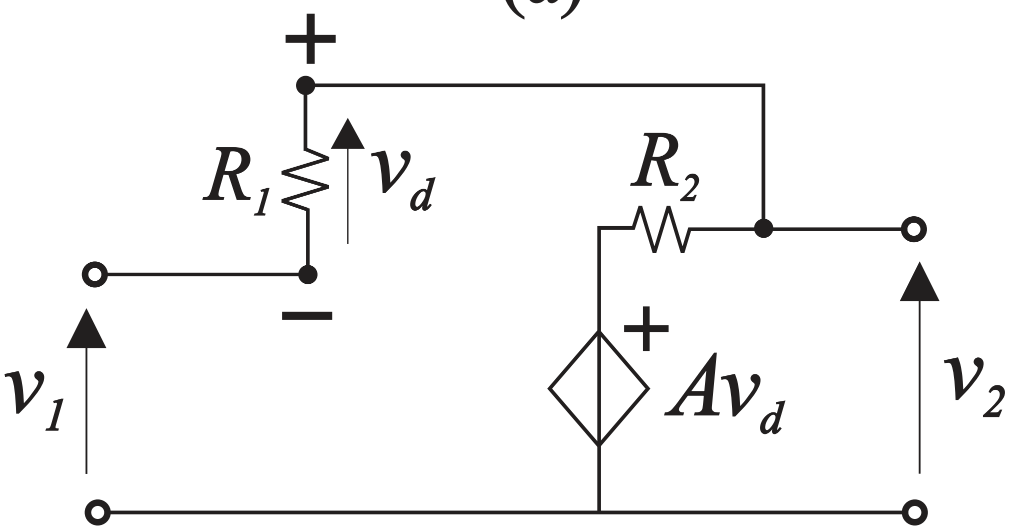


(b)

- Modelli resistivi con resistenze di ingresso e di uscita: ((a) stabile, (b) instabile)



(a)



(b)

- Equazioni del modello stabile a)

$$\begin{cases} \dfrac{v_2 - Av_d}{R_2} + \dfrac{v_2 - v_1}{R_1} = 0 \\ v_d = v_1 - v_2 \end{cases}$$

$$\frac{v_2 - Av_1 + Av_2}{R_2} + \frac{v_2 - v_1}{R_1} = 0$$

$$A_v = \frac{v_2}{v_1} = \frac{AR_1 + R_2}{(A+1)R_1 + R_2}$$

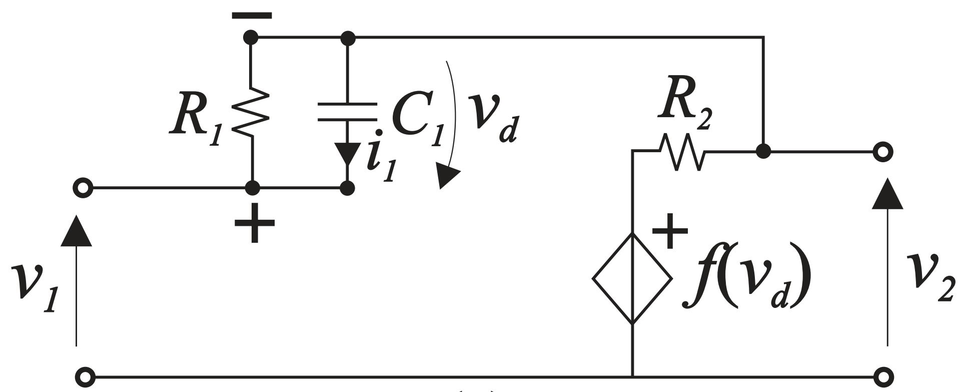
- Equazioni del modello instabile b)

$$\begin{cases} \dfrac{v_2 - Av_d}{R_2} + \dfrac{v_2 - v_1}{R_1} = 0 \\ v_d = v_2 - v_1 \end{cases}$$

