More Is Different

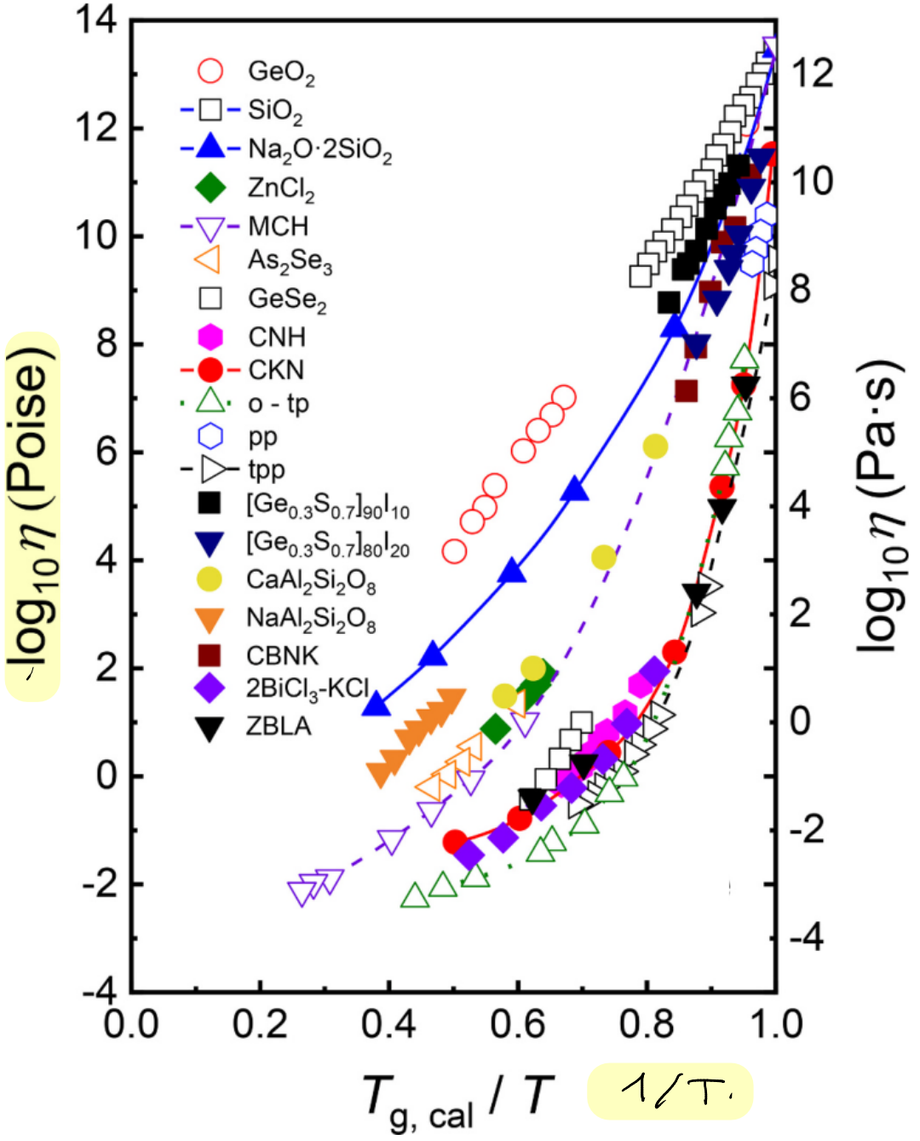
Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

1972

I do not mean to give the impression that all is settled. For instance, I think there are still fascinating questions of principle about glasses and other amorphous phases, which may reveal even more complex types of behavior. Nevertheless, the role of this type of broken symmetry in the properties of inert but macroscopic material bodies is now understood, at least in principle. In this case we can see how the whole becomes not only more than but very different from the sum of its parts.

Nobel prize 1977


GeO2
SiO2
Na2O·2SiO2
ZnCl2
MCH
As2Se3
GeSe2
CNH
CKN
o - tp
pp
tpp
[Ge0.3S0.7]90I10
[Ge0.3S0.7]80I20
CaAl2Si2O8
NaAl2Si2O8
CBNK
2BiCl3-KCl
ZBLA
log10η (Poise)
log10η (Pa·s)
Tg, cal / T
1/T

ORDINE E DISORDINE

Disordine : 1) assenza di ordine a lungo raggio
2) disordine gelato ("quenched randomness")

Sistema : dofs + interazione → hamiltoniana

1) **Dofs posizionali** $\{\vec{r}_i\}$: posizioni

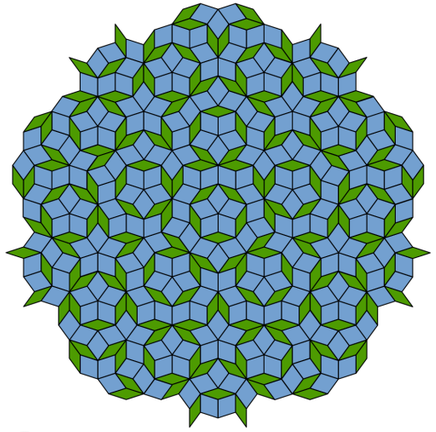
$H = K + U(\{\vec{r}_i\})$

Es.: $U = \sum_i \sum_j{}' u(|\vec{r}_i - \vec{r}_j|)$ additività a coppie

- cristallo periodico : reticolo di Bravais
- quasicristallo (ordine quasiperiodico)

1930 : cristallo → spettro di diffrazione discreto

esagonale 2d



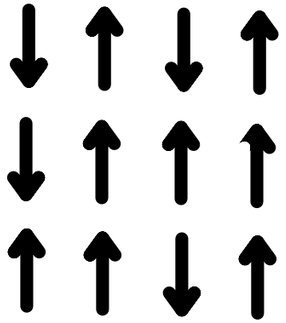
Penrose tiling

2) **Dofs orientazionali** $\{\vec{\sigma}_i\}$: spins

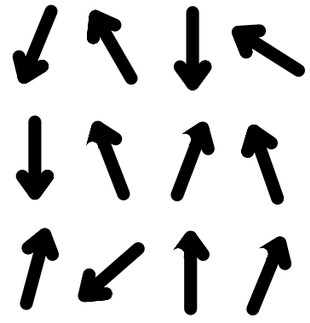
$H = H(\{\vec{\sigma}_i\})$ → ordine ferromagnetico

Es.: Ising $\sigma_i = \pm 1$ $\quad H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$

Heisenberg $\vec{\sigma}_i \in \mathbb{R}^d$ $\quad H = -J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j$



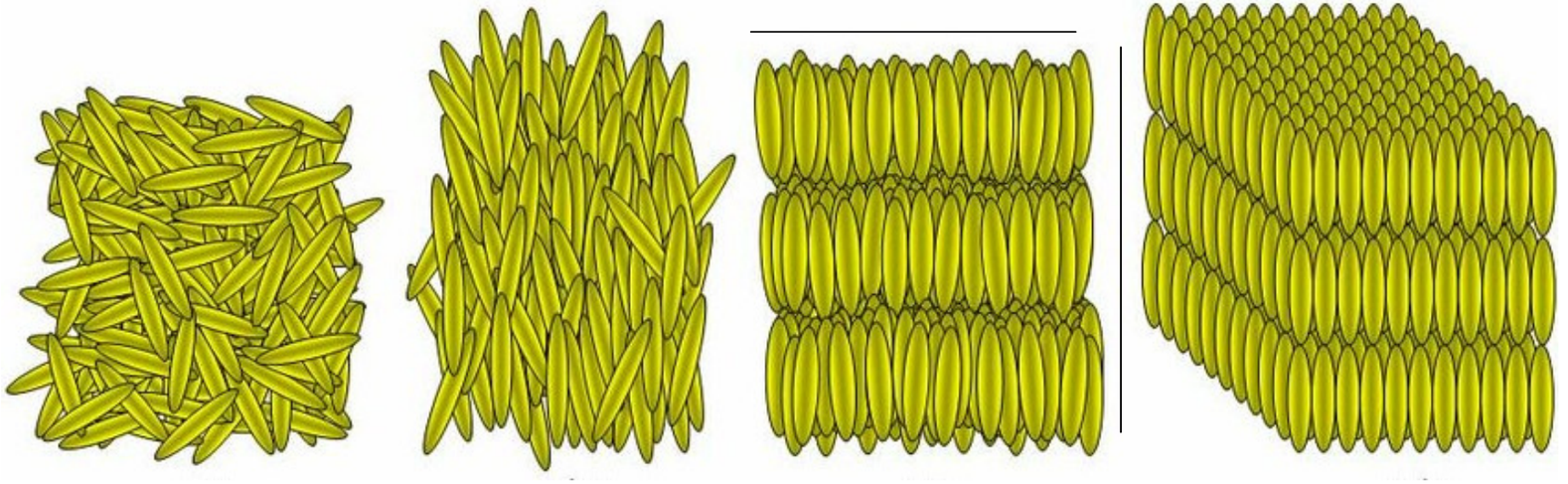
Ising



Heisenberg

3) Dofs misti $\{\bar{F}_i, \bar{T}_i\}$: molecole

Es: cristalli liquidi



liquida nematica "smectic" cristallina

SIMMETRIE E FASI DELLA MATERIA

1) Configurazione

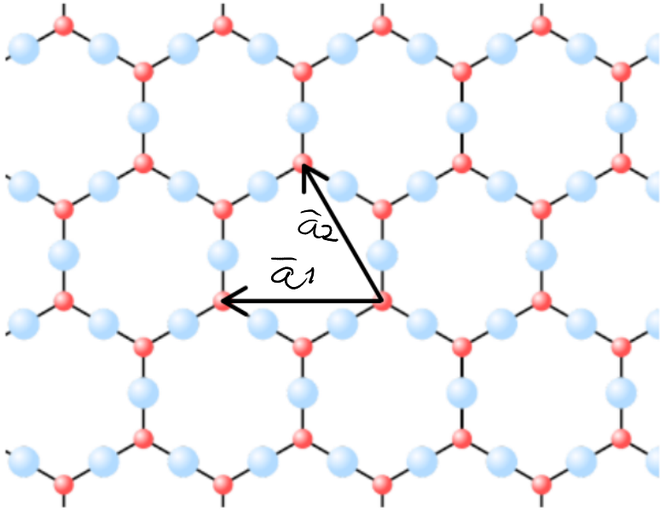
Cristallo periodico in 2d

$\exists\ \vec{a}_1, \vec{a}_2$ t.c.

$$T_\ell : \vec{r} \to \vec{r} + \vec{R}_\ell \qquad \vec{R}_\ell = \ell_1 \vec{a}_1 + \ell_2 \vec{a}_2 \qquad \ell_1, \ell_2 \in \mathbb{Z}$$

mappa la configurazione in se stessa.

Invariante per traslazione discreta.



2) Sistema

$A(\{\vec{r}_i\})$ è invariante per T se $A(\{\vec{r}_i\}) = A(T(\{\vec{r}_i\}))$

Sistema possiede la simmetria T se $H(\{\vec{r}_i\}) = H(T(\{\vec{r}_i\}))$

Traslazione continua

$$T : \vec{r} \to \vec{r} + \vec{R} \qquad \vec{R} \in \mathbb{R}^3$$

Es.: additività a coppie $u(|\vec{r}_i - \vec{r}_j|)$

$$U(T(\{\vec{r}_i\})) = U(\{\vec{r}_i\})$$

Simmetria di tipo "time-reversal"

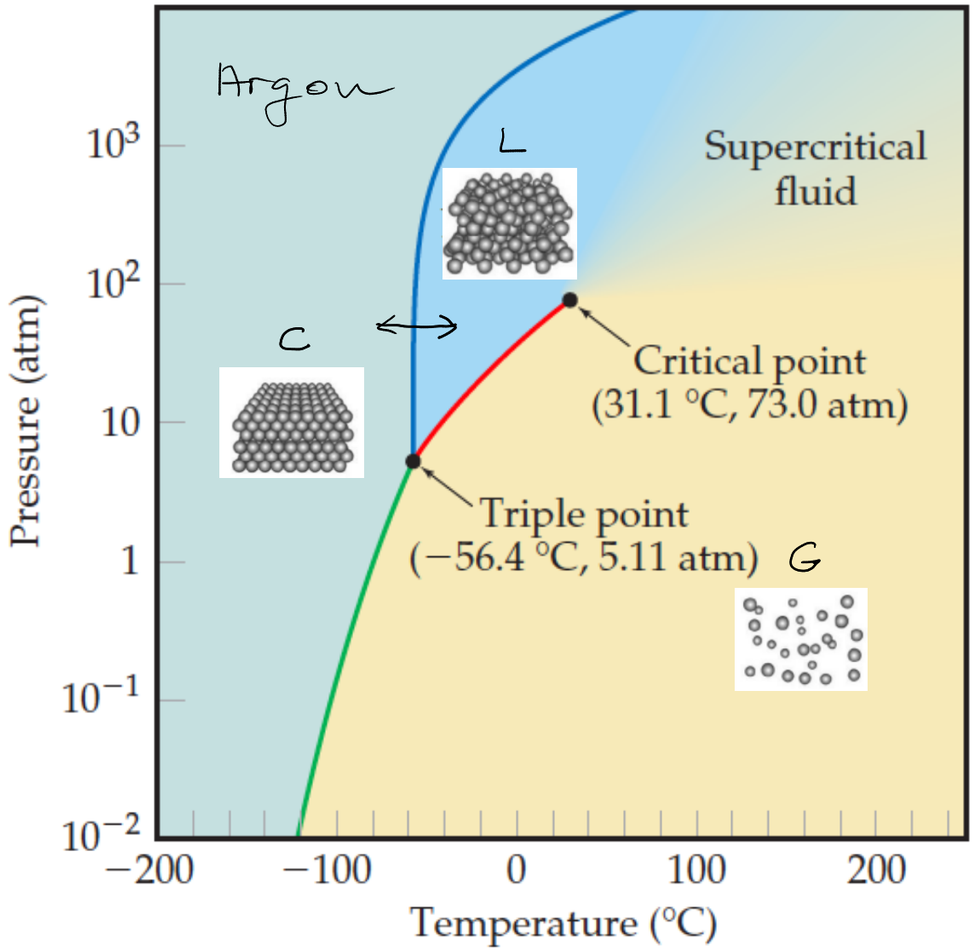
$Z_2 : \sigma \to -\sigma$ Es: Ising $U(\{\sigma_i\}) = U(\{-\sigma_i\})$

