

# The structure and evolution of stars

## Lecture 6: Nuclear reactions in stellar interiors



Fred Hoyle

# Learning Outcomes

The student will learn

- The main nuclear processes in stellar interiors
- The relative importance of each
- The temperature dependence of the the processes and likely sites of occurrence
- The relation to the cosmic abundance pattern

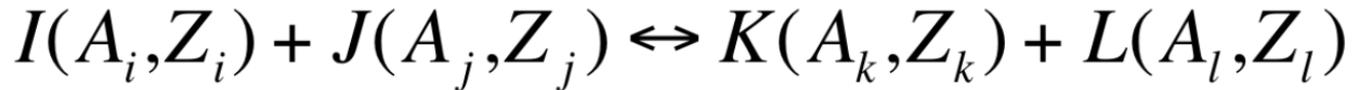
# Introduction

We have seen that the 4 equations of stellar structure must be supplemented with expressions for  $P$ ,  $\varepsilon$ ,  $\kappa$

- $\varepsilon$  will be defined by the nuclear energy source in the interiors. We need to develop a theory and understanding of nuclear physics and reactions
- $\kappa$  will be determined by the atomic physics of the stellar material
- $P$  will be given by the equation of state of the stellar matter
- This lecture will explore the physics of the nuclear energy reactions

# The binding energy of the atomic nucleus

The general description of a nuclear reaction is



Where  $A_i$  = *the baryon number, or nucleon number (nuclear mass)*

And  $Z_i$  = *the nuclear charge*

The nucleus of any element is uniquely defined by the two integers  $A_i$  and  $Z_i$

Recall that in any nuclear reaction the following must be conserved:

1. The baryon number – protons, neutrons and their anti-particles
2. The lepton number – light particles, electrons, positrons, neutrinos, and anti-neutrinos.
3. Charge

Note also that the anti-particles have the opposite baryon/lepton number to their particles.

The total mass of a nucleus is known to be less than the mass of the constituent nucleons. Hence there is a decrease in mass if a companion nucleus is formed from nucleons, and from the Einstein mass-energy relation  $E=mc^2$  the mass deficit is released as energy. This difference is known as the *binding energy* of the compound nucleus. Thus if a nucleus is composed of  $Z$  protons and  $N$  neutrons, it's binding energy is

$$Q(Z,N) \equiv [Zm_p + Nm_n - m(Z,N)]c^2$$

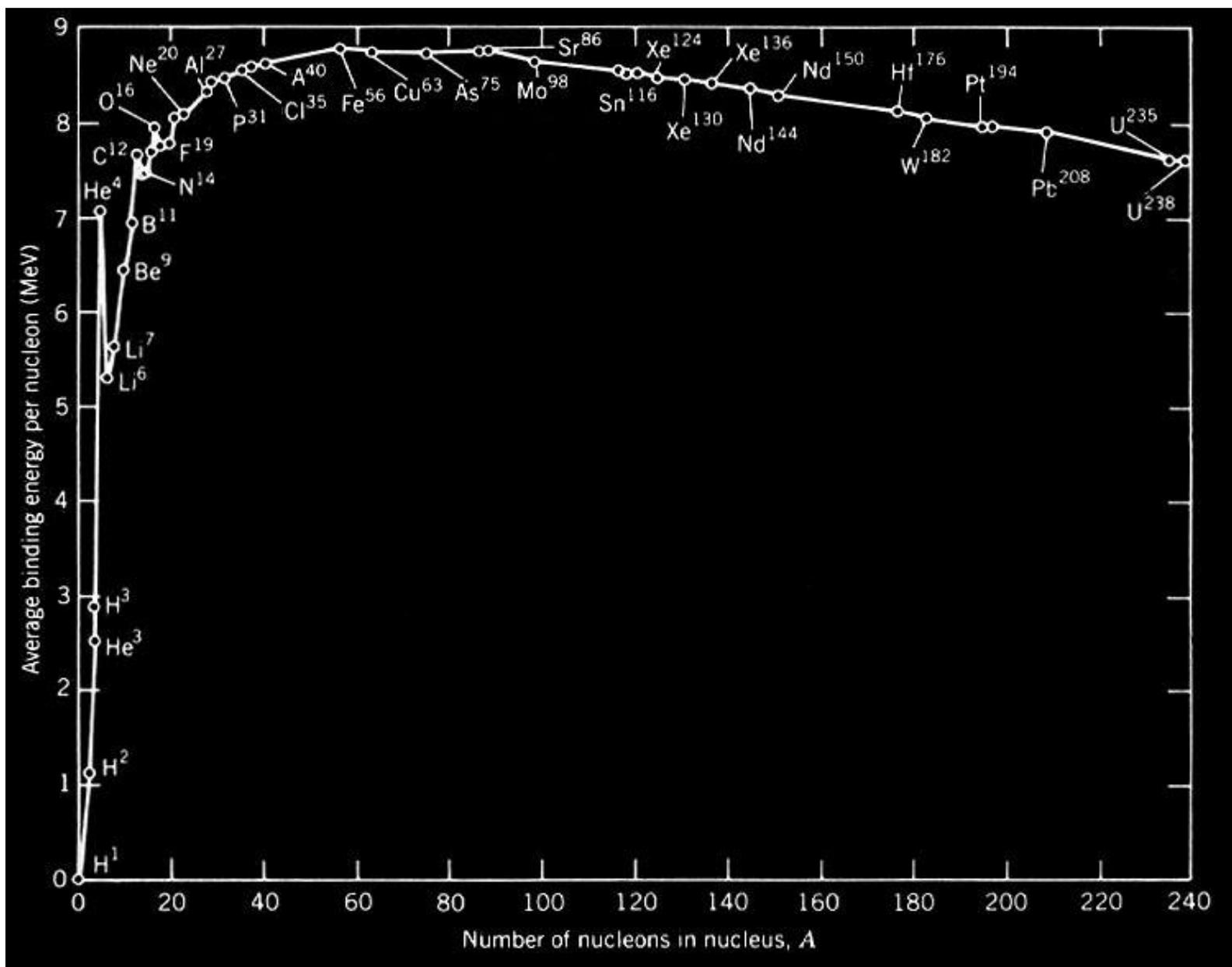
For our purposes, a more significant quantity is the total *binding energy per nucleon*. And we can then consider this number relative to the hydrogen nucleus