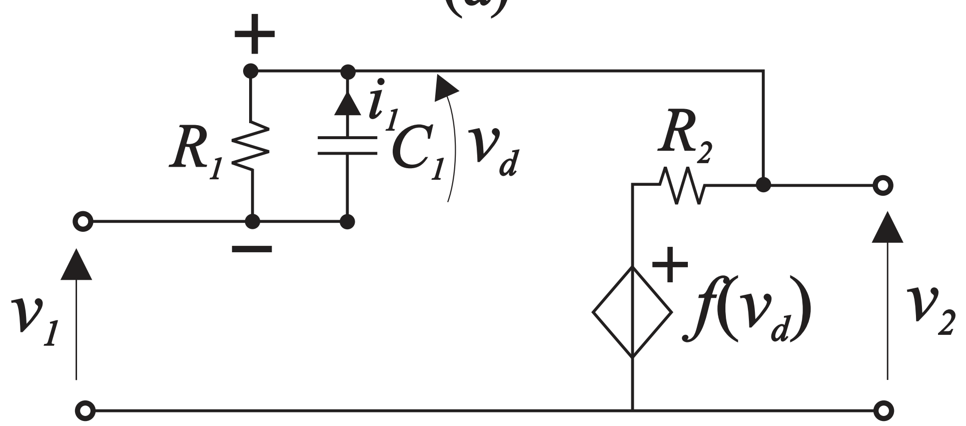
$$\frac{v_2 - Av_2 + Av_1}{R_2} + \frac{v_2 - v_1}{R_1} = 0$$

$$A_v = \frac{v_2}{v_1} = \frac{AR_1 - R_2}{(A-1)R_1 - R_2}$$

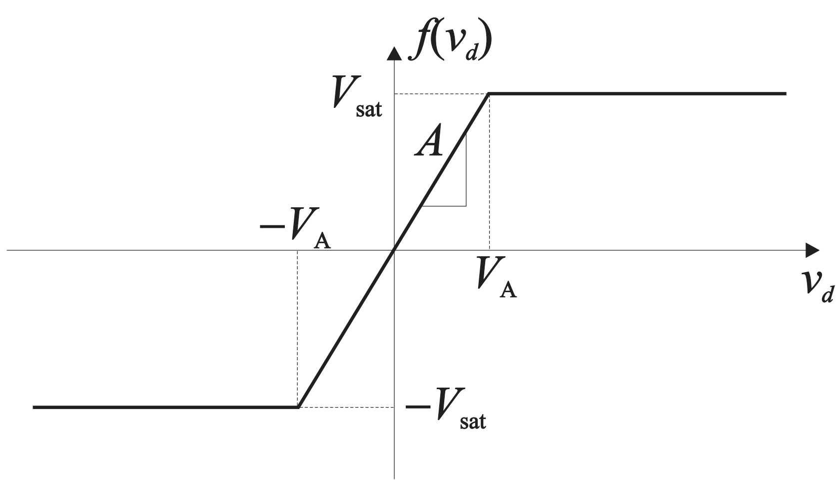
- Modello dinamico: ((a) stabile, (b) instabile)



(a)



(b)



- Equazioni del modello dinamico a):

$$\begin{cases} \dfrac{v_2(t) - f\left(v_d(t)\right)}{R_2} + \dfrac{v_2(t) - v_1(t)}{R_1} - C_1 \dfrac{\mathrm{d}v_d(t)}{\mathrm{d}t} = 0 \\ v_d(t) = v_1(t) - v_2(t) \end{cases}$$

$$\frac{v_2(t) - f\left(v_1 - v_2\right)}{R_2} + \frac{v_2 - v_1}{R_1} + C_1 \frac{\mathrm{d}(v_2 - v_1)}{\mathrm{d}t} = 0$$

$$v_1(t) = V_1; \qquad R_p = R_1 \,//\, R_2$$

$$\begin{cases} \dfrac{\mathrm{d}v_2(t)}{\mathrm{d}t} = -\dfrac{v_2(t)}{R_p C_1} + \dfrac{V_1}{R_1 C_1} + \dfrac{f\left(V_1 - v_2(t)\right)}{R_2 C_1} \\ v_2(0) = V_1 - v_d(0) = V_1 - V_{d0} \end{cases}$$

$$V_1 = 0;\ R_2 << R_1 \Rightarrow R_2 \approx R_p;\ i_1 = -C_1 \frac{\mathrm{d}v_2(t)}{\mathrm{d}t}$$

$$\Rightarrow i_1 = \frac{v_2 - f\left(-v_2\right)}{R_p} = \frac{v_2 + f\left(v_2\right)}{R_p}$$

$$\text{se}:\ i_1 > 0 \Rightarrow \frac{\mathrm{d}v_2(t)}{\mathrm{d}t} < 0; \qquad i_1 < 0 \Rightarrow \frac{\mathrm{d}v_2(t)}{\mathrm{d}t} > 0$$