3) Fasi

Regione di analiticità di $f = \frac{F}{N}$ in funzione dei parametri di controllo con $N \to \infty$ e $N/V = cost$

Fase → insieme di microstati

Diagramma di fase : gas rari



$$\hat{A}(\vec{r}\,;\{\vec{r}_i\}) = \hat{A}(\vec{r})$$

$$\langle \hat{A} \rangle_\alpha = \frac{Tr_\alpha [\hat{A} e^{-\beta H}]}{Tr_\alpha [e^{-\beta H}]} \qquad \alpha = c, L, G$$

Es.: densità microscopica

$$\hat{\rho}(\vec{r}) = \sum_{i=1}^{N} \delta(\vec{r} - \vec{r}_i)$$

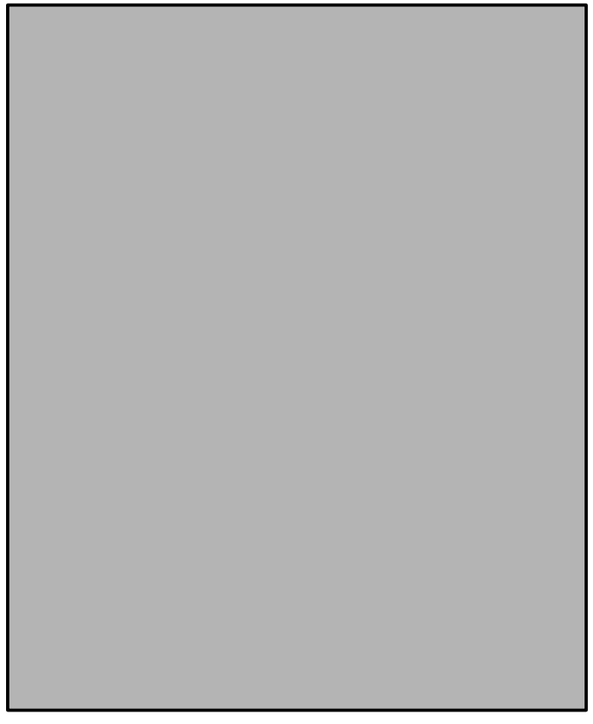
densità locale

$$\rho_\alpha(\vec{r}) = \langle \hat{\rho}(\vec{r}) \rangle_\alpha$$

T_ℓ traslazione discreta

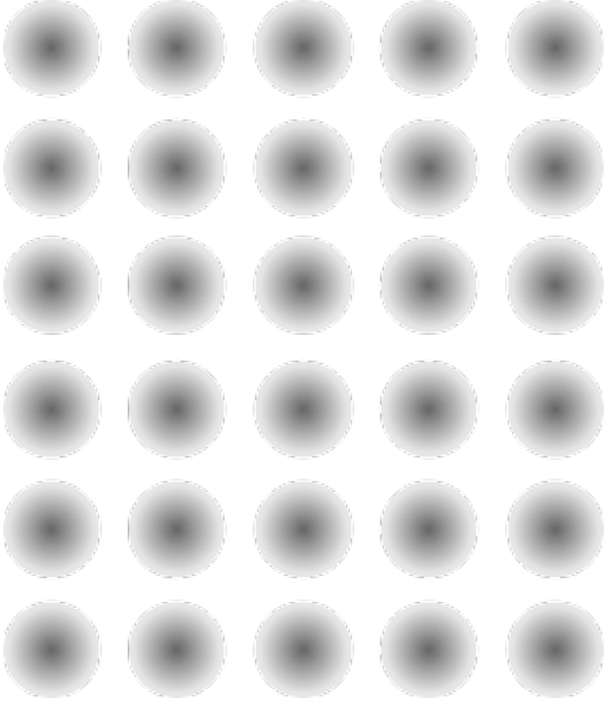
T traslazione continua

profilo di densità locale $\rho(\vec{r})$



$T = T_c$

rottura spontanea di simmetria



LIQUIDO

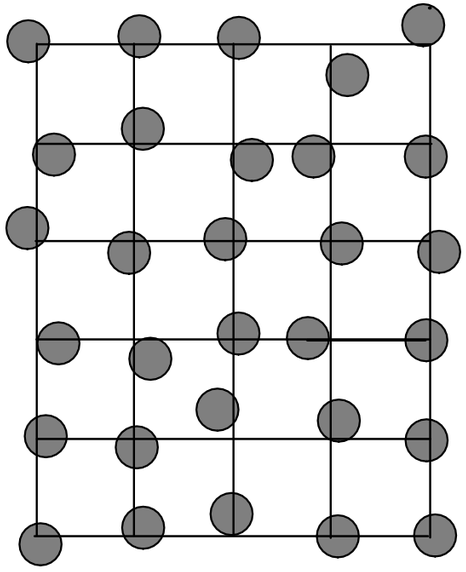
$\rho_L(\vec{r}) = cost$

CRISTALLO

$\rho_C(\vec{r}) \neq cost$

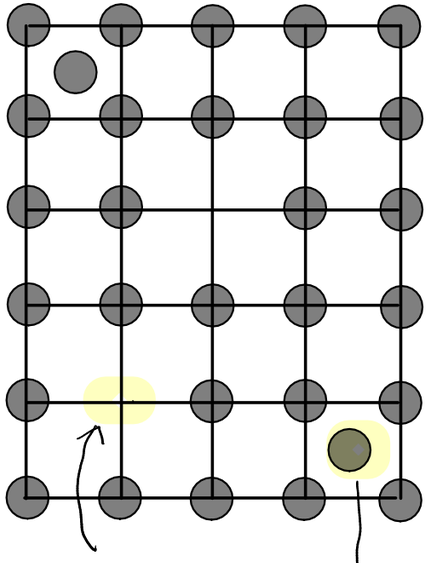
TIPOLOGIE DI DISORDINE

Disordine (o meglio: aleatorietà) nei cristalli



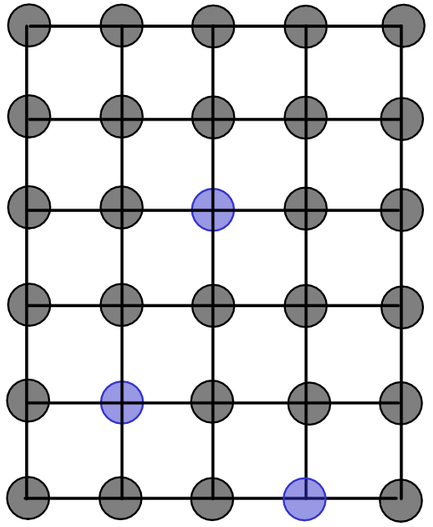
moto vibrazionale

difetti puntuali



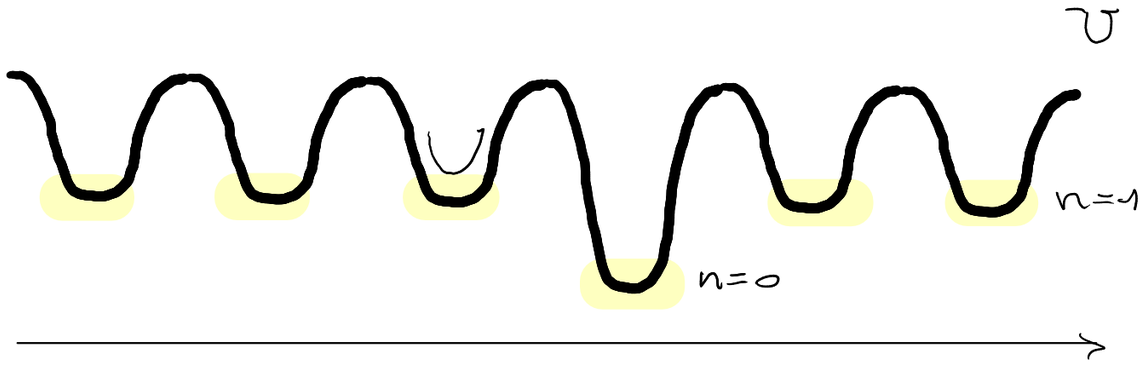
vacanze

interstiziali



disordine sostituzionale

Difetti puntuali



Reticolo $N+n$ siti

N particelle → ioni

n difetti → vacanze

$n \ll N$ → indipendenti

Termostato a temp. T , volume $V = cost$

Energia libera

$$F(n) = U_0(n) + F_{vib} - TS_c$$

- $U_0(n)$: energia potenziale
- F_{vib}: vibrazionale di un bacio
- S_c: configurazionale dei bacini

$$S_c = k_B \ln \Omega \qquad \Omega = \frac{(N+n)!}{N!\, n!}$$

$$\ln \Omega \approx (N+n)\ln(N+n) - (N+n) - N\ln N + N - n\ln n + n$$

$$\frac{\partial F}{\partial n} = \frac{\partial U_0}{\partial n} + \frac{\partial F_{vib}}{\partial n} - T\frac{\partial S_c}{\partial n} = \epsilon - k_B T \ln\left(\frac{N+n}{n}\right)$$

$$\epsilon \approx \frac{\partial U_0}{\partial n}\Big|_{n=0}$$

$\sim$ energia di legame

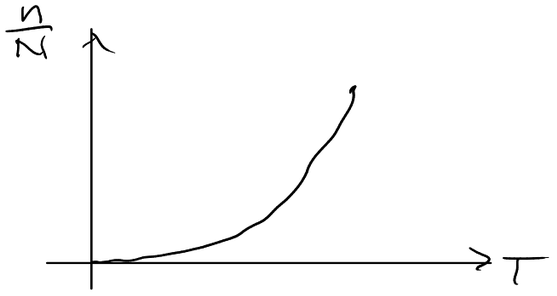
$$\frac{\partial \ln \Omega}{\partial n} = \ln(N+n) + 1 - \ln n - 1 = \ln\left(\frac{N+n}{n}\right)$$

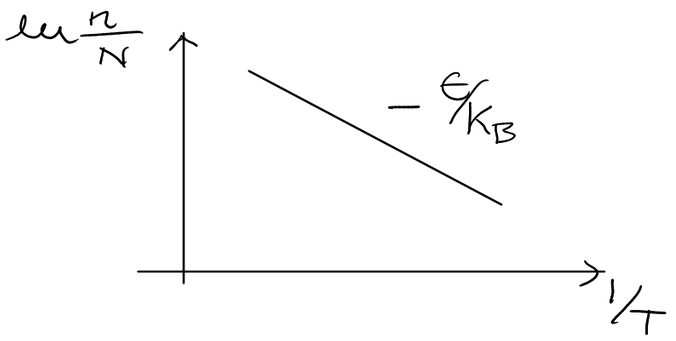
Equilibrio:

$$\frac{\partial F}{\partial n} \approx \epsilon - k_B T \ln \frac{N}{n} = 0$$

$$k_B T \ln \frac{n}{N} = -\epsilon$$

$$\frac{n}{N} = \exp\left(-\frac{\epsilon}{k_B T}\right)$$





$$\ln \frac{n}{N} = -\frac{\epsilon}{k_B}\frac{1}{T}$$

n difetti $\quad n \ll N$

Funzione di partizione per n difetti

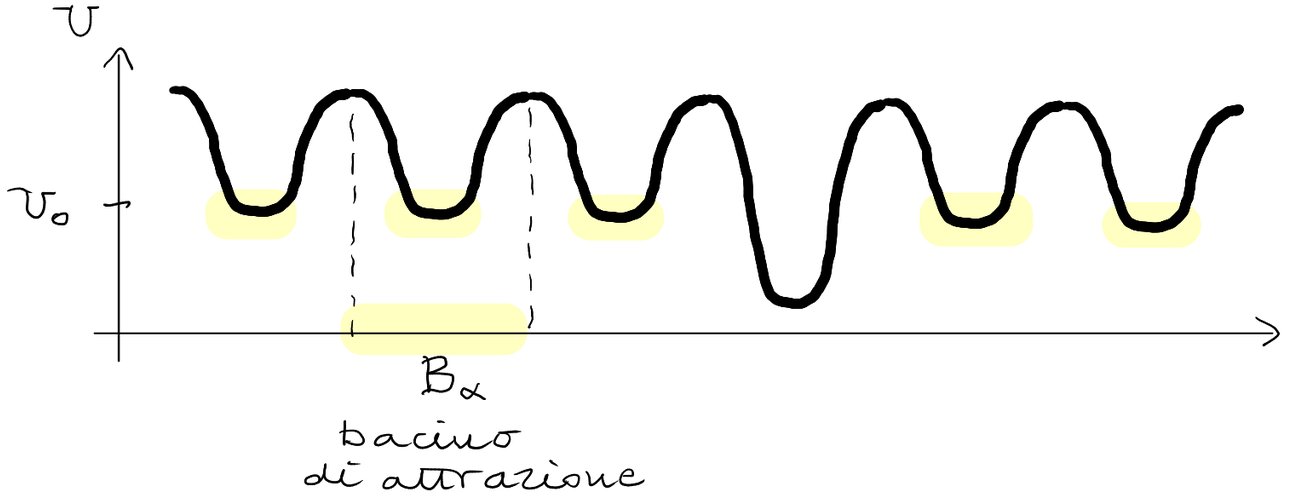
$$Z(n) = Tr_n \left[e^{-\beta H} \right]$$

$$= Tr_n \left[e^{-\beta U_0} e^{-\beta (H - U_0)} \right]$$

$$= e^{-\beta U_0} \, Tr_n \left[e^{-\beta (H - U_0)} \right]$$

$$= e^{-\beta U_0} \sum_{\alpha} \underbrace{Tr_{B_\alpha} \left[e^{-\beta (H - U_0)} \right]}$$

$$= e^{-\beta U_0} \cdot \Omega \cdot e^{-\beta F_{vib}} = e^{-\beta (U_0 + F_{vib} - TS_c)} = e^{-\beta F}$$



