COSMIC CHRONOMETERS

For a review: https://arxiv.org/pdf/2201.07241.pdf

STANDARD CLOCKS AND CHRONOMETERS

Given how the age of the Universe scales as a function of redshift:

$$t(z) = \frac{1}{H_0} \int_0^z \frac{dz'}{E(z')(1+z')}$$

$$E(z) = \frac{H(z)}{H_0} = \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}$$

finding the oldest objects at each redshift it is possible to use them to constrain the Hubble parameter. In other words, we can use standard clocks – i.e. objects whose absolute age is known – to constrain cosmology.

Or we can consider:

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt}$$

By measuring the differential age of the Universe (how much the Universe has aged between two redshifts) it is possible to obtain a direct determination of the expansion rate H(z). The main difference here is that instead of looking for some standard clocks, we will be looking for standard chronometers, a homogeneous population of objects with a synchronized formation, i.e. whose clocks started "ticking" at the same time and that are therefore optimal tracers of the differential age evolution of the Universe

STANDARD CHRONOMETERS

The pillars of the cosmic chronometers method are:

• Selection of a population of optimal cosmic chronometers, i.e. a population of objects able to trace homogeneously how much the Universe has aged between two redshifts.

• Robust measurement of the differential age dt (the advances in spectroscopic surveys makes the measurement of dz remarkably accurate when a spectroscopic redshift is available – typically $\delta z/(1 + z) \lesssim 10^{-3}$).

→ The most elementary objects in the Universe that we can date and that can be found from the local Universe up to high redshifts are galaxies. To apply the cosmic chronometer method, the idea is therefore to find at each redshift the oldest population of galaxies available. Massive passively evolving galaxies are among the best candidates: having formed most of their mass (>90%) at very high redshifts (z> 2-3), in a very quick episode of star formation, and having mostly exhausted their gas reservoir are expected to evolve passively as a function of cosmic time.

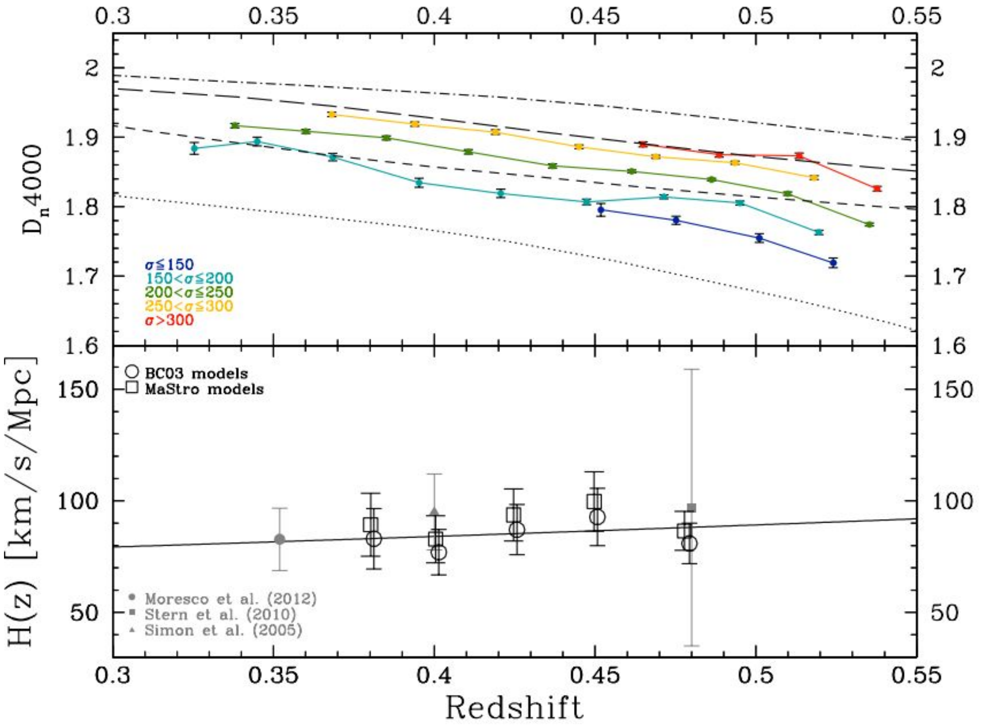
STANDARD CHRONOMETERS

When observed at cosmic times considerably later than their formation epoch, the age evolution of ETGs stars serves a clock that is synchronized with the evolution of cosmic time.

To minimize the dependence of the age estimate on evolutionary stellar population synthesis (EPS) models, one can study a direct observable of galaxy spectra, the 4000 ˚A break (D4000).

$$D_n4000 = A(SFH, Z/Z_\odot) \cdot \mathrm{age} + B,$$

$$H(z) = -\frac{1}{1+z} A(SFH, Z/Z_\odot) \frac{dz}{dD_n4000}$$



https://arxiv.org/pdf/1601.01701

STANDARD CHRONOMETERS

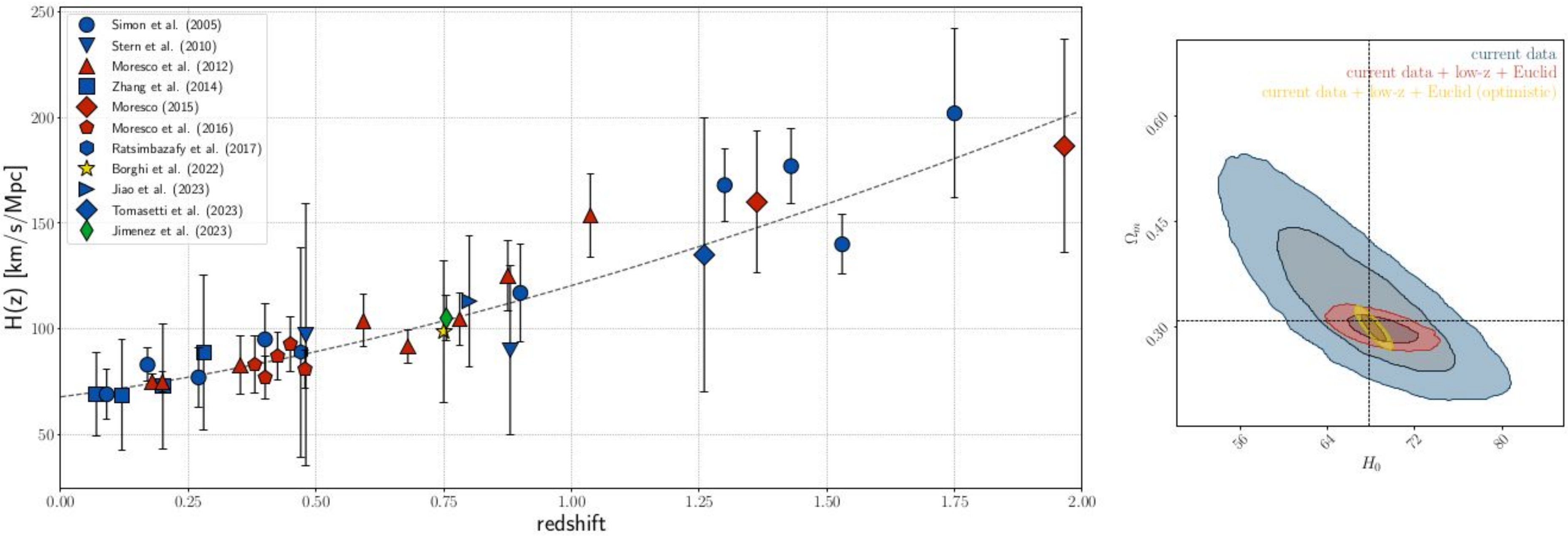


Figure 5: Hubble parameter measurements obtained with the CC method. Different colors refer to different methods adopted to estimate dt, as presented in Tab. 1. The dashed line shows the flat ΛCDM cosmological model from Planck Collaboration et al. (2020a) as a pure illustrative reference.

