ESERCIZIO 1

1) Uso la conservazione dell'energia

$$E_1 = \frac{1}{2} m v_1^2 + U_1 = \frac{1}{2} m v_2^2 + U_2$$

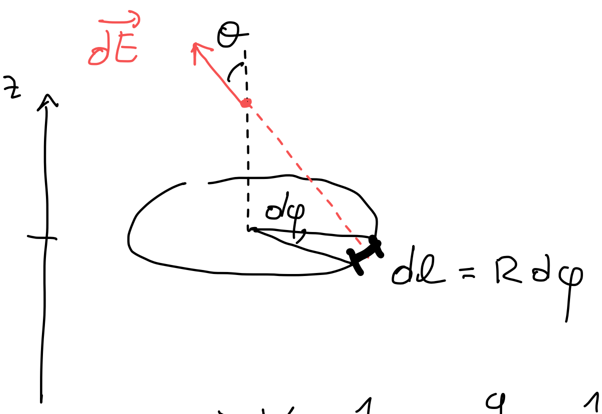
So che $v_2 = 0$ e $U_2 - U_1 = |e|(V_2 - V_1)$

$$\Rightarrow |e|(V_2 - V_1) = \frac{1}{2} m v_1^2$$

$$\boxed{V_2 - V_1 = \frac{m v_1^2}{2|e|} \approx 4.7 \times 10^2 \, V}$$

2) Ora devo calcolare $V_2 - V_1$

Calcolo il potenziale generato da un anello lungo l'asse, con $V = 0$ all'infinito

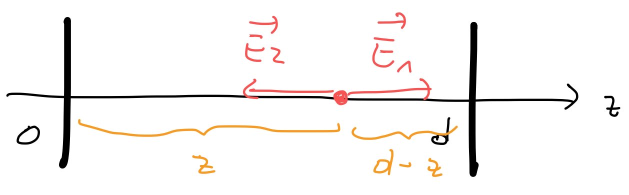


$$dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda R\, d\varphi}{(R^2+z^2)^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{2\pi R} \frac{R\, d\varphi}{(R^2+z^2)^{1/2}}$$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0} \frac{q}{2\pi} \frac{1}{(R^2+z^2)^{1/2}} \int_0^{2\pi} d\varphi = \frac{q}{4\pi\varepsilon_0} \frac{1}{(R^2+z^2)^{1/2}}$$

Sommo i campi dei due anelli. Fisso $z_1 = 0$ e $z_2 = d$.



In un punto generico z

$$V(z) = \frac{1}{4\pi\varepsilon_0} \left\{ \frac{q_1}{(R^2+z^2)^{1/2}} + \frac{q_2}{[R^2+(d-z)^2]^{1/2}} \right\}$$

e quindi

$$V_2 - V_1 = \frac{1}{4\pi\varepsilon_0}\left\{\left[\frac{q_1}{(R^2+d^2)^{1/2}} + \frac{q_2}{R}\right] - \left[\frac{q_1}{R} + \frac{q_2}{(R^2+d^2)^{1/2}}\right]\right\}$$

$$= \frac{1}{4\pi\varepsilon_0}(q_1-q_2)\left\{\frac{1}{(R^2+d^2)^{1/2}} - \frac{1}{R}\right\}$$

Questo è uguale a $\frac{m v_1^2}{2e}$ quindi

$$q_1 - q_2 = 4\pi\varepsilon_0 \frac{m v_1^2}{2e}\left\{\frac{1}{(R^2+d^2)^{1/2}} - \frac{1}{R}\right\}^{-1}$$

e $$\boxed{\lambda_2 = \frac{q_2}{R} = \frac{q_1}{R} - \frac{4\pi\varepsilon_0}{R}\frac{m v_1^2}{2e}\left\{\frac{1}{(R^2+d^2)^{1/2}} - \frac{1}{R}\right\}^{-1} \approx 356 \frac{nC}{m}}$$

e $q_2 \approx 45\ nC$

3) conservazione dell'energia:

$$\frac{1}{2}m v_1^2 - \frac{1}{2}m\left(\frac{v_1}{2}\right)^2 = e\left[V(d_1) - V(0)\right]$$

$$\frac{3}{8}m v_1^2 = eV(d_1) - eV(0)$$

$$V(d_1) = \frac{3}{8}\frac{m v_1^2}{e} + V(0)$$

$$\frac{1}{4\pi\varepsilon_0}\left\{\frac{q_1}{(R^2+d_1^2)^{1/2}} + \frac{q_2}{[R^2+(d-d_1)^2]^{1/2}}\right\} = \frac{3 m v_1^2}{8e} + V(0)$$

Conviene assumere $R \ll d_1$ e $R \ll |d-d_1|$ e trascurare R al denominatore

$$\left\{ \frac{q_1}{d_1} + \frac{q_2}{d-d_1} \right\} = \underbrace{\left(\frac{3 m v_1^2}{8e} + V(0) \right) 4\pi \varepsilon_0}_{A} \qquad \text{con } d > d_1$$

$$q_1(d-d_1) + q_2 d_1 = A\, d_1 (d-d_1)$$

$$A d_1^2 + (q_2 - q_1 - Ad) d_1 + q_1 d = 0$$

$$\boxed{d_1 = \frac{-(q_2 - q_1 - Ad) + \sqrt{(q_2 - q_1 - Ad)^2 - 4 A q_1 d}}{2A} \approx 20 \text{ cm}}$$

