

# Course of Geothermics

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## Course Outline:

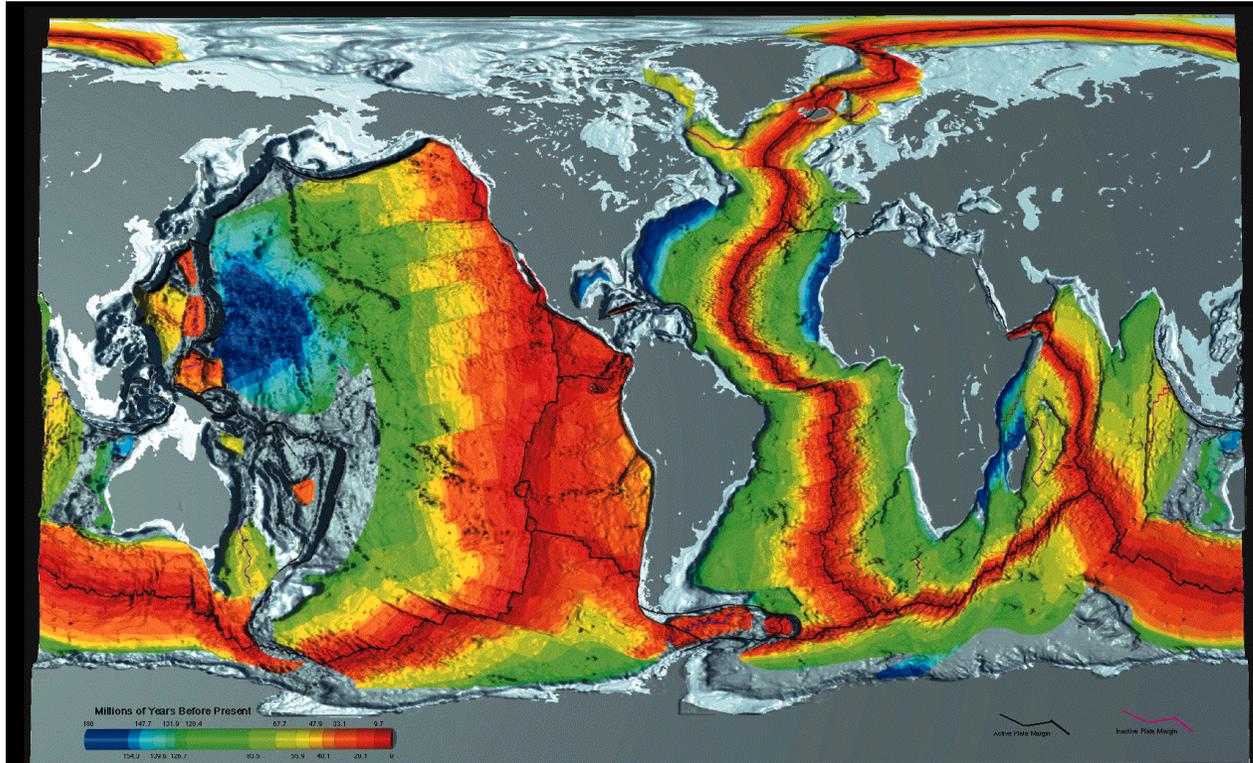
1. Thermal conditions of the early Earth and present-day Earth's structure
2. Thermal parameters of the rocks
3. Thermal structure of the lithospheric continental areas (steady state)
- 4. Thermal structure of the lithospheric oceanic areas**
5. Thermal structure of the lithosphere for transient conditions in various tectonic settings
6. Heat balance of the Earth
7. Thermal structure of the sedimentary basins
8. Thermal maturity of sediments
9. Mantle convection and hot spots
10. Magmatic processes and volcanoes
11. Heat transfer in hydrogeological settings
12. Geothermal Systems



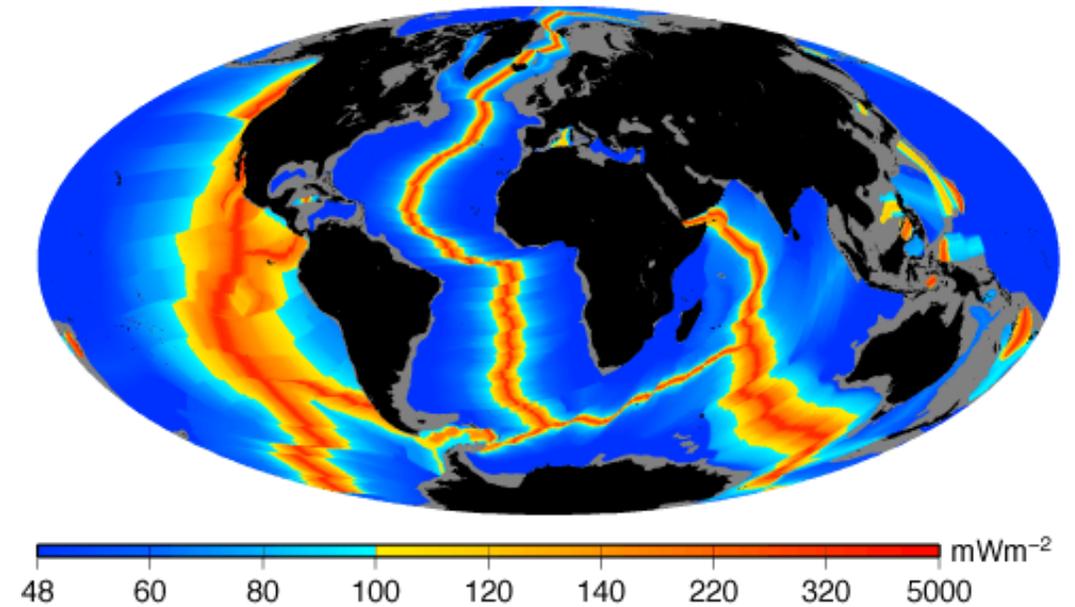
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# Age and Temperature of Oceanic Lithosphere

## Oceanic Age



## Oceanic Surface Heat Flow



Oceanic average HF: 67 mWm<sup>2</sup> (only due to conduction), 101 mWm<sup>-2</sup> (including heat loss from hot fluids)

# Eulerian and Lagrangian System

- An Eulerian point (**E**) is immobile, not connected with any material point.
- A Lagrangian point (**L**) is connected with a given material point and is moving with this point.



## Cooling models for oceanic lithosphere: Half-Space cooling model (Eulerian System)

**The cooling model predicts how heat flux decreases and how the depth of the sea floor increases with age of the sea floor**

- In the oceanic realm, flow is dominantly horizontal, but the large wavelengths of  $Q$  variations imply a predominant vertical heat transfer. If  $x$  is the distance from the ridge and  $z$  the depth from the sea floor, temperature equation is:

$$\rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right) = \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right) \quad u \frac{\partial T}{\partial x} = \text{advection of heat with moving plate}$$

$u$  = horizontal velocity,  $\rho$  = density of the lithosphere,  $C_p$  = heat capacity,  $\lambda$  = thermal conductivity

In steady state conditions (the temperature remains constant at a fixed distance from the ridge), for a constant  $\lambda$ :

$$\rho C_p u \frac{\partial T}{\partial x} = \lambda \frac{\partial^2 T}{\partial z^2}$$

For a constant spreading rate, the age  $\tau$  is:  $\tau = x/u$ ,  $\frac{\partial T}{\partial \tau} = \kappa \frac{\partial^2 T}{\partial z^2}$

$z$  = depth (m)

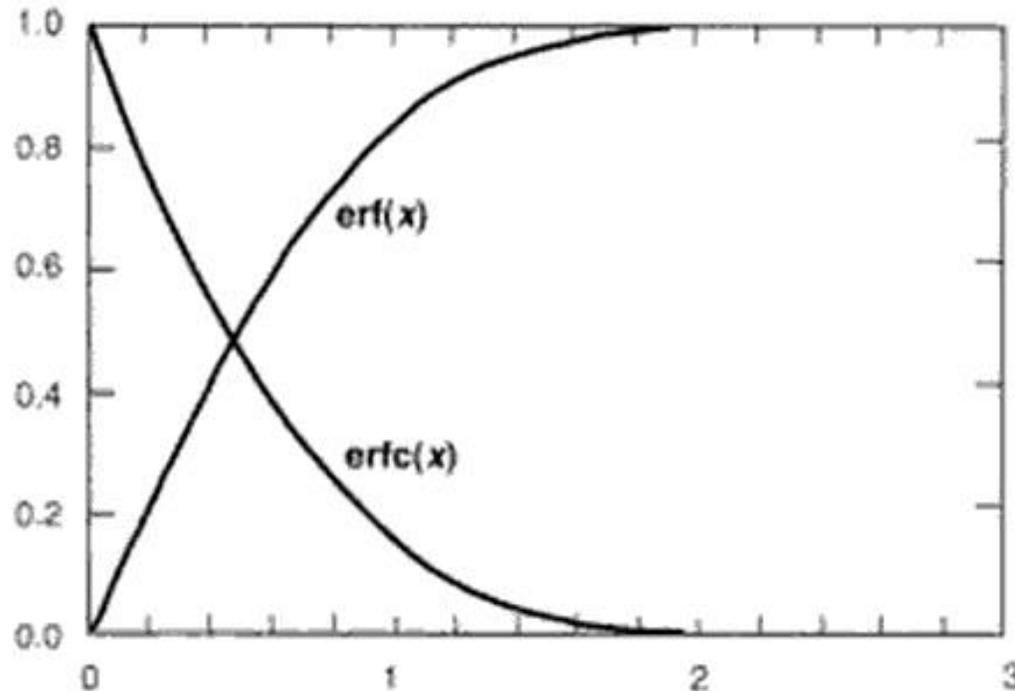
$\kappa$  = thermal diffusivity ( $10^{-6} \text{ m}^2/\text{s}$  or  $31.5 \text{ km}^2 \text{ My}^{-1}$ )

# Error Function (erf(x))

In mathematics, the **error function** (also called the **Gauss error function**) is a special function (non-elementary) of sigmoid shape that occurs in probability, statistics, and partial differential equations:

$$\operatorname{erf}(x) := \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$\operatorname{erfc}(x) := 1 - \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$$



x	erf(x)	erfc(x)	x	erf(x)	erfc(x)
0,00	0,0000000	1,0000000	1,30	0,9340079	0,0659921
0,05	0,0563720	0,9436280	1,40	0,9522851	0,0477149
0,10	0,1124629	0,8875371	1,50	0,9661051	0,0338949
0,15	0,1679960	0,8320040	1,60	0,9763484	0,0236516
0,20	0,2227026	0,7772974	1,70	0,9837905	0,0162095
0,25	0,2763264	0,7236736	1,80	0,9890905	0,0109095
0,30	0,3286268	0,6713732	1,90	0,9927904	0,0072096
0,35	0,3793821	0,6206179	2,00	0,9953223	0,0046777
0,40	0,4283924	0,5716076	2,10	0,9970205	0,0029795
0,45	0,4754817	0,5245183	2,20	0,9981372	0,0018628
0,50	0,5204999	0,4795001	2,30	0,9988568	0,0011432
0,55	0,5633234	0,4366766	2,40	0,9993115	0,0006885
0,60	0,6038561	0,3961439	2,50	0,9995930	0,0004070
0,65	0,6420293	0,3579707	2,60	0,9997640	0,0002360
0,70	0,6778012	0,3221988	2,70	0,9998657	0,0001343
0,75	0,7111556	0,2888444	2,80	0,9999250	0,0000750
0,80	0,7421010	0,2578990	2,90	0,9999589	0,0000411
0,85	0,7706681	0,2293319	3,0	0,9999779	0,0000221
0,90	0,7969082	0,2030918	3,10	0,9999884	0,0000116
0,95	0,8208908	0,1791092	3,20	0,9999940	0,0000060
1,00	0,8427008	0,1572992	3,30	0,9999969	0,0000031
1,10	0,8802051	0,1197949	3,40	0,9999985	0,0000015
1,20	0,9103140	0,0896860	3,50	0,9999993	0,0000007

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)n!} = \frac{2}{\sqrt{\pi}} \left( x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \frac{x^9}{216} - \dots \right)$$

## Half-space (HS) cooling model

If only one of the boundaries is fixed in temperature, then cooling of the oceanic lithosphere may be described as a *half space problem*

Assuming that the mantle is a uniform infinite halfspace, with the initial condition that  $T(z, 0) = T_m$ , the boundary condition that  $T(0, t) = 0$  and the assumption that radioactive heating can be neglected:

$$T(z, t) = T_m \operatorname{erf}\left(\frac{z}{2\sqrt{\kappa t}}\right) \quad \text{or we can write in the form: } T(z, \tau) = \Delta T \operatorname{erf}\left(\frac{z}{2\sqrt{\kappa \tau}}\right) = \frac{2\Delta T}{\sqrt{\pi}} \int_0^{z/2\sqrt{\kappa \tau}} \exp(-\eta^2) d\eta$$

$$x = z/2\sqrt{\kappa t} \quad \frac{d \operatorname{erf}(x)}{dx} = \operatorname{erf}'(x) = \frac{2}{\sqrt{\pi}} e^{-x^2}$$

$$\frac{\partial T}{\partial z} = T_m \frac{d \operatorname{erf}(x)}{dx} \frac{\partial x}{\partial z} = T_m \frac{2}{\sqrt{\pi}} e^{-x^2} \frac{1}{2\sqrt{\kappa t}}$$

The heat flux  $q_0$  at  $z = 0$  is

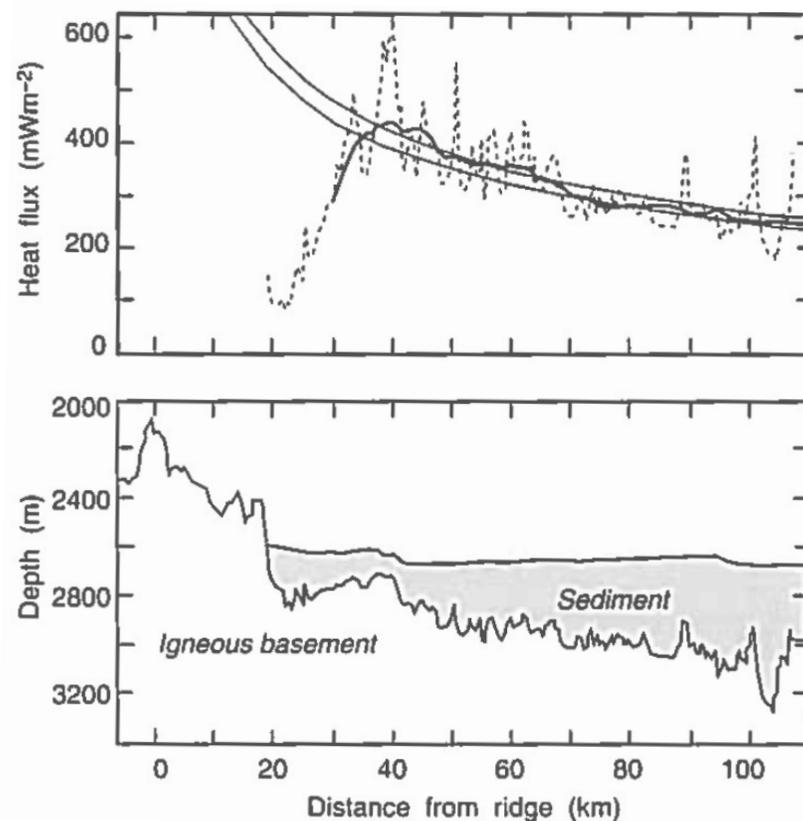
$$q_0 = -K \left. \frac{\partial T}{\partial z} \right|_{z=0} = -\frac{KT_m}{\sqrt{\pi \kappa t}}$$

# Half-space (HS) cooling model

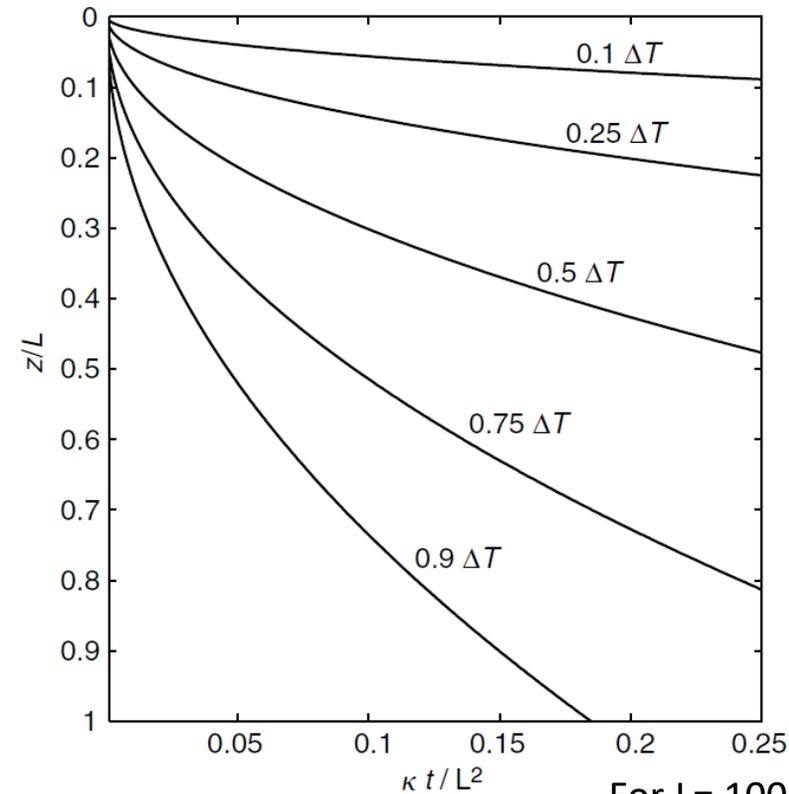
In the half-space mode, the depth of isotherms increases with age, or distance from the spreading center

$$q(\tau) = \lambda \frac{\Delta T}{\sqrt{\pi \kappa \tau}} = C_Q \tau^{-1/2} \quad C_Q = \lambda \Delta T / \sqrt{\pi \kappa} \quad \lambda = K \quad C_Q = HF \text{ cooling constant}$$

- The heat flux values can be fitted by a  $\tau^{-1/2}$  relationship with the constant  $C_Q$  between 470 and 510  $\text{mW m}^{-2} \text{Myr}^{1/2}$ .
- Heat flux data selected from sites where thick sedimentary cover is hydraulically resistive and seals off hydrothermal circulation fit the HS cooling model, with the constraint that for HF  $\rightarrow 0$  as age  $\rightarrow \infty$  and excluding ocean floor  $> 80 \text{Myr}$ .