- Equazioni del modello dinamico b):

$$\begin{cases} \dfrac{v_2(t) - f\left(v_d(t)\right)}{R_2} + \dfrac{v_2(t) - v_1(t)}{R_1} + C_1 \dfrac{\mathrm{d}\left(v_d(t)\right)}{\mathrm{d}t} = 0 \\ v_d(t) = v_2(t) - v_1(t) \end{cases}$$

$$\frac{v_2(t) - f\left(v_2 - v_1\right)}{R_2} + \frac{v_2 - v_1}{R_1} + C_1 \frac{\mathrm{d}(v_2 - v_1)}{\mathrm{d}t} = 0$$

$$v_1(t) = V_1; \qquad R_p = R_1 \,//\, R_2$$

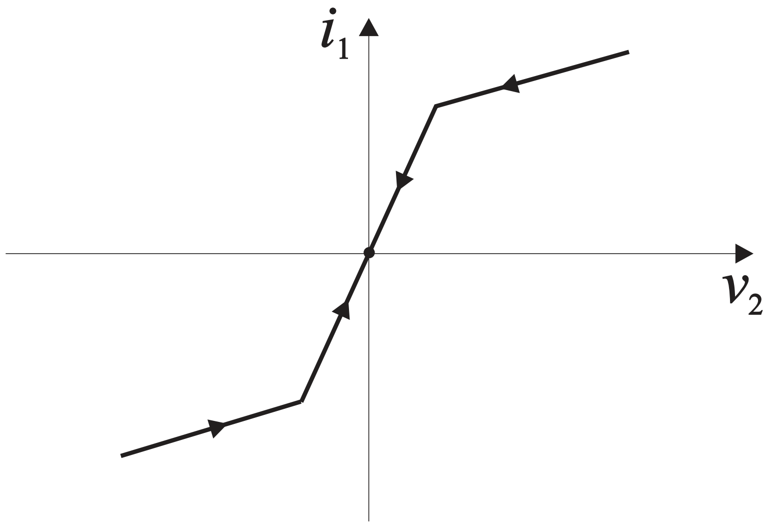
$$\begin{cases} \dfrac{\mathrm{d}v_2(t)}{\mathrm{d}t} = -\dfrac{v_2(t)}{R_p C_1} + \dfrac{V_1}{R_1 C_1} + \dfrac{f\left(v_2(t) - V_1\right)}{R_2 C_1} \\ v_2(0) = V_1 + v_d(0) = V_1 + V_{d0} \end{cases}$$

$$V_1 = 0;\ R_2 << R_1 \Rightarrow R_2 \approx R_p;\ i_1 = -C_1 \frac{\mathrm{d}v_2(t)}{\mathrm{d}t}$$

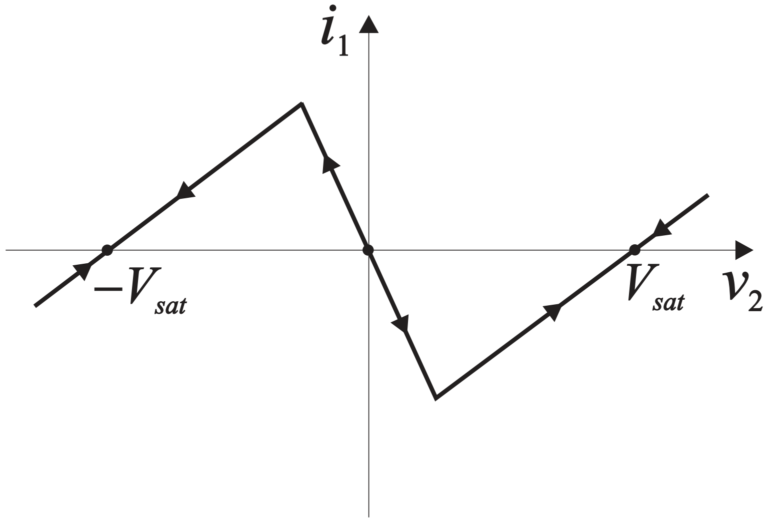
$$\Rightarrow i_1 = \frac{v_2 - f\left(v_2\right)}{R_p} = \frac{v_2 - f\left(v_2\right)}{R_p}$$

$$\text{se}:\ i_1 > 0 \Rightarrow \frac{\mathrm{d}v_2(t)}{\mathrm{d}t} < 0; \qquad i_1 < 0 \Rightarrow \frac{\mathrm{d}v_2(t)}{\mathrm{d}t} > 0$$

- La dynamic route nei due casi è:



(a)



(b)