Entropia configurazionale

$$S_c(n) = k_B \ln \Omega$$

Entropia vibrazionale

$$F_{vib} = -TS_{vib}$$

$$\Downarrow$$

$$F(n) = U_0 + F_{vib} - TS_c$$

$$= U_0 - TS_{vib} - TS_c$$

Effetti dovuti a T e P

$$\epsilon \approx \underbrace{\frac{\partial U_0}{\partial n} + \frac{\partial F_{vib}}{\partial n}}_{\approx \left.\frac{\partial U_0}{\partial n}\right|_{n=0}} \approx$$

energia di formazione di un difetto

energia di coesione per particella se vacanza

$$n/N = \exp\left(-\frac{\epsilon}{k_B T}\right)$$

- $k_B T a \approx 10^{-23} \times 300\,J \approx 4 \times 10^{-21}\,J$

$$\epsilon \begin{cases} 0.1\ eV & 10^{-20}\,J \\ 1\ eV & 10^{-19}\,J \\ 10\ eV & 10^{-18}\,J \end{cases} \Rightarrow n/N \begin{cases} 0.1 \\ 10^{-11} \\ 10^{-100} \end{cases}$$

- $\uparrow n \uparrow S_{vib} \downarrow F_{vib} \Rightarrow \frac{\partial F_{vib}}{\partial n} < 0 \Rightarrow$ aumento n/N

- $G = F + PV \qquad V = (N+n)\,v_0$

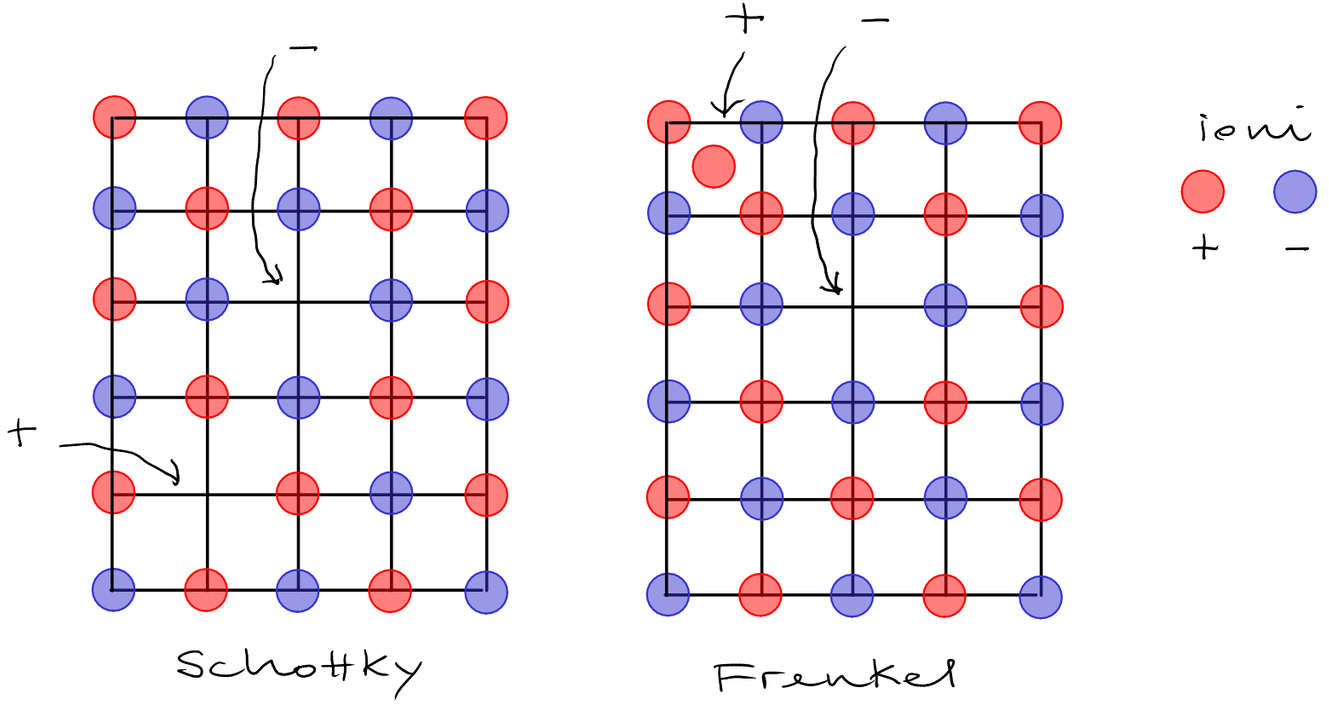
$$\epsilon' = \epsilon + P v_0$$

$P \approx 10^5\,Pa \quad v_0 \approx 10^{-29}\,m^3 \quad P v_0 \approx 10^{-24}\,J \ll \epsilon$

Difetti correlati

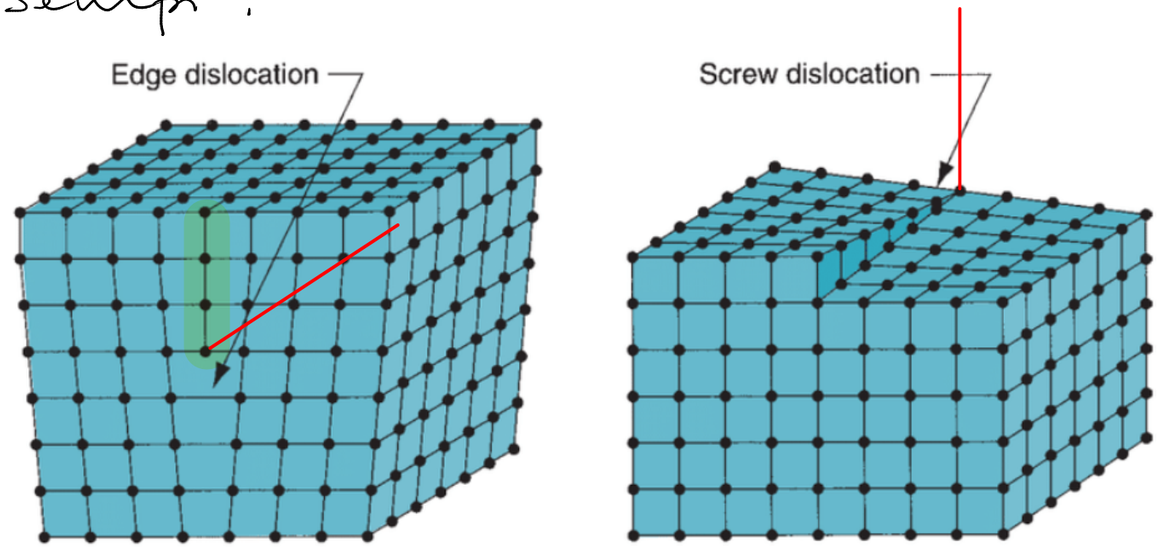
Conservazione di carica

$n_+ , n_- \rightarrow n_+ = n_-$

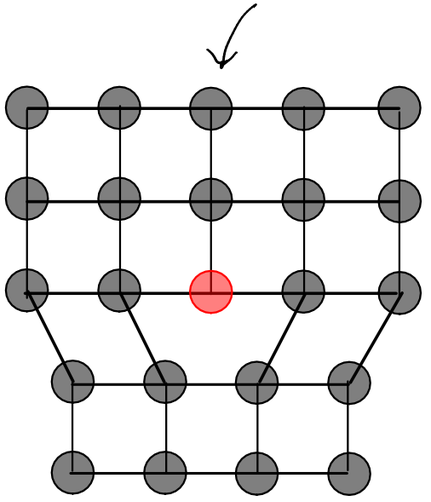


Difetti topologici

Esempi :

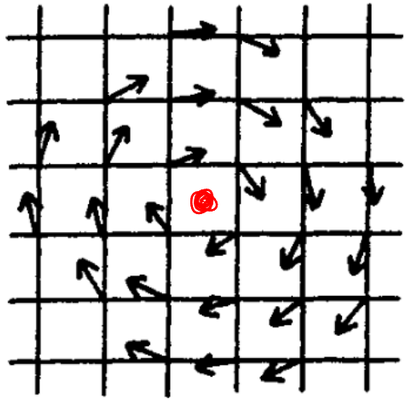


particelle 3d : dislocazioni

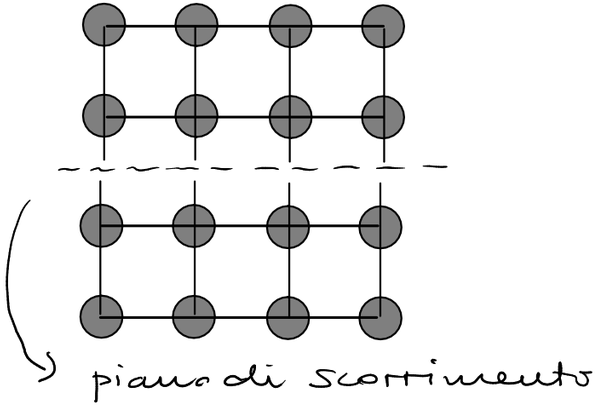


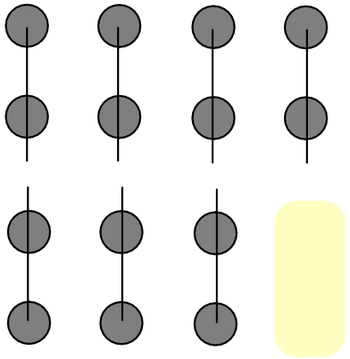
particelle 2d : dislocazioni
disclinazioni