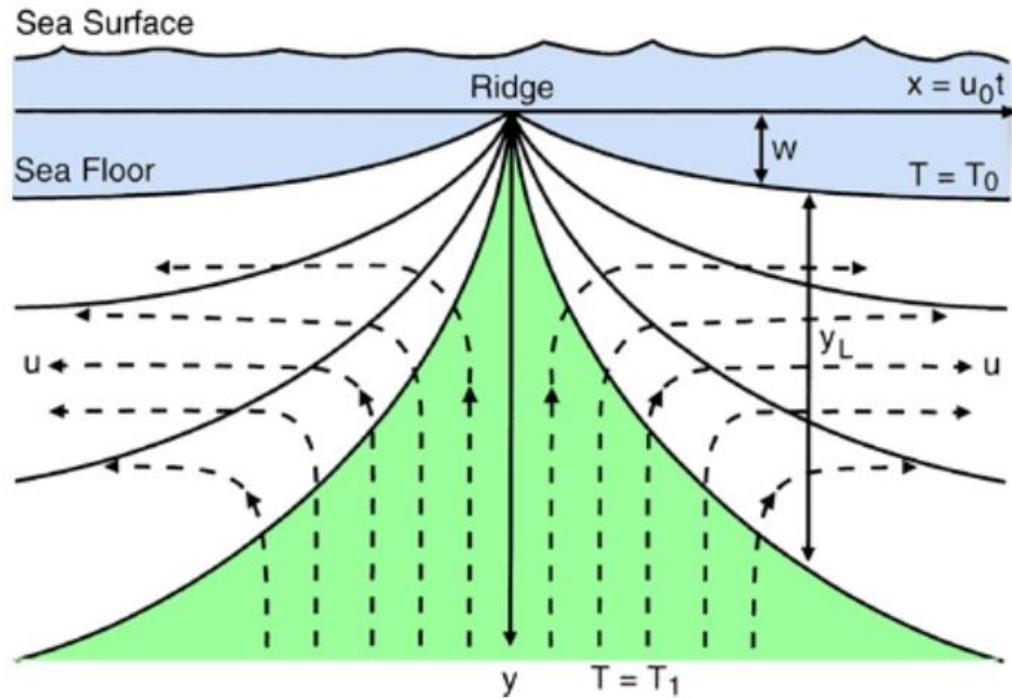


Depth of the isotherms as a function of sea-floor age  $t$

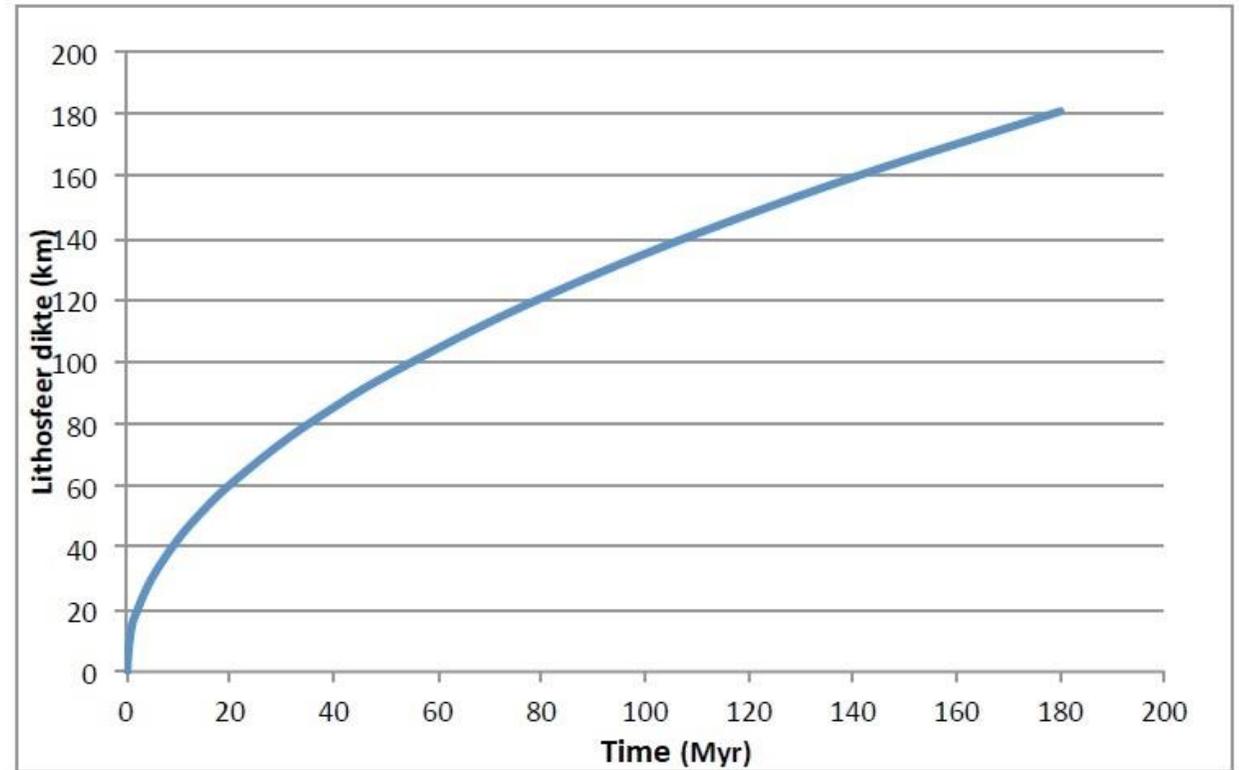


# Half-space (HS) cooling model

There is no lower (box) boundary



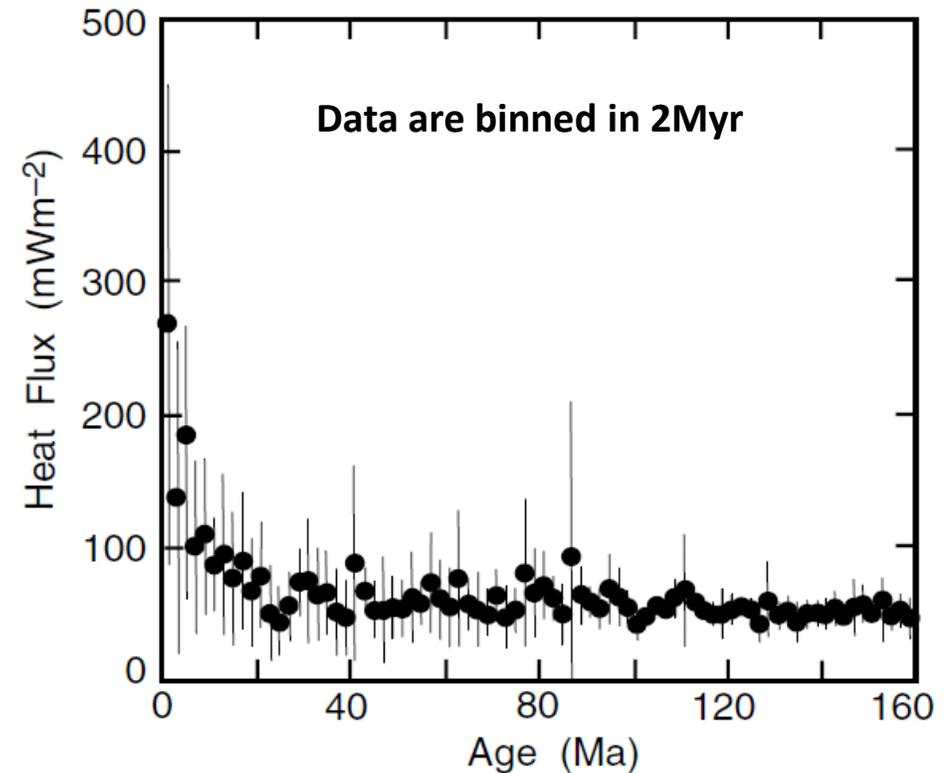
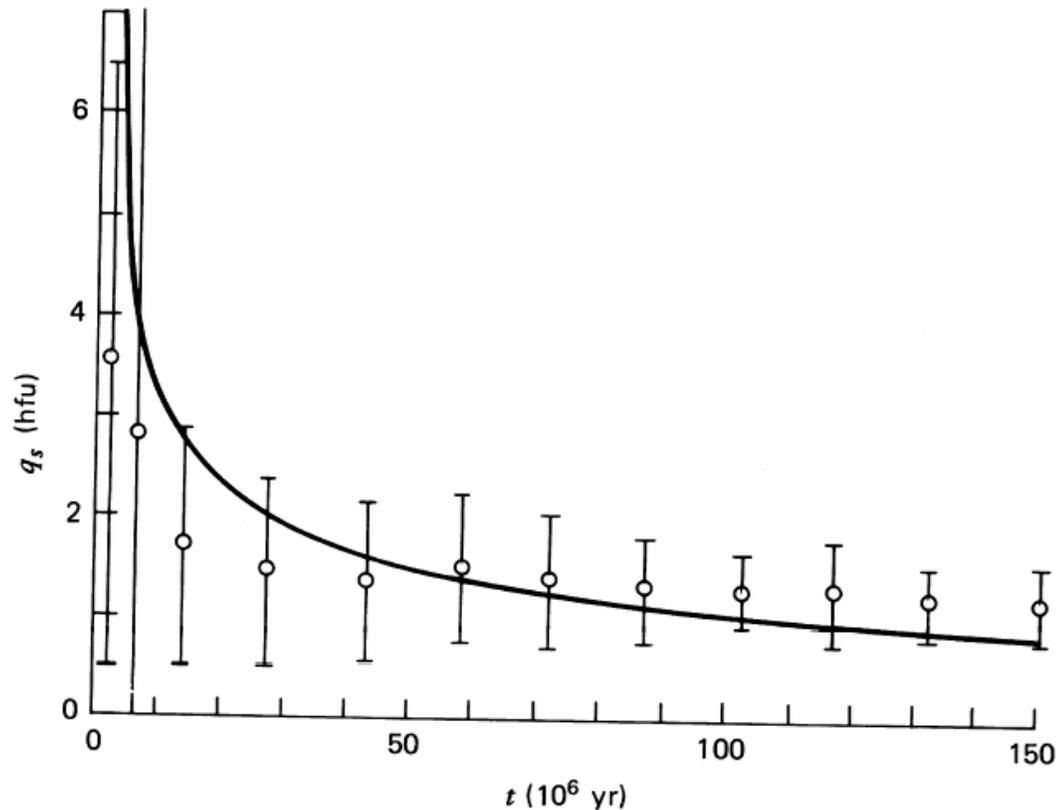
Boundary layer model: thickness of the lithosphere



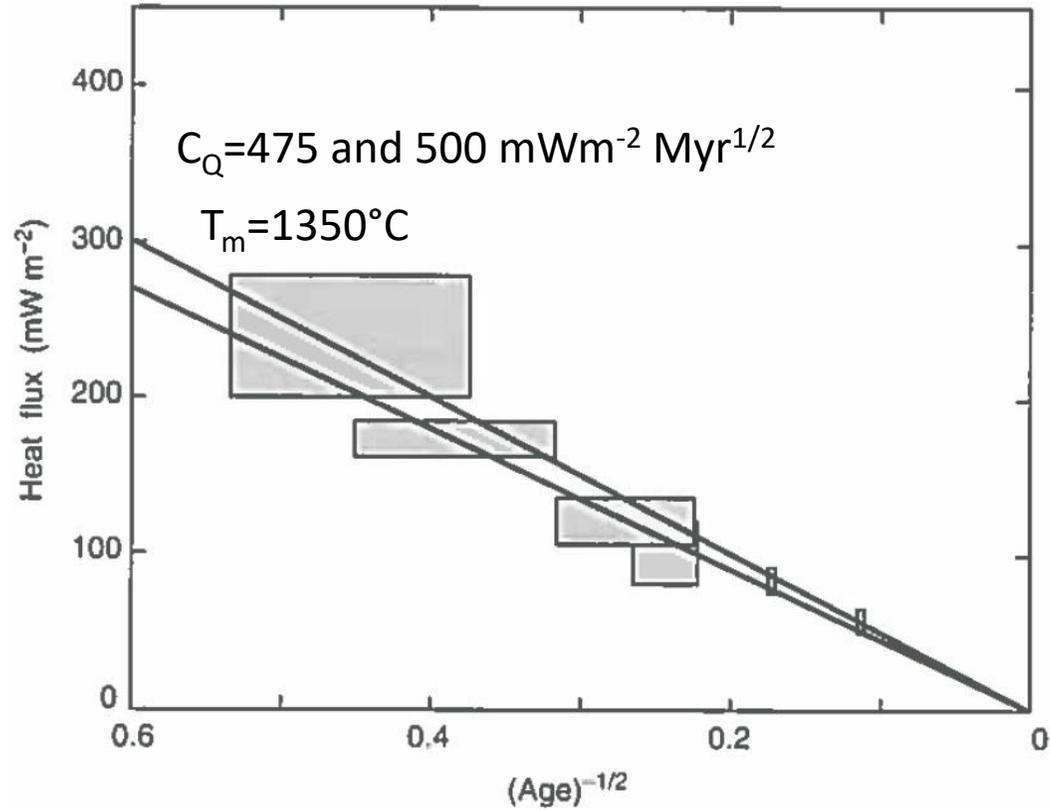
# Half-space (*HS*) cooling model

Oceanic lithospheric thickness increases with age:  $h = \sqrt{\pi \kappa \tau}$

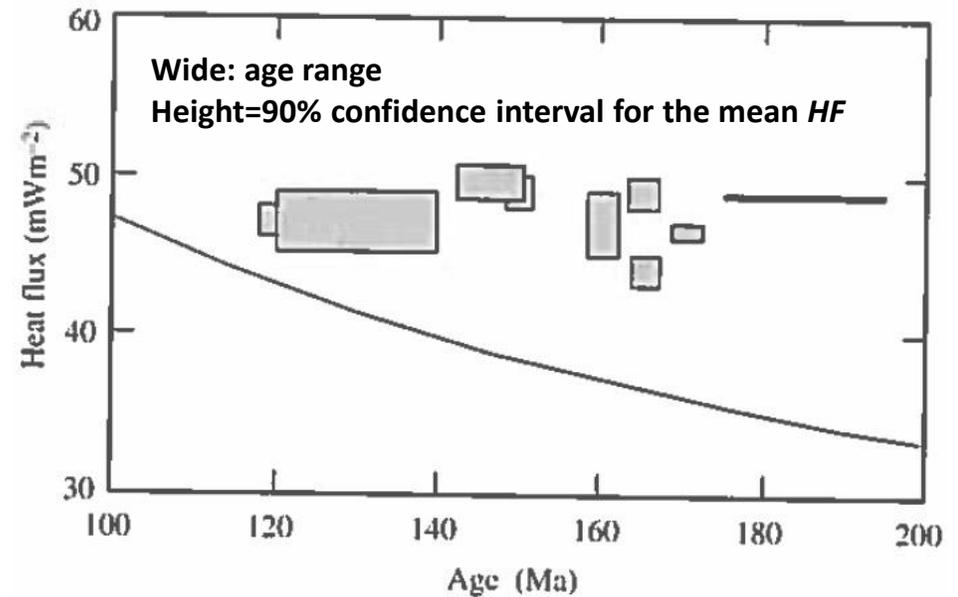
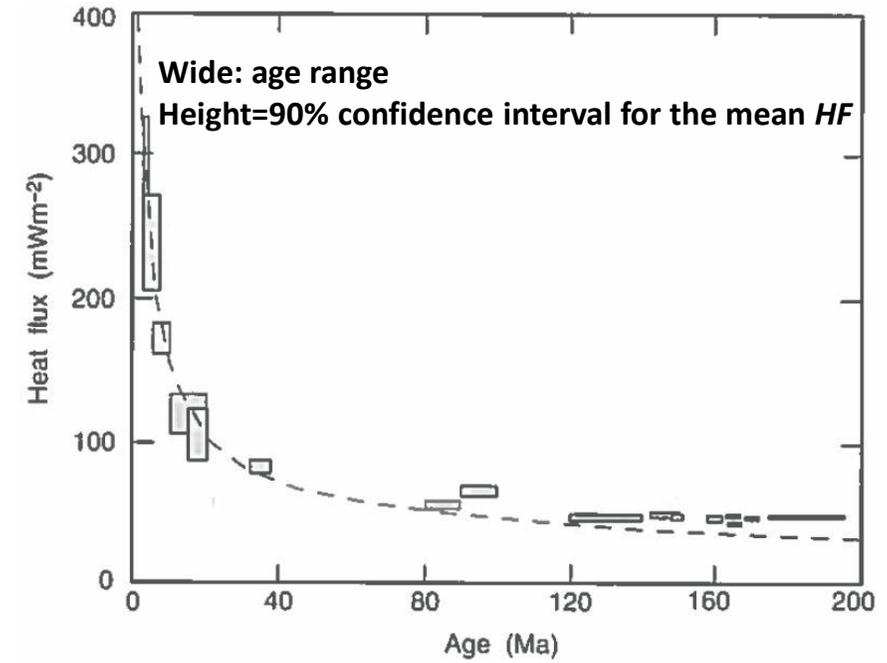
- Due to hydrothermal circulation, heat flux data close to the ridges are very scattered
- The heat loss by hydrothermal circulation ( $\sim 11$  TW) can be obtained as the difference between the predicted *HF* (*HS* cooling model) and the average measured *HF* within each age bin ( $\sim 550$  mWm<sup>2</sup>) and integrating this difference over the entire sea floor.