- Caso a): 1 punto stabile nell’origine
- Caso b): 1 punto instabile nell’origine e 2 punti stabili sugli assi

- Caso a)
- Transitorio per $V_1 = 0$ ($R_2 \cong R_p$; $C_1 R_p = \tau$):

$$\begin{cases} \dfrac{\mathrm{d}v_2(t)}{\mathrm{d}t} = -\dfrac{v_2(t)}{\tau} + \dfrac{f\left(-v_2(t)\right)}{\tau} = -\dfrac{v_2(t)}{\tau} - \dfrac{f\left(v_2(t)\right)}{\tau} \\ v_2(0) = V_{20}(= -V_{d0});\ V_{20} > V_A;\ t \geq 0 \end{cases}$$

$$\frac{\mathrm{d}v_2(t)}{\mathrm{d}t} = -\frac{v_2(t)}{\tau} - \frac{V_{sat}}{\tau}$$

$$\Rightarrow \begin{cases} v_2(t) = \left[V_{20} + V_{sat}\right] e^{-\frac{t}{\tau}} - V_{sat} \\ t \geq 0,\ v_2(t) \geq V_A \end{cases}$$

$$v_2(t_A) = V_A = \left[V_{20} + V_{sat}\right] e^{-\frac{t_A}{\tau}} - V_{sat}$$

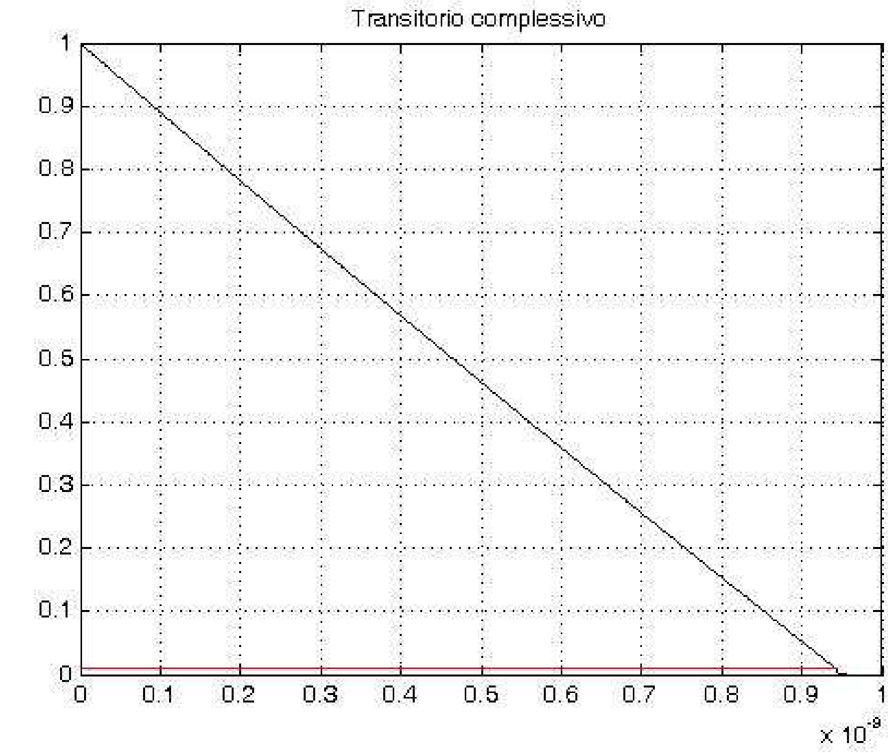
$$\Rightarrow t_A = -\tau \ln\left(\frac{V_A + V_{sat}}{V_{20} + V_{sat}}\right) \Rightarrow \Delta t = t - t_A$$

