A.A. 2025-2026 ELEMENTI DI TERMOFLUIDODINAMICA PER LE MACCHINE

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Termodinamica dei fluidi

EQUAZIONE DI BILANCIO DELL'ENERGIA

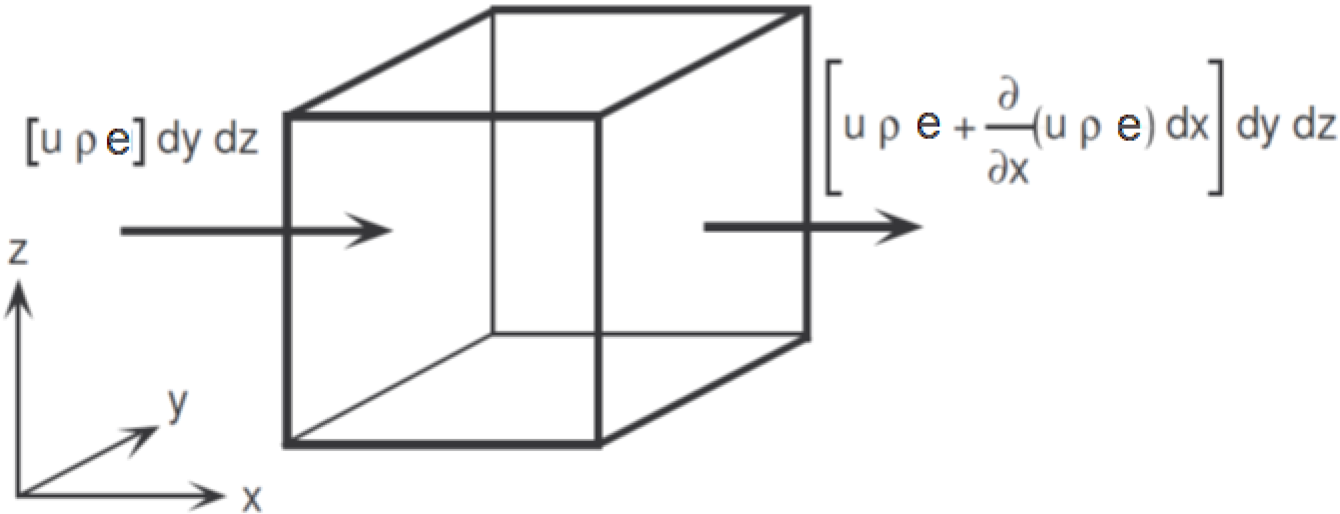
La differenza tra l'energia del fluido che in un tempo infinitesimo entra in un volume infinitesimo fisso nello spazio e l'energia che ne esce più le quantità di calore e i lavori relativi al volume infinitesimo è rappresentata dall'energia che nello stesso tempo si accumula nel volume infinitesimo.

$$e = e_i + e_c + e_p = e_i + \frac{1}{2}|\mathbf{u}|^2 + gz$$ energia per unità di massa

$$dE = dm\,e = \rho dV\,e$$ energia infinitesima relativa al volume infinitesimo dV