# Half-space (*HS*) cooling model



- Heat flux data through sea floor  $> 80$  Myr are systematically higher than model prediction.



## Half-space (*HS*) cooling model

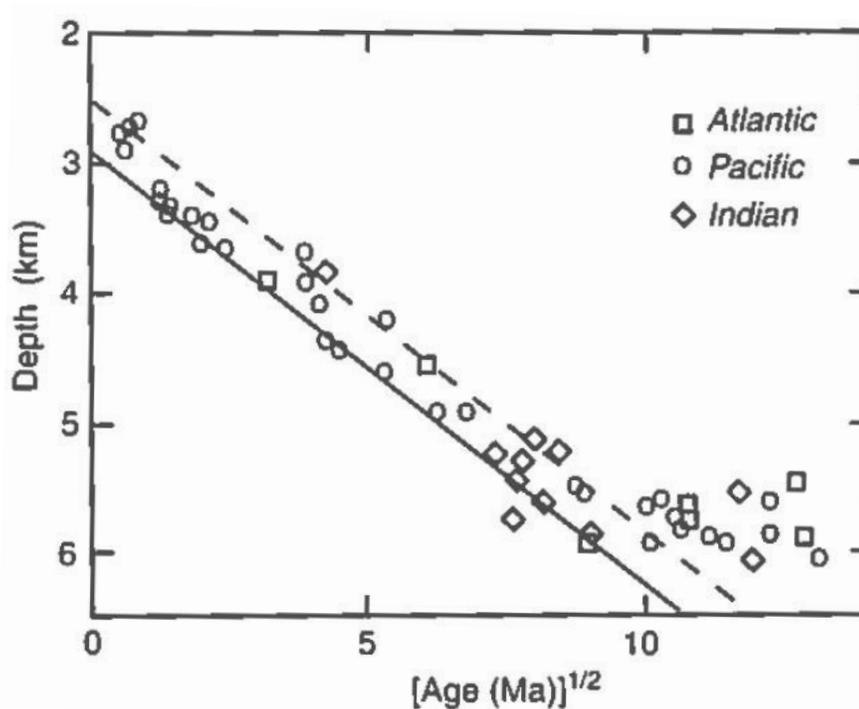
- Bathymetry records the total cooling of the oceanic lithosphere since its formation at the mid-oceanic ridges and are far less noisy than heat flux data.
- Bathymetry fits extremely well the predictions of the *HS* cooling model for oceanic lithosphere younger than  $\approx 100$  My.
- However, flattening of the bathymetry does not allow straightforward conclusions (depth values exhibit some scatter due to inaccurate estimates of sediment thickness, presence of sea-mounts, and large hot spot volcanic edifices).

After a correction for isostatic adjustments to sediment loading, the basement depth for oceanic age  $< 80$  Myr:

$$h(\tau) = (2600 \pm 20) + (345 \pm 3)\tau^{1/2}$$

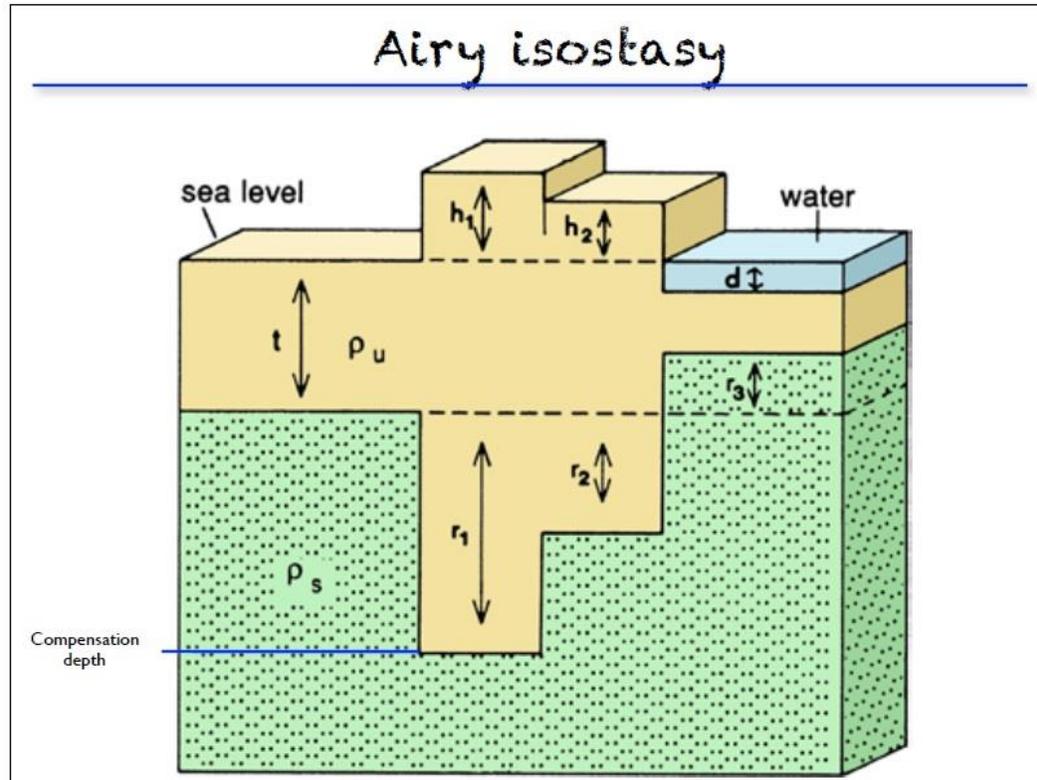
$$q(\tau) = (480 \pm 4)\tau^{-1/2}$$

$h$  (m),  $\tau$  (Myr), and  $q = \text{mWm}^{-2}$

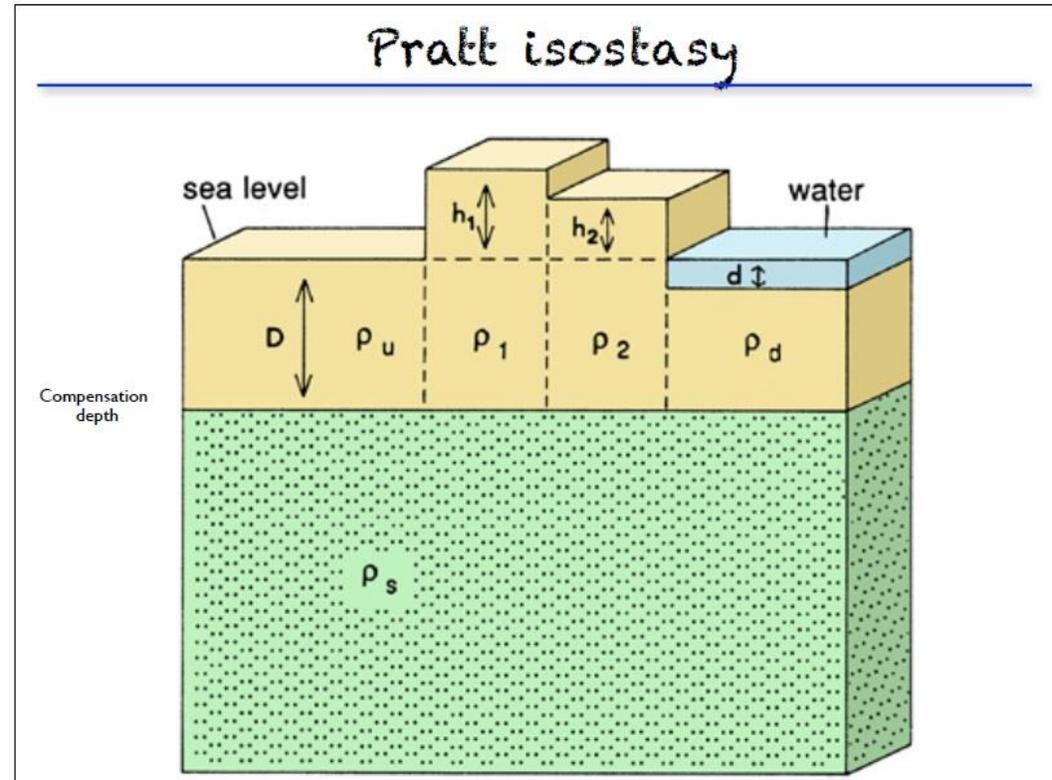


# Hydrostatic Isostasy

## Isostatic compensation depth



**Airy:** Crustal density is roughly equal, compensation is due to crustal roots



**Pratt:** Continental crust extends to a common depth, while the density is variable

Gravimetric data show that many orogens are not in isostatic equilibrium, but their topography is dynamically supported

# Hydrostatic Isostasy

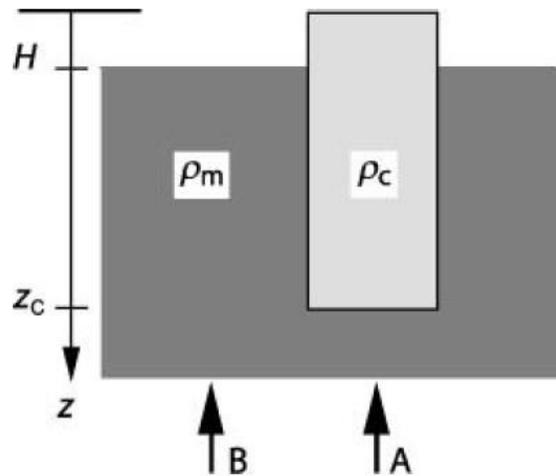
- **Hydrostatic isostatic model:** all vertical profiles through the lithosphere may be considered independently of each other (shear stress are neglected). It is applicable only for features extended for few hundred km.
- There is a depth (**isostatic compensation depth**) at which the vertical stresses of all vertical profiles are equal:

$$\sigma_{zz}^A|_{z=z_K} = \sigma_{zz}^B|_{z=z_K}$$

Downward force exerted by an entire vertical column:

$$\int_0^{z_K} \rho_A(z)g dz = \int_0^{z_K} \rho_B(z)g dz$$

Example:



$$\rho_c g z_c \Big|_0^{z_c} = g \int_0^{H_{\text{mat}}} \rho_{\text{air}} dz + g \int_{H_{\text{mat}}}^{z_c} \rho_m dz$$

$$\rho_c z_c = \rho_m z_c - \rho_m H_{\text{mat}}$$

$$H = H_{\text{mat}} = z_c \left( \frac{\rho_m - \rho_c}{\rho_m} \right)$$

$$\text{If } \rho_c = 0, H = z_c$$

$$\text{If } \rho_c = \rho_m, H = 0$$

# Depth of the Ocean

- The water depth of the oceans is a direct function of the distance to the mid-ocean ridges.
- Density variations in the oceanic lithosphere depends prevalently on temperature (progressively cooling of the lithosphere)

$$\sigma_{zz}^A|_{z=z_1} = \rho_w g w + \int_0^{z_1} \rho(z) g dz \quad \sigma_{zz}^B|_{z=z_1} = \rho_m g w + \rho_m g z_1$$

$$\rho_m z_1 + w(\rho_m - \rho_w) = \int_0^{z_1} \rho(z) dz \quad w(\rho_m - \rho_w) = \int_0^{z_1} (\rho(z) - \rho_m) dz \quad \text{Since } \rho \text{ depends on } T \quad \rho(T) = \rho_m(1 + \alpha(T_1 - T))$$

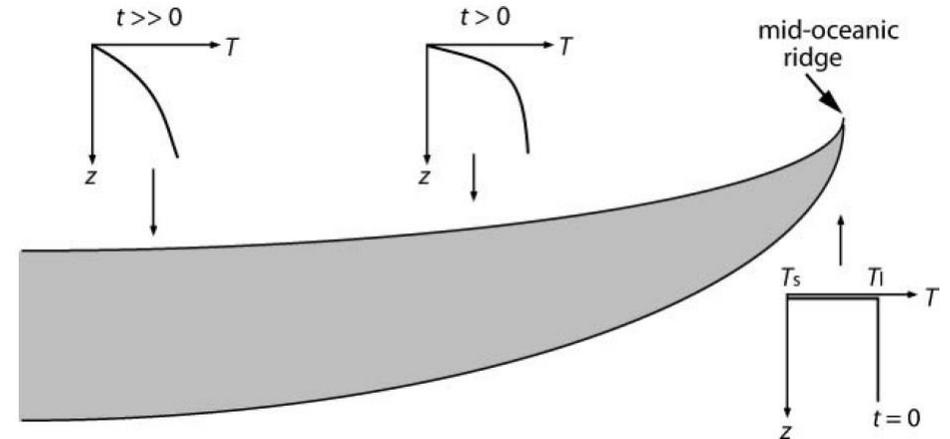
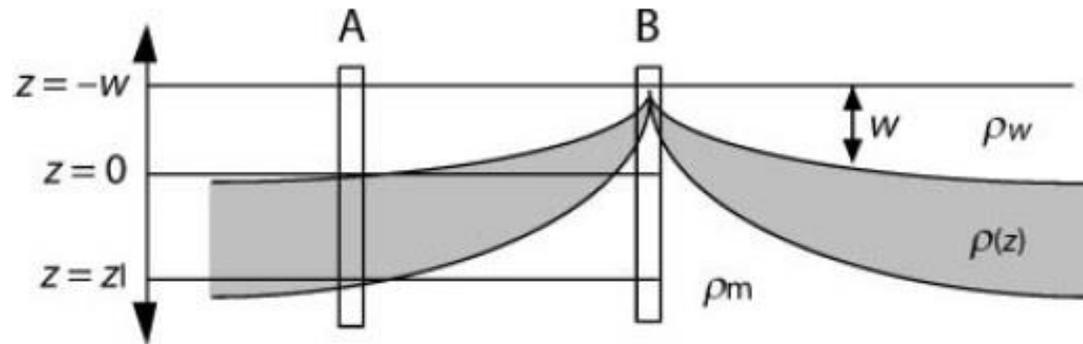
$$w(\rho_m - \rho_w) = \int_0^{z_1} \rho_m \alpha (T_1 - T(z)) dz \quad T(z) \text{ is described by the half space cooling model}$$

$$T(z, t) = T_m \operatorname{erf}\left(\frac{z}{2\sqrt{\kappa t}}\right)$$

$T = T_s$  at the depth  $z = 0$  and  $T = T_1$  in all depths  $z > 0$  at time  $t = 0$

$T = T_s$  at  $z = 0$  for all  $t > 0$  and  $T = T_1$  at  $z = \infty$  for all  $t > 0$ .

$$T = T_s + (T_1 - T_s) \operatorname{erf}\left(\frac{z}{\sqrt{4\kappa t}}\right)$$



# Depth of the Ocean

Oceanic depth is proportional to the square root of age of oceanic lithosphere

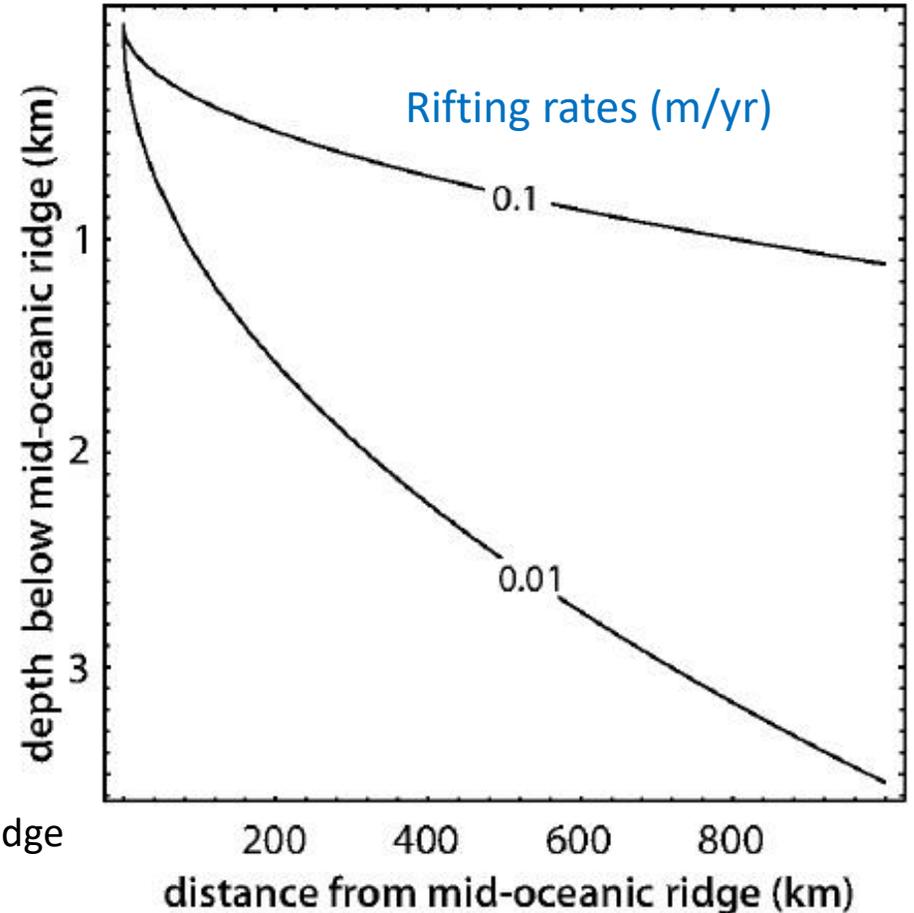
$$w(\rho_m - \rho_w) = \int_0^{z_1} \rho_m \alpha \left( T_1 - T_s - (T_1 - T_s) \operatorname{erf} \left( \frac{z}{\sqrt{4\kappa t}} \right) \right) dz$$

$$w(\rho_m - \rho_w) = \int_0^{z_1} \rho_m \alpha (T_1 - T_s) \operatorname{erfc} \left( \frac{z}{\sqrt{4\kappa t}} \right) dz$$

