

Operatore	Coordinate cartesiane (x,y,z)	Coordinate cilindriche (ρ,φ,z)	Coordinate sferiche (r,θ,φ)
Definizione delle coordinate		$\begin{bmatrix} x = \rho \cos \phi \\ y = \rho \sin \phi \\ z = z \end{bmatrix}.$	$\begin{bmatrix} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{bmatrix} \begin{matrix} 0 \leq \theta \leq \pi \\ 0 \leq \phi < 2\pi \\ 0 \leq r < +\infty \end{matrix}$
		$\begin{bmatrix} \rho = \sqrt{x^2 + y^2} \\ \phi = \arctang(y/x) \\ z = z \end{bmatrix}.$	$\begin{bmatrix} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \arccos(z/r) \\ \phi = \arctang(y/x) \end{bmatrix}.$
A Campo vettoriale A	$A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}} + A_z \hat{\mathbf{z}}$	$A_\rho \hat{\boldsymbol{\rho}} + A_\phi \hat{\boldsymbol{\phi}} + A_z \hat{\mathbf{z}}$	$A_r \hat{\mathbf{r}} + A_\theta \hat{\boldsymbol{\theta}} + A_\phi \hat{\boldsymbol{\phi}}$
Gradiente ∇f	$\frac{\partial f}{\partial x} \hat{\mathbf{x}} + \frac{\partial f}{\partial y} \hat{\mathbf{y}} + \frac{\partial f}{\partial z} \hat{\mathbf{z}}$	$\frac{\partial f}{\partial \rho} \hat{\boldsymbol{\rho}} + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z} \hat{\mathbf{z}}$	$\frac{\partial f}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\boldsymbol{\phi}}$
Divergenza $\nabla \cdot \mathbf{A}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho} \frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2} \frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}(A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$
Rotore $\nabla \times \mathbf{A}$	$\begin{pmatrix} \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \end{pmatrix} \hat{\mathbf{x}} +$ $\begin{pmatrix} \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \end{pmatrix} \hat{\mathbf{y}} +$ $\begin{pmatrix} \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \end{pmatrix} \hat{\mathbf{z}}$	$\begin{pmatrix} \frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \end{pmatrix} \hat{\boldsymbol{\rho}} +$ $\begin{pmatrix} \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \end{pmatrix} \hat{\boldsymbol{\phi}} +$ $\frac{1}{\rho} \begin{pmatrix} \frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi} \end{pmatrix} \hat{\mathbf{z}}$	$\frac{1}{r \sin \theta} \left(\frac{\partial}{\partial \theta}(A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right) \hat{\mathbf{r}} +$ $\frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(r A_\phi) \right) \hat{\boldsymbol{\theta}} +$ $\frac{1}{r} \left(\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta} \right) \hat{\boldsymbol{\phi}}$
Laplaciano $\nabla^2 f$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$
Laplaciano di un vettore $\nabla^2 \mathbf{A}$	$\nabla^2 A_x \hat{\mathbf{x}} + \nabla^2 A_y \hat{\mathbf{y}} + \nabla^2 A_z \hat{\mathbf{z}}$	$\left(\nabla^2 A_\rho - \frac{A_\rho}{\rho^2} - \frac{2}{\rho^2} \frac{\partial A_\phi}{\partial \phi} \right) \hat{\boldsymbol{\rho}} +$ $\left(\nabla^2 A_\phi - \frac{A_\phi}{\rho^2} + \frac{2}{\rho^2} \frac{\partial A_\rho}{\partial \phi} \right) \hat{\boldsymbol{\phi}} +$ $(\nabla^2 A_z) \hat{\mathbf{z}}$	$\left(\nabla^2 A_r - \frac{2A_r}{r^2} - \frac{2}{r^2 \sin \theta} \frac{\partial(A_\theta \sin \theta)}{\partial \theta} - \frac{2}{r^2 \sin \theta} \frac{\partial A_\phi}{\partial \phi} \right) \hat{\mathbf{r}} +$ $\left(\nabla^2 A_\theta - \frac{A_\theta}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{\partial A_r}{\partial \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_\phi}{\partial \phi} \right) \hat{\boldsymbol{\theta}} +$ $\left(\nabla^2 A_\phi - \frac{A_\phi}{r^2 \sin^2 \theta} + \frac{2}{r^2 \sin \theta} \frac{\partial A_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_\theta}{\partial \phi} \right) \hat{\boldsymbol{\phi}}$
Lunghezza infinitesima	$d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}$	$d\mathbf{l} = d\rho \hat{\boldsymbol{\rho}} + \rho d\phi \hat{\boldsymbol{\phi}} + dz \hat{\mathbf{z}}$	$d\mathbf{l} = dr \hat{\mathbf{r}} + r d\theta \hat{\boldsymbol{\theta}} + r \sin \theta d\phi \hat{\boldsymbol{\phi}}$
Aree infinitesime	$d\mathbf{S} = dydz \hat{\mathbf{x}} +$ $dx dz \hat{\mathbf{y}} +$ $dx dy \hat{\mathbf{z}}$	$d\mathbf{S} = \rho d\phi dz \hat{\boldsymbol{\rho}} +$ $dp dz \hat{\boldsymbol{\phi}} +$ $\rho dp d\phi \hat{\mathbf{z}}$	$d\mathbf{S} = r^2 \sin \theta d\theta d\phi \hat{\mathbf{r}} +$ $r \sin \theta dr d\phi \hat{\boldsymbol{\theta}} +$ $r dr d\theta \hat{\boldsymbol{\phi}}$
Volume infinitesimo	$dv = dx dy dz$	$dv = \rho d\rho d\phi dz$	$dv = r^2 \sin \theta dr d\theta d\phi$

Relazioni notevoli (valgono in tutti i sistemi di riferimento):

- $\text{div grad } f = \nabla \cdot (\nabla f) = \nabla^2 f$ (Laplaciano)
- $\text{rot grad } f = \nabla \times (\nabla f) = 0$
- $\text{div rot } \mathbf{A} = \nabla \cdot (\nabla \times \mathbf{A}) = 0$
- $\text{rot rot } \mathbf{A} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$
- $\nabla^2 fg = f\nabla^2 g + 2\nabla f \cdot \nabla g + g\nabla^2 f$
- Formula di Lagrange per il [prodotto vettoriale](#): $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$
- $\nabla \cdot (f\mathbf{A}) = f\nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla f$
- $\nabla \times f\mathbf{A} = f\nabla \times \mathbf{A} - \mathbf{A} \times \nabla f$
- $\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A})$,

che insieme a $\mathbf{A} = \mathbf{B} = \mathbf{v}$ segue immediatamente la chiave per il fluido di trasformazione meccanica Weber:

$$(\mathbf{v} \cdot \nabla)\mathbf{v} = \nabla \frac{\mathbf{v}^2}{2} - \mathbf{v} \times (\nabla \times \mathbf{v})$$

- $\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$
- $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$