Gravitational wave probes of new physics

Overview:

- GW introduction
- GW's from new physics in the early universe
  - Phase transitions
  - Scalar field dynamics (axions)
- GW's as indirect probes of NP
  - superradiance - BH spindown
  - new forces in mergers

What are GW's?

Any GR textbook, e.g. Carroll

Einstein equations: $G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$

You have seen this!

metric

matter

$g_{\mu\nu}$ and its derivatives

EOM for $g_{\mu\nu}$

Expand around empty, flat space-time:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \eta_{\mu\nu} = \pm \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

say

with $|h_{\mu\nu}| \ll 1$.

Generic, weak field limit → contains more than just GW's

skip

Example: gravitational field outside of star:

$$h_{00} = -2\phi \qquad \text{with } \Delta\phi(\vec{x}) = 4\pi G \rho(\vec{x})$$

(Poisson equation)

Analogy with E&M: Not every electric field is an EM-wave

Q: What are sources of EM-waves?

[A: Accelerated charges, dipole-radiation]

Gauge invariance: In GR, have general coordinate invariance

$g_{\mu\nu}, h_{\mu\nu}$ not unique

↳ can choose a gauge for $h_{\mu\nu}$

skip

Useful for GW's: Transverse, traceless gauge

$$h^{TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & 2s_{ij} & \\ 0 & & & \end{pmatrix}$$

↳ Simplifies vacuum equations of motion

$$\Box h^{TT}_{\mu\nu} = 0$$

Q: How do I solve this?

say

Plane wave ansatz: $h^{TT}_{\mu\nu} = C_{\mu\nu} e^{ik_\sigma x^\sigma}$

with $k_\sigma k^\sigma = 0$. For $k_\mu = (\omega, 0, 0, \omega)$ have transverse polarisation tensor

$$C_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & C_{11} & C_{12} & 0 \\ 0 & C_{12} & -C_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{aligned} C_{11} &= h_+ \\ C_{12} &= h_\times \end{aligned}$$

Q: Why $+$, $\times$?

say

<u>GW sources:</u> $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$

now keep this part

find $\Box \bar{h}^{TT}_{\mu\nu} = -16\pi G_N T^{TT}_{\mu\nu}$ where $\bar{h}^{TT}_{\mu\nu} = h^{TT}_{\mu\nu} - \frac{1}{2} h^{TT\sigma}_{\sigma} \eta_{\mu\nu}$

inhomogeneous wave equation

Formal solution using Green's function of d'Alembert operator, G: (drop ^{TT} superscripts) skip

$$\Box_x G(x-y) = \delta^4(x-y)$$

$$\hookrightarrow \quad \bar{h}_{\mu\nu}(x) = -16\pi G \int d^4y \, G(x-y)\, T_{\mu\nu}(y)$$

Again in full analogy with E&M.

Typically interested in the field far away from the source. In E&M, this is the dipole component. Here: say

retarded time $t_r = t - r$

$$\bar{h}_{ij}(t,\vec{x}) = \frac{2G}{r}\frac{d^2}{dt^2} I_{ij}(t_r)$$

Quadrupole moment $I_{ij}(t) = \int y^i y^j \, T^{00}(t,\vec{y})\, d^3\vec{y}$

Notes:
- $\frac{d^2}{dt^2} \leftrightarrow$ accelleration
- T^{00} appears because we assume a non-relativistic source
- quadrupole radiation (vs. dipole in E&M case).

Astrophysical sources: say

- Binary systems of black holes, neutron stars, white dwarfs etc. Also supermassive BHs !

say

- historically important: Hulse Taylor binary

↳ Change in orbital frequency due to energy loss from GW emission → indirect detection of GWs.

- these systems are now becoming laboratories for probing deviations from SM expectations → last part of lecture.

say

<u>Cosmological sources:</u>

- Inflation
- Cosmic strings
- Phase transitions
- Axions

} focus on these

Note: Cosmological sources produce a stochastic signal (i.e. noise) while e.g. binaries produce a coherent signal - though there is also a stochastic BG from unresolved binaries

$\Rightarrow$ $h_{ij}(t, \vec{x})$ is random variable, only averages known.

Like the CMB!