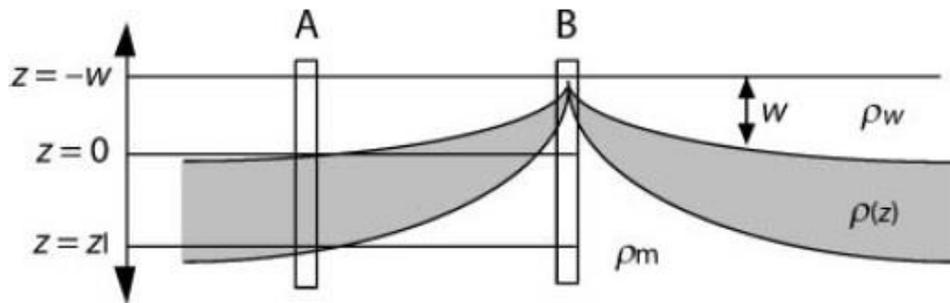
$$w = \frac{\rho_m \alpha (T_1 - T_s)}{(\rho_m - \rho_w)} \int_0^{z_1} \operatorname{erfc} \left( \frac{z}{\sqrt{4\kappa t}} \right) dz \quad n = z/\sqrt{4\kappa t},$$

$$w = \sqrt{4\kappa t} \frac{\rho_m \alpha (T_1 - T_s)}{(\rho_m - \rho_w)} \int_0^{z_1} \operatorname{erfc}(n) dn \quad \int_0^{\infty} \operatorname{erfc}(n) dn = \frac{1}{\sqrt{\pi}}$$

$$w = \frac{2\rho_m \alpha (T_1 - T_s)}{(\rho_m - \rho_w)} \sqrt{\frac{\kappa t}{\pi}} \quad w \approx 5.91 \cdot 10^{-5} \sqrt{t}$$



$w$  is only the *additional* water depth on top of the water depth at the mid-oceanic ridge

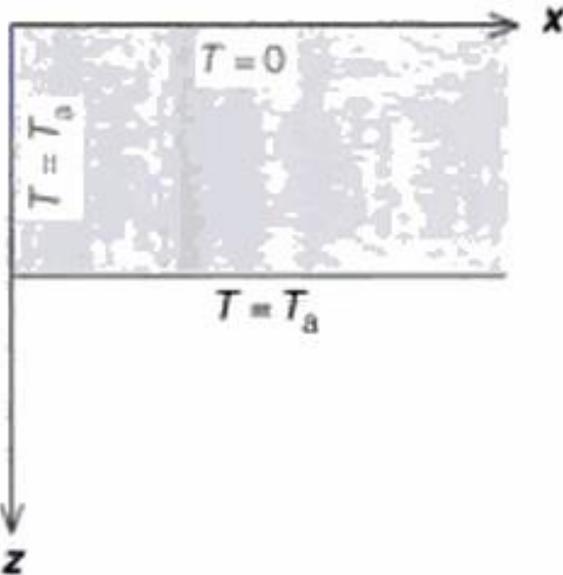


$\rho_m = 3200 \text{ kgm}^{-3}$ ,  $\rho_w = 1000 \text{ kgm}^{-3}$ ,  $\alpha = 3 \times 10^{-5} \text{ K}^{-1}$   
 $T_1 = 1280^\circ\text{C}$ ,  $T_s = 0^\circ\text{C}$ , and  $\kappa = 10^{-6} \text{ m}^2 \text{ s}^{-1}$

## Why do we need a plate model?

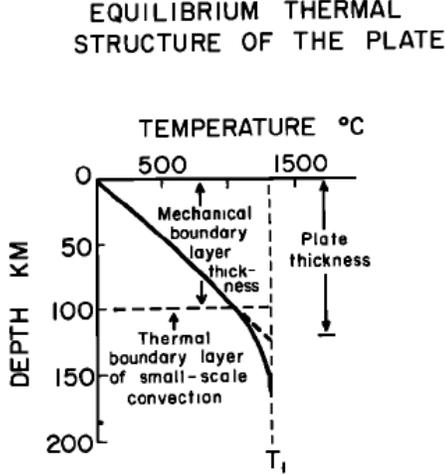
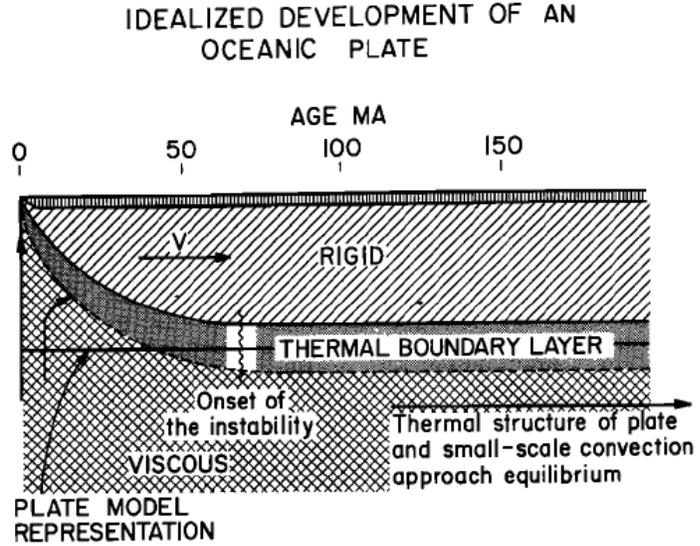
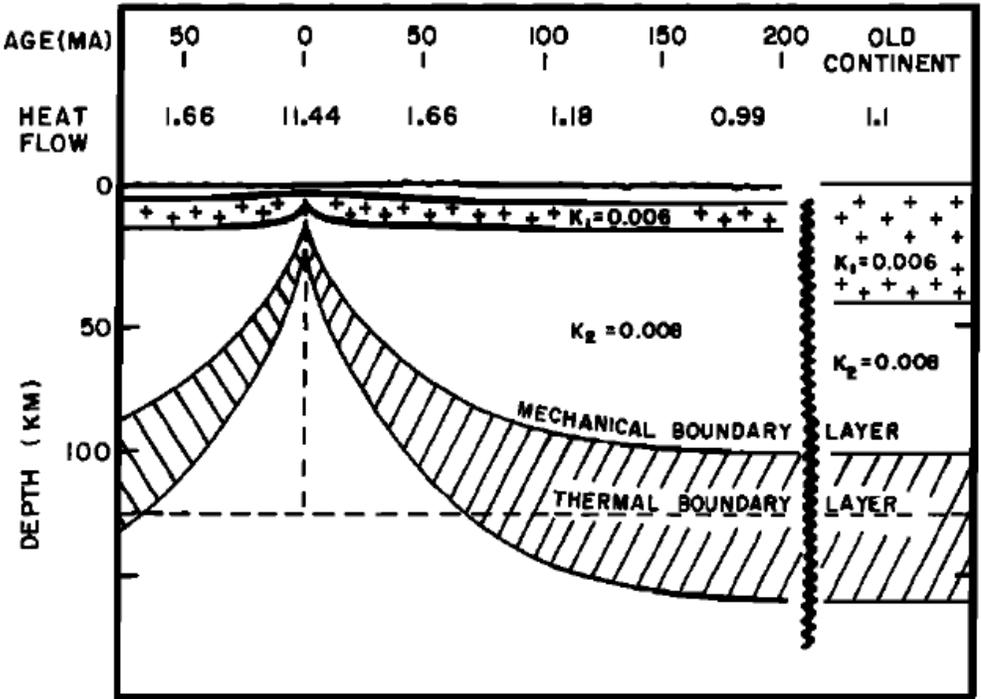
- Heat flux data exhibit no detectable variation at ages  $>80\text{-}120\text{My}$  ( $q_0$  levels off  $\sim 48\text{ mWm}^{-2}$ ).
- Departure of heat flux data from the  $1/\sqrt{\tau}$  behavior, indicates that heat is supplied to the lithosphere from below.
- Flattening of the bathymetry and heat flux implies that heat is brought into the lithosphere from below at the same rate as it is lost at the surface.
- Small scale convection occurs in the asthenosphere at the base of the old lithosphere, which would increase the  $HF$  into the base of the lithosphere and maintain a more constant lithospheric thickness.
- Then, we need a “plate” model for which a boundary condition is specified at some fixed depth (the base of the plate).

In the plate model the oceanic lithosphere is taken to be of constant thickness  $L$  and at its base  $T$  is equal to  $T_a$  (temperature of the asthenosphere) and at its top  $T=0$ .



# Unified Thermal Boundary Layer Model

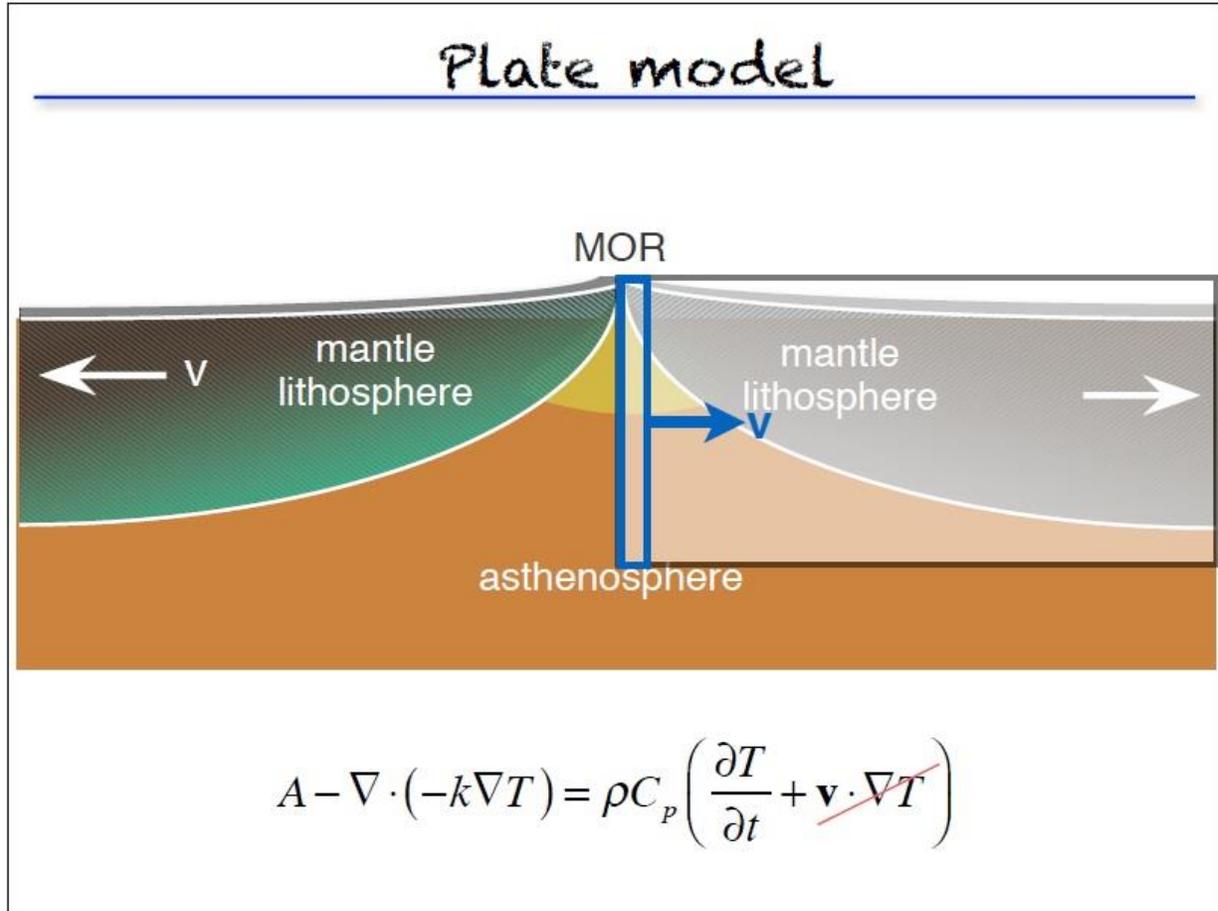
- Plate dynamics is governed by a thermomechanical boundary layer that evolves by conductive decay



Sclater et al. (1981), after Parsons & McKenzie (1978)

← TBL thickness maintained by convective instability

## Cooling models for oceanic lithosphere: Plate Model (Lagrangian System)



Vertical heat transport only:

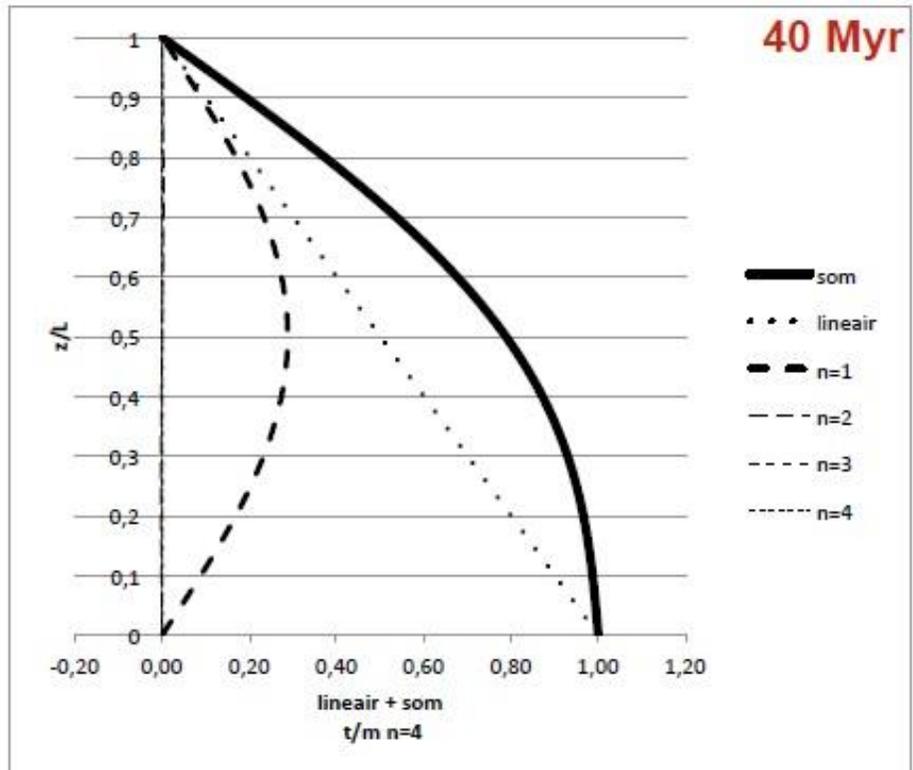
$$A - \frac{\partial}{\partial z} \left( -k \frac{\partial T}{\partial z} \right) = \rho C_p \frac{\partial T}{\partial t}$$

No heat production,  $k$  uniform:

$$\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial z^2} \quad K \equiv \frac{k}{\rho C_p}$$

- In a Lagrangian frame everything is moving with the material, then the horizontal velocity is zero (the fluid parcel moves through space and time).