STANDARD CLOCKS AND CHRONOMETERS

Advantages

- **Model-independent**: No assumption on the underlying cosmological model.
- Complementary to other probes like **BAO**, **SNe Ia**, and **CMB**.
- Useful at **intermediate redshifts (z ~ 0.1–2)**, where other probes are sparse.

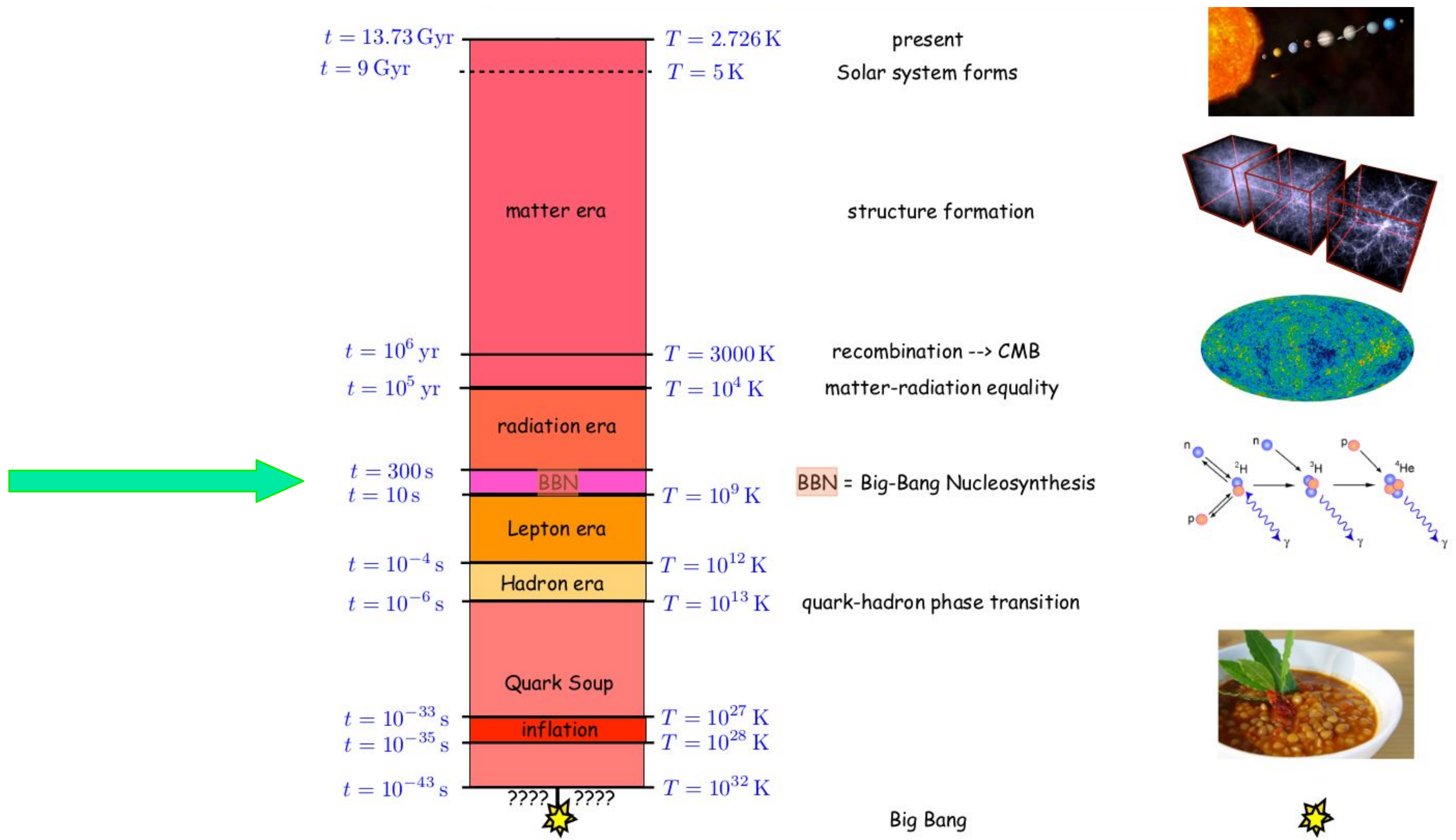
Challenges

- Requires accurate age-dating of galaxies (affected by metallicity, star formation history).
- Systematics in stellar population modeling must be controlled.

BIG BANG NUCLEOSYNTHESIS

For a review: https://arxiv.org/pdf/astro-ph/0511534.pdf or https://arxiv.org/pdf/astro-ph/0601514.pdf

THERMAL HISTORY OF THE UNIVERSE



PRIMORDIAL NUCLEOSYNTHESIS

Primordial nucleosynthesis takes place in the first 3 minutes of life of the Universe, and it is a crucial piece of evidence in favor of standard hot big bang model:

At a temperature of kT>1MeV the rate of weak interactions, $\Gamma_{n\leftrightarrow p,}$ is higher than the expansion rate of the universe, H; p, n, e are in equilibrium vai the reactions:

$$e^+ + n \longleftrightarrow p + \bar{\nu}_e$$
$$\nu_e + n \longleftrightarrow p + e^-$$

At temperatures below 1 MeV (t~1 s), neutrinos decouple and the weak interactions are frozen out, $\Gamma_{n\leftrightarrow p}$ < H; neutrons and protons cease to interconvert. The equilibrium abundance of neutrons at this temperature is about 1/6 the abundance of protons (due to the slightly larger neutron mass). The neutrons have a finite lifetime (τ = 890 s) that is somewhat larger than the age of the universe at this epoch, t(1 MeV) ≈ 1 s, but they begin to gradually decay into protons and leptons (β-decay) until the neutron-to-proton ratio has dropped to ~1/7.

PRIMORDIAL NUCLEOSYNTHESIS

Protons and neutrons can combine to form ^4He through the chain of reactions:

$$p+n \leftrightarrow {}^2H+\gamma \qquad {}^2H+n \rightarrow {}^3H+\gamma \qquad {}^3H+p \rightarrow {}^4He+\gamma$$

$$ {}^2H+p \rightarrow {}^3He+\gamma \qquad {}^3He+n \rightarrow {}^4He+\gamma$$

The bottleneck of these reactions is the formation of Deuterium, which is destroyed by energetic photons, until their number, n_γ^{diss} , becomes comparable with the number of baryons, n_b, at kT~0.1 MeV (T~10^9 K). At this epoch, t_{BBN}~150s, D is not destroyed and basically all the neutrons which are not decayed forms ^4He nuclei. The (mass) abundance of ^4He is determined mainly by:

- **The temperature at which neutrino decouples and the n-to-p ratio at frozen, $\Gamma_{n\leftrightarrow p}$ ~ H, which in turns depends on the total number of neutrino species.**
- **The mean neutron lifetime (~889 s)**
- **The baryon-to-photon ratio , $\eta = n_b / n_\gamma = 2.7 \times 10^{-8}\, \Omega_b h^2$, which determines t_{BBN}**

Further reactions lead to the formation of ^{7}Li:

$$^4He + ^3H \leftrightarrow ^7Li + \gamma \qquad ^4He + ^3He \leftrightarrow ^7Be + \gamma \qquad ^7Be + \gamma \leftrightarrow ^7Li + p$$

with a relative abundance compared to the hydrogen of ~10^{-9} - 10^{-10}. Heavier elements cannot be synthesized because:

- There are no stable isotopes with mass numbers 5 or 8, in particular ^{8}Be is unstable.

- The density and temperature is too low for the triple-alpha process that could form ^{12}C to occur.