Causality: Correlation length $\ell_p \leq \frac{1}{H_p}$ at p := production

$\hookrightarrow$ Hubble horizon

$$\ell_p^0 = \ell_p \frac{a_0}{a_p}$$

Compare: $$\frac{\ell_p^0}{H_0^{-1}} = \frac{\ell_p \; a_0}{H_0^{-1} a_p} \leq \frac{H_p^{-1} a_0}{H_0^{-1} a_p} = \frac{a_0/a_p}{\sqrt{\Omega_{mat}(z_p) + \Omega_r(z_p) + \Omega_\Lambda}}$$

$$\Omega_i(z) = \frac{\rho_i(z)}{\rho_c^0}$$

Sources in radiation era !

$$\frac{a_0}{a_p} \propto \frac{T_p}{T_0} \simeq 10^{13} \frac{T_p}{\mathrm{GeV}}$$

up to d.o.f corrections.
From entropy conservation.
$[g_s(T)\, T^3 a^3(T) = \text{const}]$

$$T_0 = 2.3 \cdot 10^{-13}\ \mathrm{GeV}$$

$$\rho_{rad} = \frac{\pi^2}{30} g_*(T)\, T^4$$

$$\Rightarrow \Omega_{rad}(T) \propto \Omega_{rad}^0 \left(\frac{a_0}{a}\right)^4$$

$$\frac{\ell_p^0}{H_0^{-1}} \approx \frac{a_0/a_p}{\sqrt{\Omega_{rad}(T_p)}} \approx \frac{a_0/a_p}{(a_0/a_p)^2} \simeq \frac{a_p}{a_0} \approx \frac{T_0}{T_p} \ll 1$$

$\Longrightarrow$ statistically homogeneous, isotropic, (unpolarised), Gaussian

Characterised via its <u>power spectrum</u>:

$$\langle h_r(\vec{k},\eta)\, h_p^*(\vec{q},\eta) \rangle = \frac{8\pi^5}{k^3} \delta^3(\vec{k}-\vec{q})\, \delta_{rp}\, h_c^2(k,\eta)$$

$\hookrightarrow$ Polarisations

h_c: characteristic strain.

$k = |\vec{k}|$

<u>Energy density</u>: $$\frac{d\rho_{GW}}{d \log k} = \frac{k^2 h_c^2(k,\eta)}{16\pi G\, a^2(\eta)}$$

The signal today:
$$f_{phys} = \frac{k}{2\pi a_0}$$
$$h_c(f) = h_c(k, \eta_0) \qquad \text{with} \quad f = \frac{k}{2\pi a_0}$$

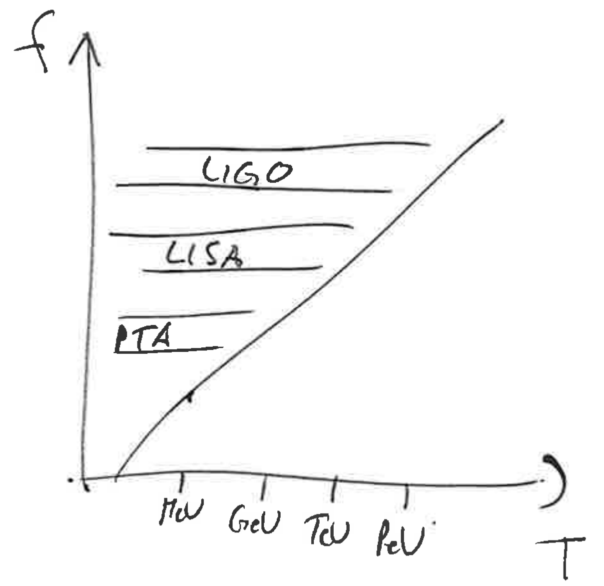
Amplitude today:
$$\Omega^{(0)}_{GW}(f) = \frac{4\pi^2}{3 H_0^2} f^3 \underbrace{\frac{h_c^2(f)}{2f}}_{S_h(f) \text{ spectral density}}$$
$$= \frac{1}{\rho_c} \frac{d\rho_{GW}}{d\log k}$$