# Plate Cooling Model (with fixed temperature at the base)



Assuming that the base of the plate is initially at a fixed temperature  $T_a$  (or  $\Delta T_T$ ), the surface is maintained at  $T = 0$ , the temperature within the plate ( $0 < z < L$ ) is:

$$T = T_a \left[ \frac{z}{L} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi z}{L}\right) \exp\left(-\frac{n^2 \pi^2 \kappa t}{L^2}\right) \right]$$

The equation defines the characteristic time  $\tau$ :  $\tau = L^2 / \kappa$

For  $t \gg \tau$

$$T = T_a \left[ \frac{z}{L} + \frac{2}{\pi} \sin\left(\frac{\pi z}{L}\right) \exp\left(-\frac{\pi^2 \kappa t}{L^2}\right) \right]$$

The surface heat flux is given by:

$$Q(t) = \frac{kT_a}{L} \left[ 1 + 2 \sum_{n=1}^{\infty} \exp\left(-\frac{n^2 \pi^2 \kappa t}{L^2}\right) \right]$$

K or  $\lambda$  = thermal conductivity  
 $T_a = \Delta T_T$        $L = a_T$

The asymptotic value for the heat flow ( $\tau \ll a_T^2 / \kappa$ ) is:  $\frac{kT_a}{L}$  which can be written as:  $\frac{\lambda \Delta T_T}{\sqrt{\pi \kappa t}}$

# Plate Cooling Model (with fixed temperature at the base)

The subsidence due to thermal contraction is obtained as:  $h(t) = \alpha \int_0^{a_T} \{T(z,t) - T(z,0)\} dz.$

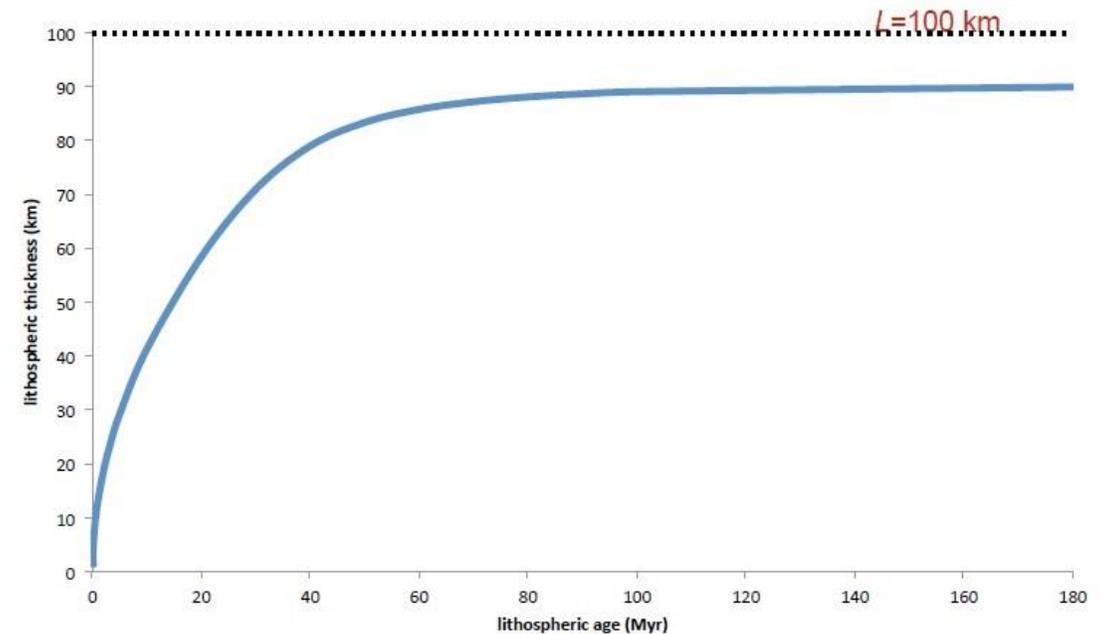
$$h(t) = \frac{\alpha \Delta T_T a_T}{2} \left( 1 - \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \exp\left(\frac{-(2n-1)^2 \pi^2 \kappa t}{a_T^2}\right) \right)$$

The asymptotic value for the depth of the lithosphere:

$$\frac{\alpha \Delta T_T a_T}{2}$$

$$T_a = \Delta T_T \quad L = a_T$$

Plate model: thickness of the lithosphere



The plate cooling model has a uniformly thick lithosphere, then  $T$  of the lithosphere approaches the equilibrium as the age increases.

## Plate Cooling Model (with fixed heat flux at the base)

The plate is initially at temperature  $\Delta T_Q$  and fixed flux  $Q(a_Q, t) = \lambda \Delta T_Q / a_Q$ , is maintained at the base  $a_Q$ :

$$T(z, t) = \frac{\Delta T_Q z}{a_Q} + \frac{4\Delta T_Q}{\pi} \sum_{n=0}^{\infty} \left( \frac{1}{(2n+1)} - \frac{2}{\pi} \frac{(-1)^n}{(2n+1)^2} \right) \times \sin \left( \frac{(2n+1)\pi z}{2a_Q} \right) \exp \left( \frac{-(2n+1)^2 \pi^2 \kappa t}{4a_Q^2} \right)$$

The surface heat flux is given by:

$$q(t) = \frac{\lambda \Delta T_Q}{a_Q} \left( 1 + 2 \sum_{n=0}^{\infty} \left( 1 + \frac{(-1)^n 2}{(2n+1)\pi} \right) \exp \left( \frac{-(2n+1)^2 \pi^2 \kappa t}{4a_Q^2} \right) \right)$$

for  $t \ll a_Q^2 / \kappa$ ,  $q(t) = \frac{\lambda \Delta T_Q}{\sqrt{\pi \kappa t}}$

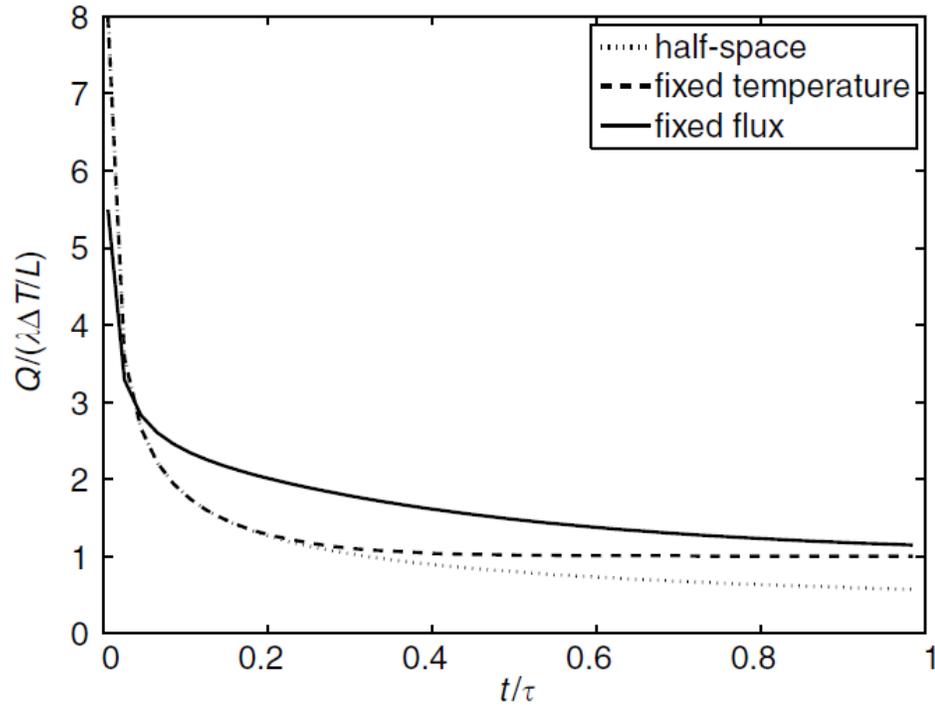
The temperature difference  $\Delta T_Q$  corresponds to steady state in the plate with basal heat flux  $Q_b$ :

$$\Delta T_Q = \frac{Q_b a_Q}{\lambda} \quad \tau_Q = 4 \frac{a_Q^2}{\kappa} \quad \tau_Q = \text{relaxation time}$$

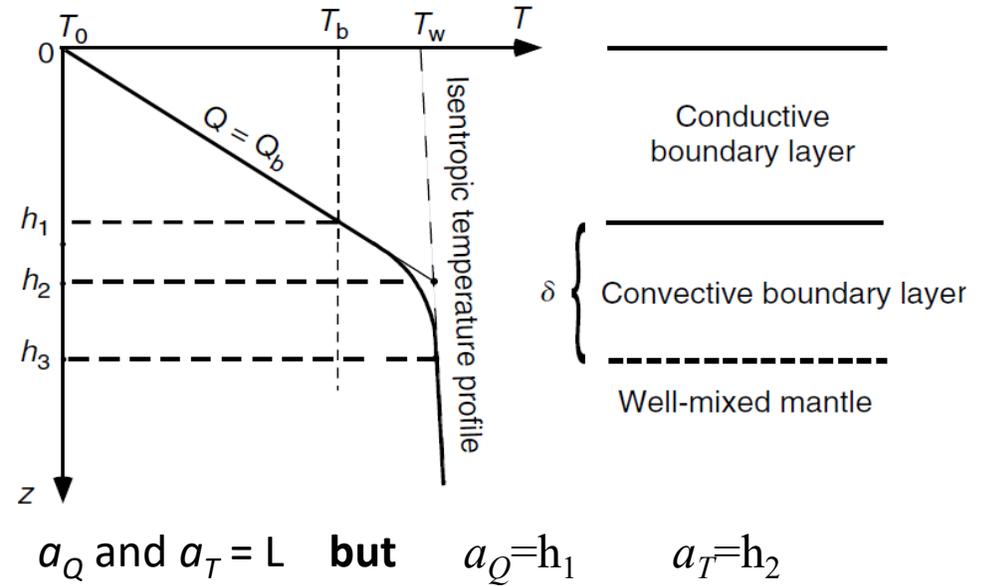
The two plate models exhibit the same type of thermal relaxation, but the value of the characteristic relaxation time depends on the boundary condition. The two relaxation times must have the same value of about 80 My (the age at which the subsidence departs from boundary layer cooling subsidence), implying a different depth to the lower boundary:

$$a_Q = \frac{a_T}{2} \approx 50 \text{ km}$$

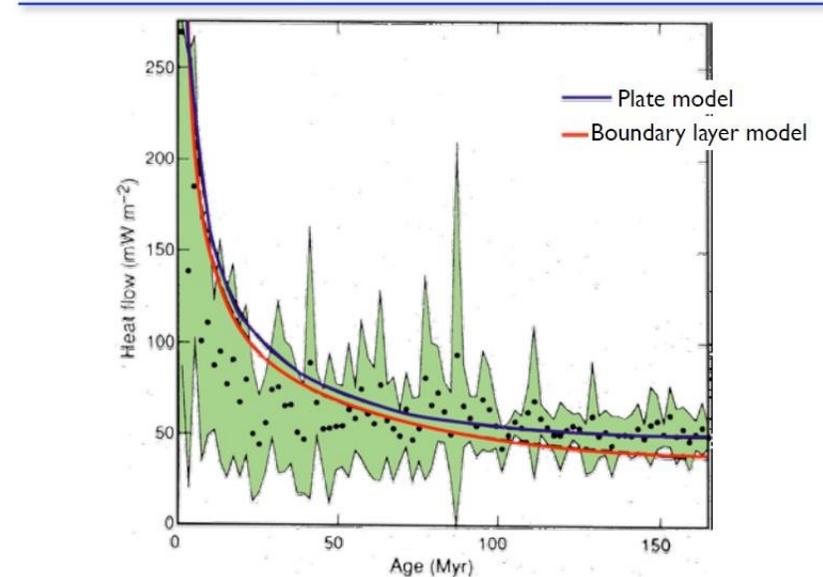
# Plate cooling model vs Half-Space cooling model



The same plate thickness is used for the flux and temperature boundary conditions (i.e.  $a_Q = a_T$ ). This leads to much longer equilibration time for the fixed flux condition.



## Surface heat flow

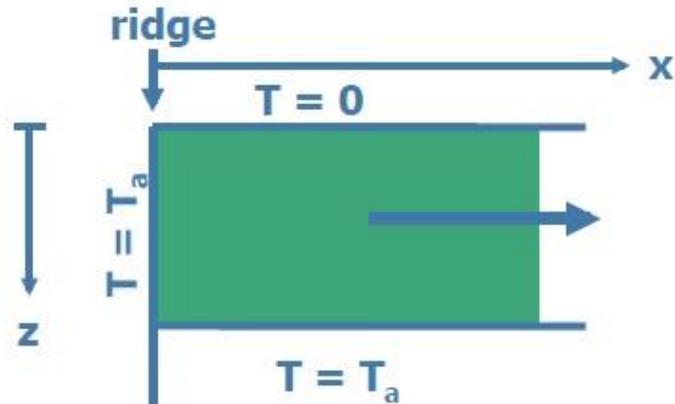


# Plate Cooling model vs Half-Space cooling model

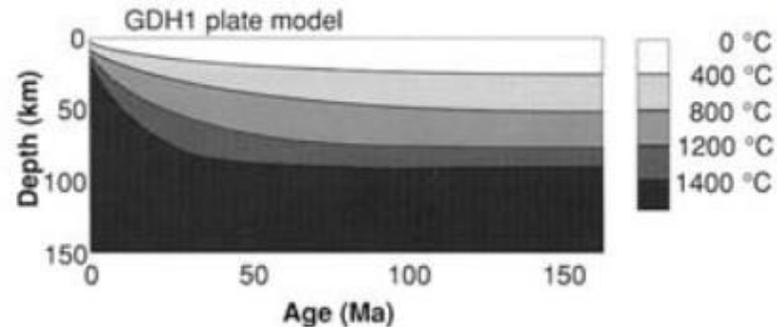
## The "plate" model

The lithosphere has a fixed thickness at the ridge and cools with time

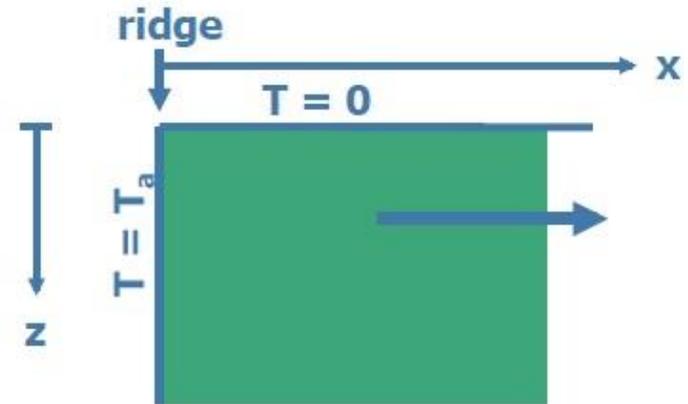
The asthenosphere below is constant temperature



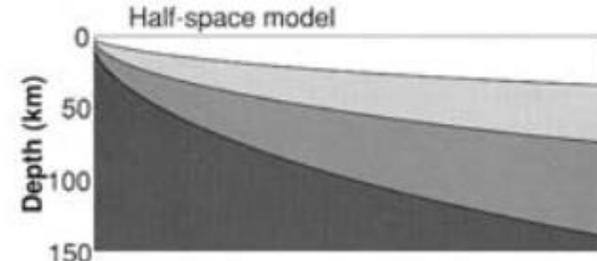
...asymptotic values of  $Q$ , depth etc.



