

Program of the course
DIFFERENTIAL GEOMETRY I

Academic Year 2024-2025 – Fabio Vlacchi

Preliminaries

Introduction to differentiable manifolds: local coordinates, charts, atlases.
Differentiable mappings between differentiable manifolds.
Immersion and embeddings.
Vector fields and Tangent bundle. Lie brackets, distributions and Frobenius Theorem (only statement).
Jacobi identity. Lie groups and Lie algebras.
Riemannian metrics on a differentiable manifold. Length of a curve on a differentiable manifold. Existence and examples of Riemannian metrics.
Isometries.

Intrinsic derivative on a manifold

Linear connections. Parallel transport and covariant derivative.
Torsion and symmetric connections. Levi-Civita connection associated with a Riemannian manifold.
Geodesics and the corresponding differential equations.
Gauss Lemma and the exponential map. Local characterization of geodesics as curves minimizing the distance between two points.

Tensors on manifolds

Introduction to the notion of curvature of a differentiable manifold: definition, main properties and Bianchi identity.
Introduction to Jacobi fields: definition, main properties and its geometric interpretation as rate of spreading of geodesics.
Conjugate points of Jacobi fields.
Ricci and sectional curvatures: definition, main properties and the Schur Lemma (only statement).
(Covariant) tensors on a differentiable manifold.
Complete manifolds. Hopf-Rinow Theorem.
Covering spaces. Hadamard Theorem and applications.
Fundamental equation: Gauss, Ricci and Codazzi equations.

Short overview on Comparison Theorems (Rauch - Bonnet Myers and Cartan Theorems) and on (Almost) Complex Manifolds.

Main references

Beside the material on the Moodle page

<https://moodle2.units.it/course/view.php?id=15340>

Riemmanian Geometry, M. P. do Carmo, Birkhäuser, (1992)

Riemmanian Geometry, S. Gallot – D. Hulin – J. Lafontaine, Springer–Verlag, (1987)

Notes on Differential Geometry, N.J. Hicks, Van Nostrand, (1965)

Calculus in Manifolds, M. Spivak, Benjamin, (1965)

Foundations of Differential Geometry, S. Kobayashi – K. Nomizu, Wiley–Interscience, (1969)

Differential Geometry, Lie Groups and Symmetric Spaces, S. Helgason, Academic Press, (1978)