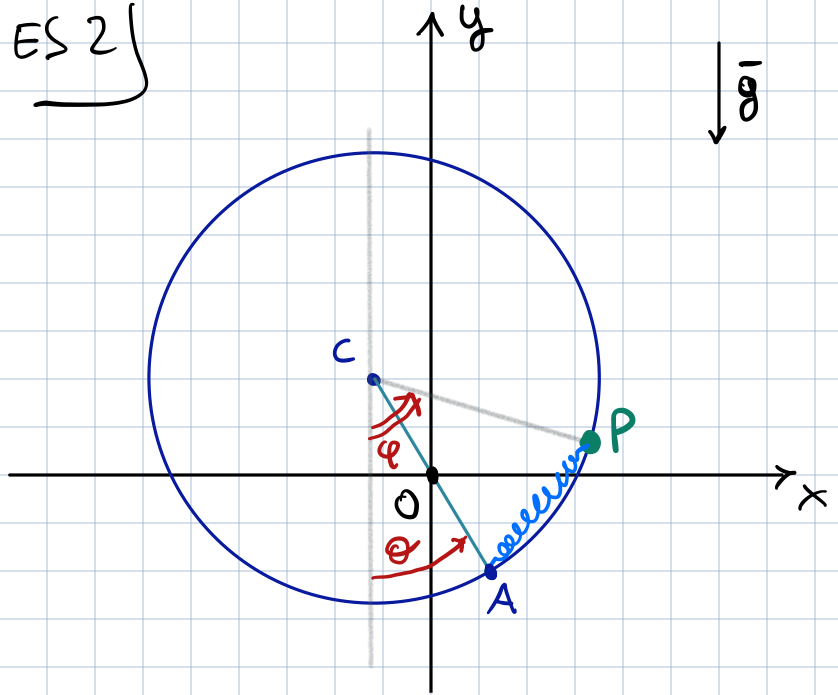
ES 2



$$d_{AP}^2 = \left(2R \,\text{sen}\left(\frac{\varphi-\theta}{2}\right)\right)^2 =$$
$$= 2R^2\left(1-\cos(\theta-\varphi)\right)$$

$$\text{sen}^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

$$x_C = -\frac{R}{2}\,\text{sen}\,\theta$$
$$y_C = \frac{R}{2}\cos\theta$$

$$x_P = -\frac{R}{2}\,\text{sen}\,\theta + R\,\text{sen}\,\varphi \qquad \dot{x}_P = -\frac{R}{2}\dot{\theta}\cos\theta + R\dot{\varphi}\cos\varphi$$

$$y_P = \frac{R}{2}\cos\theta - R\cos\varphi \qquad \dot{y}_P = -\frac{R}{2}\dot{\theta}\,\text{sen}\,\theta + R\dot{\varphi}\,\text{sen}\,\varphi$$

$$T_P = \frac{m}{2}\left(\dot{x}_P^2+\dot{y}_P^2\right) = \frac{mR^2}{2}\left(\frac{\dot{\theta}^2}{4} + \dot{\varphi}^2 - \dot{\theta}\dot{\varphi}\,\underbrace{(\cos\theta\cos\varphi + \text{sen}\,\theta\,\text{sen}\,\varphi)}_{\cos(\theta-\varphi)}\right)$$

$$T_C = \frac{1}{2}I\dot{\theta}^2 = \frac{mR^2}{2}\left(\frac{5}{4}\dot{\theta}^2\right)$$

$$\mathcal{Q} = mR^2\begin{pmatrix} \frac{3}{2} & -\frac{1}{2}\cos(\theta-\varphi) \\ -\frac{1}{2}\cos(\theta-\varphi) & 1 \end{pmatrix}$$

$$T = \frac{mR^2}{2}\left(\frac{3}{2}\dot{\theta}^2 + \dot{\varphi}^2 - \dot{\theta}\dot{\varphi}\cos(\theta-\varphi)\right)$$

$$V = mg\,y_C + mg\,y_P + \frac{k}{2}d_{AP}^2 =$$
$$= mg\frac{R}{2}\cos\theta + mg\frac{R}{2}\cos\theta - mgR\cos\varphi + kR^2\left(1-\cos(\theta-\varphi)\right)$$