## Simple half-space model

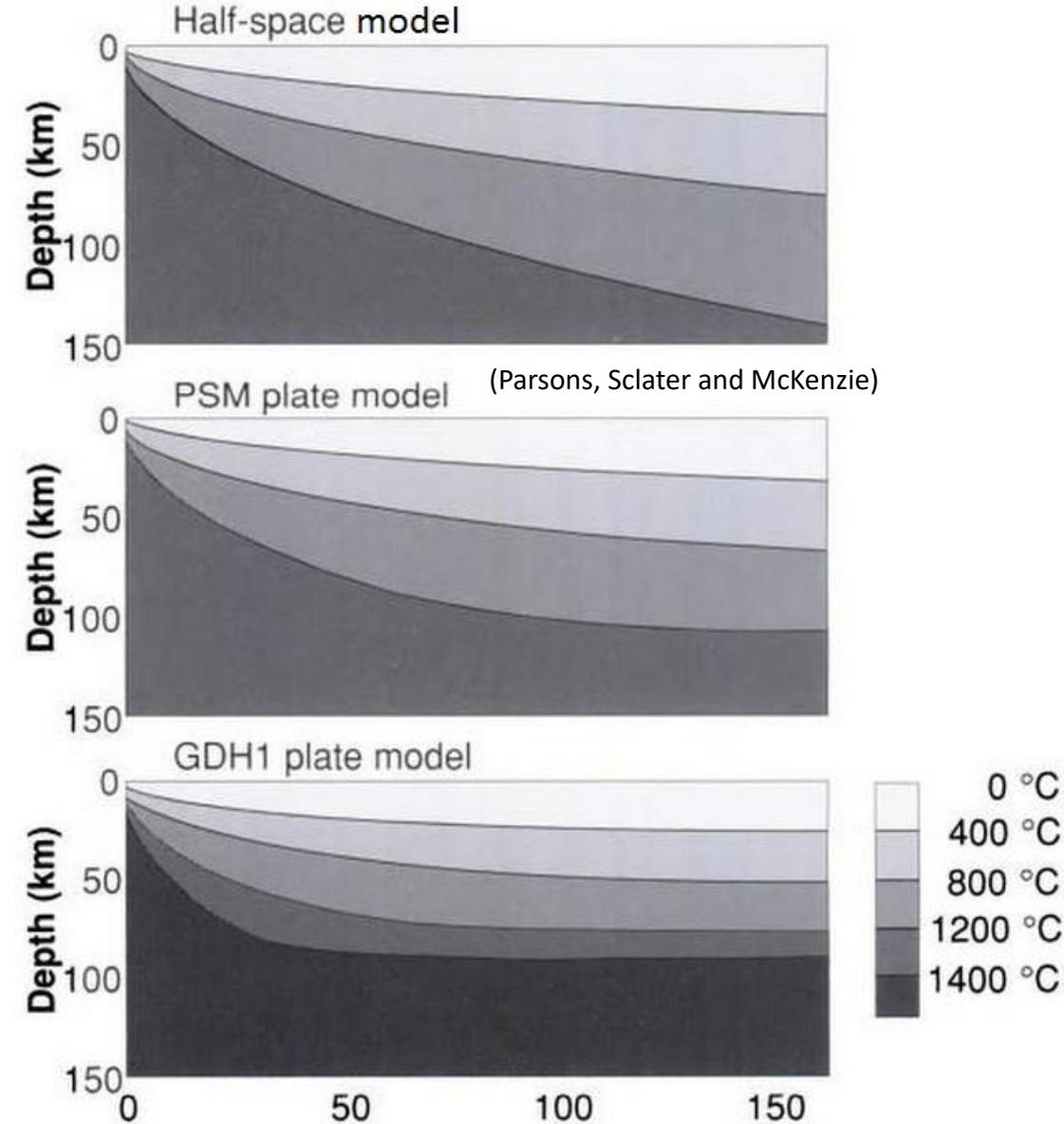


...cools and thickens for ever

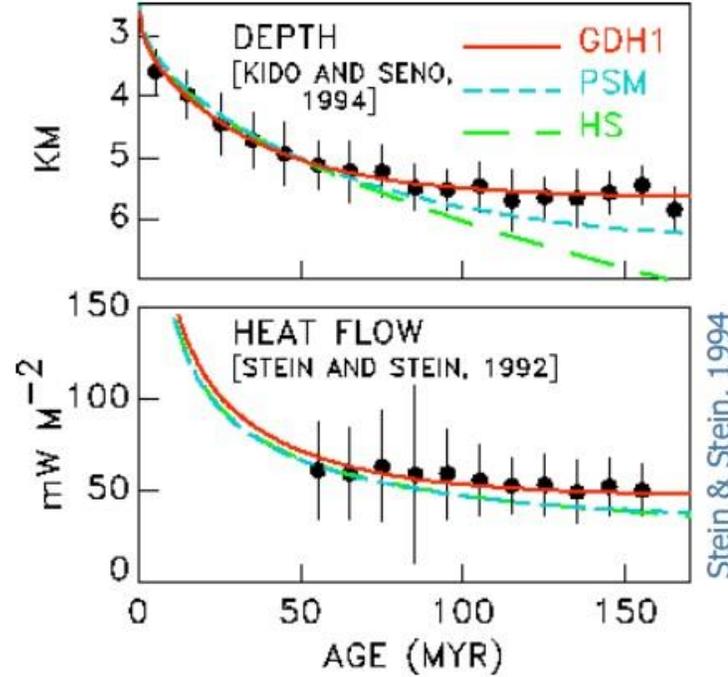


# Plate cooling model vs Half-Space cooling model

## Depth and heat flow – observations



Which model(s) fit the data?



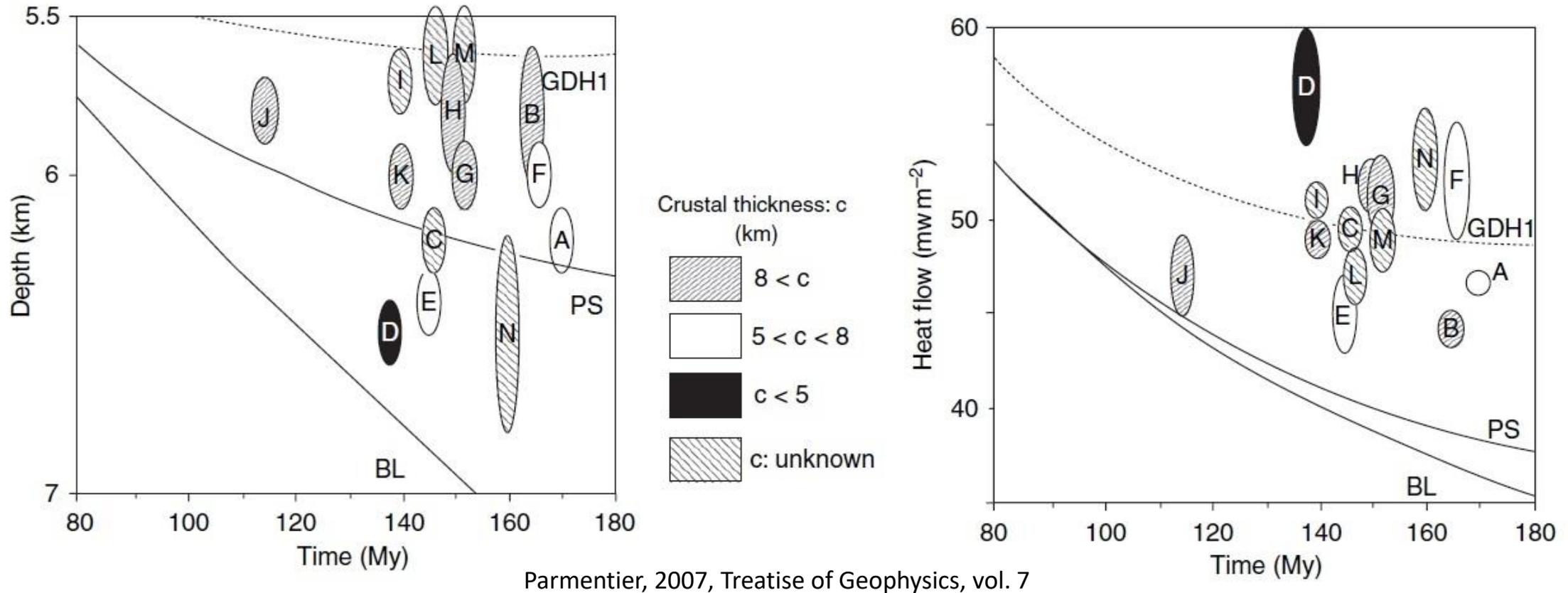
HS – Half-space model  
GDH1 – plate model  
PSM – plate model

The GDH1 “plate” model does a better job of fitting the depth data (which is better constrained)

All fit the heat flow data (within error)

- *HS* cooling model fits the ocean-depth datasets for young lithosphere better than plate cooling model.
- GDH1 has a thinner plate and higher temperatures than the other models

## Plate cooling model vs Half-Space cooling model



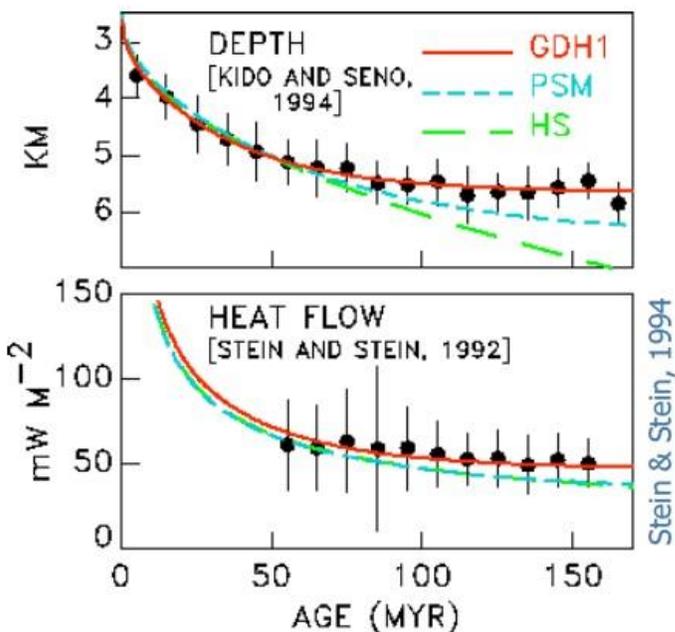
Seafloor basement depth, corrected for sediment load and crustal thickness and heat flow at sites in the western North Pacific (A–F) and northwestern Atlantic (G–N) as functions of age

- The plate model of parameters of Parsons and Sclater (1977) with a plate thickness 125 km and a mantle temperature 1350°C fits the depths well but underestimates the heat flow.
- In contrast, the hotter and thinner plate model of Stein and Stein (1992) with a plate thickness 95 km and a mantle temperature 1450°C fits the heat flow well but underestimates old seafloor depth.

# Plate cooling model vs Half-Space cooling model

## Depth and heat flow – observations

Which model(s) fit the data?



HS – Half-space model  
 GDH1 – plate model  
 PSM – plate model

The GDH1 “plate” model does a better job of fitting the depth data (which is better constrained)

All fit the heat flow data (within error)

Thermal parameters for oceanic-lithosphere models

		GDH1	PSM	HS
$L_r$	plate thickness (km)	$95 \pm 10$	$125 \pm 10$	—
$T_a$	temperature at base of plate ( $^{\circ}\text{C}$ )	$1450 \pm 100$	$1350 \pm 275$	$1365 \pm 10$
$\alpha_r$	coefficient of thermal expansion ( $^{\circ}\text{C}^{-1}$ )	$3.1 \times 10^{-5}$	$3.28 \times 10^{-5}$	$3.1 \times 10^{-5}$
$k_r$	thermal conductivity ( $\text{W m}^{-1}$ )	3.138	3.138	3.138
$c_p$	specific heat ( $\text{kJ kg}^{-1}$ )	1.171	1.171	1.171
$\kappa_r$	thermal diffusivity ( $\text{m}^2 \text{s}^{-1}$ )	$0.804 \times 10^{-6}$	$0.804 \times 10^{-6}$	$0.804 \times 10^{-6}$
$\rho_m$	mantle density ( $\text{kg m}^{-3}$ )	3330	3330	3330
$\rho_w$	water density ( $\text{kg m}^{-3}$ )	1000	1000	1000
$d_r$	ridge depth (km)	2.6	2.5	2.6

for  $\tau < 20$  Myr

$$d = 2.6 + 0.365t^{1/2}$$

for  $< 55$  Myr

$$Q = 510t^{-1/2}$$

for  $\tau > 20$  Myr

$$d = 5.65 - 2.47e^{-t/36}$$

for  $> 55$  Myr

$$Q = 48 + 96e^{-t/36}$$

## Oceanic cooling models

- If the same plate thickness is used for the flux and temperature boundary conditions, for fixed heat flux conditions the re-equilibration time is much longer than for fixed temperature conditions.

*Variation of depth and heat flow with age for oceanic-lithosphere models*

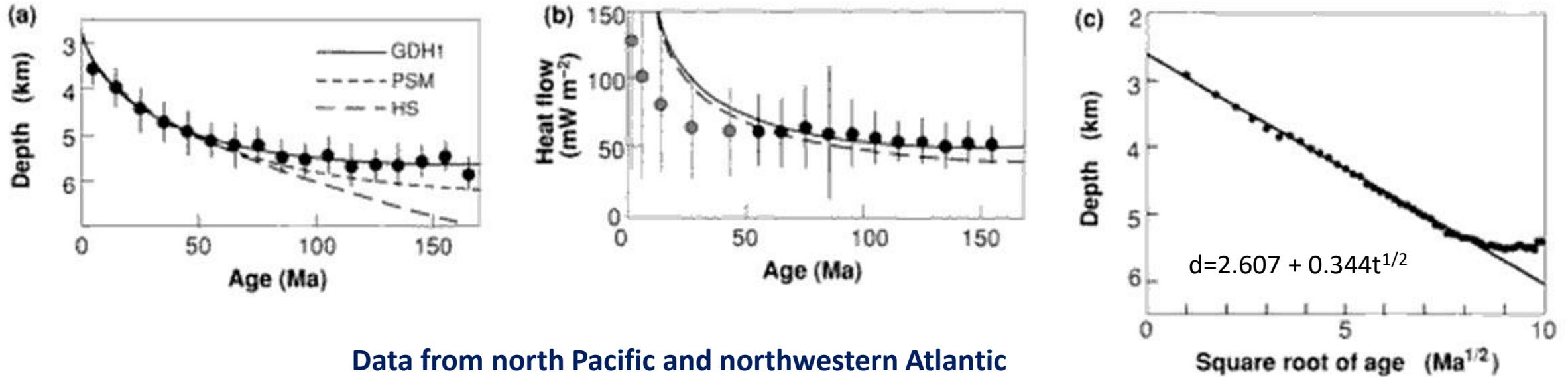
	Ocean depth (km)	Heat flow (mW m <sup>-2</sup> )
Half-space	$2.6 + 0.345t^{1/2}$	$480t^{-1/2}$ < 80Myr
PSM	$2.5 + 0.350t^{1/2}$ , $t < 70$ Ma	$473t^{-1/2}$ , $t < 120$ Ma
	$6.4 - 3.2e^{-t/62.8}$ , $t > 70$ Ma	$33.5 + 67e^{-t/62.8}$ , $t > 120$ Ma
GDH1	$2.6 + 0.365t^{1/2}$ , $t < 20$ Ma	$510t^{-1/2}$ , $t < 55$ Ma
	$5.65 - 2.47e^{-t/36}$ , $t > 20$ Ma	$49 + 96e^{-t/36}$ , $t > 55$ Ma

*Different estimates of the parameters of the oceanic cooling model*

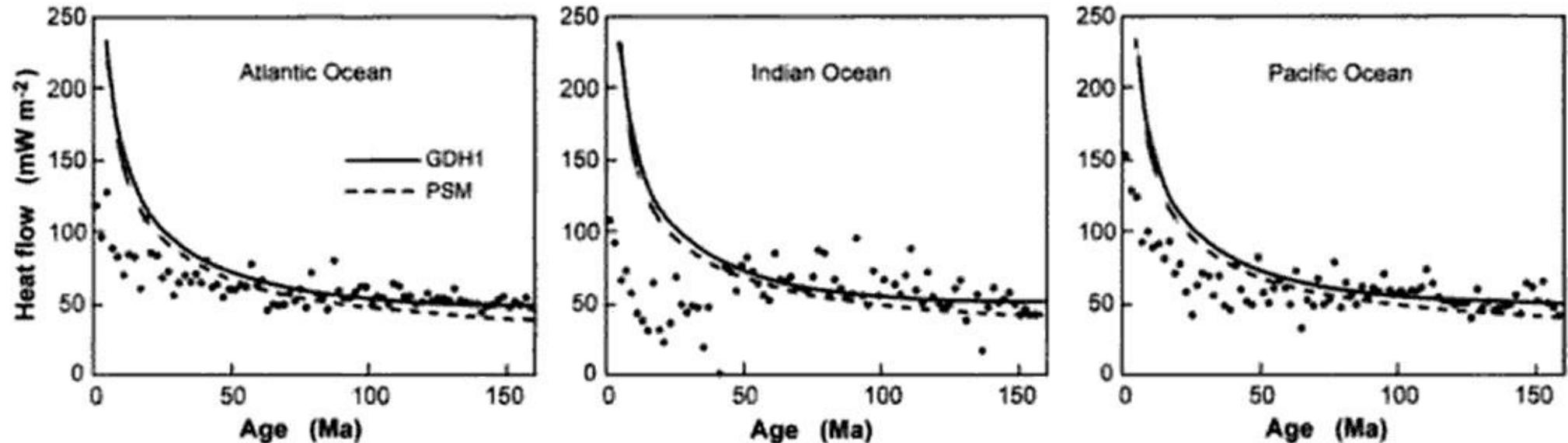
ΔT (°C)	a (km)	Method	Reference
1370	–	bathymetry (age < 80 My)	
		Half-space model	Johnson and Carlson (1992)
1333	125	Constant properties – fixed T	Parsons and Sclater (1977)
1450	95	Constant properties – fixed T	Stein and Stein (1992)
1350	118	T-dependent properties	
		fixed Q at variable depth†	Doin and Fleitout (1996)
1315	106	T-dependent properties – fixed T	McKenzie <i>et al.</i> (2005)

† In this model, heat flux is fixed at the base of the growing thermal boundary layer.

# Oceanic depth and heat flow data variations with age

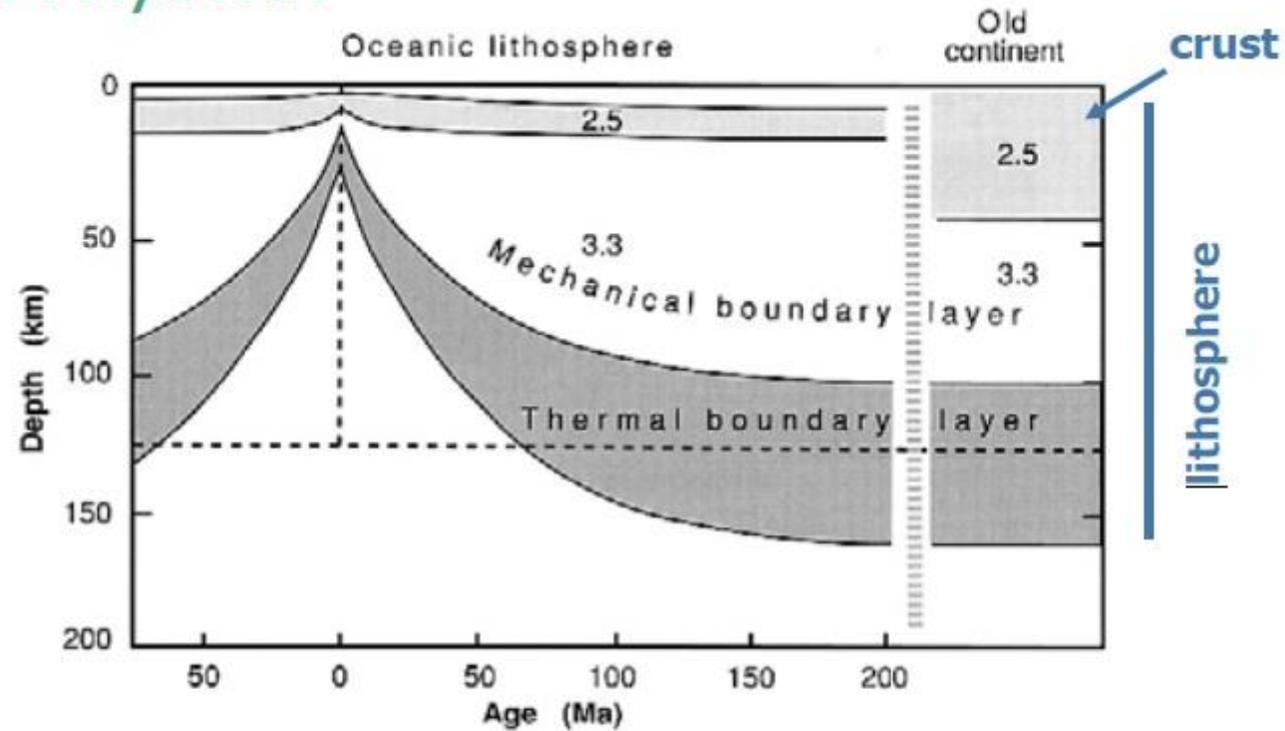


Data from north Pacific and northwestern Atlantic  
(in grey data < 50Myr affected by hydrothermal circulation)



# Plate cooling model vs Half-Space cooling model

A hybrid?