Temperature, frequency relation:

$$f = \frac{1}{2\pi} \frac{k}{a_0}$$

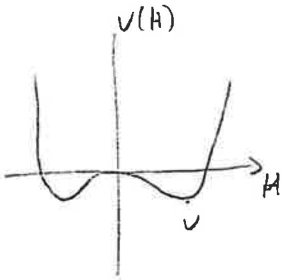
Def: $x_k = \dfrac{k/a_p}{H_p}$ ← physical wave number at production; ← Hubble radius at production

$$\Rightarrow f = \frac{x_k}{2\pi} \frac{a_p}{a_0} H_p \qquad x_k \geq 1 \text{ depends on source}$$
$$= 2.6 \cdot 10^8 \, Hz \; x_k \frac{T_p}{GeV}$$



Phase transitions in the early universe

The Higgs potential of the SM is given by $V(H) = -\mu^2 |H|^2 + \lambda |H|^4$



At tree level the minimum $\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ is given by

$$v^2 = \frac{\mu^2}{\lambda} \quad (\text{if } \mu^2 > 0) .$$

In QFT the potential receives radiative corrections. To incorporate them one introduces the effective potential $V_{eff}(\varphi_{cl})$, where φ_{cl} is the classical background field. The minimum is then found by imposing

$$\frac{\partial V_{eff}}{\partial \varphi_{cl}} = 0 .$$

Methods for computing the one loop corrected eff. potential can be found e.g. in Peskin.

For the SM, the renormalized 1-loop result is

$$V(\varphi_{cl}) = V_0(\varphi_{cl}) + \frac{1}{64\pi^2} \sum_{\substack{i=W,Z \\ h,t,\chi}} \underbrace{n_i}_{\#\text{dof}} m_i^4(\varphi_{cl}) \Big[\log \frac{m_i^2(\varphi_{cl})}{\mu^2} - C_i \Big]$$

C_i: $\frac{5}{6}$ gauge bosons, $\frac{3}{2}$ else.

Literature:

- M. Quiros: Finite T field theory and PT's (hep-ph/9901312)
- Laine, Vuorinen: 1701.01554
- Books by Kapusta, Le Bellac
- Arxiv: 2008.09136 Hindmarsh et al

This is the vacuum case. The early universe is a messy place however, a hot plasma with all SM particles in thermal equilibrium. This induces additional, <u>finite temperature</u> contributions to V_{eff}.

Intuitive picture: A particle moving through the plasma is constantly hit and scattered. This makes it more difficult to accelerate it $\leftrightarrow$ the particle gets an additional <u>thermal mass</u>.

The contributions to V_{eff} separate into a $T=0$ and a finite temperature part. One finds:

$$\Delta V^{(1,\text{loop})}(\varphi_{cl},T) = \frac{T^4}{2\pi^2}\Big[\sum_{i=W,Z} n_i \underbrace{J_B\Big(\frac{m_i^2(\varphi_{cl})}{T^2}\Big)}_{\to\, -\frac{\pi^4}{45}+\frac{\pi^2}{12}\frac{m^2}{T^2}+\dots} + n_t \underbrace{J_F\Big(\frac{m_t^2(\varphi_{cl})}{T^2}\Big)}_{\to\, \frac{7\pi^4}{360}-\frac{\pi^2}{24}\frac{m^2}{T^2}+\dots}\Big]$$

Finally the total eff. potential at finite T can be written as

$$V(\varphi_{cl},T) = D(T^2-T_0^2)\varphi_{cl}^2 - ET\varphi_{cl}^3 + \frac{\lambda(T)}{4}\varphi_{cl}^4$$

$$D = \frac{1}{8v^2}(2m_W^2 + m_Z^2 + 2m_t^2)$$

$$E = \frac{1}{4\pi v^3}(2m_W^3 + m_Z^3) \qquad [\text{no fermion contribution}]$$

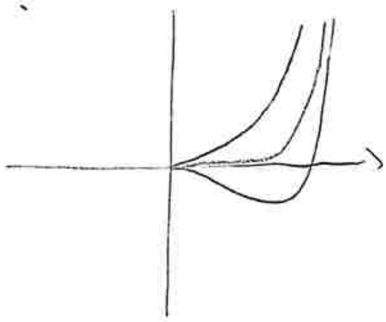
$$T_0^2 = \frac{m_h^2 - 8Bv^2}{4D} \qquad B = \frac{3}{64\pi^2 v^4}(2m_W^4 + m_Z^4 - 4m_t^4)$$

The important point here is that at $T > T_0$, the global minimum of $U_{eff}(T)$ is at $\varphi_{cl} = 0$, i.e. the electroweak symmetry is restored.