All the other heavier elements (C, N, O, Fe) are formed by thermonuclear processes inside stars

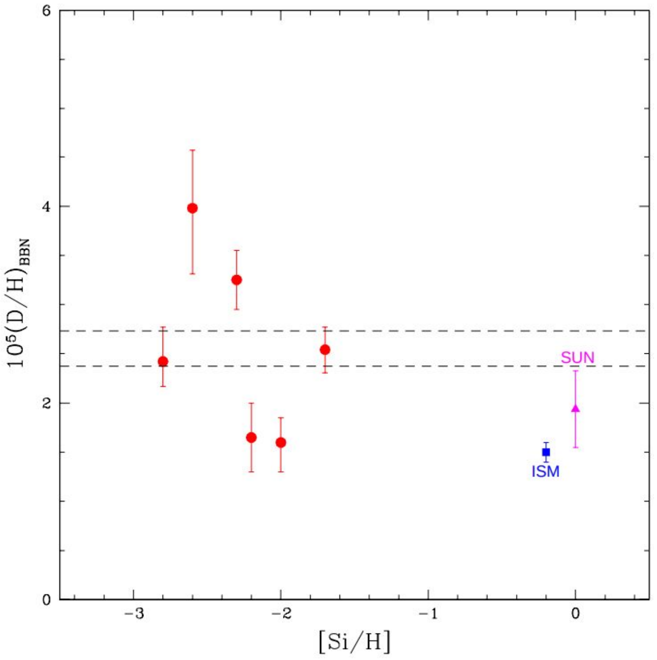
PRIMORDIAL NUCLEOSYNTHESIS

Deuterium is the baryometer of choice since its post-BBN evolution is simple (and monotonic!):

There are no astrophysical process that produce D and, as the most weakly bound of the light nuclides, any deuterium cycled through stars is burned to ^{3}He. Thus, deuterium observed anywhere, anytime, should provide a lower bound to the primordial D abundance.

For "young" systems at high redshift and/or with very low metallicity, which have experienced very limited stellar evolution, the observed D abundance should be close to the primordial value.

Thus, although there are observations of deuterium in the solar system and the interstellar medium (ISM) of the Galaxy which provide interesting lower bounds to the primordial abundance, it is the observations of relic D in a few, high redshift, low metallicity, quasars absorption line systems which are of most value in enabling estimates of its primordial abundance



Deuterium abundance in intergalactic neutral hydrogen clouds at high redshift (observed as absorption lines in quasar spectra). Dashes lines indicate what is expected from WMAP CMB analysis.

PRIMORDIAL NUCLEOSYNTHESIS

- Observations of ^{3}He, are restricted to the solar system and HII region of our Galaxy. The post-BBN evolution of ^{3}He, involving competition among stellar production, destruction, and survival, is considerably more complex and model dependent than that of D.

- The post-BBN evolution of ^{4}He is quite simple. As gas cycles through generations of stars, hydrogen is burned to helium-4 (and beyond), increasing the ^{4}He abundance above its primordial value. The key data for inferring its primordial abundance are provided by observations of helium and hydrogen emission (recombination) lines from low-metallicity, extragalactic H II regions

- In the post-BBN universe ^{7}Li is produced in the Galaxy by cosmic ray spallation and (at least in some) stars. Therefore, in order to probe the BBN yield of ^{7}Li, it is necessary to restrict attention to the oldest, most metal-poor halo stars in the halo of our galaxy

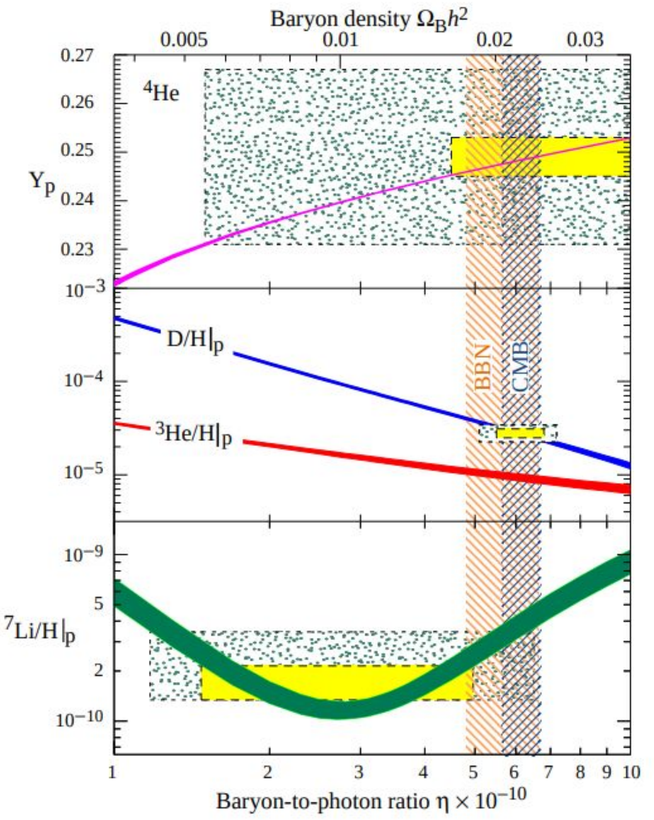
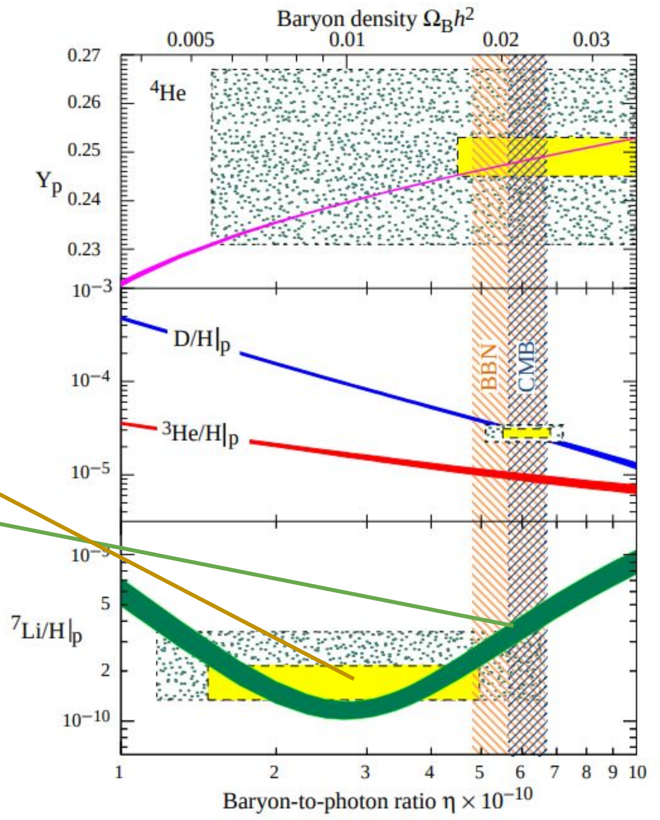


Figure 1.1: The abundances of ^{4}He, D, ^{3}He and ^{7}Li as predicted by the standard model of big-bang nucleosynthesis. Boxes indicate the observed light element abundances (smaller boxes: 2σ statistical errors; larger boxes: $\pm 2\sigma$ statistical and systematic errors). The narrow vertical band indicates the CMB measure of the cosmic baryon density. See full-color version on color pages at end of book.

PRIMORDIAL NUCLEOSYNTHESIS

- **^{7}Li problem: BBN predictions** (based on the standard ΛCDM model and baryon density from the CMB) **overestimate the primordial abundance of ^{7}Li** by a factor of **~3** compared to what we **observe in old, metal-poor (Population II) stars** in the galactic halo:

$$\left(\frac{^7\mathrm{Li}}{\mathrm{H}}\right)_{\mathrm{BBN}} \sim 5 \times 10^{-10} \qquad \left(\frac{^7\mathrm{Li}}{\mathrm{H}}\right)_{\mathrm{obs}} \sim 1.6 \times 10^{-10}$$



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Proposed Solutions (none fully successful yet):

- **Astrophysical**: ^{7}Li depletion in stars (e.g. diffusion, mixing, destruction)

  → but no known stellar process explains the uniform low abundance in old stars (Spite plateau).