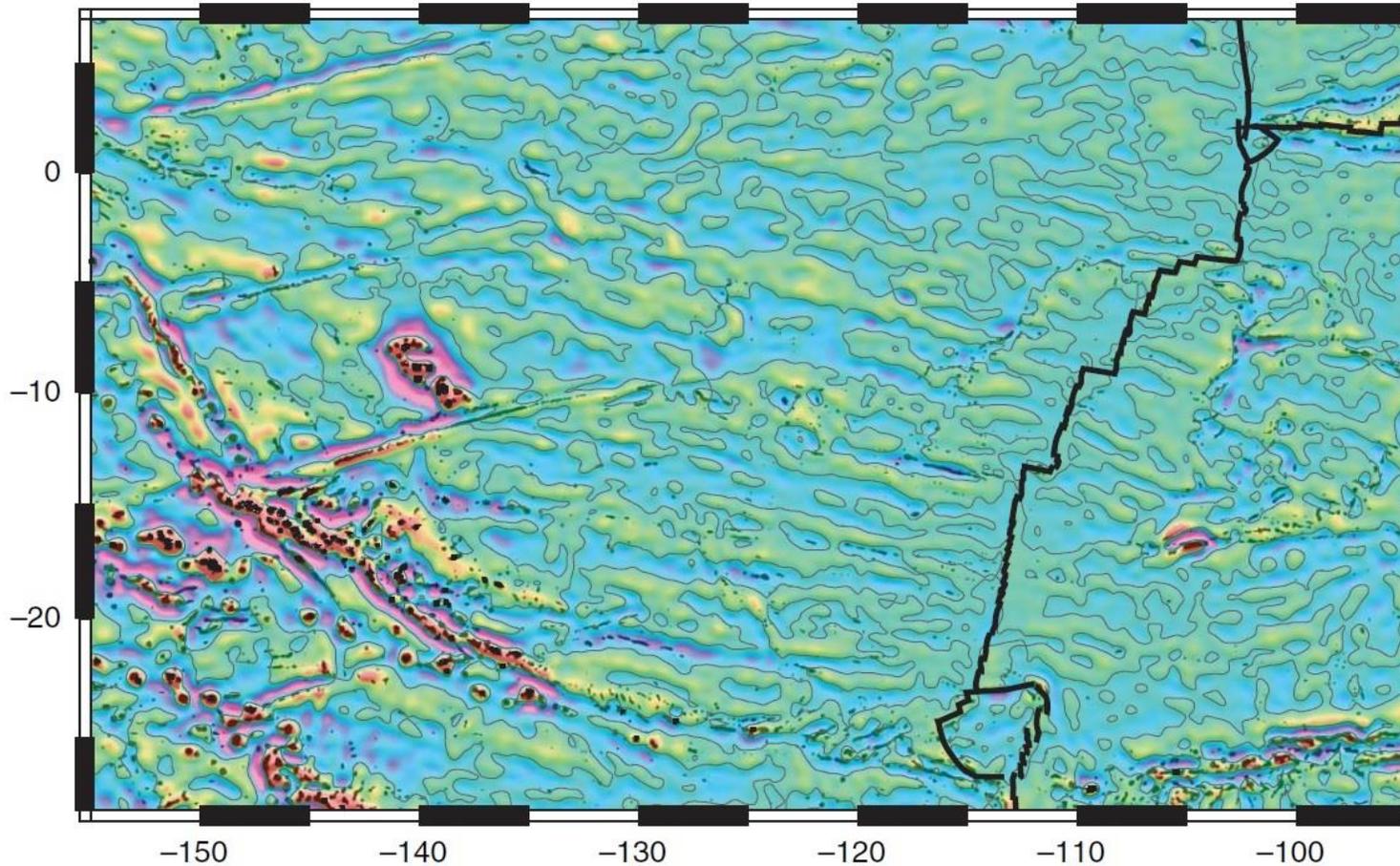
“Plate” model fits depth and Q best

but there is other geophysical evidence for a thickening lithosphere

- increasing elastic thickness
- increasing depth to low velocity asthenosphere

→ thermal boundary layer with small-scale convection

## Gravity lineations in the oceanic plates

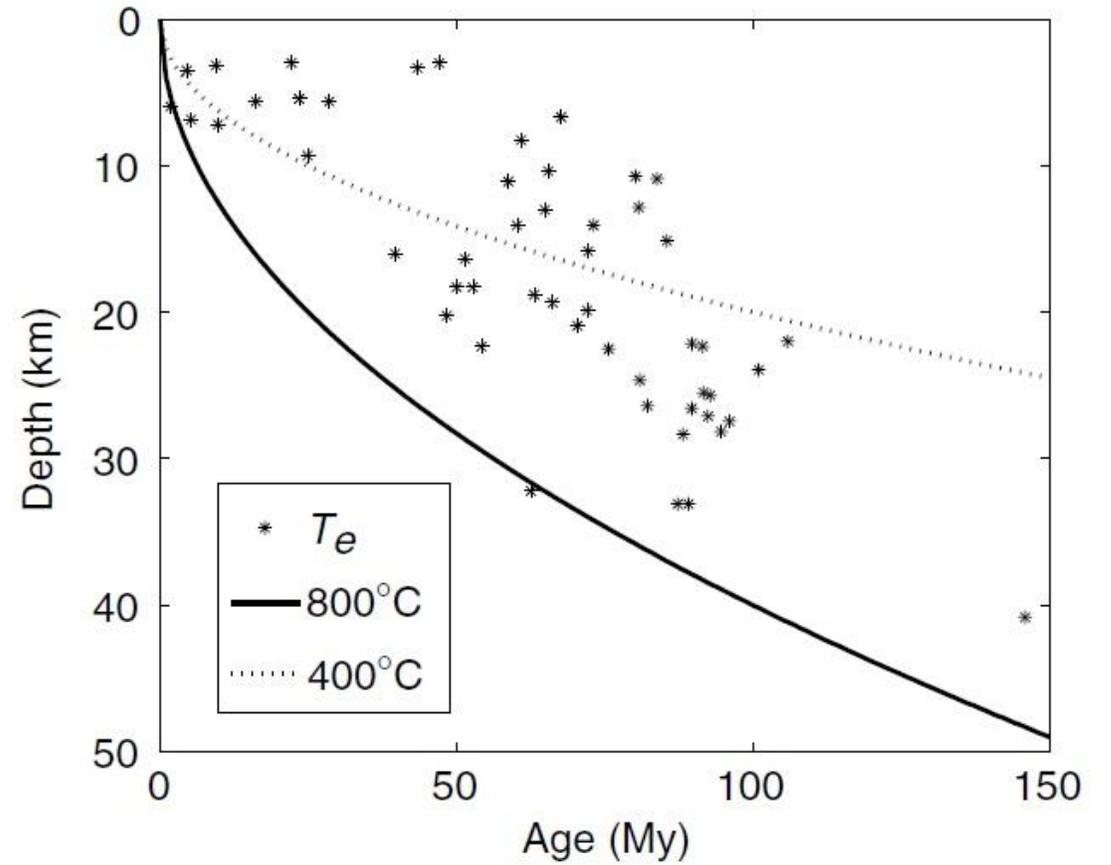
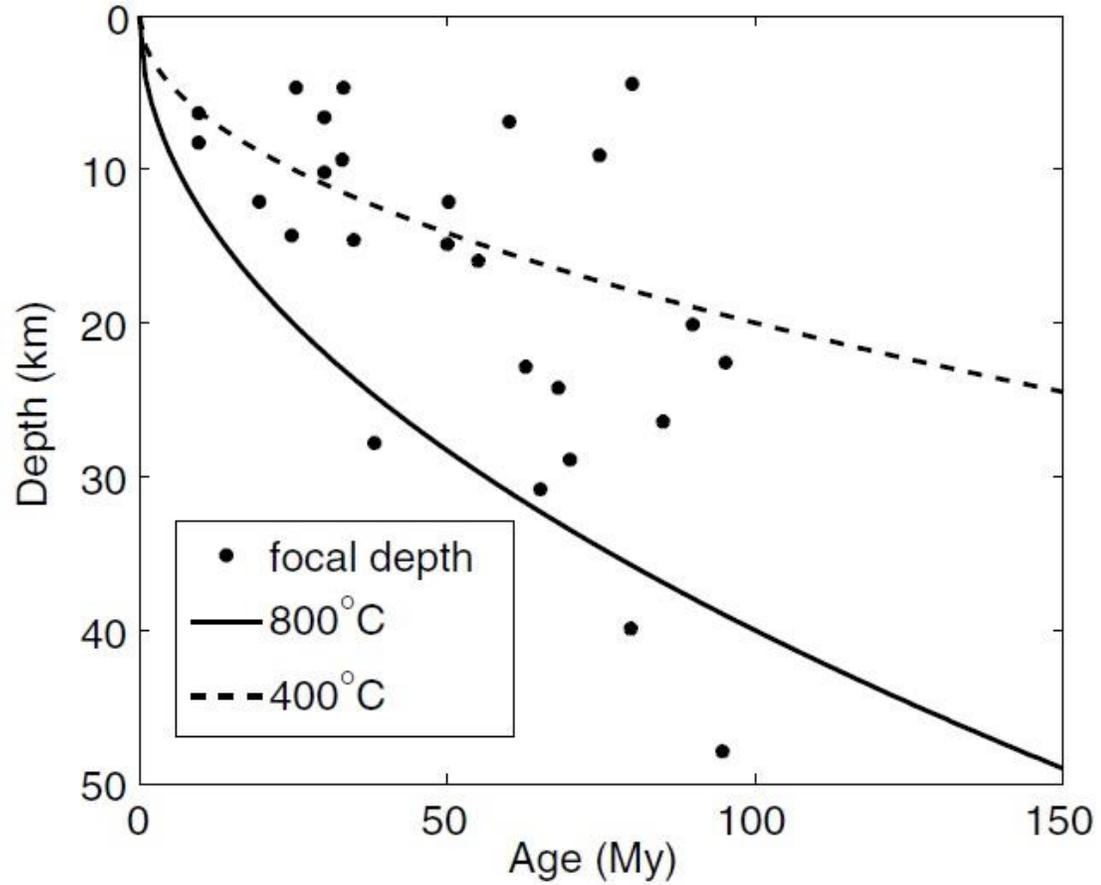


Gravity lineaments with 140 km wavelength develop between the ridge axis and 6 Myr and are oriented in the direction of absolute plate motion.

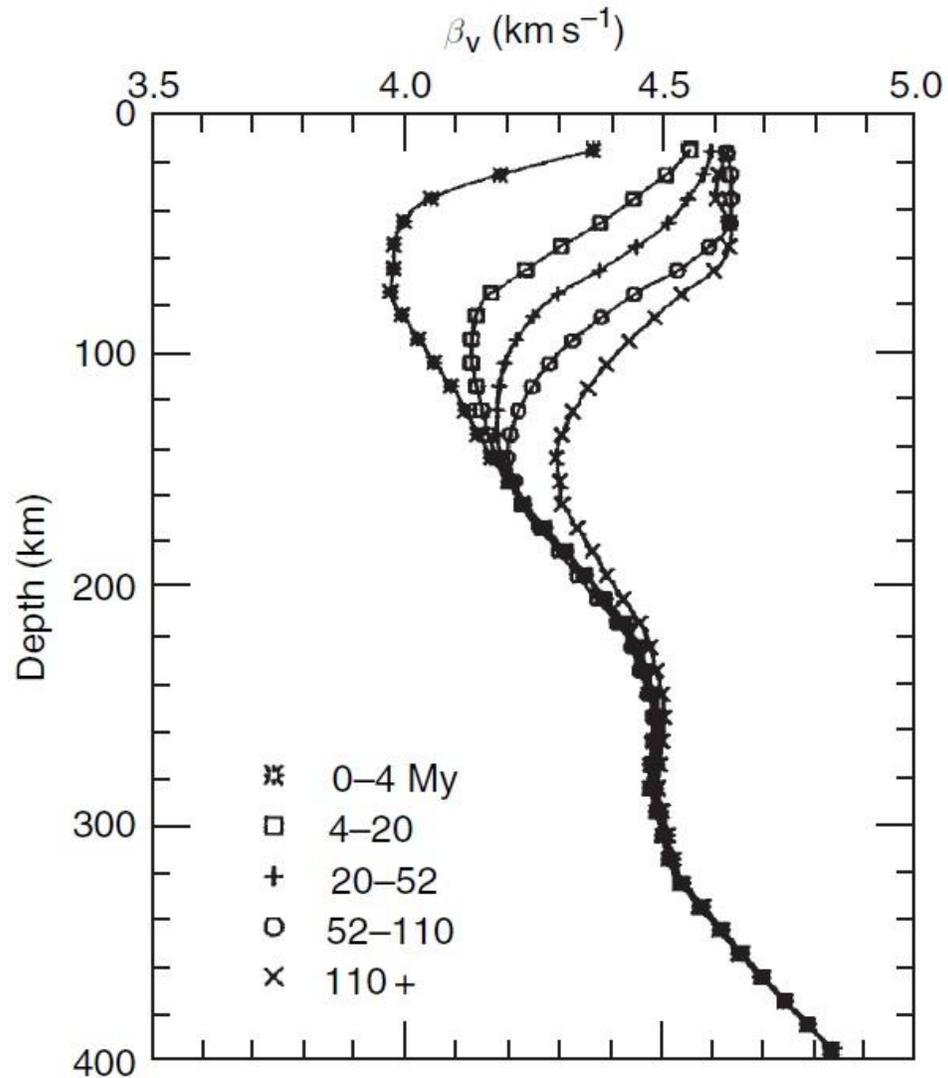
Parmentier, 2007, Treatise of Geophysics, vol. 7

- The presence of gravity lineations due to convective instability beneath the lithosphere of the Pacific and Nazca plates, only a few million years old, contrasts with the view that convective instability at ages 70 Myr explained the flattening of old seafloor.
- Formation of gravity lineations in young plates can be favored by their high speed: higher strain rates could create smaller grain size, resulting in in more rapid diffusion creep.