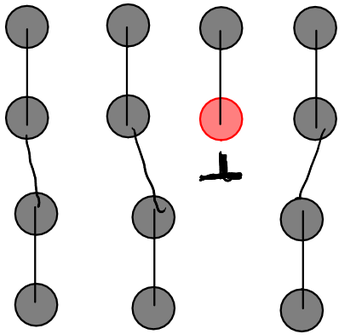
modello XY



spin 2d
vortici





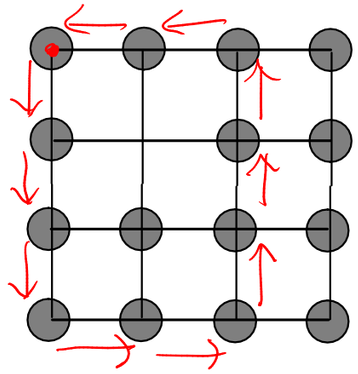


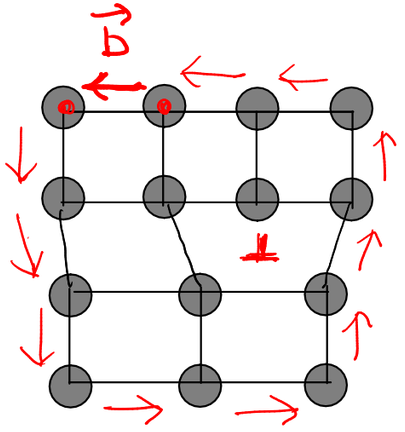
Puntuali : $O(1)$

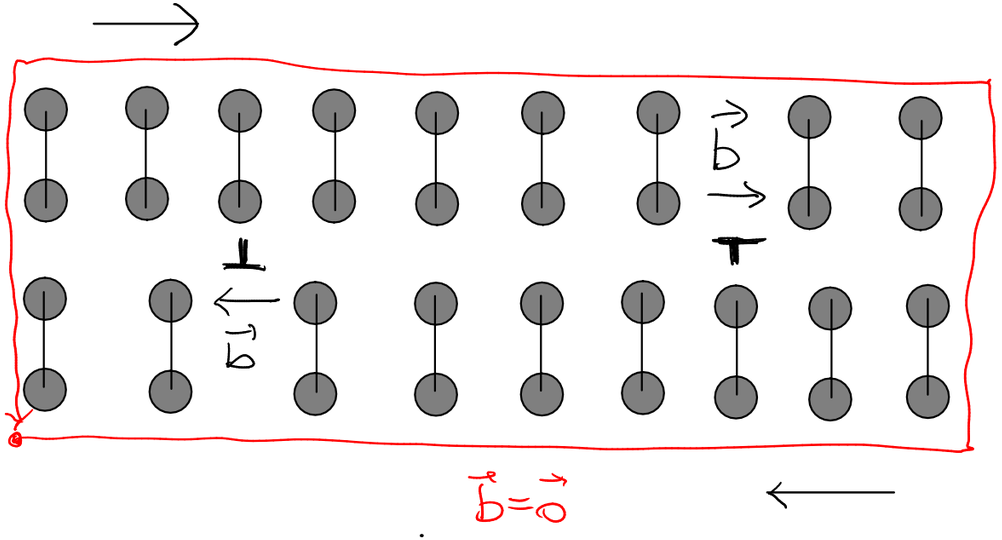
Linea : $O(N^{1/3}) \sim L$

Superficie : $O(N^{2/3}) \sim L^2$

Circuito di Burgers







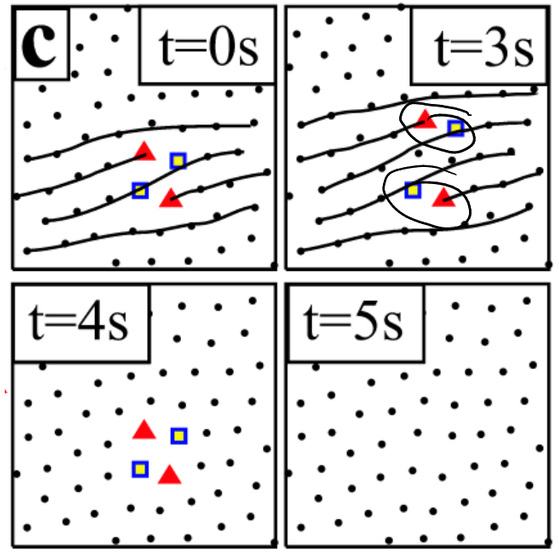
vettore di Burgers : $\vec{b}$

$\vec{u}$ deformazione

$b_i = -\oint_L du_i$

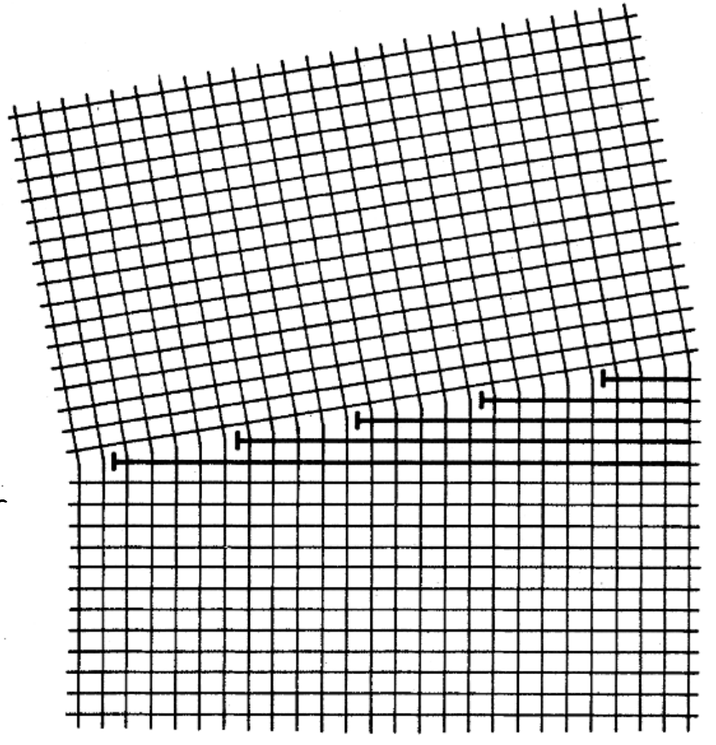
elasticità

2d colloidal crystal

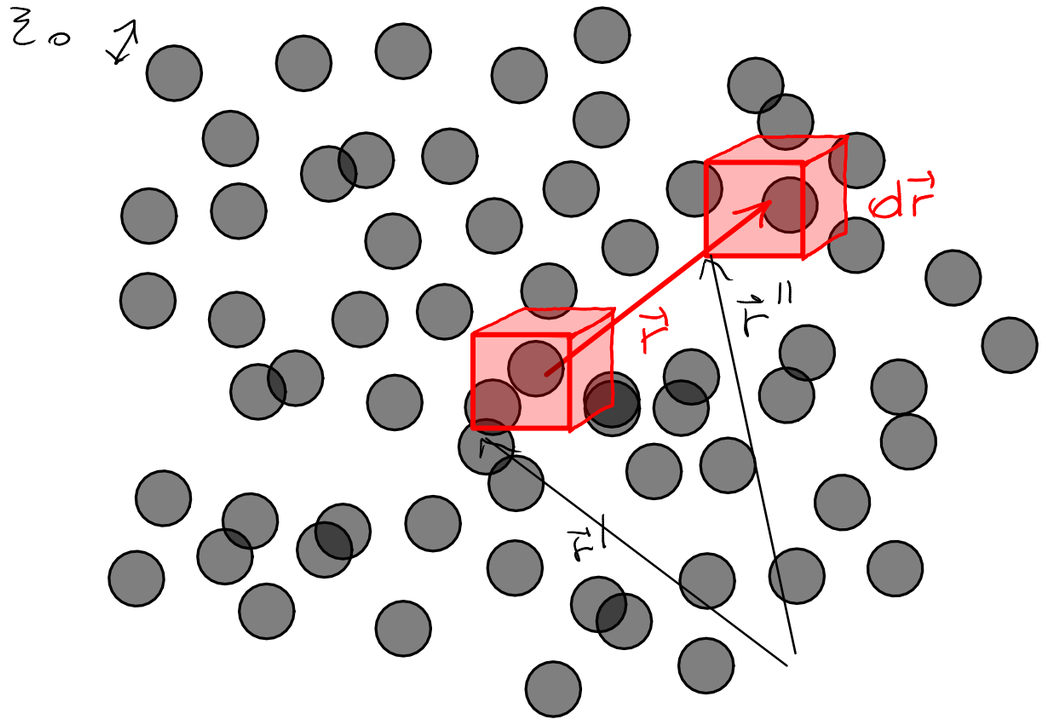


Eisenman et. al PRL 2005

grain boundary



ORDINE E CORRELAZIONI



Densità microscopica

$$\hat{\rho}(\vec{r}) = \sum_{i=1}^{N} \delta(\vec{r} - \vec{r}_i)$$

$$\int_V d\vec{r}\, \hat{\rho}(\vec{r}) = N$$

$$\int_{\xi_0^3} d\vec{r}\, \hat{\rho}(\vec{r}) = \begin{cases} 1 \\ 0 \end{cases}$$

Densità locale: omogeneo

$$\rho(\vec{r}) = \langle \hat{\rho}(\vec{r}) \rangle \overset{\downarrow}{=} N/V$$

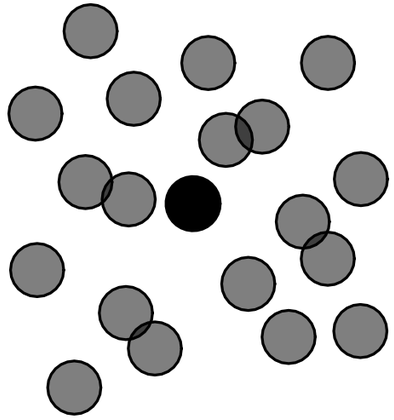
Funzione di correlazione

$$G(\vec{r}', \vec{r}'') = \langle (\hat{\rho}(\vec{r}') - \langle \hat{\rho} \rangle)(\hat{\rho}(\vec{r}'') - \langle \hat{\rho} \rangle) \rangle \qquad \langle \hat{\rho} \rangle = \frac{N}{V}$$

Se omogeneo: $G(\vec{r}', \vec{r}'') = G(\vec{r}) \qquad \vec{r} = \vec{r}'' - \vec{r}' \qquad G(x,y,z) \to G(x)$

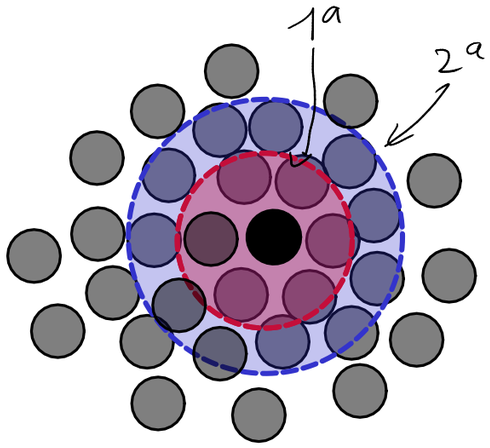
gas

corto raggio



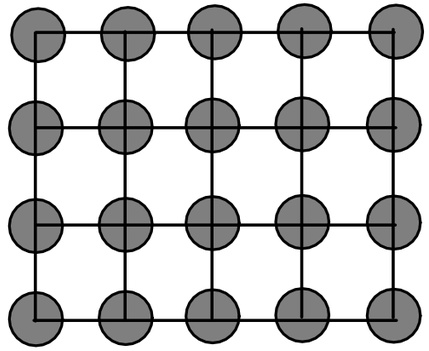
liquido

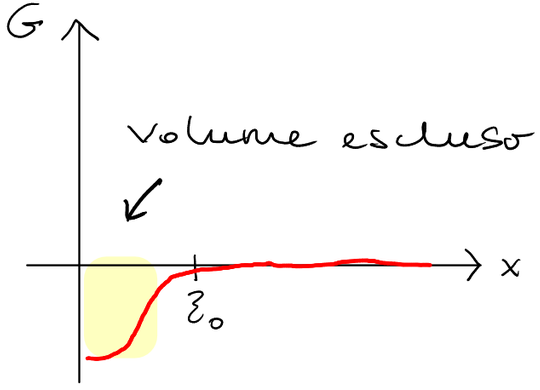
$\xi \approx \xi_0$ corto ; $\xi \gtrsim \xi_0$ medio

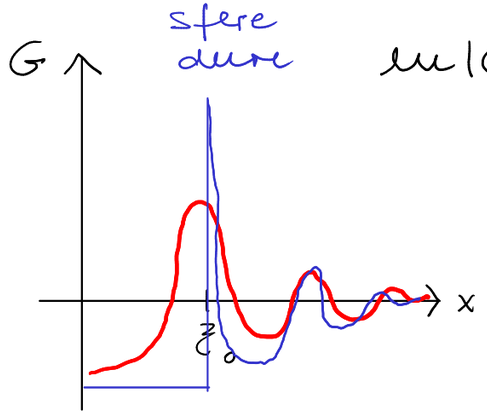


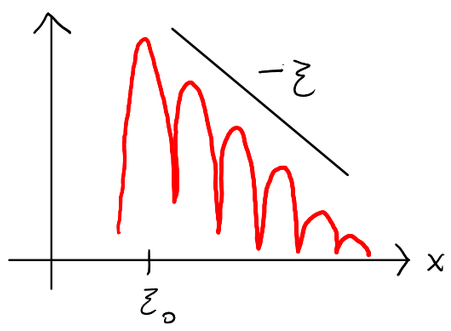
cristallo

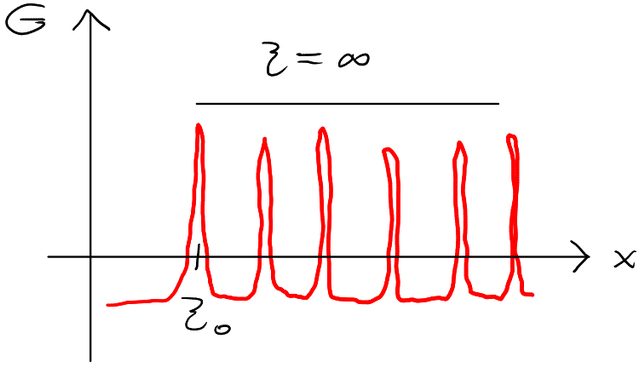
lungo raggio







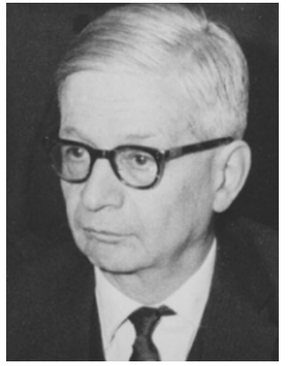




$G(x) \sim \exp\left(-\frac{x}{\xi}\right)$ ξ = lunghezza di correlazione

ORDINE E DIMENSIONALITÀ

Rottura spontanea di simmetria → dimensioni spaziali d lowest critical dimension

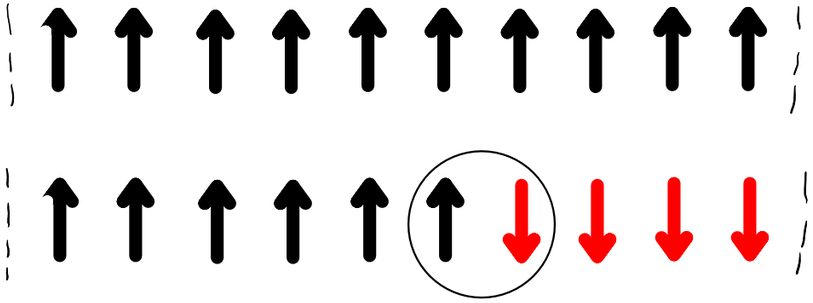

Peierls


Landau

1) **Simmetrie discrete** cf. Goldenfeld ch.2

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j \qquad \sigma_i = \pm 1 \qquad i = 1, \dots, N \qquad \text{PBC}$$

- **d = 1** Ising



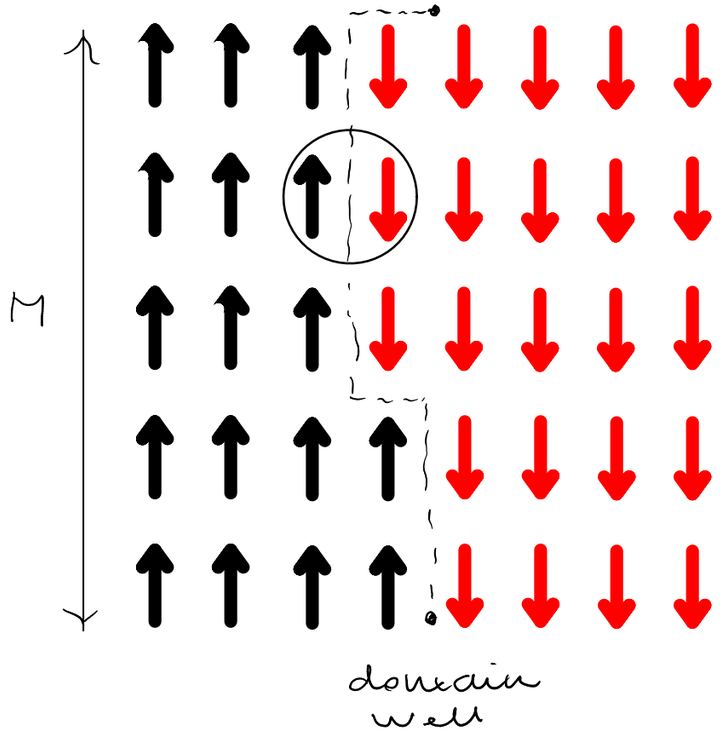
$F = E - TS \approx \sim NJ$

$\Delta E = 2J \qquad \Delta S = k_B \ln N \qquad \Delta F = 2J - k_B T \ln N$

$\forall T \; \exists N$ t.c. $\Delta F < 0$ $\Rightarrow$ non c'è ordine a lungo raggio in d = 1

- $d=2$ Ising

n. di coordinazione z



n = n. coppie di spin a cavallo dell'interfaccia

$$\Delta E = nJ + MJ \approx 2nJ$$

$$\Delta S \approx n k_B \ln(z-1)$$

$$\underbrace{z(z-1)\cdots(z-1)}_{n} \approx (z-1)^n \qquad n \to \infty$$

$$\Delta F = \Delta E - T\Delta S = \underbrace{[2J - k_B T \ln(z-1)]}_{} n$$

$$T_c = \frac{2J}{k_B \ln(z-1)}$$

$T > T_c$: $\Delta F \to -\infty$ $n \to \infty$, $N \to \infty$ $\Rightarrow$ destabilizzo il cristallo

$T < T_c$: ΔF minimo se $n=0$ per $N \to \infty$ $\Rightarrow$ ordine a lungo raggio