$$L = \frac{mR^2}{2}\left(\frac{3}{2}\dot\theta^2 + \dot\varphi^2 - \dot\theta\dot\varphi\cos(\theta-\varphi) \right)$$

$$- mgR\cos\theta + mgR\cos\varphi - KR^2\left(1-\cos(\theta-\varphi)\right)$$

2) $$\frac{d}{dt}\frac{\partial L}{\partial\dot\varphi} = \frac{d}{dt}\left(mR^2\dot\varphi - \frac{mR^2}{2}\dot\theta\cos(\theta-\varphi)\right) =$$

$$= mR^2\left(\ddot\varphi - \ddot\theta\cos(\theta-\varphi) + \frac{\dot\theta^2}{2}\operatorname{sen}(\theta-\varphi) \cancel{+\frac{\dot\theta\dot\varphi}{2}\operatorname{sen}(\varphi-\theta)}\right)$$

$$\frac{\partial L}{\partial\varphi} = \cancel{\frac{mR^2}{2}\dot\theta\dot\varphi\operatorname{sen}(\theta-\varphi)} - mgR\operatorname{sen}\varphi + KR^2\operatorname{sen}(\varphi-\theta)$$

eq. Lap.: $$\ddot\varphi - \ddot\theta\cos(\theta-\varphi) = \frac{\dot\theta^2}{2}\operatorname{sen}(\varphi-\theta) - \frac{g}{R}\operatorname{sen}\varphi + \frac{K}{m}\operatorname{sen}(\varphi-\theta)$$

3) $$V = mgR\cos\theta - mgR\cos\varphi + \cancel{KR^2} - KR^2\cos(\theta-\varphi))$$

$$\frac{\partial V}{\partial\theta} = -mgR\operatorname{sen}\theta + KR^2\operatorname{sen}(\theta-\varphi)$$

$$\frac{\partial V}{\partial\varphi} = +mgR\operatorname{sen}\varphi - KR^2\operatorname{sen}(\theta-\varphi)$$

$$\Rightarrow \quad \operatorname{sen}\theta = \operatorname{sen}\varphi \quad \rightarrow \quad \theta = \varphi \;, \quad \theta = \pi - \varphi$$

$$\operatorname{sen}(\theta-\varphi) = \frac{mg}{KR}\operatorname{sen}\varphi$$

$\theta = \varphi$: $\quad \operatorname{sen}\varphi = 0 \to (\theta, \varphi) = (0,0) \ , \ (\pi, \pi)$

$\theta = \pi - \varphi$: $\operatorname{sen}(\pi - 2\varphi) = \operatorname{sen} 2\varphi = 2 \operatorname{sen}\varphi \cos\varphi \quad \to (\theta - \varphi = \pi - 2\varphi)$

$$0 = mgR \operatorname{sen}\varphi - 2KR^2 \operatorname{sen}\varphi \cos\varphi = 2KR^2 \left(\frac{mg}{2KR} - \cos\varphi \right) \operatorname{sen}\varphi$$

$$\varphi = 0, \pi, \pm\varphi_* \qquad \text{con} \qquad \cos\varphi^* = \frac{mg}{2KR}$$

$\uparrow$

$\exists$ solo se $\frac{mg}{2KR} \leq 1$!

$$(\theta, \varphi) = (\pi, 0), \ (0, \pi), \ (\pi \mp \varphi_*, \pm\varphi_*)$$

$$\partial^2 V = \begin{pmatrix} -mgR\cos\theta + KR^2\cos(\theta - \varphi) & -KR^2\cos(\theta - \varphi) \\ -KR^2\cos(\theta - \varphi) & mg\cos\varphi + KR^2\cos(\theta - \varphi) \end{pmatrix}$$

$$\partial^2 V(0,0) = \begin{pmatrix} KR^2 - mgR & -KR^2 \\ -KR^2 & KR^2 + mgR \end{pmatrix}$$

$Tr = 2KR^2 > 0$

$\det = \cancel{(KR^2)^2} - (mgR)^2 - \cancel{(KR^2)^2} < 0$

INST.

$$\partial^2 V(\pi, \pi) = \begin{pmatrix} mgR + KR^2 & -KR^2 \\ -KR^2 & -mgR + KR^2 \end{pmatrix}$$

