Kuramoto model

$$\dot{\theta}_i(t) = \omega_k - \sum_j k_{ij} \sin(\theta_i(t) - \theta_j(t))$$

AI

Kuramoto Model

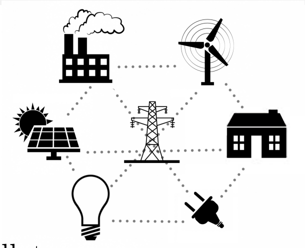
Acebrón, Bonilla, Vicente, Ritort, Spigler
Rev Mod Phys 77:137–185, 2005

- Biological Systems: Modeling the rhythmic behavior and synchronization in biological oscillators such as cardiac pacemaker cells, circadian rhythms in the brain, and the synchronous flashing of fireflies.
- Neuroscience: Investigating neuronal synchronization, which is crucial for various brain functions and is implicated in neurological disorders like epilepsy and Parkinson's disease.
- Power Grids: Analyzing the stability and synchronization of interconnected generators and loads, crucial for preventing blackouts and ensuring efficient power delivery, especially with the integration of renewable energy sources.

...Kuramoto Model (cont.)

- Social Dynamics: Studying opinion formation, flocking behavior in animals (birds, fish), and coordination in human crowds, where individual agents adjust their ”phase” (e.g., opinion, direction) based on interactions with others.
- Physics: Examining synchronization in coupled lasers, Josephson junctions, chemical oscillators, mechanical systems, modelling vehicle traffic

Oscillator networks as power grids



Network of coupled oscillators

$$
\begin{aligned}
M_i\ddot{\theta}_i + D_i\dot{\theta}_i &= \omega_i - \sum_{j=1} a_{ij}\sin(\theta_i - \theta_j), && i \text{ is a producer} \\
D_k\dot{\theta}_k &= \omega_k - \sum_{j=1} a_{kj}\sin(\theta_k - \theta_j), && k \text{ is a user}
\end{aligned}
$$

- M_i large rotational inertia of individual turbine generators
- $a_{ij}\sin(\theta_i - \theta_j)$ power flow along the transmission lines
- $\omega_i > 0$ injects power into the system, $\omega_k < 0$ absorbs power

  F. Dörfler, M. Chertkov, and F. Bullo, PNAS 2013

Coupled Overdamped Oscillators under external fields

Network of N coupled oscillators, at constant temperature T, with natural frequency f_i, mean natural frequency f_0

$$\dot{\phi}_i(t) = f_i - \frac{K}{N}\sum_j \sin(\phi_i(t) - \phi_j(t)) + \eta_i(t), \quad \langle \eta_i \eta'_j \rangle = 2\delta_{ij} T_i \delta(t-t')$$

$$\sigma e^{i\psi_0} = \frac{1}{N}\sum_j e^{i\phi_j}, \qquad \theta_i = \phi_i - f_0 t$$

$$\dot{\theta}_i = h_i - K\sigma \sin(\theta_i) + \eta_i(t), \qquad h_i = f_i - f_0$$

- Given the natural frequency (*force*) distribution, $g(f)$, one can calculate σ as a function of T and K.
- Given σ one can determine the mean velocity $v_i = < \dot{\theta}_i >$ as a function of T, K and h_i.

see, e.g., *H. Sakaguchi Progr. Theo. Phys., 1988*