$z = 4$ $T_c \approx 1.82\ J/k_B$

1941: Kramers-Wannier duality $T_c = 2.269\ J/k_B$

1943: Onsager prima soluzione esatta del modello di Ising 2d

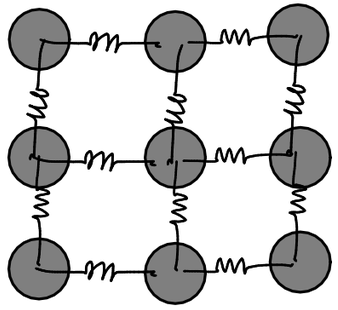
1953: Kac & Ward metodo combinatorio

Lowest critical dimension: ordine a lungo raggio possibile solo se

$d > d_c = 1$ in sistemi con simmetria discreta

2) Simmetrie continue

- Particelle, N, PBC, $H = K + U$, d dimensioni spaziali, equilibrio a T



approssimazione armonica

$$U \approx \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{\alpha=1}^{d} \sum_{\beta=1}^{d} \frac{\partial^2 U}{\partial r_{i\alpha} \partial r_{j\beta}} \Delta r_{i\alpha} \Delta r_{j\beta}$$

$\rightarrow$ $n = 1, \dots, dN$ modi normali

Spostamento

$$\Delta \vec{r}_i = \sum_{n=1}^{dN} \vec{e}_{in} \xi_n$$

$\lambda_n = m\omega_n^2$, autovalori

$\{\vec{e}_{in}\}_{i=1,\dots N}$ autovettori

della matrice hessiana $\frac{\partial^2 U}{\partial r_{i\alpha} \partial r_{j\beta}}$

Spostamento quadratico medio

$$\langle |\Delta \vec{r}|^2 \rangle = \frac{1}{N} \left\langle \sum_{i=1}^{N} |\Delta \vec{r}_i|^2 \right\rangle = d \, \frac{1}{dN} \left\langle \sum_{n=1}^{dN} \xi_n^2 \right\rangle$$

oscillatore armonico a temp. T $\rightarrow$ equipartizione dell'energia

$$\left\langle \frac{1}{2} m\omega^2 \xi^2 \right\rangle = \frac{1}{2} k_B T \quad \rightarrow \quad \langle \xi^2 \rangle = \frac{k_B T}{m\omega^2} \quad \rightarrow \quad \langle \xi_n^2 \rangle = \frac{k_B T}{m\omega_n^2}$$

$$\langle |\Delta \bar{F}|^2 \rangle = d \frac{K_B T}{m} \underbrace{\frac{1}{dN} \sum_{n=1}^{dN} \frac{1}{\omega_n^2}}_{} = \frac{d K_B T}{m} \int_0^\infty d\omega \, D(\omega) \frac{1}{\omega^2}$$

$D(\omega)$: densità di stati vibrazionali

Bassa frequenza: $\omega = cK$ modi acustici

Modello di Debye: $D(K) \sim K^{d-1} \rightarrow D(\omega) \sim \omega^{d-1}$

$$\langle |\Delta \bar{F}|^2 \rangle \sim \int_{\frac{2\pi c}{L}}^{\infty} d\omega \, \omega^{d-3} \qquad K_{min} = \frac{2\pi}{L}$$

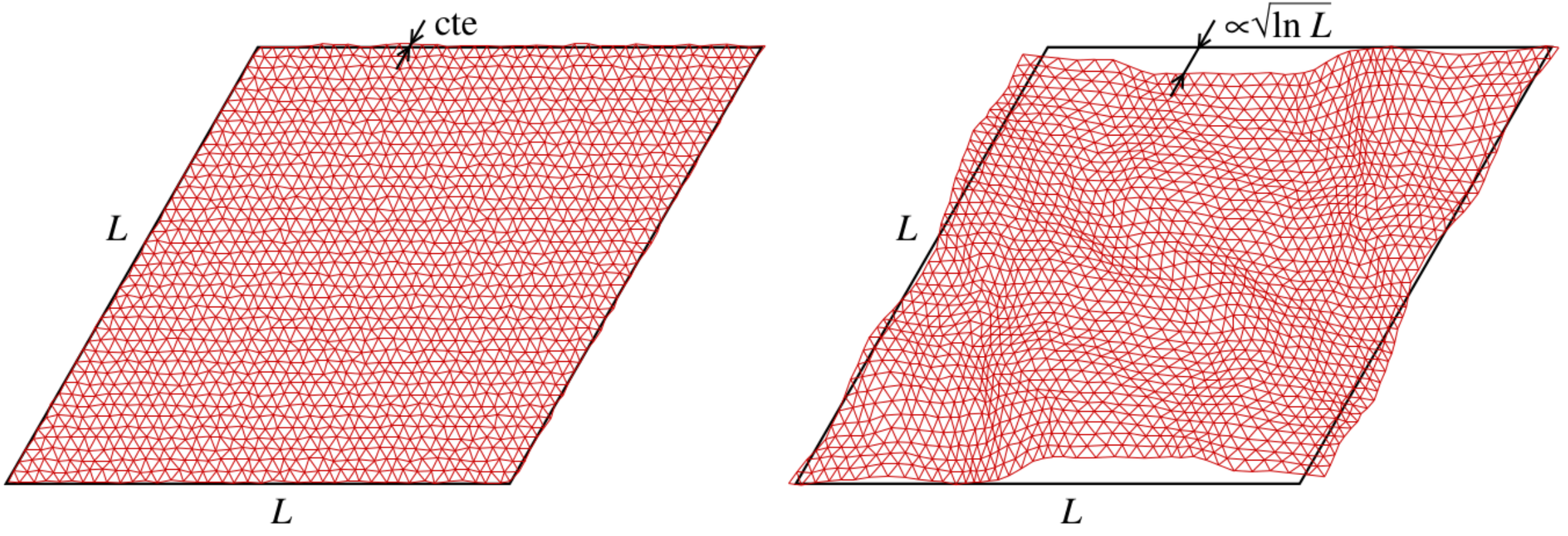
$d \geq 3$: $\langle |\Delta F|^2 \rangle \rightarrow$ cost

$d = 2$: $\langle |\Delta F|^2 \rangle \sim -\ln \frac{1}{L} \sim \ln L$

$d = 1$: $\langle |\Delta F|^2 \rangle \sim L$

$\rightarrow \infty$ se $L \rightarrow \infty$

$\Rightarrow$ lowest critical dimension: $d_c = 2$



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Teorema di Mermin-Wagner

Sotto ipotesi abbastanza generali sulla natura delle interazioni e dei gradi di libertà, una simmetria continua non può essere rotta spontaneamente in 1d e 2d.

Lowest critical dimension : 2

(*) Spins : ordine orientazionale (Mermin-Wagner PRL 1966)
particelle : ordine posizionale (Mermin PR 1968)


Mermin


Wagner

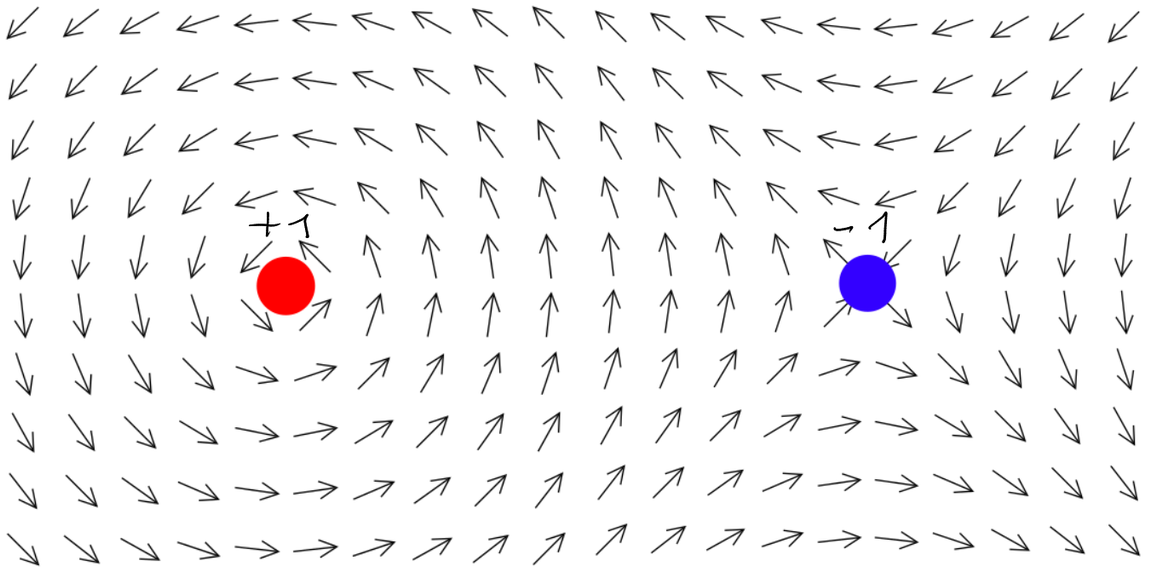

Berezinsky


Kosterlitz


Thouless

Transizioni di Berezinsky - Kosterlitz - Thouless

Goldenfeld ch 11



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Modello XY → simmetria continua

$\vec{\sigma}_i \quad i = 1, \dots, N \qquad |\vec{\sigma}_i| = 1$

$$H = -J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

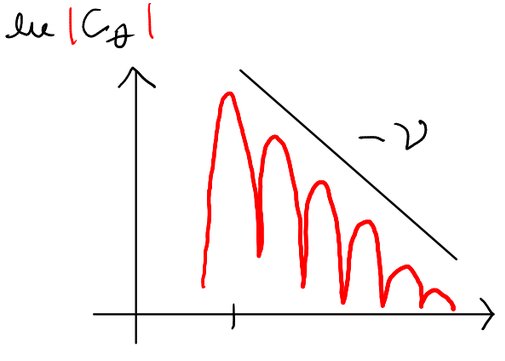
$\vec{\sigma}_i = (\cos\vartheta_i, \sin\vartheta_i) \quad , \quad d = 2$

Energia di un vortice

$$\Delta E_v = J' \ln N + E_c \qquad J' \sim J$$

$$\Delta S_v = k_B T \ln N$$

$$\Delta F = (J' - k_B T) \ln N \qquad T_c = \frac{J'}{k_B}$$



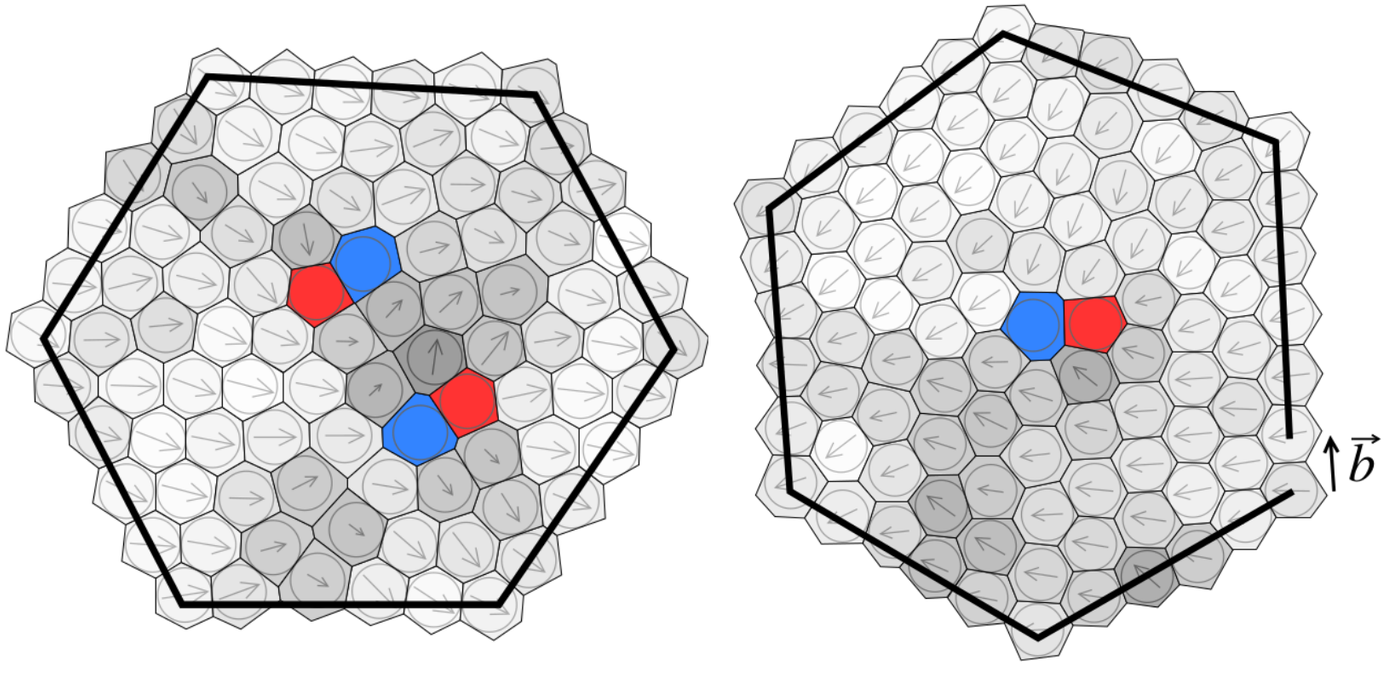
$$C_\theta(\vec{r}) \sim r^{-\nu}$$

$\nu = 1/4$

$T > T_c$: $\Delta F \to -\infty$ $N \to \infty$ disordinato

$T < T_c$: $\Delta F \to +\infty$ $\Rightarrow$ no vortici isolati

dislocazione



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Particelle in 2d : dischi

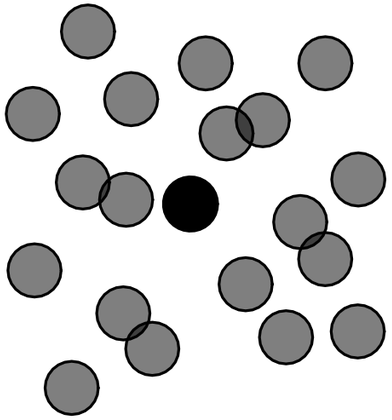
Difetti topologici : dislocazioni

ciascuna associata a una coppia di difetti di coordinazione con 5 / 7 vicini

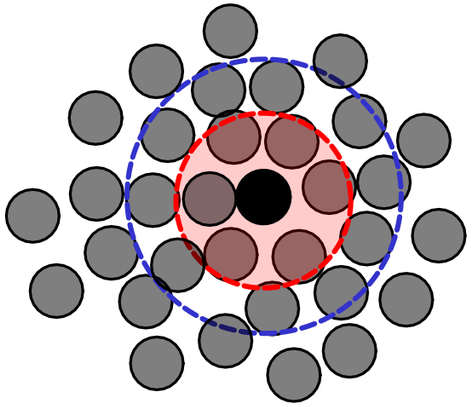
$T > T_c$: dislocazioni libere, fase disordinata