- **Nuclear physics**: Re-evaluating nuclear reaction rates (e.g., ^{7}Be destruction)

  → but updated measurements haven't fixed the problem.

See e.g.: https://arxiv.org/pdf/1801.08023

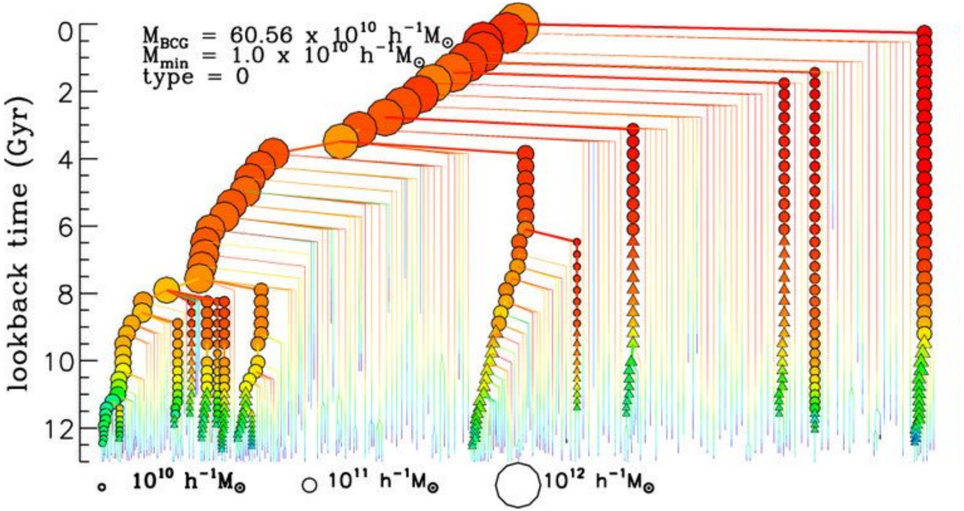
STATISTICAL PROPERTIES OF THE LARGE SCALE STRUCTURES:

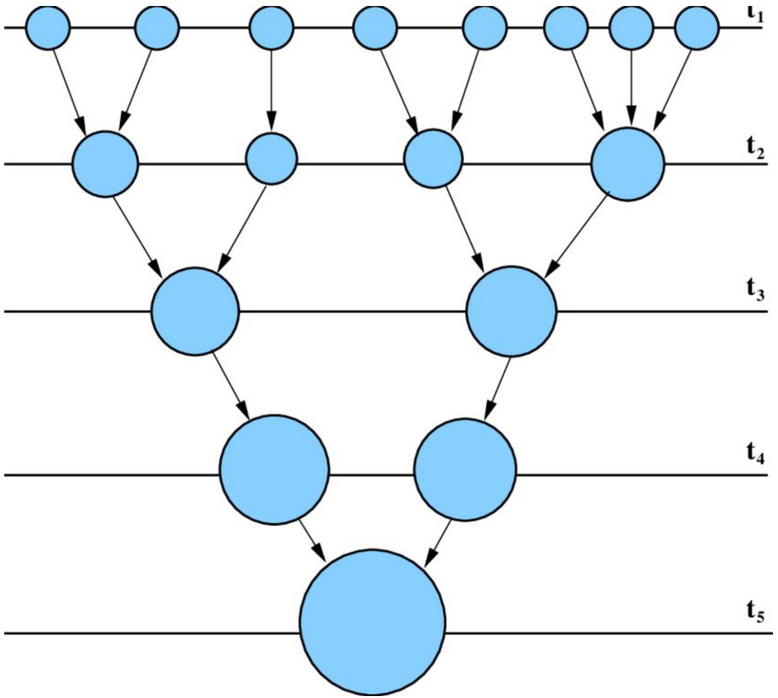
CLUSTER NUMBER COUNTS

For a review: **Allen+2011** **or** **Kravtsov+2012**

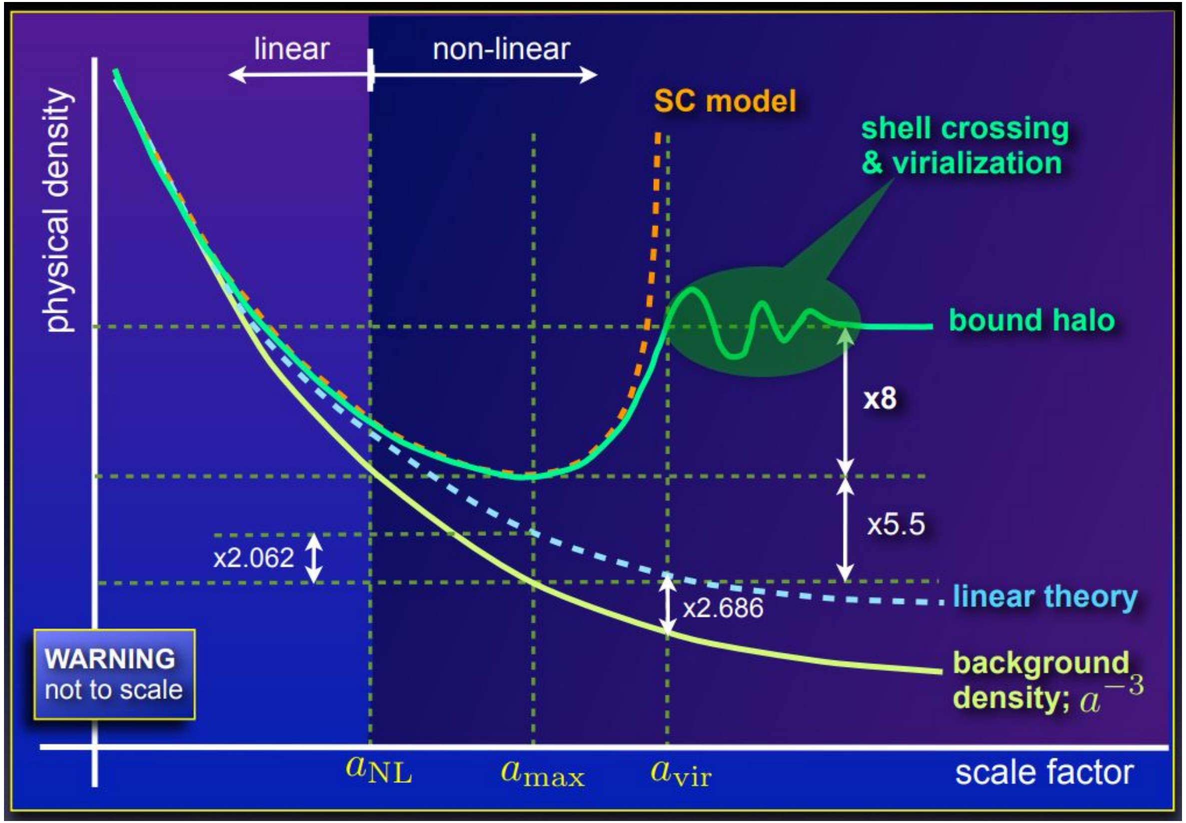
STRUCTURE FORMATION: DARK MATTER HALOS

In the LCDM scenario, structures grow *hierarchically*: Small overdensities are able to overcome the cosmological expansion and collapse first, and the resulting dark matter "halos" merge together to form larger halos which serve as sites of galaxy and galaxy cluster formation