Termodinamica dei fluidi

Portata di energia passante in direzione x



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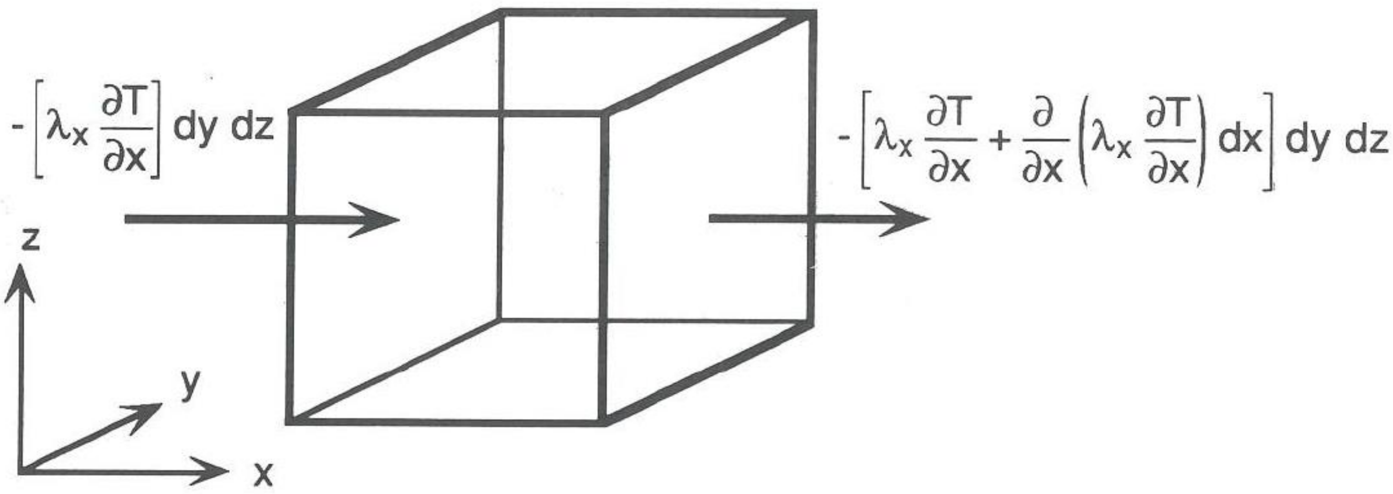
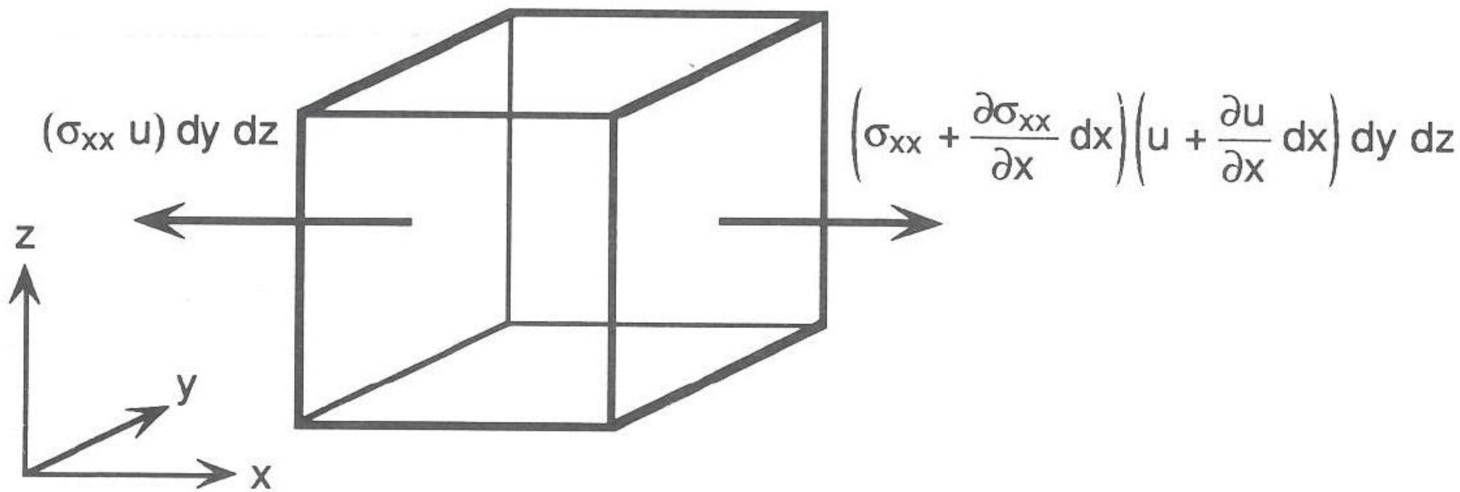
Calore passante per conduzione in direzione x attraverso un cubetto infinitesimo