$T < T_c$: dislocazioni a coppie, $C(\vec{r}) \sim r^{-\gamma}$ → ordine a quasi-lungo raggio

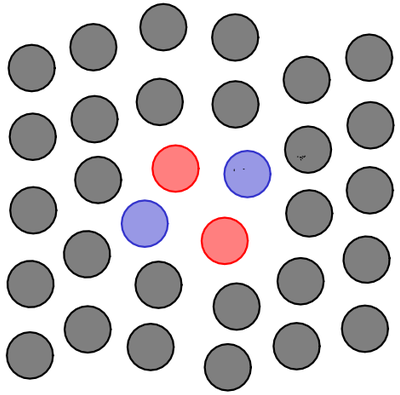
gas



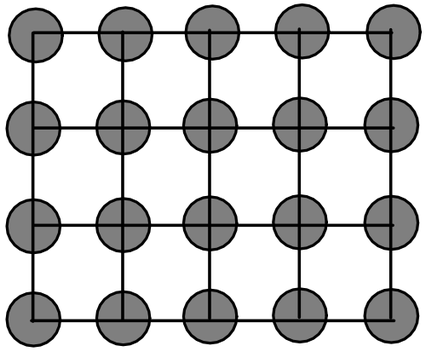
liquido

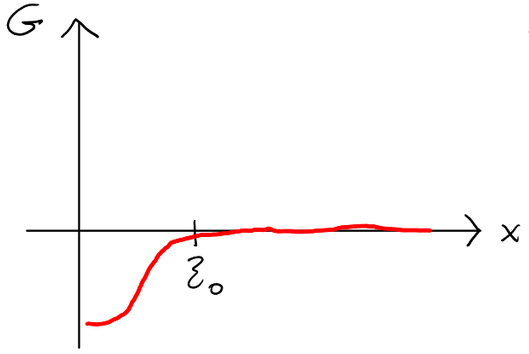


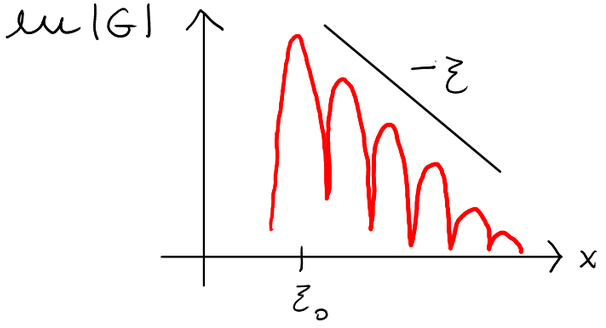
cristallo 2d



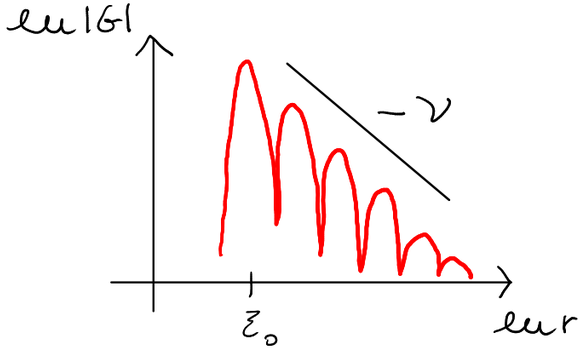
cristallo



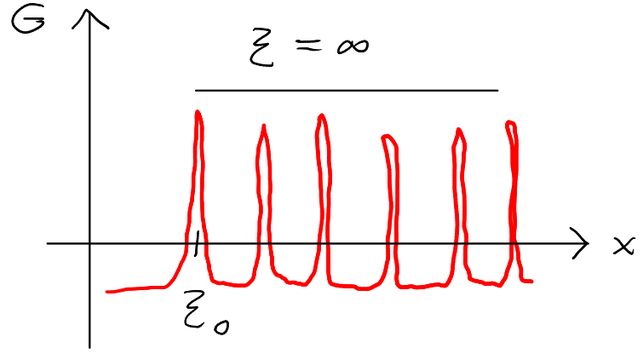




corto raggio

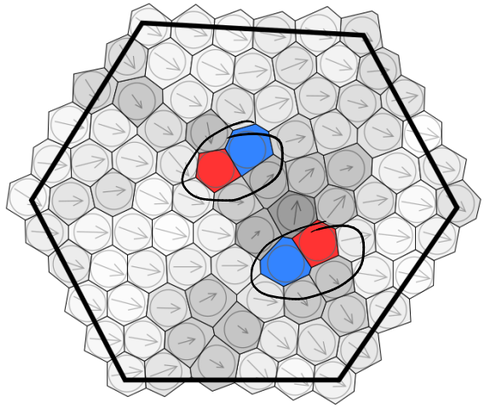


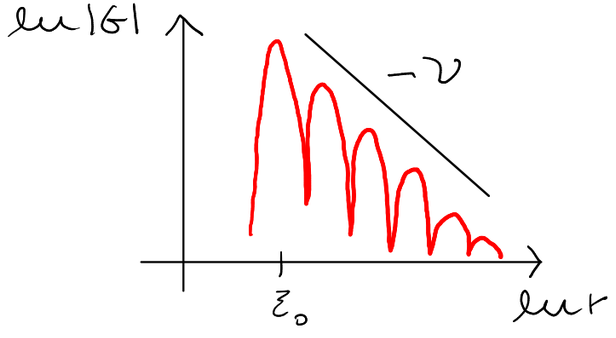
quasi-lungo raggio



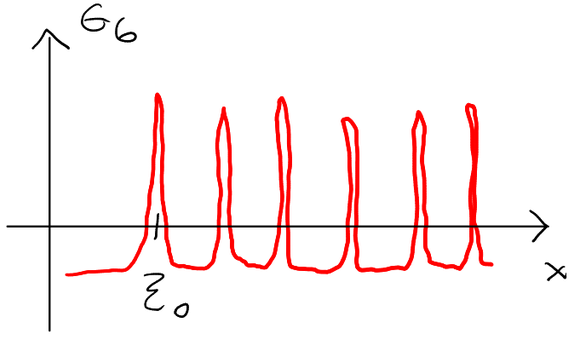
lungo raggio

Fase cristallina 2d

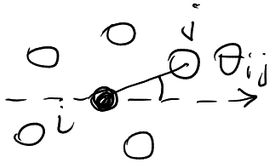




posizionale QLR



orientazionale LR

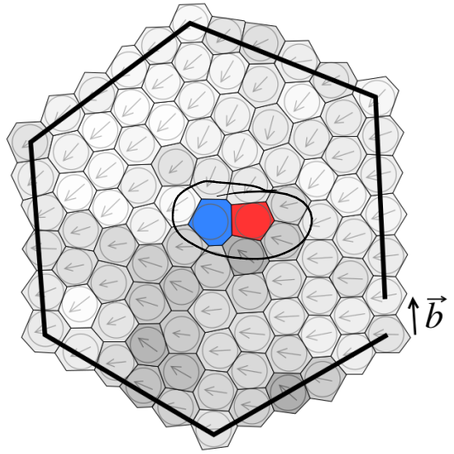


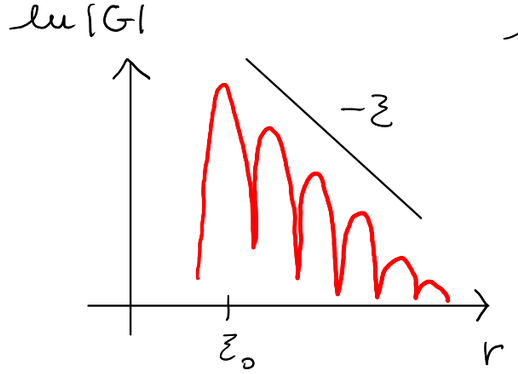
$$\psi_{6i} = \frac{1}{n_i} \sum_{j=1}^{n_i} e^{i6\theta_{ij}}$$

$$\hat{\psi}_6(\vec{r}) = \sum_{i=1}^{N} \delta(\vec{r}-\vec{r}_i)\,\psi_{6i}$$

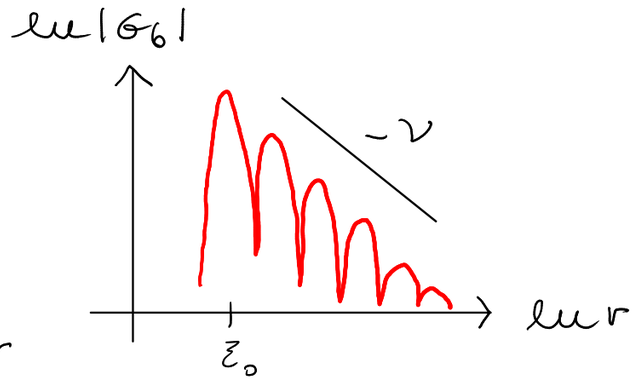
$$G_6(\vec{r}) = \langle \hat{\psi}_6(\vec{r}') \hat{\psi}_6^*(\vec{r}'+\vec{r}) \rangle$$

Fase esatica ("hexatica")



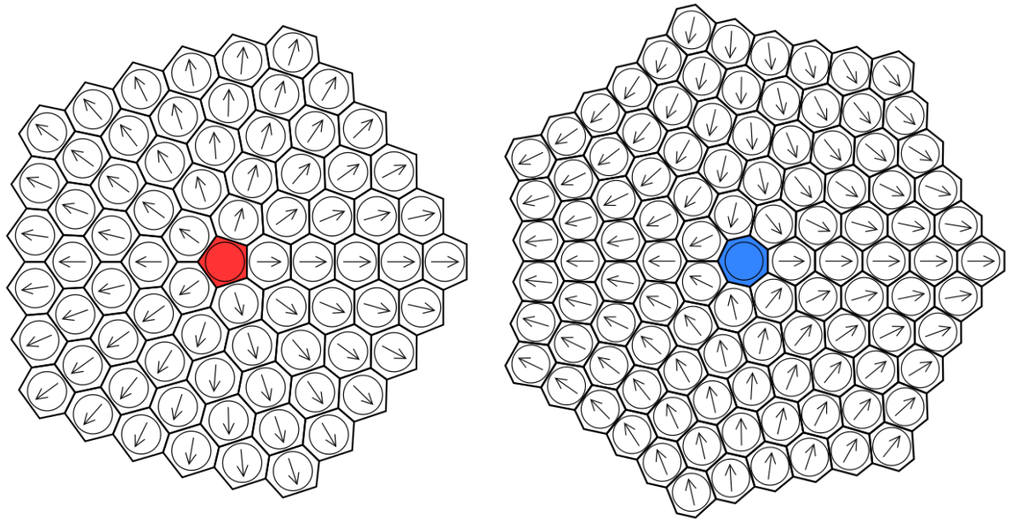


posizionale SR



orientazionale QLR

Fase liquida



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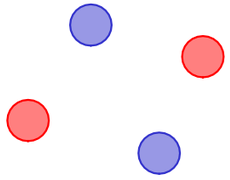
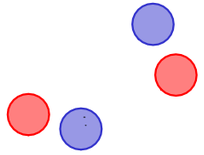
Disclinazioni → 5 o 7 isolati

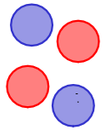
Posizionale & orientazionale SR

Scenario KTHNY (Kosterlitz, Thouless, Halperin, Nelson, Young, 1978 - 1980)

	LIQUID	HEXATIC	2D CRYSTAL
ordine posizionale	corto	corto	quasi-lungo
ordine orientazionale	corto	quasi-lungo	lungo
dislocazioni	libere	libere	coppie
disclinazioni	libere	coppie	quartetti