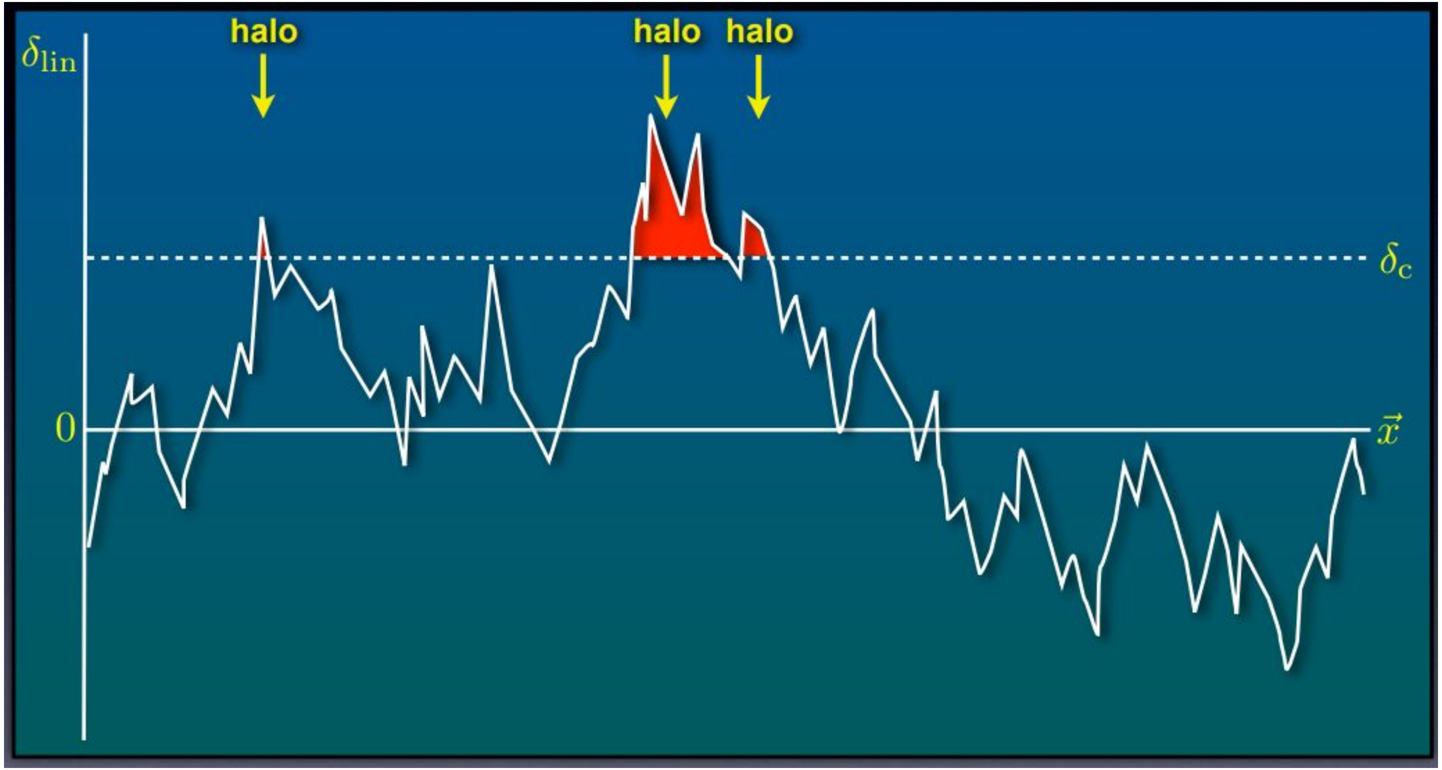
STRUCTURE FORMATION: SPHERICAL COLLAPSE MODEL



We can follow the collapse of a spherical overdensity in a homogeneous universe. SC model becomes inaccurate (brakes down) shortly after turn-around it is still a useful model to identify important epochs in the linearly evolved density field.

- **The linearly extrapolated density field collapses when $\delta_{lin} = \delta_c = 1.686$**
- **Virialized dark matter haloes have an average overdensity of $\Delta_{vir} = 178$**

STRUCTURE FORMATION: SPHERICAL COLLAPSE MODEL



According to the spherical collapse model, regions with $\delta(x,t) > \delta_c \simeq 1.686$ will have collapsed to produce dark matter haloes by time *t*.

Assuming a Gaussian density field, the fraction of cosmic volume with $\delta(x,t) > \delta_c$ depends only on the variance of the matter density field:

$$\sigma^2 = \langle \delta^2 \rangle$$

GALAXY CLUSTERS

- **Most massive bound objects in the Universe**:

  $M \simeq 10^{13} - 10^{15}\, M_{\odot}$ and $R \simeq 1 - 5$ Mpc

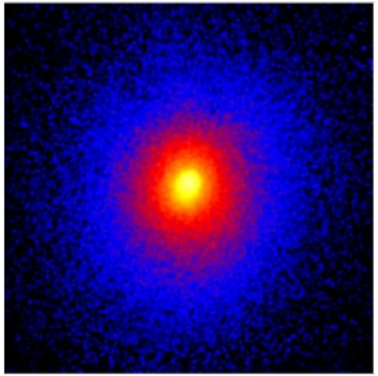
- **Multi-component systems**:

  Galaxies and stars (~5%), ICM (~15%), DM (~80%)

OPTICAL

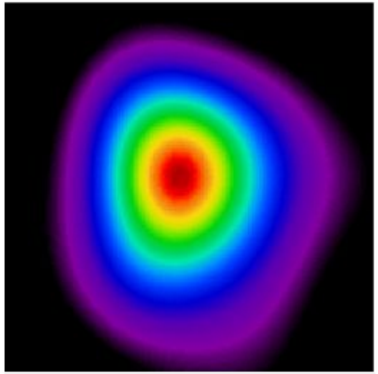
RICHNESS, LENSING EFFECTS

X-RAYS

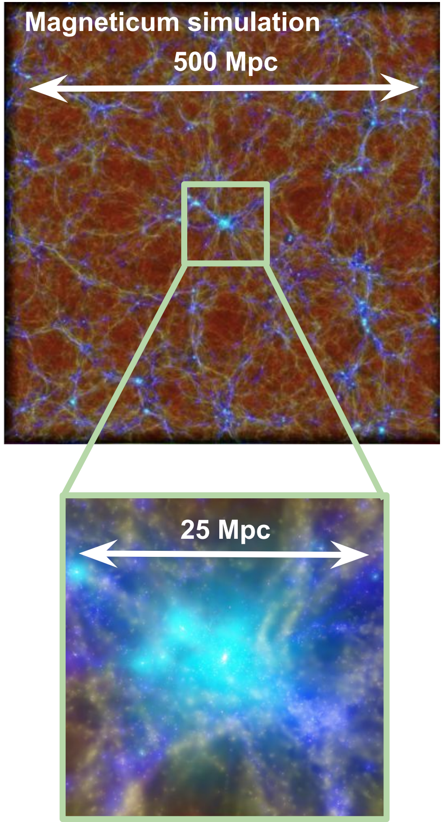


LUMINOUS AND EXTENDED X-RAY SOURCES

MICROWAVES



SUNYAEV-ZEL'DOVICH EFFECT



From Hirschmann+2014

GALAXY CLUSTERS AS COSMOLOGICAL PROBE

The abundance and spatial distribution of galaxy clusters are sensitive to the growth rate of cosmic structures and expansion history of the Universe

