ONDE D'URTO OBLIQUE

RELAZIONE $\theta-\beta-M$

$$\frac{\rho_2}{\rho_1}=\frac{(\gamma+1)M_1^2\,\text{sen}^2\beta}{2+(\gamma-1)M_1^2\,\text{sen}^2\beta}$$

$$\tan\beta=\frac{U_{n1}}{U_{t1}}\ ,\quad \tan(\beta-\theta)=\frac{U_{n2}}{U_{t2}}$$

$$U_{t1}=U_{t2}\ \longrightarrow\ \frac{\tan\beta}{\tan(\beta-\theta)}=\frac{U_1}{U_2}=\frac{\rho_2}{\rho_1}=\frac{(\gamma+1)M_1^2\,\text{sen}^2\beta}{2+(\gamma-1)M_1^2\,\text{sen}^2\beta}$$

$$\tan(\beta-\theta)=\frac{\tan\beta-\tan\theta}{1+\tan\beta\tan\theta}\ ,\quad \text{sen}^2\beta+\cos^2\beta=1\ ,\ \cos^2\beta-\text{sen}^2\beta=\cos2\beta$$

$$\Longrightarrow\ \frac{\tan\beta\,(1+\tan\beta\tan\theta)}{\tan\beta-\tan\theta}=\frac{(\gamma+1)M_1^2\,\text{sen}^2\beta}{2+(\gamma-1)M_1^2\,\text{sen}^2\beta}$$

$$\tan\beta+\tan^2\beta\tan\theta=\frac{(\gamma+1)M_1^2\,\text{sen}^2\beta}{2+(\gamma-1)M_1^2\,\text{sen}^2\beta}\left(\tan\beta-\tan\theta\right)$$

$$\tan\theta\left(\tan^2\beta+\frac{(\gamma+1)M_1^2\,\text{sen}^2\beta}{2+(\gamma-1)M_1^2\,\text{sen}^2\beta}\right)=\tan\beta\left(\frac{(\gamma+1)M_1^2\,\text{sen}^2\beta}{2+(\gamma-1)M_1^2\,\text{sen}^2\beta}-1\right)$$

$$\tan\theta=\frac{\tan\beta}{\tan^2\beta}\left(\frac{\dfrac{(\gamma+1)M_1^2\,\text{sen}^2\beta}{2+(\gamma-1)M_1^2\,\text{sen}^2\beta}-1}{1+\dfrac{(\gamma+1)M_1^2\,\text{sen}^2\beta/\tan^2\beta}{2+(\gamma-1)M_1^2\,\text{sen}^2\beta}}\right)=$$

$$=\frac{1}{\tan\beta}\ \frac{(\gamma+1)M_1^2\,\text{sen}^2\beta-\left(2+(\gamma-1)M_1^2\,\text{sen}^2\beta\right)}{2+(\gamma-1)M_1^2\,\text{sen}^2\beta+(\gamma+1)M_1^2\cos^2\beta}$$

$$=\cot\beta\ \frac{2M_1^2\,\text{sen}^2\beta-2}{2+\gamma M_1^2\left(\text{sen}^2\beta+\cos^2\beta\right)+M_1^2\left(\cos^2\beta-\text{sen}^2\beta\right)}$$

$$=\cot\beta\ \frac{2\left(M_1^2\,\text{sen}^2\beta-1\right)}{2+\gamma M_1^2+M_1^2\cos2\beta}=\cot\beta\ \frac{2\left(M_1^2\,\text{sen}^2\beta-1\right)}{2+M_1^2\left(\gamma+\cos2\beta\right)}$$