$$\frac{Q(Z,N)}{A}$$

# The binding energy per nucleon

The variation of binding energy per nucleon with baryon number  $A$

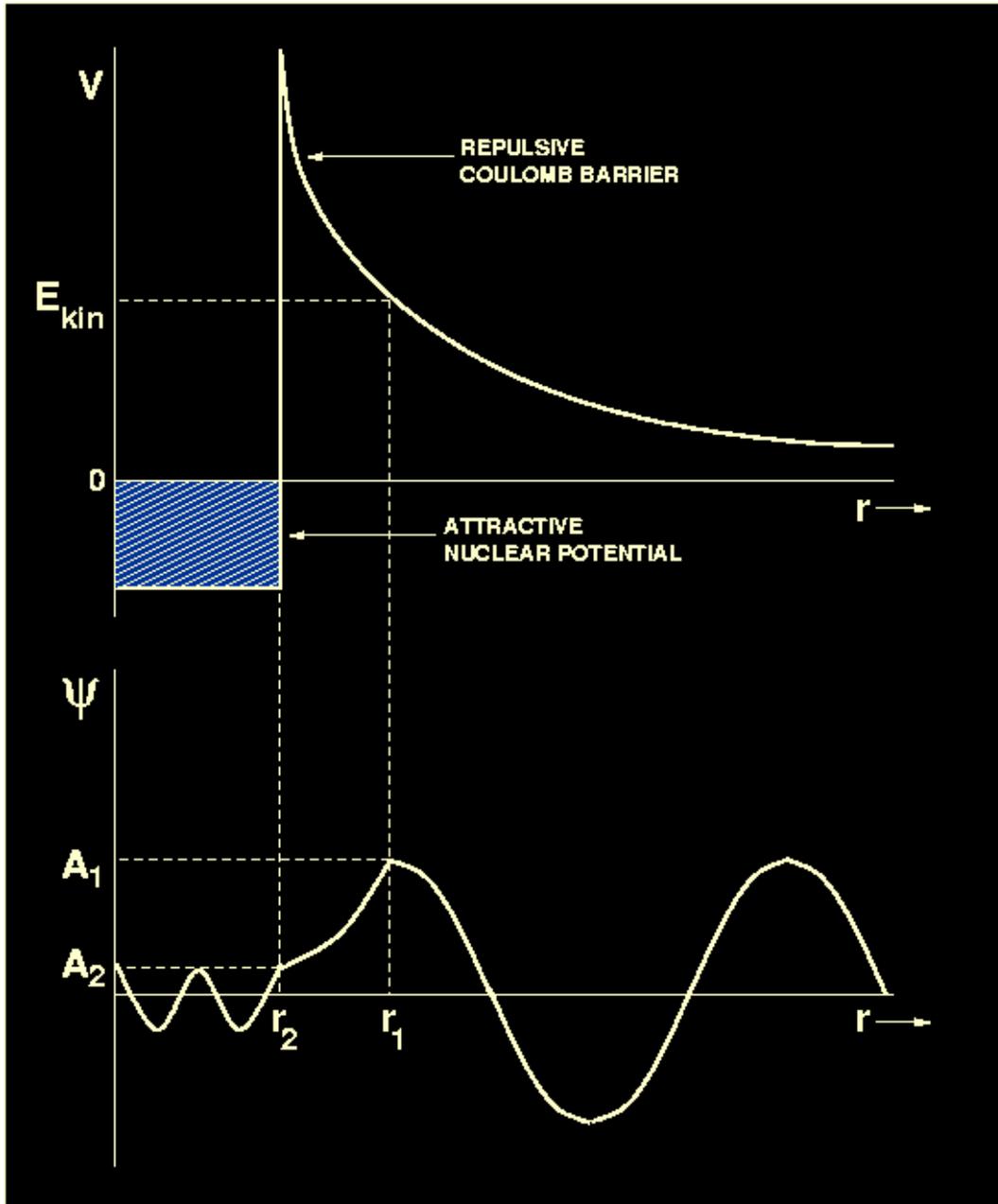
- General trend is an increase of  $Q$  with atomic mass up to  $A= 56$  (Fe). Then slow monotonic decline
- There is steep rise from H through  $^2\text{H}$ ,  $^3\text{He}$ , to  $^4\text{He}$   
→ fusion of H to He should release larger amount of energy *per unit mass* than say fusion of He to C
- Energy may be gained by *fusion* of light elements to heavier, up to iron
- Or from *fission* of heavy nuclei into lighter ones down to iron.