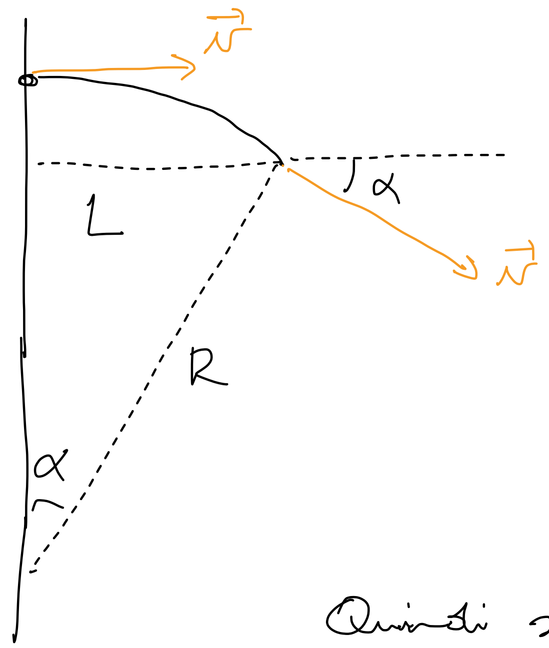
ESERCIZIO 2

1) Il campo magnetico non compie lavoro sul protone, quindi posso usare la conservazione dell'energia

$$E_k = \frac{1}{2} m v^2 \Rightarrow v = \left(\frac{2E_k}{m}\right)^{1/2} \approx 9.8 \times 10^7 \frac{m}{s}$$

$$\Rightarrow \boxed{p = (2 m E_k)^{1/2} \approx 1.6 \times 10^{-19} \, kg \frac{m}{s}}$$

2) L'orbita è circolare.



$$L = R \sin\alpha$$

Il raggio della traiettoria è dato da

$$\frac{m v^2}{R} = q v B$$

$$\Rightarrow R = \frac{m v}{q B} \approx 2.0 \, m$$

Quindi $\sin\alpha = \frac{L}{R} = \frac{q B L}{m v} \approx 0.49$

3) $y = R\cos\alpha - R \approx -0.26 \, m$

ESERCIZIO 3

1) $B(t_1) = \mu_0 n I = \mu_0 n I_0 e^{-\alpha t_1} \approx 1.1\ mT$

2) $\phi = B \pi r^2 \approx 8.8\ \mu Wb$

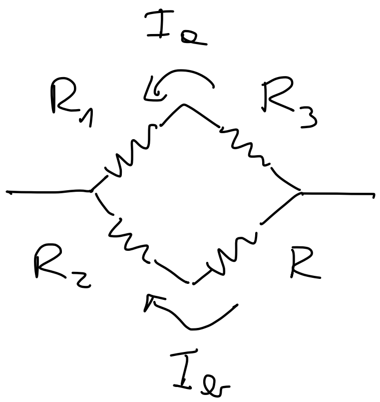
3) $\mathcal{E} = -\frac{d\phi}{dt} = -\pi r^2 \mu_0 n \frac{dI}{dt} = \pi r^2 \mu_0 n \alpha I_0 e^{-\alpha t}$

$\mathcal{E}(t_1) \approx 0.44\ mV$

4) $I_{anello}(t_1) = \frac{\mathcal{E}(t_1)}{R} \approx 2.2\ mA$

ESERCIZIO 4

1)



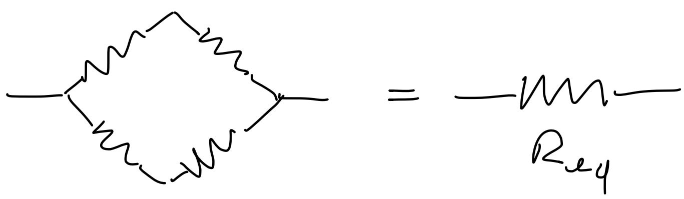
$$\begin{cases} (R_1+R_3)\, I_a = (R_2+R)\, I_b \\ R_1 I_a = R_2 I_b \Rightarrow I_a = \dfrac{R_2}{R_1} I_b \end{cases}$$

$$\Rightarrow (R_1+R_3)\,\frac{R_2}{R_1} = R_2 + R$$

$$R = \frac{(R_1+R_3)R_2}{R_1} - R_2 = \frac{R_1R_2 + R_2R_3 - R_1R_2}{R_1} = \frac{R_2R_3}{R_1}$$

$$\boxed{R = \frac{R_2R_3}{R_1} = 159\ \Omega}$$

2) La resistenza equivalente è



con $R_{eq} = \left(\dfrac{1}{R+R_2} + \dfrac{1}{R_1+R_3}\right)^{-1} \approx 123.6\ \Omega$

L'impedenza del circuito è pari a

$$|Z| = \left[R_{eq}^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2} \approx 2.0 \times 10^5\ \Omega$$

e $\operatorname{Arg} Z = -89.96°$

3) $$I_{eff} = \frac{V_{eff}}{|Z|} = \frac{V_0/\sqrt{2}}{|Z|} \approx 0.35\,mA$$

4) $$\frac{1}{\omega C} \approx 2.0 \times 10^5\,\Omega$$
$$\omega L \approx 4\,\Omega$$

$Z_C \gg Z_L, Z_R$

il circuito è in regime capacitivo

$$\tan\varphi = \frac{\Im m\, Z}{Re\, Z} = \frac{\omega L - \frac{1}{\omega C}}{R_{eq}} = -1.6 \times 10^3 < 0$$

Siccome $I(t) = |I_0| \cos(\omega t + \varphi)$

la corrente è in ritardo rispetto alla tensione