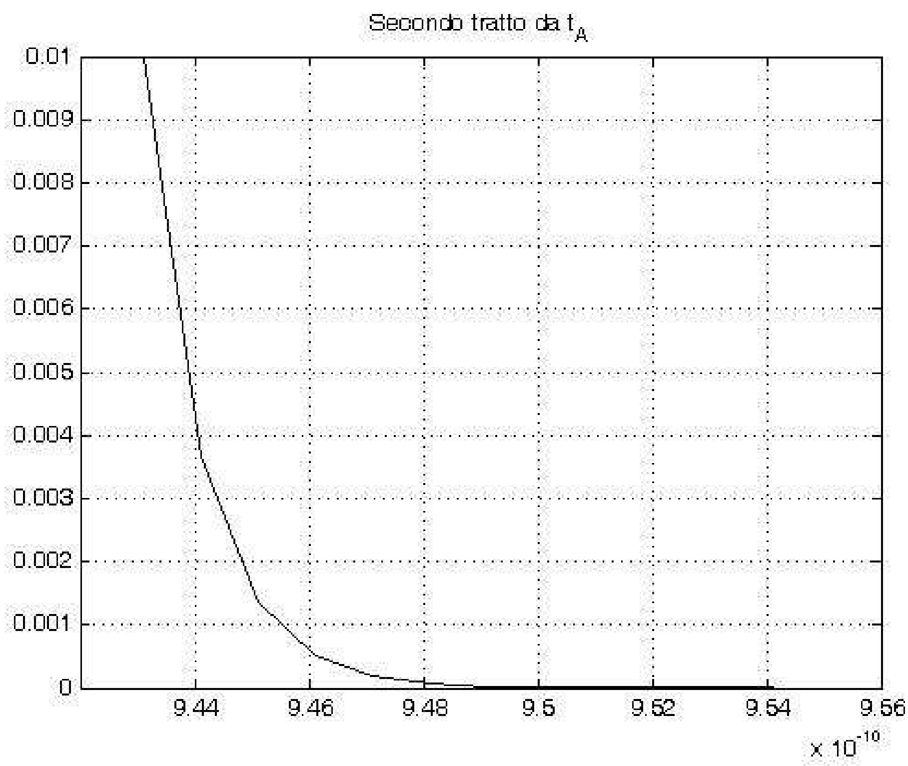
$$\begin{cases} \dfrac{\mathrm{d}v_2(\Delta t)}{\mathrm{d}t} = -\dfrac{v_2(\Delta t)}{\tau} - \dfrac{A v_2(\Delta t)}{\tau} = -\dfrac{v_2(\Delta t)}{\tau}\left(1 + A\right) \\ v_2(0) = V_A;\ \Delta t \geq 0 \end{cases}$$

$$\Rightarrow \begin{cases} v_2(t - t_A) = V_A\, e^{-\frac{(t - t_A)}{\tau}(1+A)} \\ t \geq t_A \end{cases}$$

- Caso a): esempio di transitorio per:
- A = 10'000; V_{sat} = 10; V_A = 0.01, V_{20} = 1;

  $C = 1$nF,R_p = 10 Ω; $\tau = 10^{-8} \Rightarrow t_A = 9.4\ 10^{-10}$





- Caso b)
- Transitorio per $V_1 = 0$ ($R_2 \cong R_p$; $C_1 R_p = \tau$):

$$\begin{cases} \dfrac{\mathrm{d}v_2(t)}{\mathrm{d}t} = -\dfrac{v_2(t)}{\tau} + \dfrac{f\left(v_2(t)\right)}{\tau} \\ v_2(0) = V_{20} (= V_{d0}); \; 0 < V_{20} < V_A; \; t \geq 0 \end{cases}$$

$$\frac{\mathrm{d}v_2(t)}{\mathrm{d}t} = -\frac{v_2(t)}{\tau} + \frac{Av_2(t)}{\tau} = \frac{v_2(t)}{\tau}\left(A-1\right)$$

$$\Rightarrow \begin{cases} v_2(t) = V_{d0}\, e^{\frac{t}{\tau}(A-1)} \\ t \geq 0, \; v_2(t) \leq V_A \end{cases}$$

$$v_2(t_A) = V_A = V_{d0} e^{\frac{t_A}{\tau}(A-1)} \Rightarrow t_A = \frac{\tau}{A-1} \ln\left(\frac{V_A}{V_{d0}}\right)$$

$$\Delta t = t - t_A \Rightarrow \begin{cases} \dfrac{\mathrm{d}v_2(\Delta t)}{\mathrm{d}\Delta t} = -\dfrac{v_2(\Delta t)}{\tau} + \dfrac{V_{sat}}{\tau} \\ v_2(0) = V_A; \; \Delta t \geq 0 \end{cases}$$

$$\Rightarrow \begin{cases} v_2(t - t_A) = \left[V_A - V_{sat}\right] e^{-\frac{(t-t_A)}{\tau}} + V_{sat} \\ t \geq t_A \end{cases}$$

- Caso b): esempio di transitorio per:
- A = 10'000; V_{sat} = 10; V_A = 0.01, V_{20} = 0.001; $C = 1$nF,R_p = 10 Ω;τ = $10^{-8} \Rightarrow t_A = 2.3\ 10^{-12}$