# Occurrence of fusion reactions

Now will discuss the conditions under which fusion can occur – and whether such conditions exist in stellar interiors

- Nuclei interact through four forces of physics – only electromagnetic and strong nuclear important here
- Two positively charged nuclei must overcome coulomb barrier (long range force  $\propto 1/r^2$ ), to reach separation distances where strong force dominates ( $10^{-15}$  m, typical size of nucleus)



## Schematic plots

- $V$  (potential energy) vs nuclei separation distance
- Wave function representing penetration of a potential barrier by nucleus with kinetic energy of approach  $E_{kin}$  (below barrier height).

To fuse nuclei, must surmount the coulomb barrier. Height of barrier is estimated by:

$$\frac{z_1 z_2 e^2}{4\pi\epsilon_0 r}$$

$z_1, z_2$  = number of protons in each nuclei

$e$  = charge on electron =  $1.6 \times 10^{-19}$  C

$\epsilon_0$  = permittivity of free space =  $8.85 \times 10^{-12}$  C<sup>2</sup>N<sup>-1</sup> m<sup>-2</sup>

Class task:

Calculate the potential energy required for fusion of 2 H nuclei (set  $r = 10^{-15}$  m )

Compare this to the average kinetic energy of a particle ( $3kT/2$  : Lecture 4)

What  $T$  do we require for fusion ?

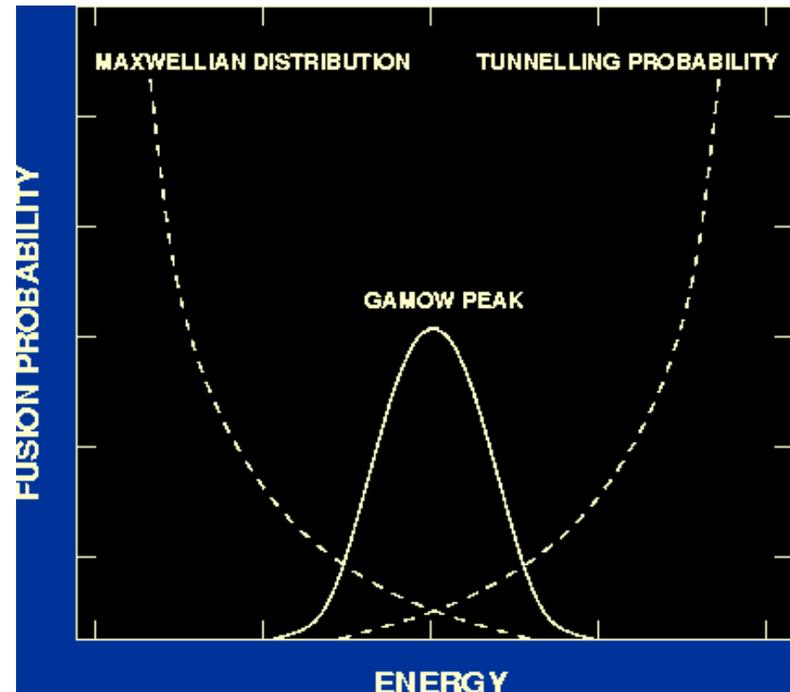
How does this compare with the minimum mean  $T$  of sun (Lecture 3) ?