$\det < 0$

$$\partial^2 V(\pi, 0) = \begin{pmatrix} mgR - KR^2 & KR^2 \\ KR^2 & mgR - KR^2 \end{pmatrix}$$

$Tr = 2KR^2\left(\frac{mg}{KR} - 1\right)$

$\det = mgR^2(mg - 2KR)$

STAB se $\frac{mg}{2KR} > 1$ (sia det che tr > 0)

INST. altrim.

$$\partial^2 V(0,\pi) = \begin{pmatrix} -mgR-KR^2 & KR^2 \\ KR^2 & -mgR-KR^2 \end{pmatrix} \quad tr<0 \qquad \text{INSTAB}$$

$$\partial^2 V(\pi\mp\varphi_*, \pm\varphi_*) = \begin{pmatrix} KR^2 & \frac{m^2g^2}{2K}-KR^2 \\ \frac{m^2g^2}{2K}-KR^2 & KR^2 \end{pmatrix}$$

$$Tr = 2KR^2 > 0 \quad \det = -\frac{m^2g^2}{4R^2}(mg-2KR)(mg+2KR)$$

>0 quando esistono

STAB quando esistono.

4) $R = \frac{mg}{4K}$ $\left(\frac{mg}{2KR} = 2 > 1\right)$ $\frac{mg}{KR} = 4$

$\to$ Pto stab. è $(\theta,\varphi) = (\pi, 0)$

$$B = \begin{pmatrix} mgR-KR^2 & KR^2 \\ KR^2 & mgR-KR^2 \end{pmatrix} = mR^2 \begin{pmatrix} 3\frac{K}{m} & \frac{K}{m} \\ \frac{K}{m} & 3\frac{K}{m} \end{pmatrix}$$

$$A = mR^2 \begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$$

$$0 = \det(B-\lambda A) = (mR^2)^2 \det \begin{pmatrix} \frac{3K}{m} - \frac{3}{2}\lambda & \frac{K}{m} - \frac{\lambda}{2} \\ \frac{K}{m} - \frac{\lambda}{2} & 3\frac{K}{m} - \lambda \end{pmatrix}$$

$$9\left(\frac{K}{m}\right)^2 - \frac{3K}{m}\frac{5}{2}\lambda + \frac{3}{2}\lambda^2 - \left(\frac{K}{m}\right)^2 - \frac{\lambda^2}{4} - \frac{K}{m}\lambda = 0$$

$$\frac{5}{\cancel{4}}\lambda^2 - \frac{26}{\cancel{4}}\left(\frac{k}{m}\right)\lambda + \frac{32}{\cancel{4}}\left(\frac{k}{m}\right)^2 = 0$$

$$\frac{\Delta}{4} = \frac{k^2}{m^2}(169 - 160) = 9\left(\frac{k}{m}\right)^2$$

$$\lambda_{1,2} = \frac{13 \pm 3}{5}\,\frac{k}{m} \quad \begin{cases} \frac{2k}{m} \equiv \lambda_1 \\ \frac{16}{5}\frac{k}{m} \equiv \lambda_2 \end{cases}$$

5) $$\begin{pmatrix} \theta(t) \\ \varphi(t) \end{pmatrix} = \begin{pmatrix} \pi \\ 0 \end{pmatrix} + A_1 \vec{u}_1 \cos\left(\sqrt{\lambda_1}\, t + \varphi_1\right)$$

$$+ A_2 \vec{u}_2 \cos\left(\sqrt{\lambda_2}\, t + \varphi_2\right)$$

Determiniamo $\vec{u}_1$ e $\vec{u}_2$

$$\frac{1}{mR^2}(B - \lambda_1 A)\vec{u}_1 = 0 \rightarrow \left[\frac{k}{m}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} - \frac{k}{m}\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}\right]\vec{u}_1 = 0$$

$$\rightarrow \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}\vec{u}_1 = 0 \qquad \vec{u}_1 \propto \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\frac{1}{mR^2}(B - \lambda_2 A)\vec{u}_2 = 0 \rightarrow \left[\frac{k}{m}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} - \frac{k}{m}\begin{pmatrix} 3\cdot\frac{8}{5} & \frac{8}{5} \\ \frac{8}{5} & \frac{8}{5} \end{pmatrix}\right]\vec{u}_2 = 0$$

$$\rightarrow \begin{pmatrix} -\frac{9}{5} & -\frac{3}{5} \\ -\frac{3}{5} & -\frac{1}{5} \end{pmatrix}\vec{u}_2 = 0 \qquad \vec{u}_2 \propto \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