In cosmology, the universe after the big bang (and inflation) reheats to high temperatures ($T \gg T_c$) so that EW symmetry is unbroken initially. The universe undergoes a transition from $\varphi_{cl} = 0$ to $\varphi_{cl} \sim v$ around $T_0 \sim 10^2$ GeV
[also QCD PT near GeV scale]

Consider $U(\phi, T) = D(T^2 - T_0^2)\phi^2 + \frac{\lambda(T)}{4}\phi^4$.

stationary points: $\frac{dU}{d\phi} = 0$



$$\phi_1(T) = 0$$

$$\phi_2(T) = \sqrt{\frac{2D(T_0^2 - T^2)}{\lambda(T)}} \qquad (T < T_0)$$

At $T > T_0$, $\phi = 0$ is the only solution.

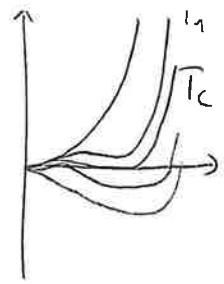
At $T = T_0$, both solutions are at $\phi = 0$. For $T < T_0$, ϕ_2 is the global minimum and ϕ_1 becomes a local maximum. There is no barrier between the minima, the field can adiabatically follow the vacuum state.

$\rightarrow$ ~~2nd order PT~~

Crossover.

For more complex potentials, a barrier might exist. E.g.

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda(T)}{4}\phi^4$$



$T > T_1$: $\phi = 0$ is min.

At T_1 : Second min. appears

At T_c : Both minima have equal value

$T < T_c$: $\phi = 0$ becomes meta-stable

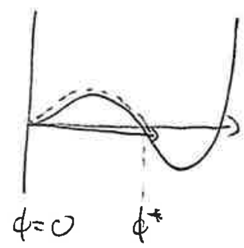
$T < T_0$: barrier goes away, $\phi = 0$ is max.

The PT can start at $T \lesssim T_c$. If the tunneling probability is small, $T < T_c$. Also models with tree level barriers exist $\Rightarrow T_0 = 0$.

Thermal tunneling

Bubble formation

Tunneling rate : $\frac{\Gamma}{V} \sim A(T)\, e^{-S_3/T}$



$$S_3 = \int d^3x \left(\frac{1}{2} (\nabla\phi)^2 + U(\phi, T) \right)$$

with ϕ being the <u>bounce</u> solution

path from $\phi = 0$ to ϕ^* that minimises action.

Explain bubble expansion

↳ energy gain vs. surface tension

Compete with expansion of universe $\rightarrow \frac{S_3}{T} \sim 140$

Motivations:

1. First order PT = deviation from th. equ $\Rightarrow$ Baryogenesis
2. GWs in range of planned expts.

GW's as probes of the early Universe

- Around 300.000 years after the big bang, electrons and protons combine to form hydrogen

  ↳ Matter becomes neutral gas, the Universe becomes transparent

  The photons that were part of the plasma at that time are what we see now in the CMB.

  Earlier times are not observable with telescopes.

- The coupling of matter to gravity is suppressed by $8\pi G = \frac{1}{M_{Pl}^2}$

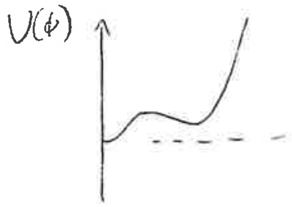
  ↳ GW's produced any time after the big bang propagate freely
  (even before big bang if you want)

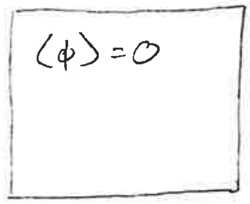
- GW's are a new way to probe the dynamics of the very early Universe → e.g. phase transitions

GW's from PT's

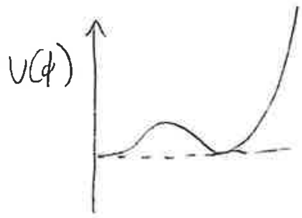
Potential | Universe (1 Hubble volume)