Any comments on these two temperatures ?

# The Gamow peak

Schematically this is plotted, and the fusion most likely occurs in the energy window defined as the Gamow Peak.

The Gamow peak is the product of the Maxwellian distribution and tunnelling probability. The area under the Gamow peak determines the reaction rate.



The higher the electric charges of the interacting nuclei, the greater the repulsive force, hence the higher the  $E_{\text{kin}}$  and  $T$  before reactions occur.

Highly charged nuclei are obviously the more massive, so reactions between light elements occur at lower  $T$  than reactions between heavy elements.

# Quantum Tunnelling

As derived in your Quantum Mechanics courses, there is a finite probability for a particle to penetrate the Coulomb barrier as if “tunnel” existed.

Quantum effect discovered by George Gamow (1928) in connection with radioactivity.

Penetration probability (calculated by Gamow) is given as:

- tunneling probability:  $P = \exp(-2\pi\eta)$

$$\text{with } \eta = \frac{Z_x Z_y e^2}{\hbar v}$$

$$\Rightarrow 2\pi\eta = 31.29 \cdot Z_x Z_y \left(\frac{A}{E}\right)^{\frac{1}{2}}$$

where  $A$  is the reduced mass in units of  $m_u$  and  $E$  is in keV

Hence this increases with  $v$  (particle velocity), but we know  $v$  will be Maxwellian distribution for ideal gas. Hence fusion probability is product

$$\text{prob}(fusion) \propto e^{\frac{-\pi Z_1 Z_2 e^2}{\varepsilon_0 \hbar v}} e^{-\frac{mv^2}{2kT}}$$