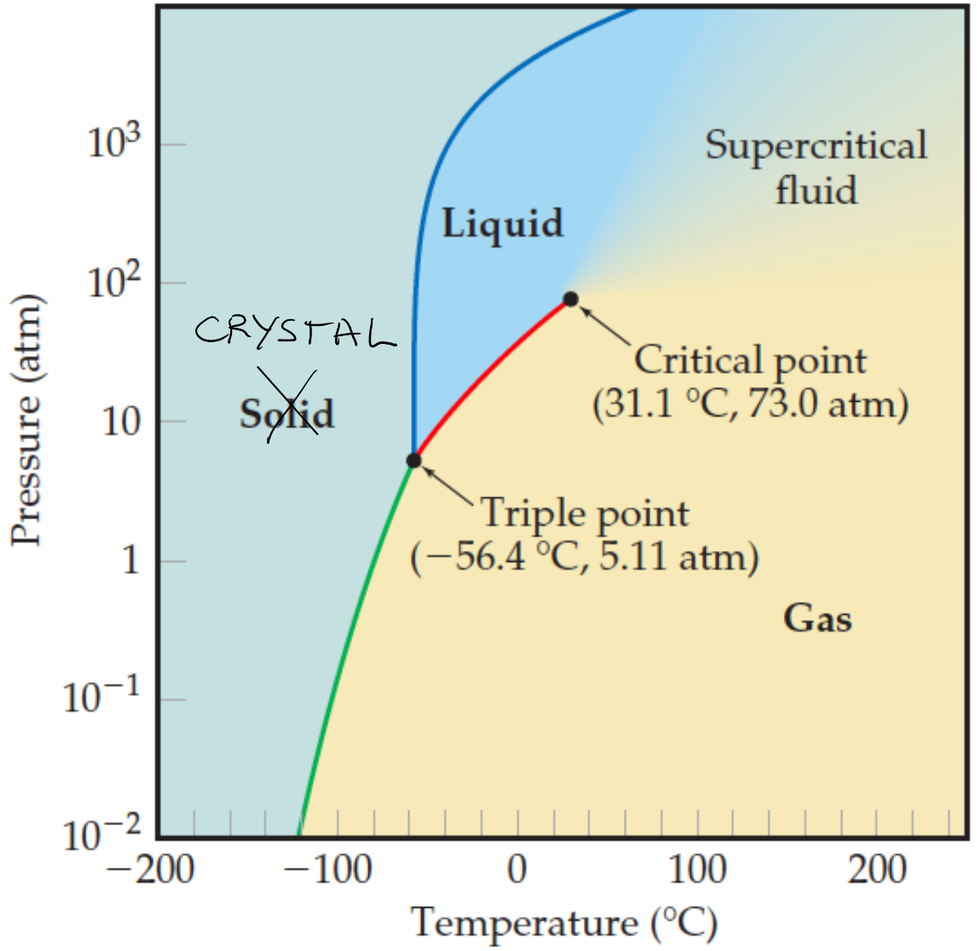




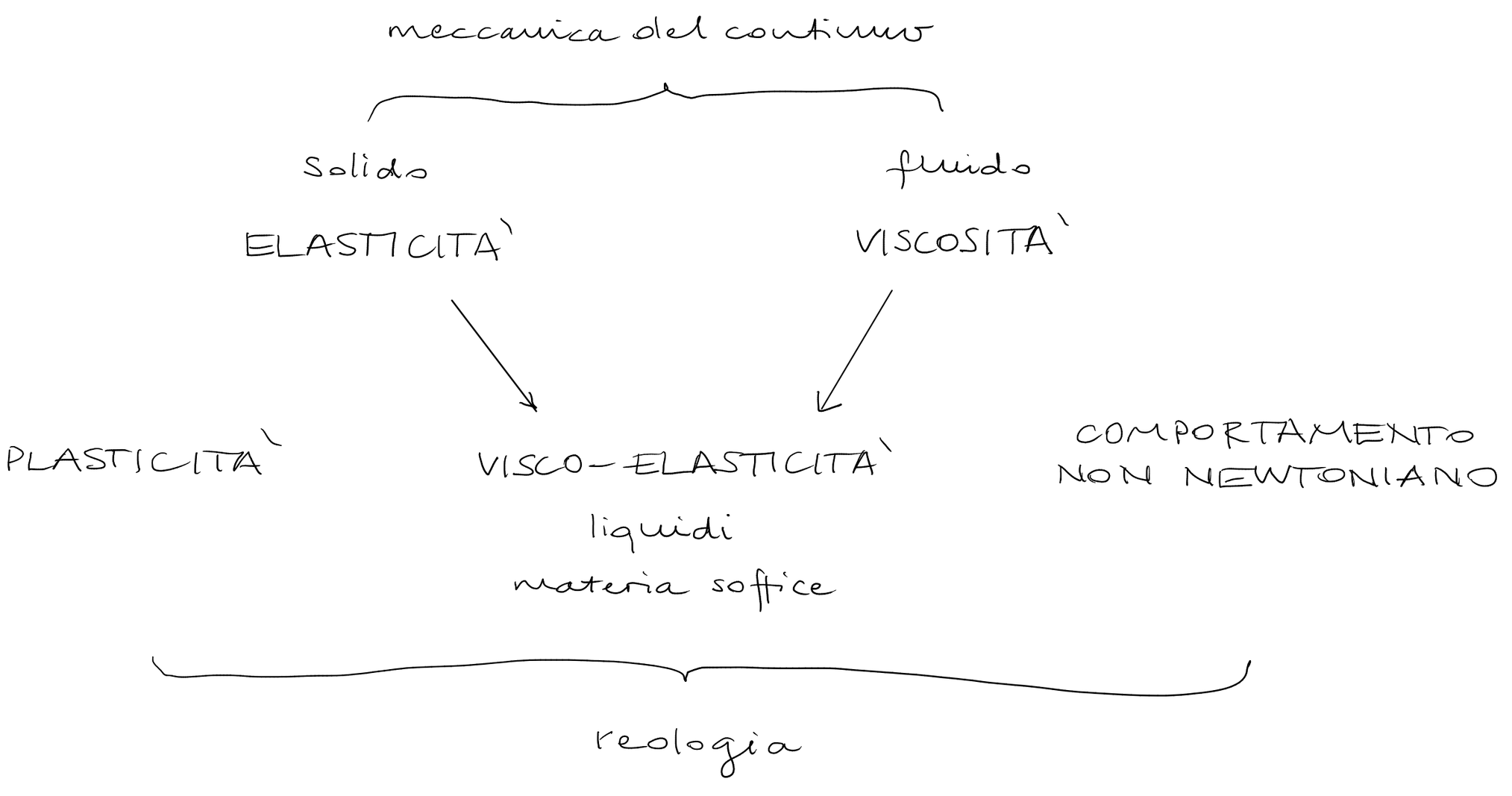


Verifica
- computazionale : Bernard & Krauth PRL 2011
- sperimentale : Thorneywork et al. PRL 2017

Argon


Supercritical fluid
Liquid
CRYSTAL
Solid
Critical point
(31.1 °C, 73.0 atm)
Triple point
(−56.4 °C, 5.11 atm)
Gas
Pressure (atm)
10^3
10^2
10
1
10^{-1}
10^{-2}
−200
−100
0
100
200
Temperature (°C)

ORDINE E RISPOSTA MECCANICA



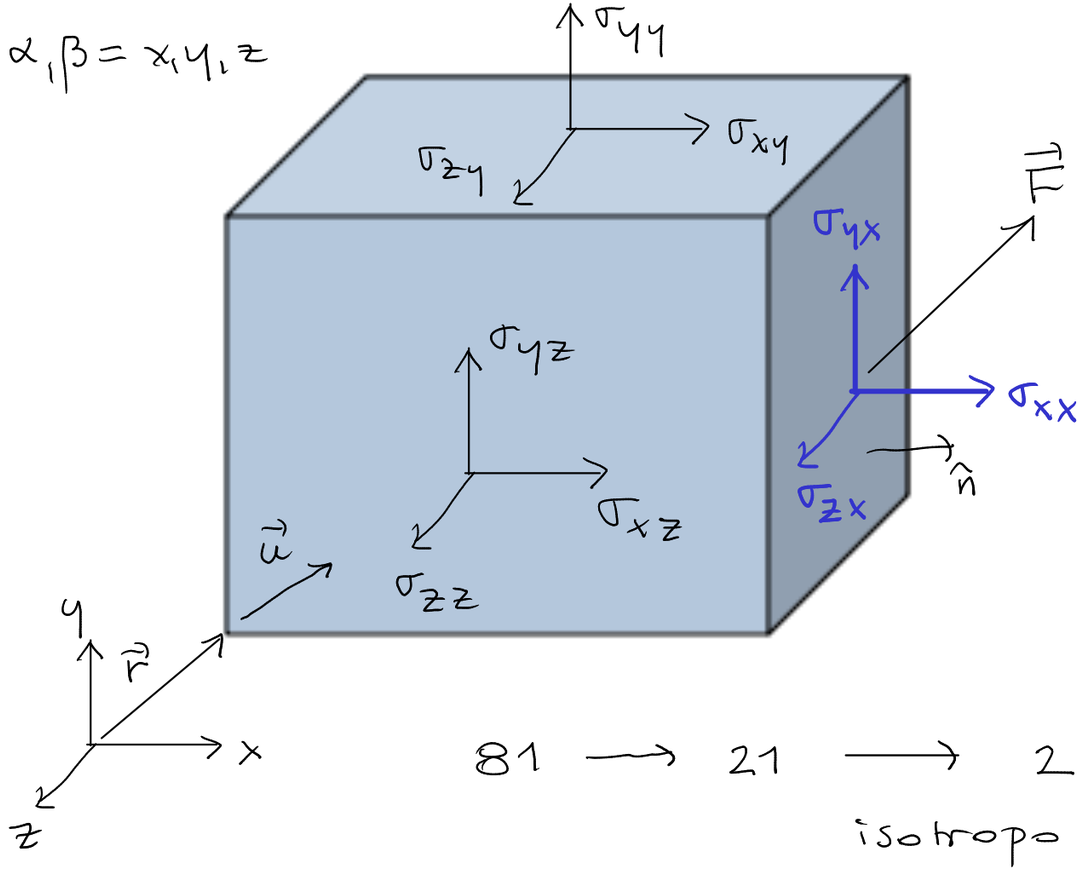
Elasticità

sforzo = modulo x deformazione

F/A SI: Pa relativa

$F = -K\Delta x$ Hooke

Solido hookiano



81 → 21 → 2

isotropo, omogeneo

Sforzo: $\sigma_{\alpha\beta}$ $\alpha \leftrightarrow \beta$

Spostamento: $\vec{u}(\vec{r})$

Deformazione: $\gamma_{\alpha\beta} = \frac{\partial u_\alpha}{\partial r_\beta}$

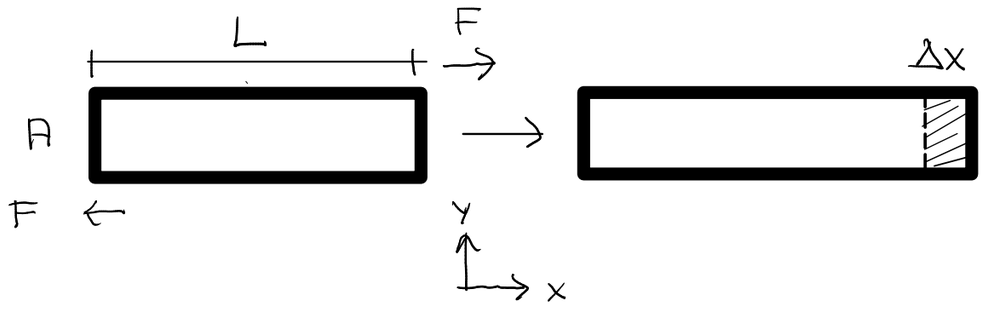
$\rightarrow \frac{1}{2}(\gamma_{\alpha\beta} + \gamma_{\beta\alpha})$

$\sigma_{\alpha\beta} = \sum_{\gamma\theta} C_{\alpha\beta\gamma\theta}\, \gamma_{\gamma\theta}$

modulo elastico

(costanti elastiche)

1) Sforzo di trazione



$$\left.\begin{aligned}\sigma_{xx} &= \sigma = \frac{F}{A}\\ \gamma_{xx} &= \gamma = \frac{\Delta x}{L}\end{aligned}\right\}$$

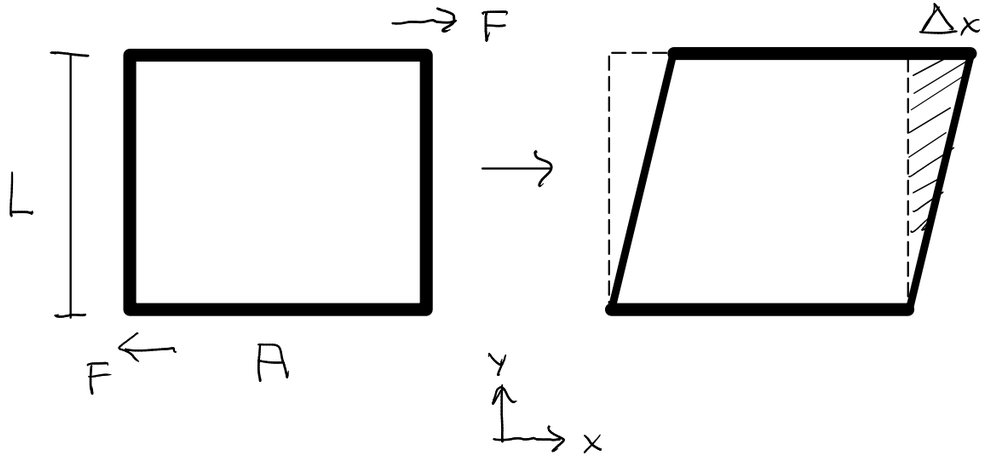
$$\sigma = Y\gamma$$

↑ modulo di Young

Es.: gomma @ Ta 0.1 GPa

diamante ~ 1000 GPa

2) Sforzo di taglio



$$\left.\begin{aligned}\sigma_{xy} &= \sigma = \frac{F}{A}\\ \gamma_{xy} &= \gamma = \frac{\Delta x}{L}\end{aligned}\right\}$$

$$\sigma = G\gamma$$

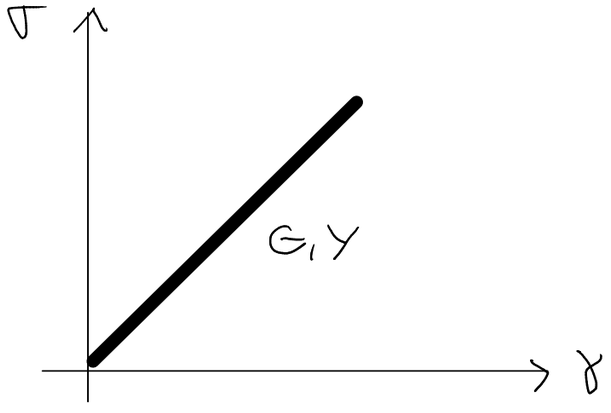
↑ modulo di taglio

("shear modulus")