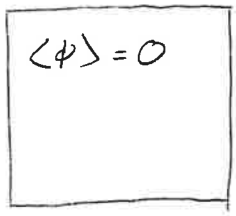
$T > T_c$



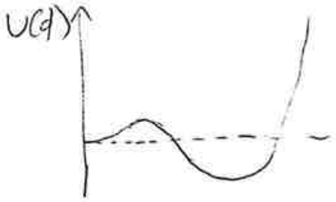


$T = T_c$

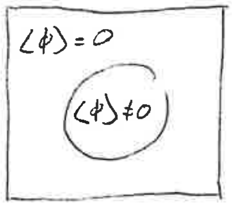




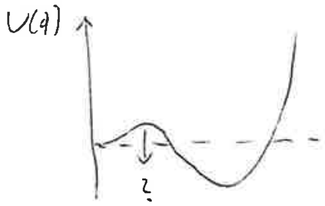
$T = T_n < T_c$

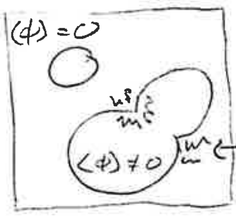


ΔU Energy budget



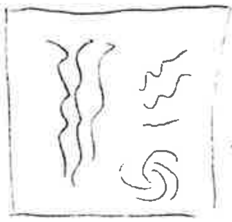
$T < T_n$





GW production

$T \ll T_n$



sound waves,
turbulence in plasma
more GW production

T_c from $U(\phi,T)$. How to find T_N?

Vacuum decay rate $\Gamma(T) \propto T^4 e^{-S_3/T}$

$S_3(T)$: Action of the $O(3)$ symmetric tunneling "bounce" solution

$\Leftrightarrow$ How much energy is needed to cross the barrier

The <u>nucleation temperature</u> T_N is defined by the requirement that one bubble per Hubble volume should be nucleated:

$$\frac{\Gamma(T)}{H^4(T)} \overset{!}{=} 1$$

Now $H \sim \frac{T^2}{M_{pl}} \Rightarrow \frac{\Gamma}{H^4} \sim \frac{M_{pl}^4}{T^4} e^{-S_3/T} \Rightarrow$

$$\frac{S_3}{T} \sim -\log\left(\frac{T^4}{M_{pl}^4}\right) \sim 140 \quad \text{for } T \sim \text{weak scale}$$

Technical details: Coleman, PRD 15, 10 p. 2929 (1977)
(see. Exercises)

$$S_3(\phi_b) = \int d^3x \left(\frac{1}{2}(\nabla\phi_b)^2 + U(\phi_b)\right)$$

where ϕ_b is the <u>bounce</u> solution, i.e. solves $\frac{d^2\varphi}{dr^2} + \frac{2}{r}\frac{d\varphi}{dr} = V'(\varphi)$

with $\varphi \to 0$ at $r \to \infty$, $\frac{d\varphi}{dr} = 0$ at $r = 0$. (and $U = T(\varphi, T)$)

How fast does the transition complete?

$$\beta \equiv -\frac{dS}{dt}\Big|_{t_N} \rightsquigarrow \frac{\beta}{H}\Big|_{T_N} = T_N \frac{dS}{dT}\Big|_{T_N}$$

Note: This is x_p !

For large β, $\frac{\Gamma}{H}$ increases rapidly and the PT is fast

Energy budget: $\alpha \approx \frac{\Delta U}{\rho_{tot}} = \frac{\text{vacuum energy}}{\text{total energy}}$

Bubble wall speed ... difficult. Most PT's of interest have $v_w \to 1$.

Nucleation temperature $T_N \sim \langle\phi\rangle$!

↳ Caveat: For slow, supercooled, vacuum dominated transitions, the PT might complete later ...

How to obtain the GW signal?

