Lezione 8 maggio 2025

giovedì 8 maggio 2025 09:52

Visto:

$$M^{\{\partial_u f, \partial_v f\}}_{\{\partial_u f, \partial_v f\}}(-dN_p) = -\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$= \frac{1}{EG-F^2}\begin{pmatrix} G & -F \\ -F & E \end{pmatrix} \cdot \begin{pmatrix} e & f \\ f & g \end{pmatrix}$$

$$K = \det\left(\frac{1}{EG-F^2} \cdot \begin{pmatrix} G & -F \\ -F & E \end{pmatrix} \cdot \begin{pmatrix} e & f \\ f & g \end{pmatrix}\right)$$

$$= \frac{1}{(EG-F^2)^2} \cdot \det\begin{pmatrix} G & -F \\ -F & E \end{pmatrix} \cdot \det\begin{pmatrix} e & f \\ f & g \end{pmatrix}$$

$$= \frac{(EG-F^2)}{(EG-F^2)^2} \cdot (eg-f^2) = \frac{eg-f^2}{EG-F^2}$$

$$e = -\partial_u N \cdot \partial_u f = \boxed{N \cdot \partial_{uu} f}$$

$$f = -\partial_u N \cdot \partial_v f = -\partial_v N \cdot \partial_u f = N \cdot \partial_{uv} f$$

$$g = -\partial_v N \cdot \partial_v f = N \cdot \partial_{vv} f$$

$N = N(u,v)$

OSS.: $N \cdot \partial_u f = 0$

$N \cdot \partial_v f = 0$

$$\Rightarrow \partial_u (N \cdot \partial_u f) = 0 = \partial_u N \cdot \partial_u f + N \cdot \partial_{uu} f \Rightarrow \underbrace{-\partial_u N \cdot \partial_u f}_{e} = N \cdot \partial_{uu} f$$

∂_v —"— $= 0$

$\partial_u (N \cdot \partial_v f) = 0$

∂_v (—"—) $= 0$

$E = \partial_u f \cdot \partial_u f$

$F = \partial_u f \cdot \partial_v f$

$G = \partial_v f \cdot \partial_v f$

OSS.: $$e = N \cdot \partial_{uu} f = \frac{\partial_u f \wedge \partial_v f}{\underbrace{\|\partial_u f \wedge \partial_v f\|}_{=\sqrt{EG-F^2}}} \cdot \partial_{uu} f$$

$$= \frac{1}{\sqrt{EG-F^2}} \underbrace{(\partial_u f \wedge \partial_v f) \cdot \partial_{uu} f}_{= \det(\partial_u f \;\; \partial_v f \;\; \partial_{uu} f)}$$

$$f = N \cdot \partial_{uv} f = \frac{1}{\sqrt{EG-F^2}} \det(\partial_u f \;\; \partial_v f \;\; \partial_{uv} f)$$

$g =$ - - - - - - - - $\partial_{vv} f$

Esempio: paraboloide ellittico

$$z = x^2 + y^2$$

$$f(u,v) = (u, v, u^2 + v^2)$$

$\partial_u f = (1, 0, 2u)$

$\partial_v f = (0, 1, 2v)$

$\partial_{uu} f = (0, 0, 2)$

$\partial_{uv} f = (0, 0, 0)$

$\partial_{vv} f = (0, 0, 2)$

$E = \partial_u f \cdot \partial_u f = 1 + 4u^2$

$F = \partial_u f \cdot \partial_v f = 4uv$

$G = \partial_v f \cdot \partial_v f = 1 + 4v^2$

$$e = \frac{1}{\sqrt{EG-F^2}} \det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2u & 2v & 2 \end{pmatrix} = \frac{2}{\sqrt{EG-F^2}} = \frac{2}{\sqrt{1+4u^2+4v^2}}$$

$f = 0$

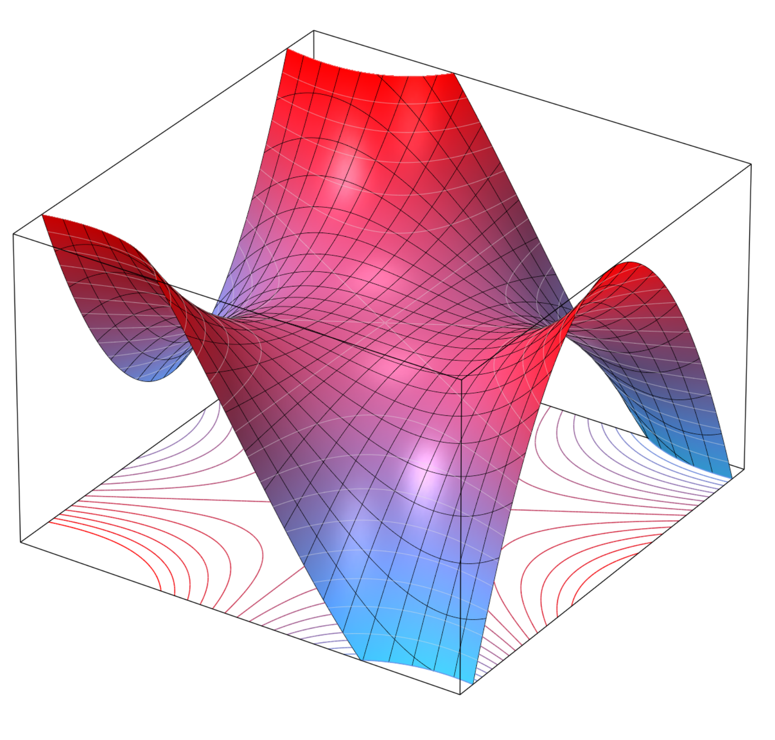
$$g = \frac{2}{\sqrt{1+4u^2+4v^2}}$$

Nel nostro caso

$$K = \frac{4}{(1+4u^2+4v^2)^2} > 0$$

Esempio SELLA DI SCIMMIA

$$f(u,v) = (u, v, u^3 - 3v^2u)$$



Esercizio $(0,0,0)$ è PLANARE