ES 1

6)
$$\begin{cases} q_1 = \tilde{q}_1 + \frac{\tilde{p}_1}{m} t & p_1 = \tilde{p}_1 \\ q_2 = \tilde{q}_2 + \frac{\tilde{p}_2}{m} t - \frac{1}{2} g t^2 & p_2 = \tilde{p}_2 - mgt \end{cases}$$

Inversa $\tilde{q}_1 = q_1 - \frac{p_1}{m} t$ $\tilde{p}_1 = p_1$

$\tilde{q}_2 = q_2 - \frac{p_2}{m} t - \frac{1}{2} g t^2$ $\tilde{p}_2 = p_2 + mgt$

$\{q_1, p_1\} = 1$ $\{q_i, p_j\} = 0$ perché i due argomenti non contengono

$\{q_2, p_2\} = 1$ $\{q_i, q_j\} = 0$ $i =$ mai variabili canonicam. coniugate

7)

$$E \nabla_{\tilde{x}} K_0 = \frac{\partial \tilde{w}}{\partial t}$$

$x = \tilde{w}(x, t)$

$$\tilde{w} = \begin{pmatrix} p_1 \\ p_2 + mgt \\ q_1 - \frac{p_1}{m} t \\ q_2 - \frac{p_2}{m} t - \frac{1}{2} g t^2 \end{pmatrix} \qquad \frac{\partial \tilde{w}}{\partial t} = \begin{pmatrix} 0 \\ mg \\ -p_1/m \\ -\frac{p_2}{m} - gt \end{pmatrix}$$

$$\frac{\partial \tilde{w}}{\partial t}(w(\tilde{x}, t)) = \begin{pmatrix} 0 \\ mg \\ -\tilde{p}_1/m \\ -\tilde{p}_2/m \end{pmatrix}$$

$$-\frac{p_2}{m} = -\frac{\tilde{p}_2}{m} + gt$$

$$\nabla_{\tilde{x}} K_0 \overset{E^2 = \mathbb{1}}{=} -E \frac{\partial \tilde{w}}{\partial t} = -\begin{pmatrix} 0 & -\mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \begin{pmatrix} 0 \\ mg \\ -\tilde{p}_1/m \\ -\tilde{p}_2/m \end{pmatrix}$$

$$\begin{pmatrix} \partial_{\tilde{p}_1} K_0 \\ \partial_{\tilde{p}_2} K_0 \\ \partial_{\tilde{q}_1} K_0 \\ \partial_{\tilde{q}_2} K_0 \end{pmatrix} = \begin{pmatrix} -\tilde{p}_1/m \\ -\tilde{p}_2/m \\ 0 \\ -mg \end{pmatrix} \Rightarrow K_0 = -\frac{\tilde{p}_1^2}{2m} - \frac{\tilde{p}_2^2}{2m} - mg\tilde{q}_2$$

8) Il flusso Ham. (1) è il moto di un corpo in caduta libera in campo gravitaz. costante lungo direz. q_2.

L'Ham. che lo genera è allora

$$H = \underbrace{\frac{p_1^2 + p_2^2}{2m}}_{T} + \underbrace{m g q_2}_{V}$$

$$\Bigg[\; K = \tilde{H} + K_0 = \frac{\cancel{\tilde{p}_1^2} + \cancel{\tilde{p}_2^2} - \cancel{2 m g t \tilde{p}_2} + \cancel{(m g t)^2}}{2m} + \cancel{m g \tilde{q}_2} + \cancel{g \tilde{p}_2 t} - \cancel{\frac{m g^2 t^2}{2}} - \cancel{\frac{\tilde{p}_1^2}{2m}} - \cancel{\frac{\tilde{p}_2^2}{2m}} - \cancel{m g \tilde{q}_2} = 0 \;.\Bigg]$$