Termodinamica dei fluidi

Lavoro totale fatto dalla tensione σ_{xx} in direzione x



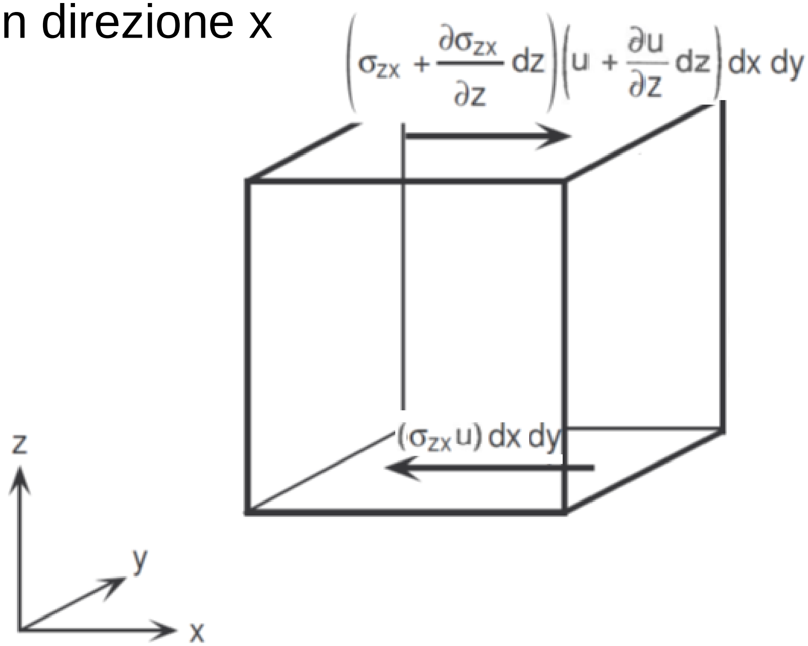
$$\frac{dL_{xx}}{dt} = \left(u + \frac{\partial u}{\partial x} dx\right)\left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx\right) dydz - u\sigma_{xx} dydz =$$

$$= u\sigma_{xx} dydz + u\frac{\partial \sigma_{xx}}{\partial x} dxdydz + \frac{\partial u}{\partial x}\sigma_{xx} dxdydz + \frac{\partial u}{\partial x}\frac{\partial \sigma_{xx}}{\partial x} dx^2 dydz - u\sigma_{xx} dydz =$$

$$= u\frac{\partial \sigma_{xx}}{\partial x} dV + \frac{\partial u}{\partial x}\sigma_{xx} dV = \frac{\partial}{\partial x}\left(u\sigma_{xx}\right) dV$$

Termodinamica dei fluidi

Lavoro totale fatto dalla tensione σ_{zx} in direzione x



$$\frac{dL_{zx}}{dt} = \left(u + \frac{\partial u}{\partial z} dz\right)\left(\sigma_{zx} + \frac{\partial \sigma_{zx}}{\partial z} dz\right) dxdy - u\sigma_{zx} dxdy =$$

$$= u\sigma_{zx} dxdy + u \frac{\partial \sigma_{zx}}{\partial z} dxdydz + \frac{\partial u}{\partial z} \sigma_{zx} dxdydz + \frac{\partial u}{\partial z} \frac{\partial \sigma_{zx}}{\partial z} dxdydz^2 - u\sigma_{zx} dydy =$$

$$= u \frac{\partial \sigma_{zx}}{\partial z} dV + \frac{\partial u}{\partial z} \sigma_{zx} dV = \frac{\partial}{\partial z}\left(u\sigma_{zx}\right) dV$$