# Depth of Earthquakes in the Oceans vs Age of the Sea Floor (Isotherms for a HS cooling model)



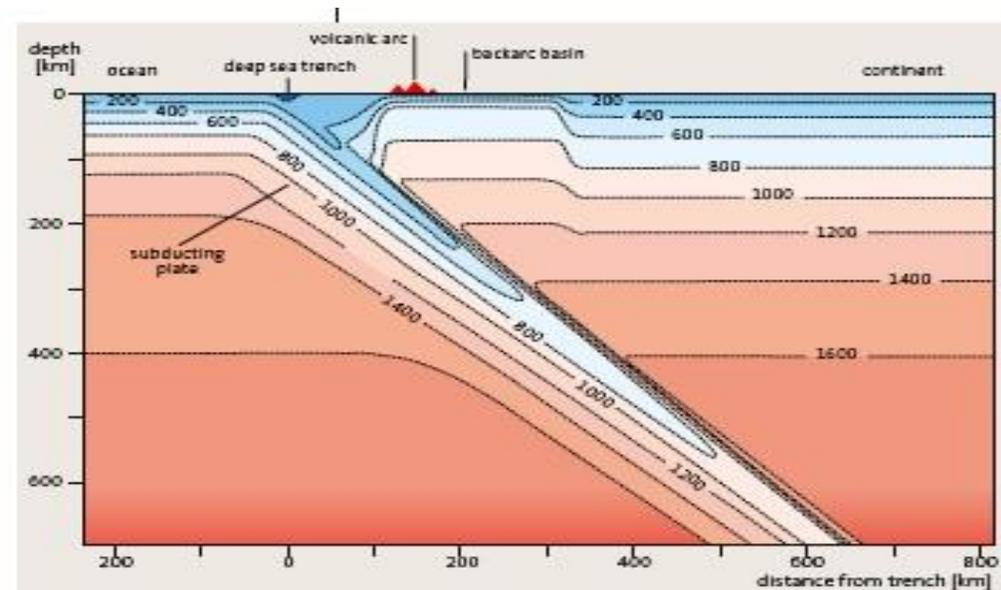
# Seismic velocity in the oceanic upper mantle



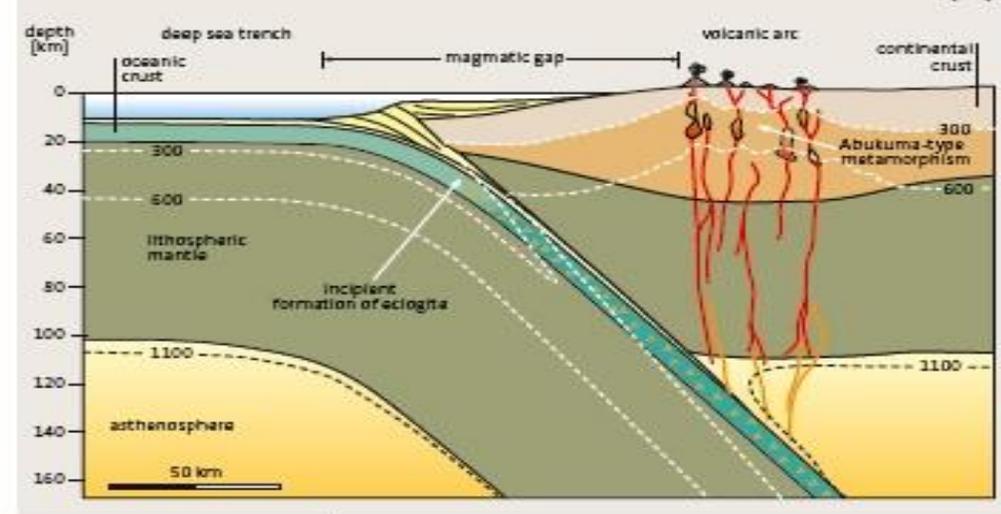
Vertically polarized shear-wave velocity in the Pacific as a function of depth in various age intervals determined from Rayleigh wave dispersion

- Seismic shear-wave velocities in the upper mantle generally decrease with depth at a given age and increase with age at a given depth.
- Seismic velocities, likely because of T, continue to change with age at depths exceeding 150 km, although the plate model implicitly assumes that mantle temperature at depths greater than the plate thickness do not change with age.

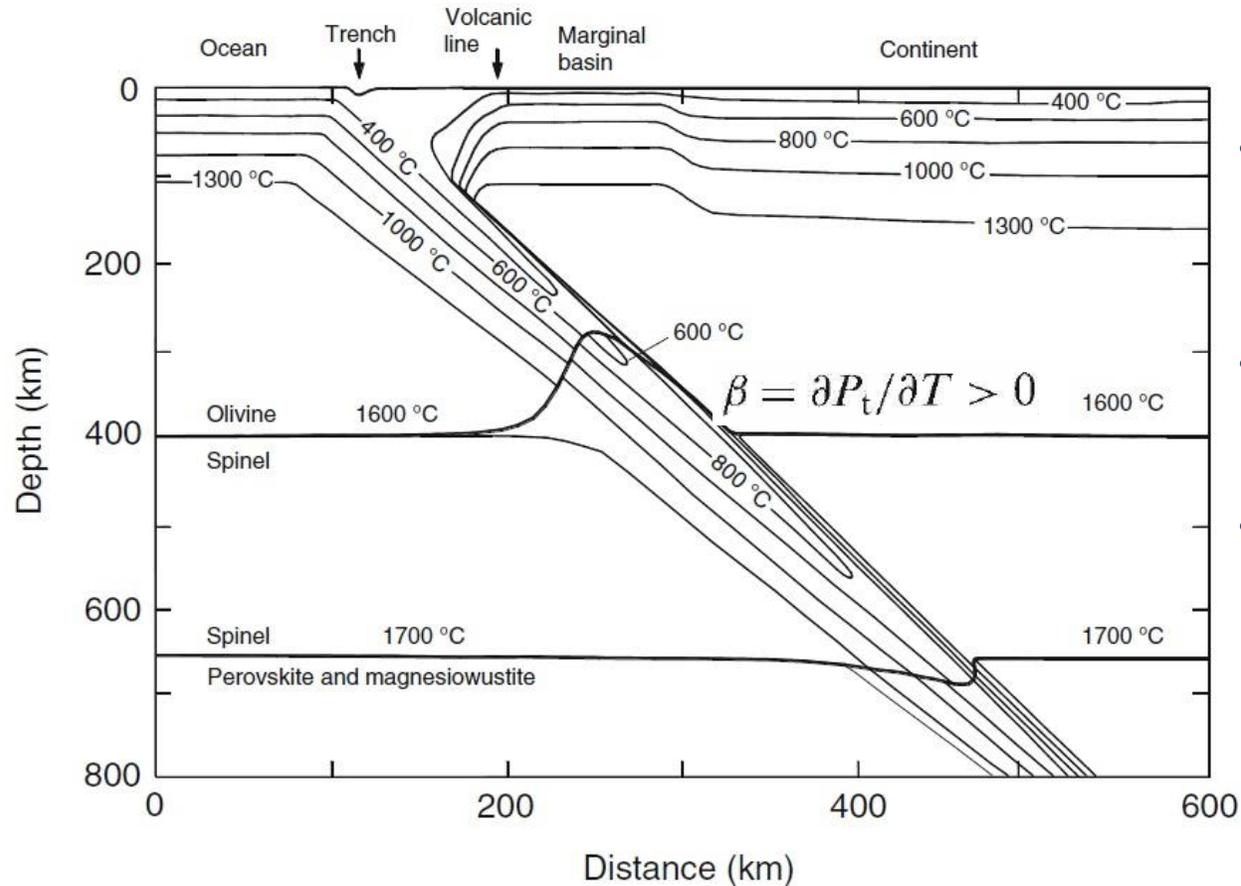
# Anomalous temperatures in oceanic realm: thermal conditions of a subduction zone system



- Rate of subduction.
- Age and thickness of descending slab.
- Frictional heating of the upper and lower slab surfaces.
- Conduction of heat into the slab from the asthenosphere.
- Adiabatic heating associated with slab compression.
- Heat derived from radioactive decay of minerals in the oceanic lithosphere.
- Latent heat associated with phase transitions of minerals (olivine-spinel transition, exothermic; spinel-oxide transitions, endothermic).



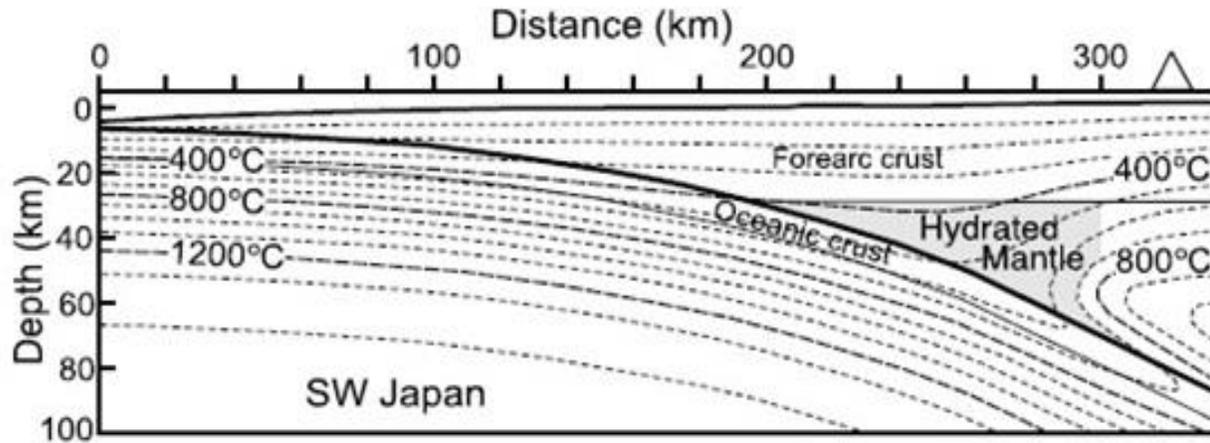
# Anomalous temperatures in oceanic realm: thermal conditions of a subduction zone system



- The transformation olivine-spinel (410 km) occurs at a lower pressure (shallower depths) in subducted lithosphere, where the temperature is lower than in surrounding mantle (Clapeyron slope,  $\beta$ , is positive =  $4\text{MPaK}^{-1}$ ).
- Therefore, it would be a region in and near the lithosphere within which the higher density phase existed at the same depth as the low-density phase in surrounding mantle.
- The transformation Spinel-Perovskite and magnesiowustite (660 km) occurs at a higher pressure (higher depths) in subducted lithosphere (Clapeyron slope is negative).

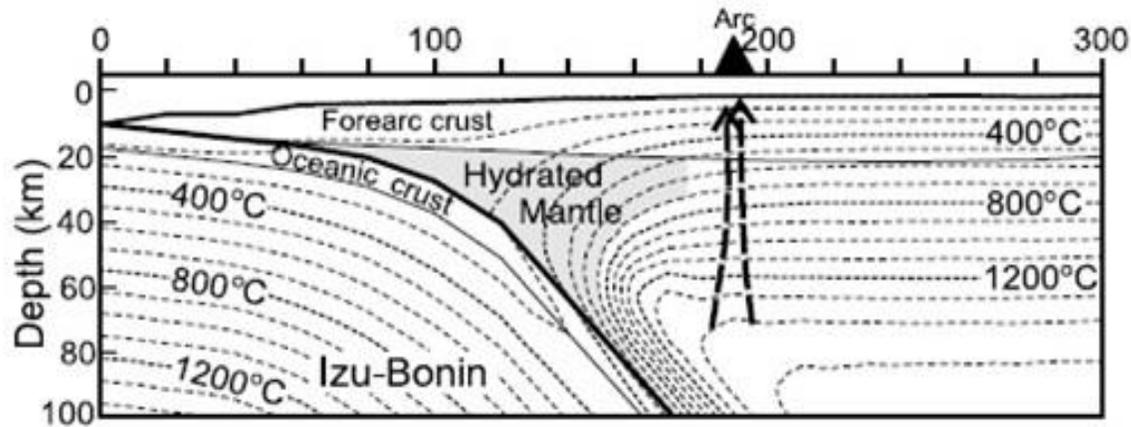
Claius-Clapeyron equation describes the variation of  $P$  with respect to  $T$  along the equilibrium curve between two phases of the same material

# Anomalous temperatures in oceanic realm: thermal conditions of a subduction zone system



## Warm continental subduction zone:

- Calculated forearc temperatures are 400-600°C for warm continental subduction zones.
- Less extended zone of serpentized forearc mantle, shallow metamorphic transformation from blueschist facies to eclogite (50 km).



## Cold oceanic subduction zone:

- In cool continental subduction zone forearcs the calculated temperatures in the uppermost forearc mantle are 150-250°C.
- More extended zone of serpentized forearc mantle, deep metamorphic transformation from blueschist facies to eclogite (100 km).

# Surface temperature (off-shore)

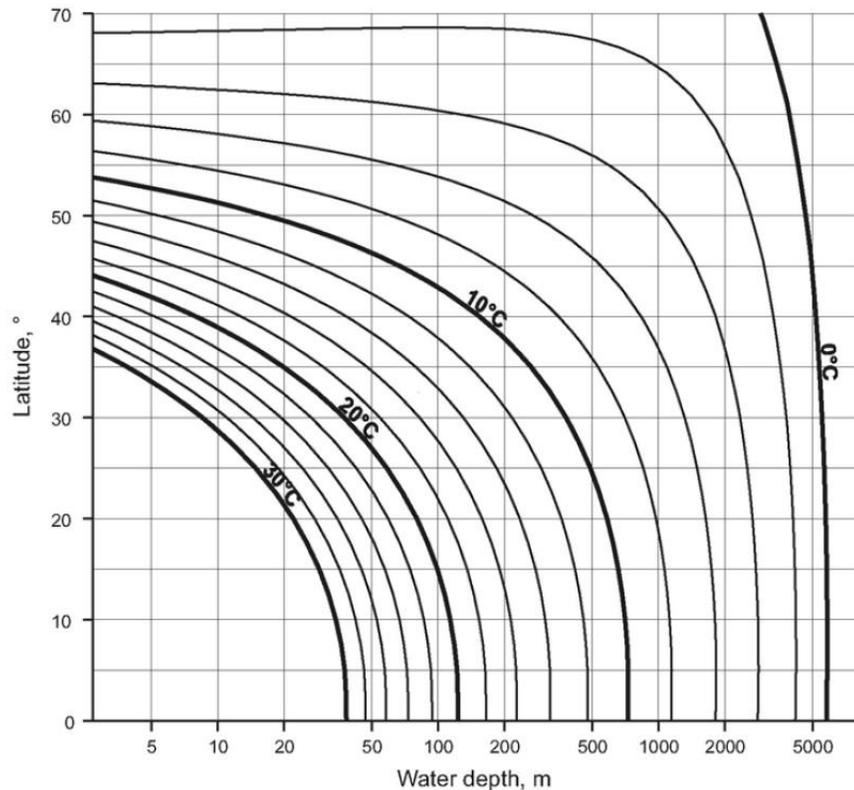
- Bottom-water temperature (surface temperature for off-shore thermal profiles) or BWT have to be used as the top boundary of the conductive heat flow models.
- BWT are related to latitude ( $L$ ) and water depth ( $z$ ):

$$\ln(T_{sf}) = A + B \times \ln(z) \quad A = 4.63 + 8.84 \times 10^{-4}L - 7.24 \times 10^{-4}L^2 \quad B = -0.32 + 1.04 \times 10^{-4}L + 7.08 \times 10^{-5}L^2$$

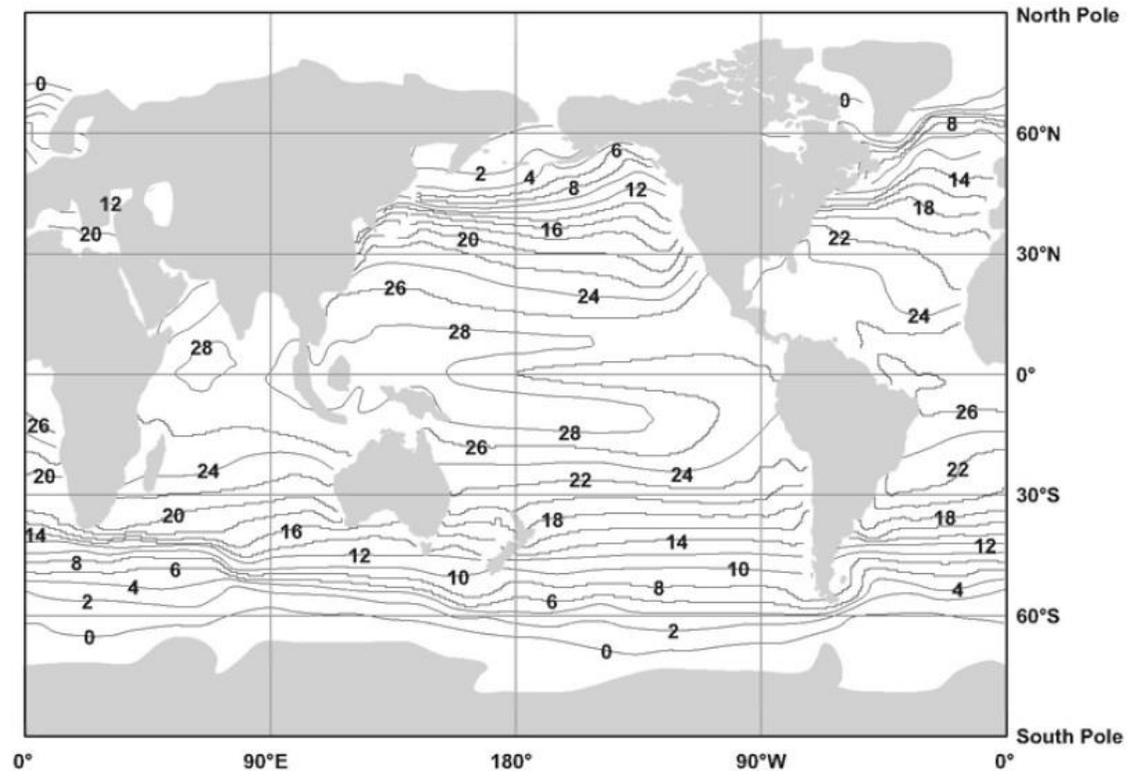
The equation works better if  $T_{sf}$  is expressed as degrees above the freezing point ( $T_f$ ):

$$T_{sf} = \text{BWT} - T_f \quad T_f \approx -1.90 - 7.64 \times 10^{-4}z \text{ } ^\circ\text{C}$$

**BWT ( $^\circ\text{C}$ )**



**Average sea surface temperature model ( $^\circ\text{C}$ )**



# References

## Main Readings:

### Books:

- Jaupart and Mareshal, 2011, Heat Generation and Transport in the Earth, Chapter 6: Thermal structure of the oceanic lithosphere, 146-175.
- Beardsmore and Cull, 2001: Crustal Heat Flow, Chapter 3, Thermal Gradient, 47-89.
- Davies, 1999, Chapter 7, Heat, Dynamic Earth Plates, Plumes and Mantle Convection, Cambridge and University Press.

### Articles:

- Parmentier, 2007, The Dynamics and Convective Evolution of the Upper Mantle, 305-323.

### Further Readings:

- Hyndman and Peacock, 2013, Serpentinization of the forearc mantle EPSL, 212, 417-432.