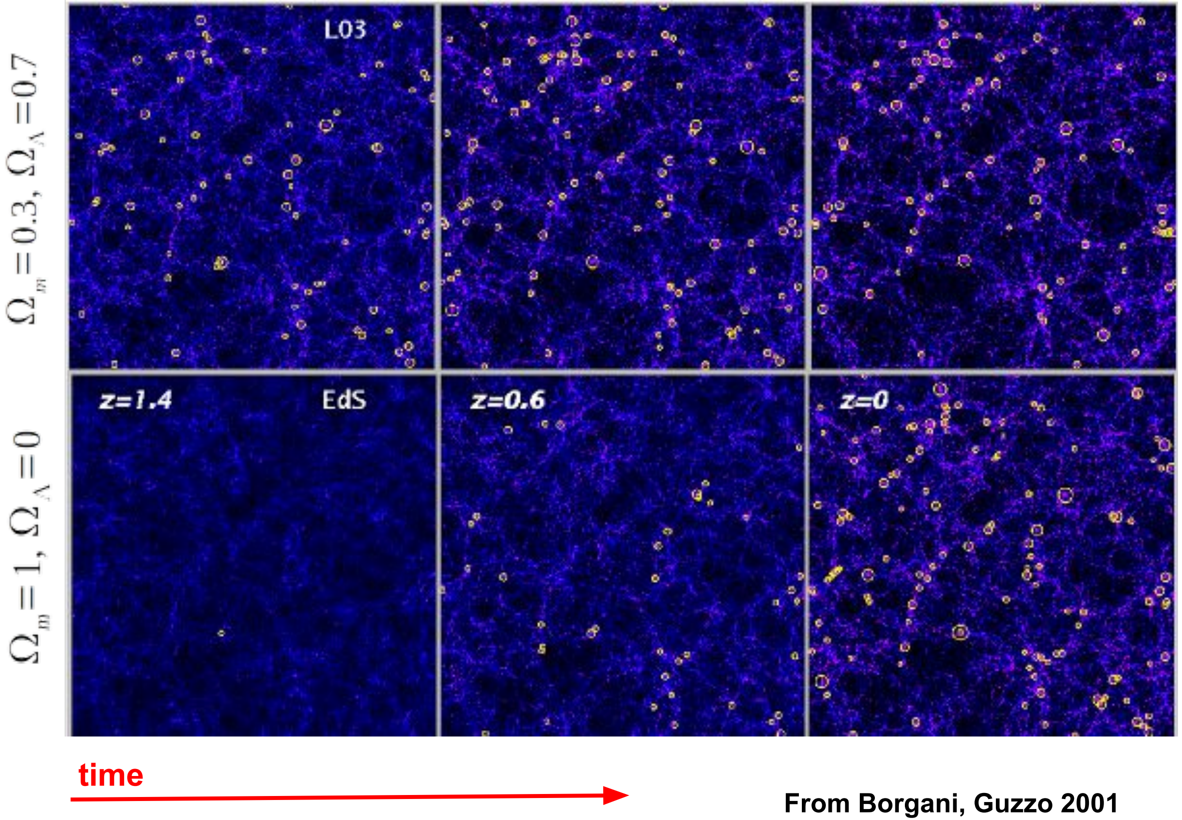
σ_8: Amplitude of the matter power spectrum

Ω_m: Present-day total matter density

$$\boxed{S_8 = \sigma_8 \, (\Omega_m / 0.3)^{0.5}}$$

Dark energy equation of state parameter *w*
Total neutrino mass
Deviation from GR
....

Evolution of the clusters population in 2 N-body simulations



From Borgani, Guzzo 2001

THE HALO MASS FUNCTION

Cluster abundance:

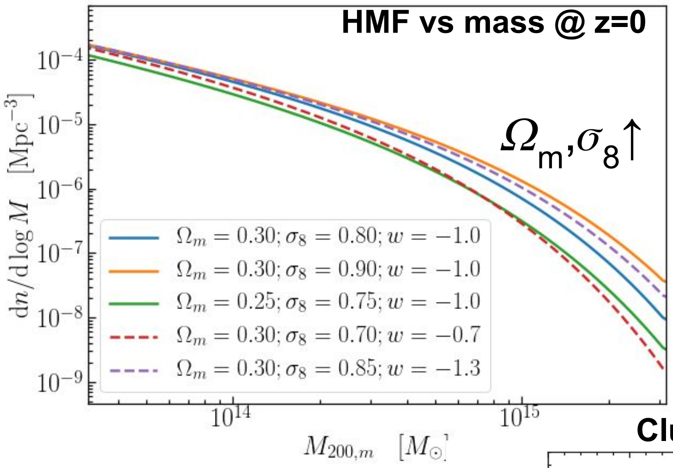
$$\frac{dN}{dzd\Omega} = \underbrace{\frac{dV}{dzd\Omega}}_{\text{geometry}} \underbrace{n(M, z)}_{\text{growth}}$$

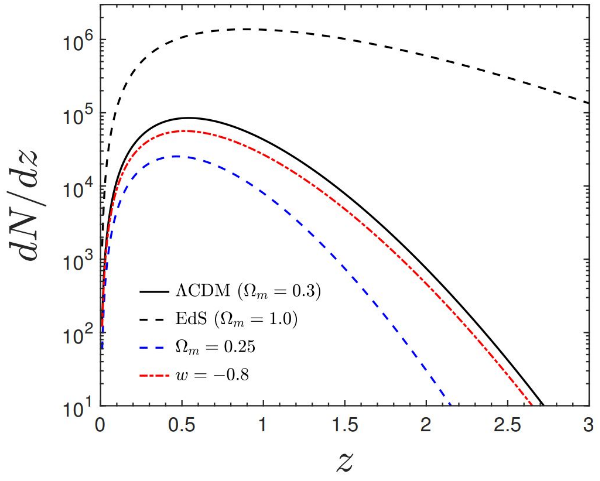
The halo mass function:

$$n(z, M) = \frac{\rho_m}{M} f(\sigma) \frac{d\ln(\sigma^{-1})}{dM}$$

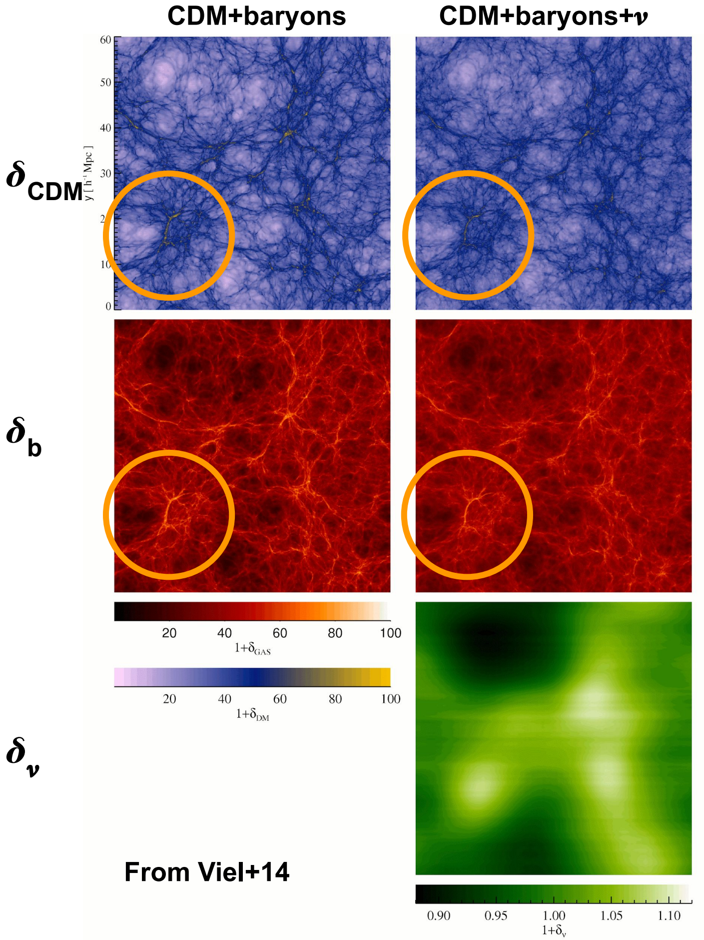
Variance of the density field:

$$\sigma(z, R) = \frac{1}{2\pi^2}\int_0^\infty dk\, k^2 \underbrace{\boxed{P_m(z, k)}}_{\text{Matter power spectrum}} |W(kR)|^2$$