Termodinamica dei fluidi

$$\frac{dL}{dt}=\frac{dl}{dt}dV=\left[\frac{\partial}{\partial x}\left(u\sigma_{xx}+v\sigma_{xy}+w\sigma_{xz}\right)+\frac{\partial}{\partial y}\left(u\sigma_{yx}+v\sigma_{yy}+w\sigma_{yz}\right)+\frac{\partial}{\partial z}\left(u\sigma_{zx}+v\sigma_{zy}+w\sigma_{zz}\right)\right]dV$$

$$\frac{dl}{dt}=\frac{dl_M}{dt}+\frac{dl_D}{dt}=\nabla\cdot\left(\mathbf{u}\cdot\bar{\bar{\boldsymbol{\sigma}}}\right)=\mathbf{u}\cdot\left(\nabla\cdot\bar{\bar{\boldsymbol{\sigma}}}\right)+\left(\nabla\mathbf{u}\right):\bar{\bar{\boldsymbol{\sigma}}}=\rho\frac{D}{Dt}\left(e_c+e_p\right)-p\nabla\cdot\mathbf{u}+\mu\Phi$$

$$\frac{dl_M}{dt}=\mathbf{u}\cdot\left(\nabla\cdot\bar{\bar{\boldsymbol{\sigma}}}\right)=\mathbf{u}\cdot\left(\nabla\cdot\left(-p\mathbf{I}+\bar{\bar{\boldsymbol{\sigma}}}^{*}\right)\right)=\rho\frac{D}{Dt}\left(e_c+e_p\right)$$

Termodinamica dei fluidi

$$\frac{dI_D}{dt}=(\nabla\mathbf{u}):\bar{\bar{\boldsymbol{\sigma}}}=(\nabla\mathbf{u}):\left(-p\mathbf{I}+\bar{\bar{\boldsymbol{\sigma}}}^*\right)=$$

$$=\begin{pmatrix}\frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x}\\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y}\\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z}\end{pmatrix}:\begin{pmatrix}-p+\mu\left(2\frac{\partial u}{\partial x}-\frac{2}{3}\nabla\cdot\mathbf{u}\right) & \mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right) & \mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\\ \mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) & -p+\mu\left(2\frac{\partial v}{\partial y}-\frac{2}{3}\nabla\cdot\mathbf{u}\right) & \mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\\ \mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) & \mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) & -p+\mu\left(2\frac{\partial w}{\partial z}-\frac{2}{3}\nabla\cdot\mathbf{u}\right)\end{pmatrix}=$$

$$=\frac{\partial u}{\partial x}\left[-p+\mu\left(2\frac{\partial u}{\partial x}-\frac{2}{3}\nabla\cdot\mathbf{u}\right)\right]+\frac{\partial v}{\partial x}\left[\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+\frac{\partial w}{\partial x}\left[\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right]+$$

$$+\frac{\partial u}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial v}{\partial y}\left[-p+\mu\left(2\frac{\partial v}{\partial y}-\frac{2}{3}\nabla\cdot\mathbf{u}\right)\right]+\frac{\partial w}{\partial y}\left[\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)\right]+$$

$$+\frac{\partial u}{\partial z}\left[\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]+\frac{\partial v}{\partial z}\left[\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right]+\frac{\partial w}{\partial z}\left[-p+\mu\left(2\frac{\partial w}{\partial z}-\frac{2}{3}\nabla\cdot\mathbf{u}\right)\right]=$$

$$=-p\nabla\cdot\mathbf{u}+\mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2+\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)^2\right\}-\frac{2}{3}\mu(\nabla\cdot\mathbf{u})^2=$$

$$=-p\nabla\cdot\mathbf{u}+\mu\Phi$$

Termodinamica dei fluidi

Il bilancio di energia dà:

$$\rho \frac{D}{Dt}\left(e_i + e_c + e_p\right) = \dot{q} + \lambda \Delta T + \rho \frac{D}{Dt}\left(e_c + e_p\right) - p\nabla \cdot \mathbf{u} + \mu\Phi + \rho \mathbf{F}_m \cdot \mathbf{u}$$

$$\boxed{\rho \frac{D}{Dt}\left(c_p T\right) - \dot{q} - \lambda \Delta T - \frac{Dp}{Dt} - \mu\Phi - \rho \mathbf{F}_m \cdot \mathbf{u} = 0}$$