Difficult, requires numerical simulations (summary e.g. 1512.06239)
1910.13125
2008.09136

Qualitative:

Peak frequency at time of emission $f_* \sim \frac{1}{\lambda_*}$ ← wavelength

Characteristic length scale = bubble radius at time of collision

$$\lambda_* \sim \underset{\substack{\uparrow \\ \text{size of Hubble patch}}}{\frac{1}{H_*}} \left(\frac{H_*}{\beta}\right) \cdot \underset{\approx 1}{v_w}$$

$\left(\frac{H_*}{\beta}\right)$ ← how fast is the transition

Now redshift:

$$f_0 \equiv f_{today} = \frac{a_*}{a_0} \cdot f_* \approx \frac{T_0}{T_*} \cdot f_* \qquad \text{(entropy conservation)}$$

$$= \frac{T_0}{T_*} \cdot \frac{1}{H_*} \cdot \left(\frac{\beta}{H_*}\right)$$

$$\approx \frac{T_0}{M_{pl}} \cdot T_* \cdot \left(\frac{\beta}{H_*}\right)$$

$T_0 \sim 3K \sim 10^{-4}\,eV$

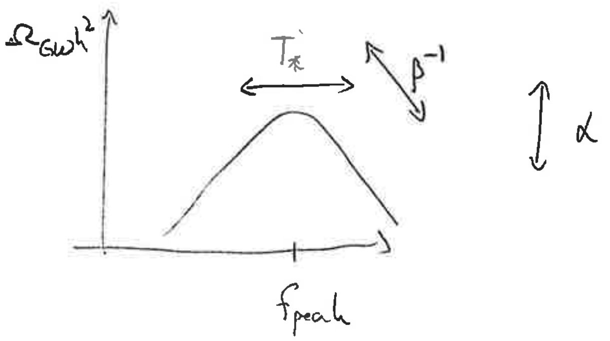
$M_{pl} \sim 10^{18}\,GeV$

$$T_* = \frac{T_*}{100\,GeV} \cdot 100\,GeV \cdot \frac{s}{s}$$

$$= \frac{T_*}{100\,GeV} \cdot 10^{26}\,Hz$$

$$\approx 10^{-5}\,Hz \cdot \left(\frac{T_*}{100\,GeV}\right) \cdot \left(\frac{\beta}{H_*}\right)$$

Signal shape:

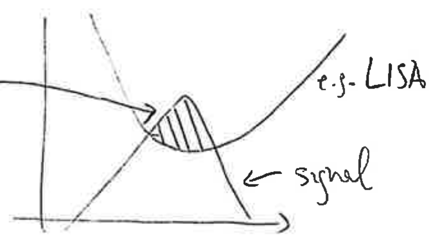


What detector?

$$10^{-3}\,Hz \rightarrow \lambda \sim 10^{10}\,m \sim 10^{7}\,km = 10\,Mkm$$

$\hookrightarrow$ space $\rightarrow$ LISA

Sensitivity $\rightarrow$ SNR



GW's from axion dynamics

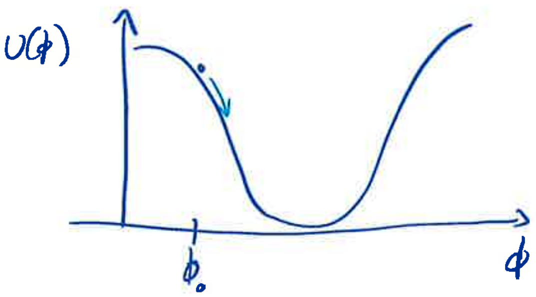
What is an axion? For all practical purposes, a very light scalar field ϕ.

Useful: Compact field range (+ shift symmetry), i.e. $\phi + 2\pi f = \phi$.

PQ scale

If m_ϕ is sufficiently small, then any initial value is equally likely, i.e.

$$\phi_0 = \Theta f \quad \text{with} \quad \Theta \in [-\pi, \pi]$$



Axion EOM:
$$\phi'' + 2aH\phi' + a^2 \underbrace{\frac{\partial V}{\partial \phi}}_{\approx m_\phi^2 \phi} = 0$$

Note: $\phi' = \frac{d\phi}{d\tau}$ with τ conformal time, i.e. $d\tau = a\,dt$

a is the scale factor, H the Hubble rate.

Rolling prevented by friction if $H \gg m_\phi$.

↳ Starts when $m_\phi \sim H$.