THE HALO MASS FUNCTION: MASSIVE NEUTRINOS

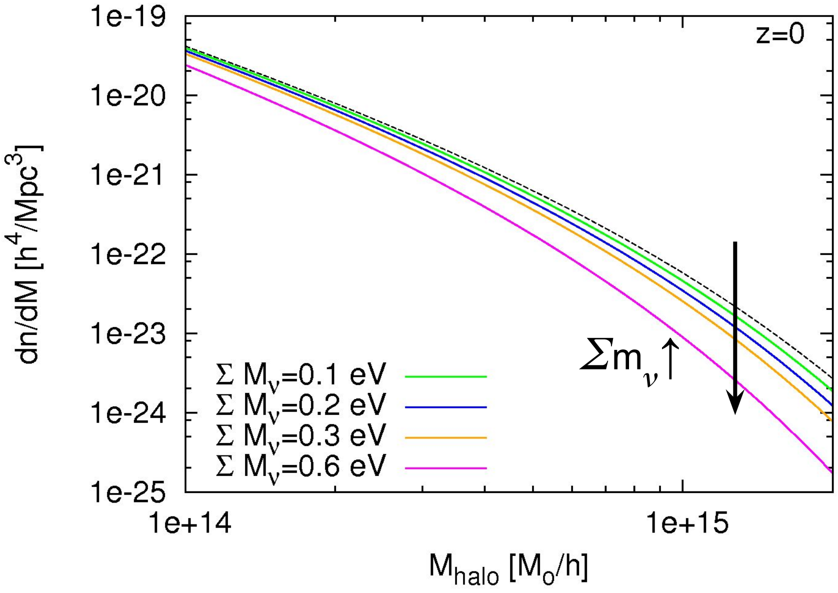


From Viel+14

Massive neutrinos:

- Delay the epoch of matter-radiation equality
- Suppress the growth of density fluctuation on scale smaller than the free-streaming length

Effects on the number density of halos as a function of mass



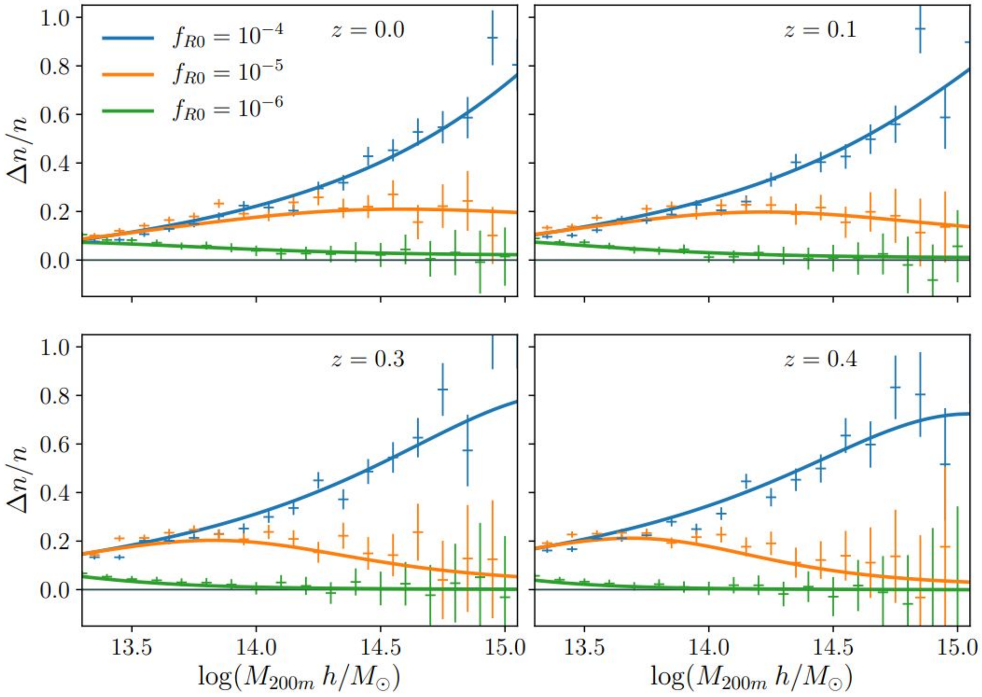
THE HALO MASS FUNCTION: MODIFIED GRAVITY

Modified gravity models, e.g. *f*(R):

$$S = \frac{1}{16\pi G}\int \sqrt{-g}[R + f(R)]d^4x$$

- Give rise to accelerated expansion and enhance gravity
- Introduce screening mechanism that restores GR in high density environments

Relative effect on the Halo Mass Function compared to ΛCDM



From Hagstotz+18

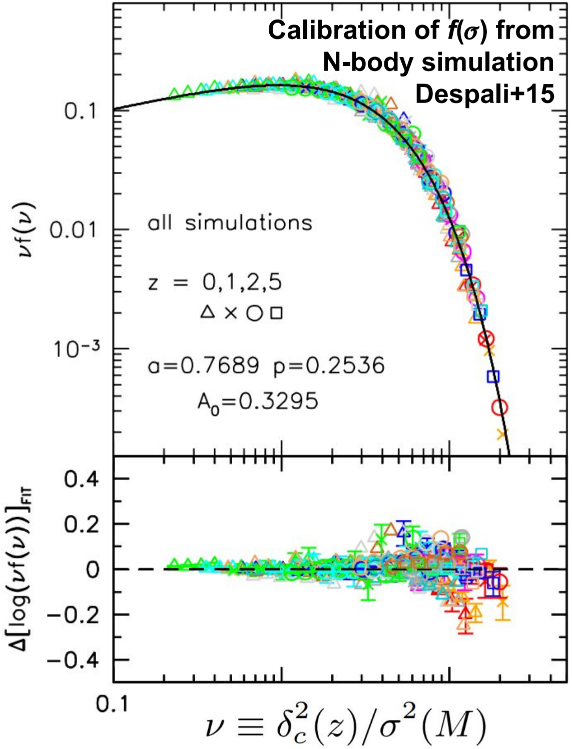
THE MULTIPLICITY FUNCTION: $f(\sigma)$

Halo mass function: $n(z,M)=\frac{\rho_m}{M} f(\sigma) \frac{d\ln(\sigma^{-1})}{dM}$

- **$f(\sigma)$ "universal" function:**
  - **Press & Schechter (1974) approximated from spherical collapse of Gaussian density field**
  - **Improved modeling using ellipsoidal collapse, e.g. Sheth & Tormen (1999)**
  - **Nowadays calibrated against N-body simulations**

Reference	Functional form
Press & Schechter (1974)	$f_{\rm PS}(\sigma) = \sqrt{\frac{2}{\pi}}\ \frac{\delta_c}{\sigma}\ \exp\left(-\frac{\delta_c^2}{2\sigma^2}\right)$
Sheth & Tormen (1999)	$f_{\rm ST}(\sigma) = A\sqrt{\frac{2a}{\pi}}\ \frac{\delta_c}{\sigma}\ \exp\left(-\frac{a\ \delta_c^2}{2\sigma^2}\right)\ \left[1+\left(\frac{\sigma^2}{a\ \delta_c^2}\right)^p\right]$
Jenkins et al. (2001)	$f_{\rm J}(\sigma) = A\exp\left(-\lvert\ln\sigma^{-1}+{\rm B}\rvert^{\rm p}\right)$
Reed et al. (2003)	$f_{\rm R}(\sigma) = f_{\rm ST}(\sigma)\exp\left(\frac{-a}{\sigma(\cosh 2\sigma)^b}\right)$
Warren et al. (2006)	$f_{\rm W}(\sigma) = A\left(\sigma^{-a}+b\right)\exp\left(-\frac{c}{\sigma^2}\right)$
Tinker et al. (2008)	$f_{\rm T}(\sigma) = A\left[\left(\frac{\sigma}{b}\right)^{-a}+1\right]\exp\left(-\frac{c}{\sigma^2}\right)$