Es.: gomma @ Ta 0,0006 GPa

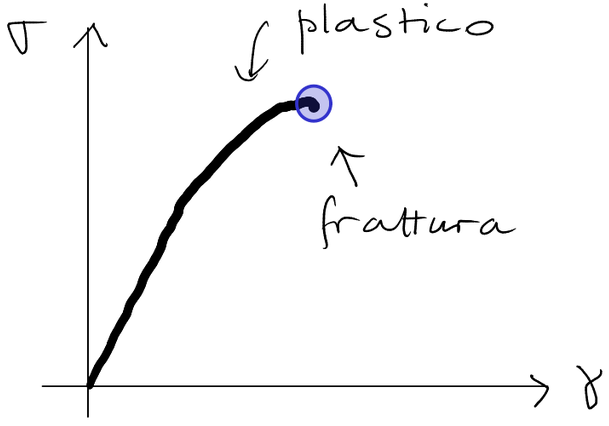
diamante @ Ta 500 GPa

Plasticità e frattura

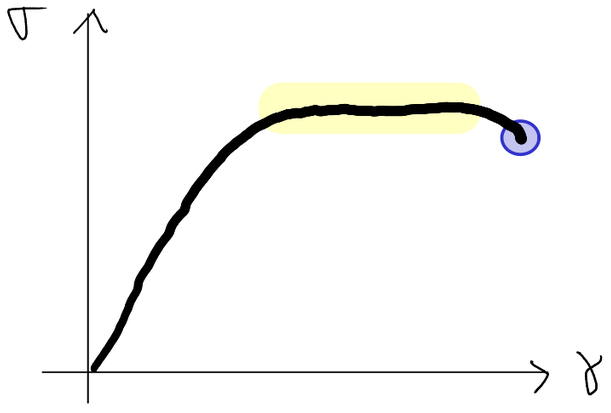


solido hookiano

fragili



duttili



Brittle Fracture

Ductile Fracture

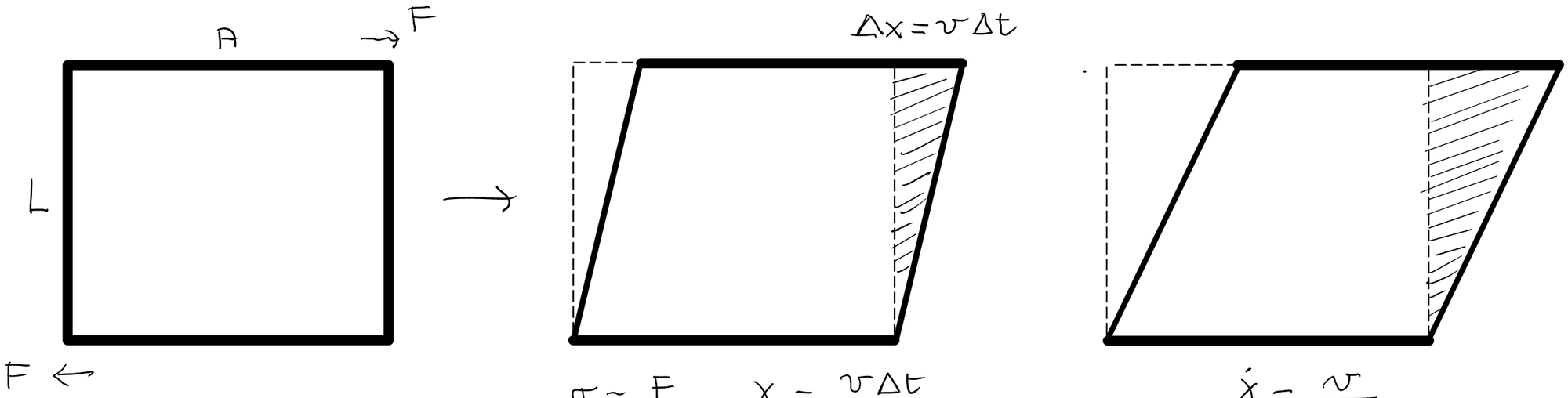
Viscosità

Hooke : equilibrio

$\sigma \sim \gamma$

Newton : stazionario

$\sigma \sim \dot{\gamma}$



$\sigma = \frac{F}{A} \qquad \gamma = \frac{v \Delta t}{L}$

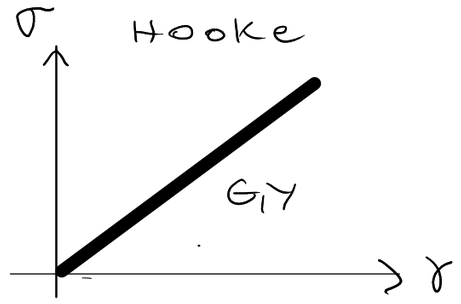
$\dot{\gamma} = \frac{v}{L}$

fluido Newtoniano : $\sigma = \eta \dot{\gamma}$ SI: Pa·s

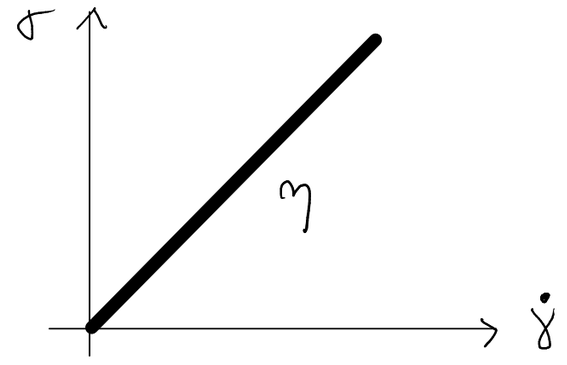
η viscosità

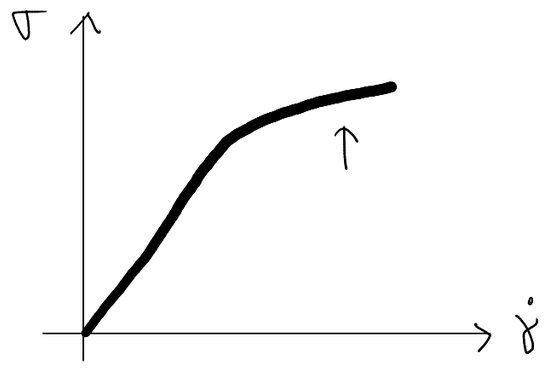
Es.: H_2O @Ta $\eta \sim 10^{-3}$ Pas @100°c $\eta \sim 10^{-4}$ Pas

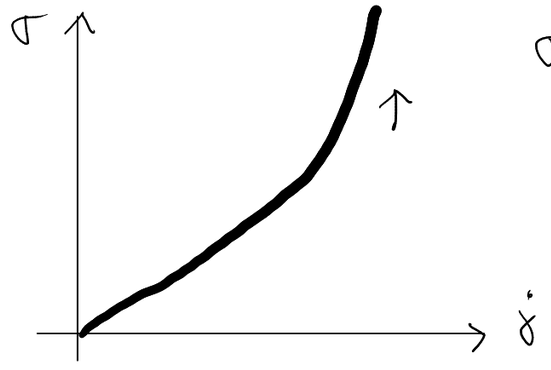
miele $\eta \sim 1-10$ Pas @T_g $\eta \sim 10^{12}$ Pas



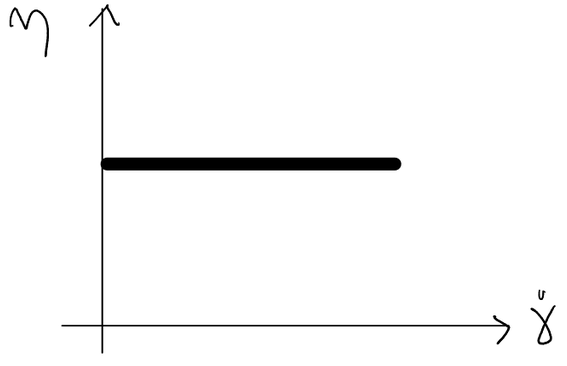
Comportamento non Newtoniano

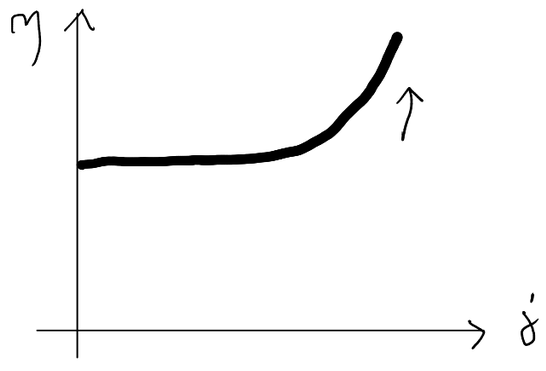






$$\sigma = \eta(\dot{\gamma})\,\dot{\gamma}$$





fluido Newtoniano

assottigliamento al taglio

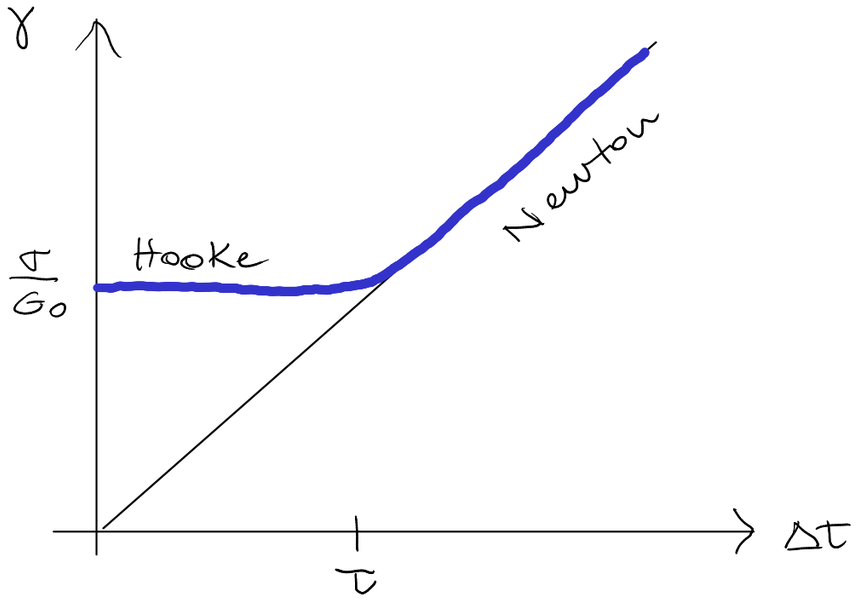
"shear thinning"

ispessimento al taglio

"shear thickening"

Visco-elasticità

Δt : intervallo di tempo durante il quale applico lo sforzo $\rightarrow$ $\nu = \frac{1}{\Delta t}$



Modello di Maxwell

Solido : $\sigma = G_0 \gamma$ $\quad \Delta t < \tau$

G_0 modulo istantaneo

fluido : $\sigma = \eta \dot{\gamma}$ $\quad \Delta t > \tau$

τ : tempo di rilassamento

$$\begin{cases} \dot{\gamma} = \frac{\sigma}{\eta} \\ \dot{\gamma} = \frac{\sigma}{G_0 \tau} \end{cases} \Rightarrow \eta = G_0 \tau \rightarrow \tau = \frac{\eta}{G_0}$$

relazione di Maxwell

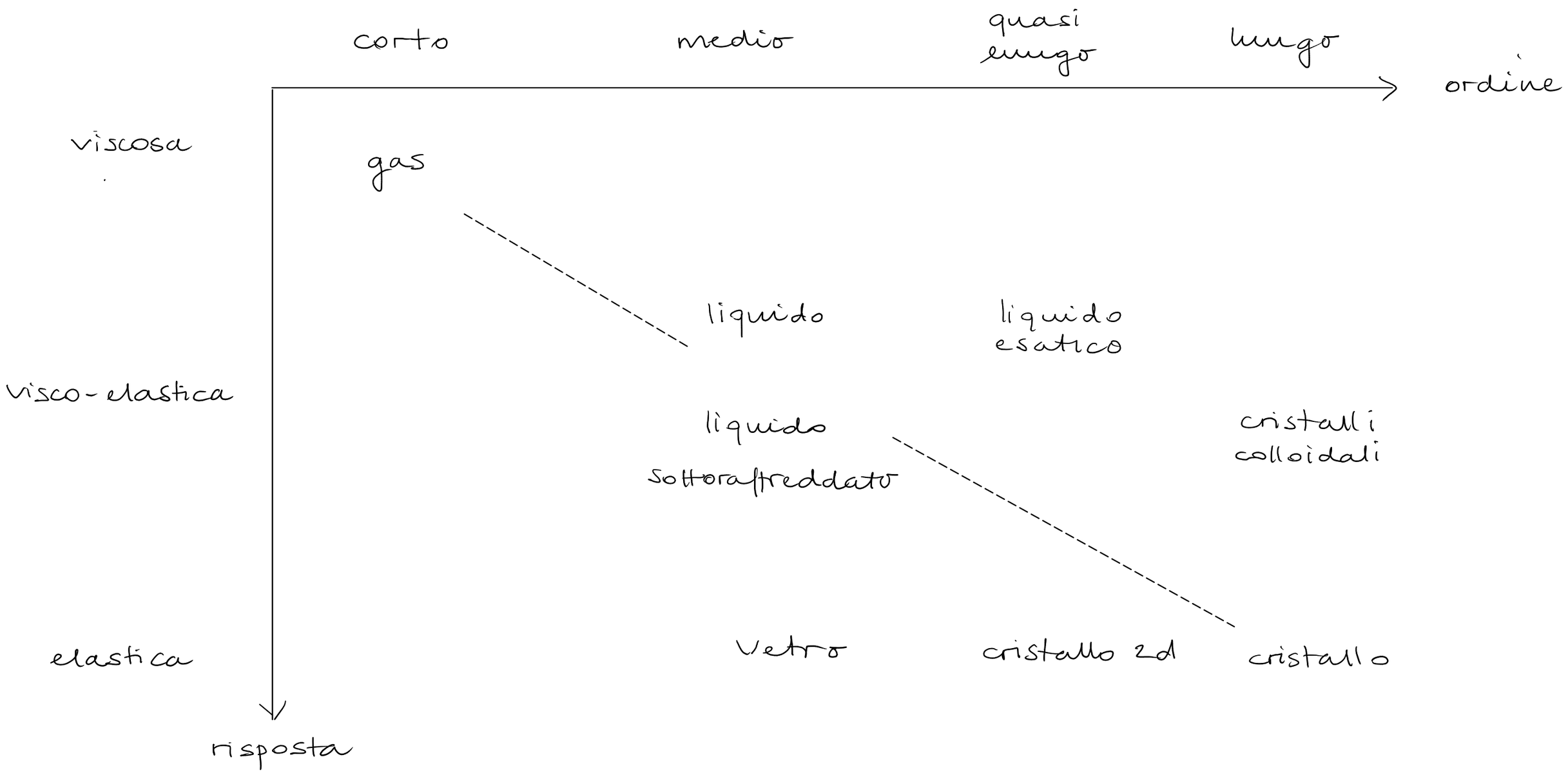
Interpretazione microscopica

$G \rightarrow$ energia elastica / volume $\qquad [G] = \frac{J}{m^3}$

$$\frac{dW}{dt} = F v = \sigma A \cdot \dot{\gamma} L = \sigma \dot{\gamma} V$$

$$\dot{w} = \sigma \dot{\gamma} = \eta \dot{\gamma}^2 \sim \eta \sim \dot{\gamma}^2$$

$\dot{w}$: tasso di dissipazione di energia (per unità di volume)


corto
medio
quasi
lungo
lungo
ordine
viscosa
gas
liquido
liquido
esatico
visco-elastica
liquido
sottoraffreddato
cristalli
colloidali
elastica
Vetro
cristallo 2d
cristallo
risposta