Radiation domination : $H \sim \frac{T^2}{M_{pl}}$ ← Temperature, ← Planck scale $\sim 10^{18}$ GeV

ϕ starts oscillating in Potential at

$$m_\phi \approx \frac{T_{osc}^2}{M_{pl}} \iff T_{osc} \approx \sqrt{M_{pl}\, m_\phi}$$

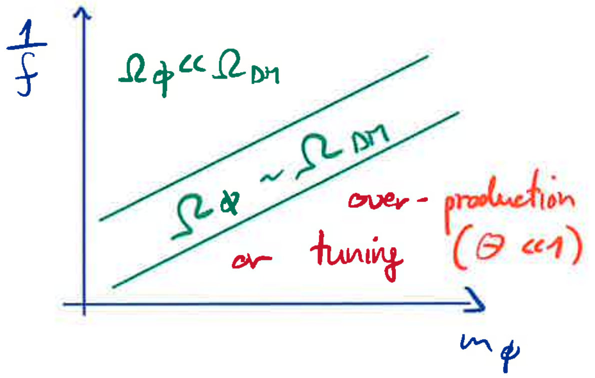
We can compute the energy density that is in "axions" at T_{osc}

$$\Omega_\phi^{osc} = \frac{\rho_\phi^{osc}}{\rho_{radiation}^{osc}} \sim \frac{m_\phi^2 \phi_0^2}{T_{osc}^4} \sim \frac{\theta^2 f^2}{M_{pl}^2}$$

It redshifts like matter : $\rho_\phi = \left(\frac{a_{osc}}{a}\right)^3 \rho_\phi^{osc}$

↳ DM candidate . Non-thermal, also <u>non-particle</u>

Parameter space :



Idea to avoid tuning : Add more friction, e.g. from particle production

add $\frac{\phi}{f} X_{\mu\nu} \tilde{X}^{\mu\nu} \longleftrightarrow \phi$ - - - $\gamma_D\ \gamma_D$ "dark photon"

Normal decays not efficient. However this specific coupling adds a "tachyonic instability" for the dark photon

$$u_\pm''(k) + \underbrace{\left(k^2 \mp k \frac{\phi'}{f}\right)}_{\text{harmonic oscillator with } \omega_\pm^2 = k^2 \mp k \frac{\phi'}{f}} u_\pm(k) = 0$$

occupation number of momentum mode "k"

$\Rightarrow$ Imaginary frequencies for some momenta, while ϕ' is non-zero

$$e^{i\omega t} \longrightarrow e^{\pm|\omega_\pm| t}$$

exponential growth

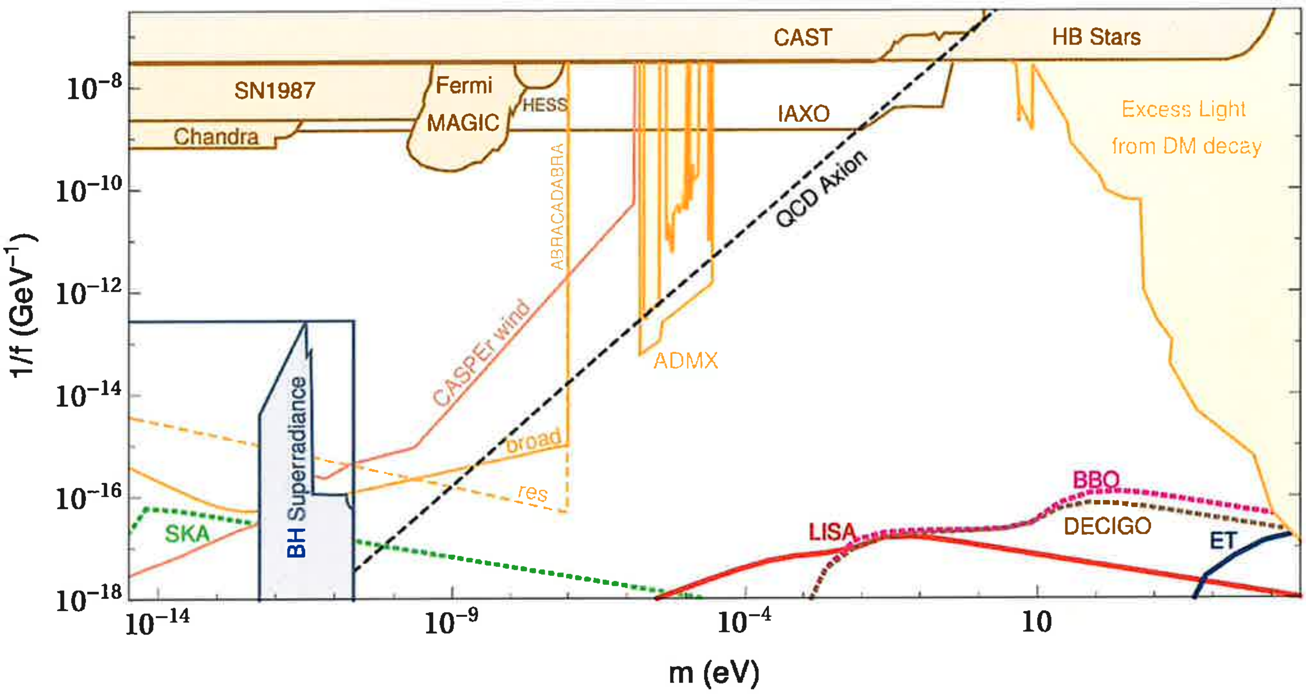
↳ Quantum fluctuations become macroscopic