Pillepich+10

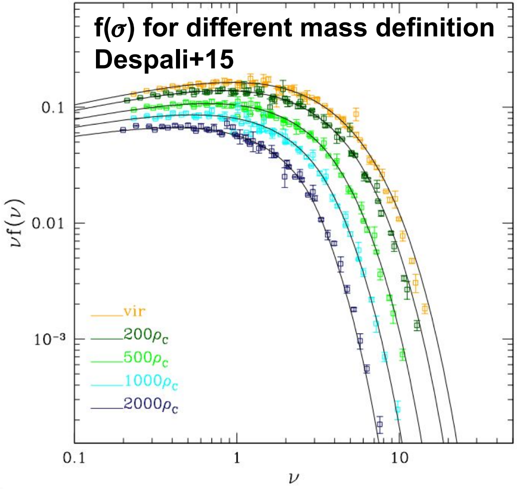


See also: Despali+15 ; Castro+22

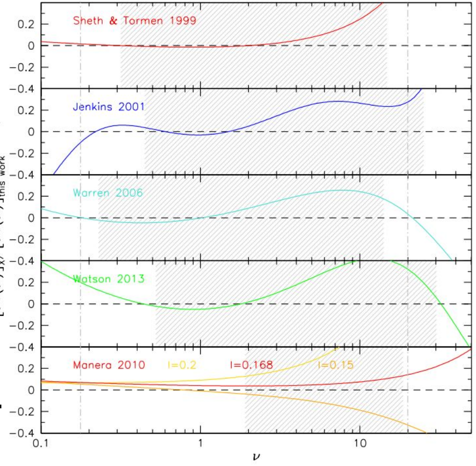
HALO MASS FUNCTION: UNIVERSALITY

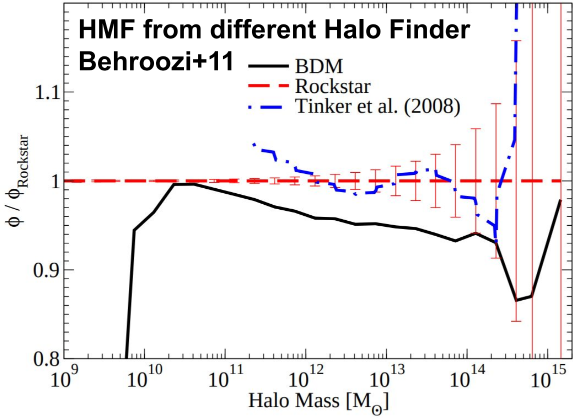
How accurate is the calibration of f(σ)? Is it universal?

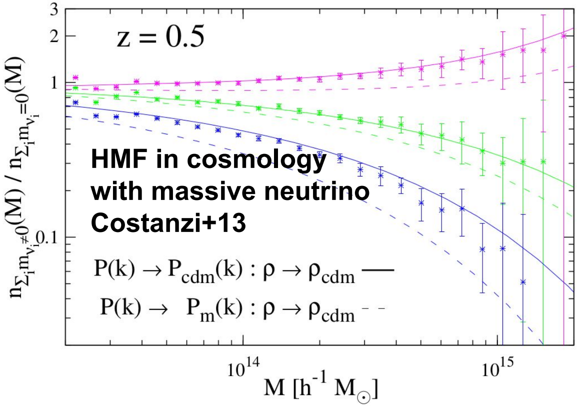
- **Specific of the simulation (e.g. box size, number of particles, softening length)**
- **Halo finder (e.g. linking length, FoF, SO)**
- **Mass definition (e.g. $M_{200,m}$, $M_{500,c}$)**
- **Redshift dependence**
- **Cosmological model (e.g. LCDM, wCDM, massive neutrino)**



Comparison different HMF calibration
Despali+15

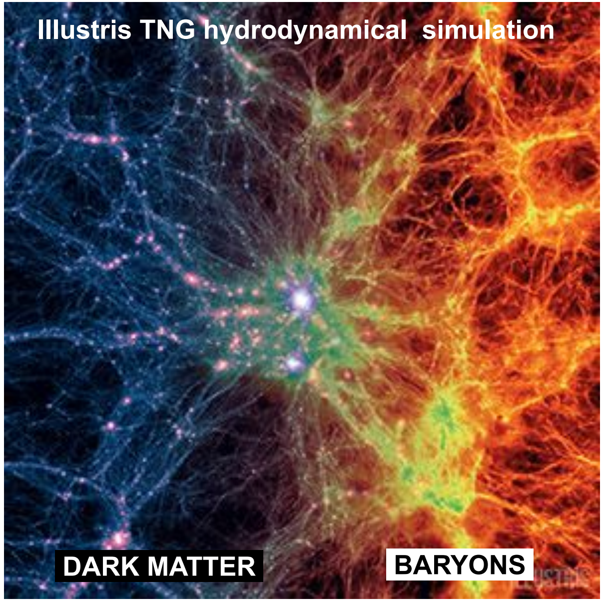


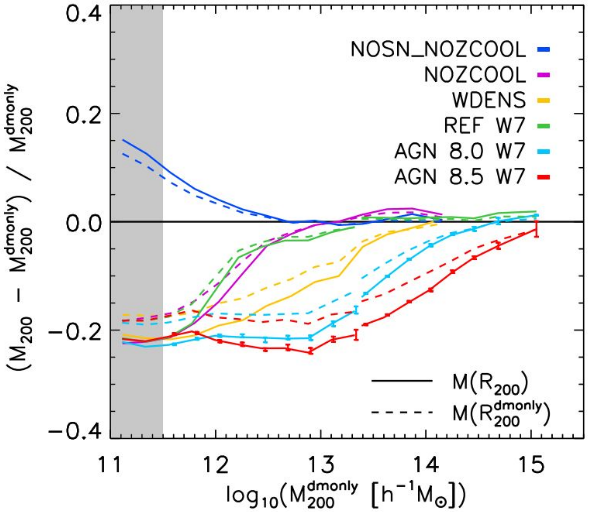


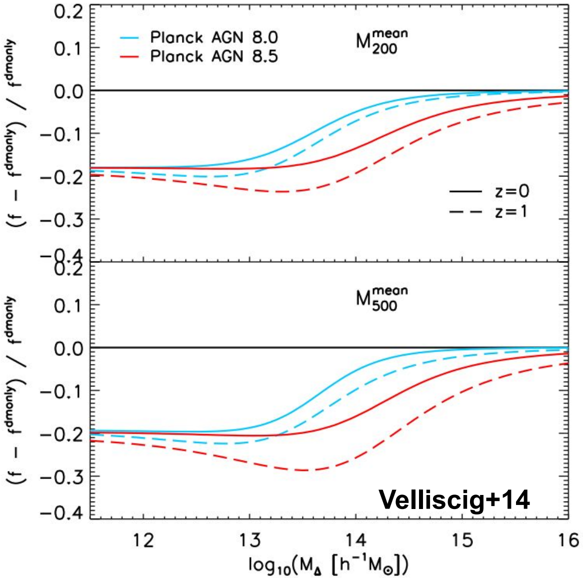


HALO MASS FUNCTION: BARYONIC EFFECTS

Baryonic feedbacks (radiative cooling, star formation, AGN feedback) redistribute and expel mass from galaxy clusters







Baryonic feedbacks most effective in the inner the regions of the halo and in low mass systems

See also Castro+21