↳ Large anisotropies

↳ GW production

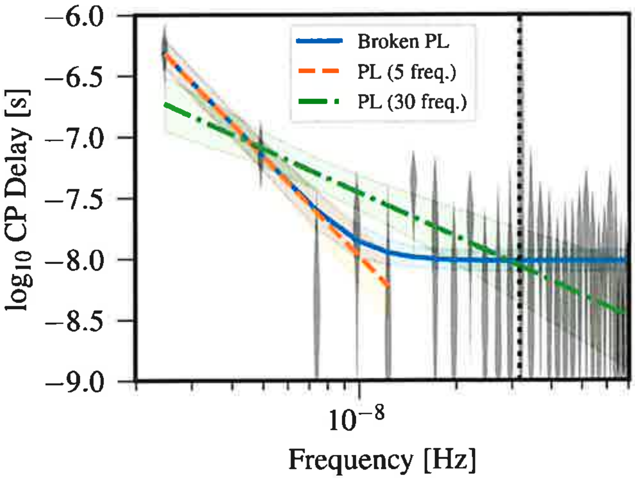
GW spectrum can be computed directly from dark photon spectrum, e.g. 1811.01950 ; 2012.11584 "Audible Axion"

(20)



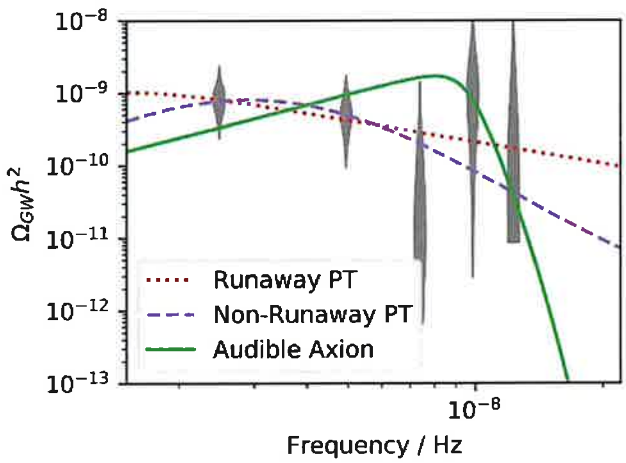
NANOGrav

2009.06607



Fit with BSM

2009.11875



Black hole superradiance

Ref: Brito, Cardoso, Pani ; 1501.06570

Light particles can form bound states with black holes, similar to Hydrogen atoms.
Through the "Penrose" process, particles can extract energy from spinning black holes, thereby reducing the BH spin.

If the Compton wavelength of the particle $\lambda \sim \frac{\hbar}{mc} \left(= \frac{1}{m}\right)$ is similar to the Schwarzschild radius, this superradiant instability is very efficient at spinning down black holes.

		r_s		m_ϕ
stellar BH	$\leftrightarrow$	30 km	$\leftrightarrow$	10^{-12} eV
SMBH	$\leftrightarrow$	10^8 km	$\leftrightarrow$	10^{-17} eV

LIGO could more firmly establish the existence of BHs with large spin. Strong bounds on existence of very light bosons.
(only apply for case of very weak interactions.)