# The Gamow peak

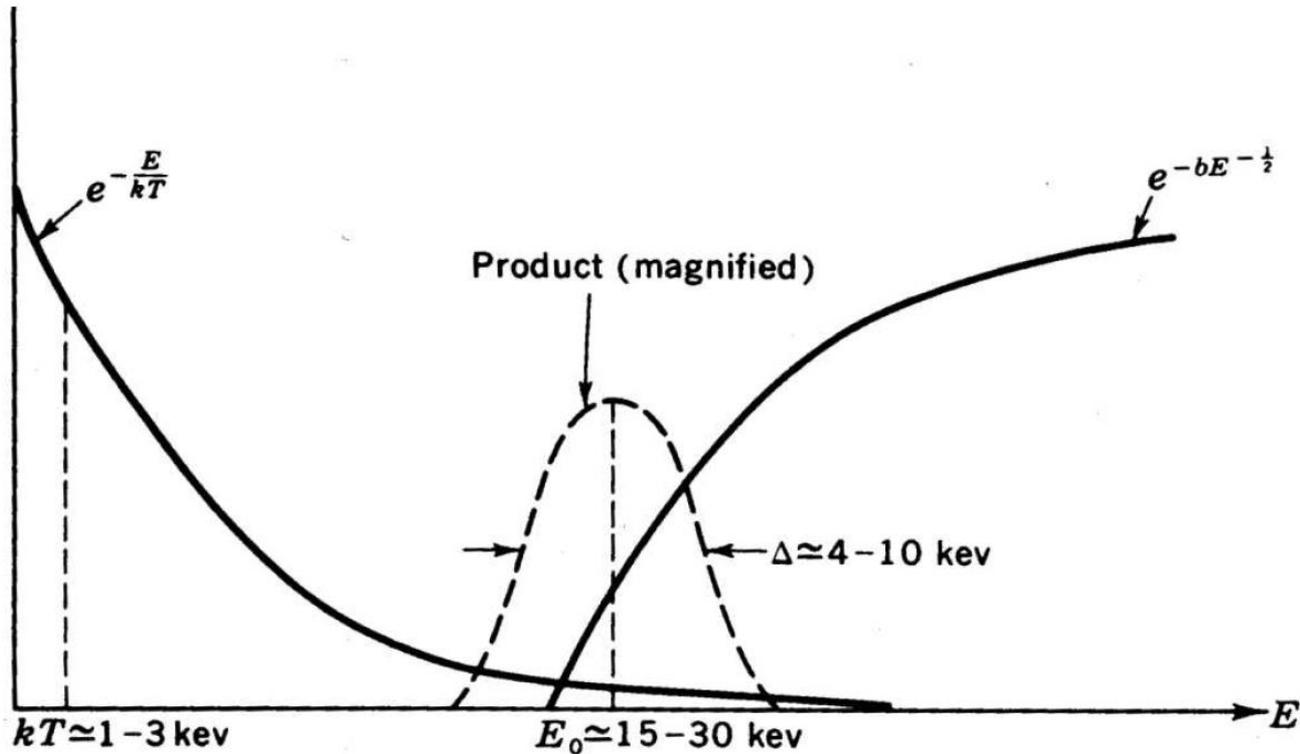


Figure 2.4: Dominant energy dependent factors in thermonuclear reactions. Most reactions occur in the overlap between the high- $E$  tail of the Maxwell-Boltzmann distribution, giving a factor  $\exp(-E/kT)$ , and the probability of tunneling through the Coulomb barrier, giving a factor  $\exp(-b/\sqrt{E})$ . Their product gives a fairly sharp peak called the **Gamow peak** at an energy  $E_0$  which is generally much larger than  $kT$ . Figure from CLAYTON.

## C. Nuclear reaction rate

Definition: nuclear reaction rate  $r_{xy} := N_x \cdot N_y \cdot v \cdot \sigma(v)$

with:  $N_x, N_y$  number density of particles  $x, y$  (i.e., particles per  $\text{cm}^3$ )

$v$  relative velocity between  $x$  and  $y$

$\sigma(v)$  cross section

$[r] = \text{reactions per cm}^3 \text{ per s} = \text{cm}^{-3} \text{ s}^{-1}$

- in stellar gas: Maxwell-Boltzmann distribution of velocities  $\Phi(v)$

$$\Rightarrow r_{xy} = N_x N_y \langle \sigma v \rangle$$

$$\text{with } \langle \sigma v \rangle := \int_0^\infty \Phi(v) v \sigma(v) dv$$

$$\Phi(v) = 4\pi v^2 \left( \frac{m}{2\pi kT} \right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) = f(T)$$

$$\text{with } m = \frac{m_x m_y}{m_x + m_y} = \text{“reduced mass”}$$

$$E = \frac{1}{2} m v^2$$

$$\Rightarrow \langle \sigma v \rangle = \left( \frac{8}{\pi m} \right)^{\frac{1}{2}} \frac{1}{(kT)^{\frac{3}{2}}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{kT}\right) dE$$

# Astrophysical S Factor

Cross section (**B**)  $\Rightarrow \sigma(E) \sim \pi\lambda^2 \sim 1/E$

Tunnel effect (**D**)  $\Rightarrow \sigma(E) \sim \exp(-2\pi\eta), \eta \sim 1/\sqrt{E}$

$\Rightarrow$  define  $S(E)$  such that

$$\sigma(E) = \frac{1}{E} \exp(-2\pi\eta) \cdot S(E)$$

$$\Rightarrow \langle \sigma v \rangle = \left( \frac{8}{\pi m} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp \left[ -\frac{E}{kT} - \frac{b}{\sqrt{E}} \right] dE$$

$$\begin{aligned} \text{with } b &:= (2m)^{1/2} \pi e^2 Z_x Z_y / \hbar \\ &= 0.989 Z_x Z_y A^{1/2} \quad [(\text{MeV})^{1/2}] \end{aligned}$$

( $b^2$  = “Gamow energy”)

• often:  $S(E)$  varies slowly with  $E$

$\rightarrow$  **Gamow-peak** at energy  $E_0 > kT$

$\rightarrow$  for narrow  $T$ -range:  $S(E) \simeq S(E_0) = \text{const.}$

$$\Rightarrow \langle \sigma v \rangle = \left( \frac{8}{\pi m} \right)^{1/2} \frac{1}{(kT)^{3/2}} S(E_0) \int_0^\infty \exp \left( -\frac{E}{kT} - \frac{b}{\sqrt{E}} \right) dE$$

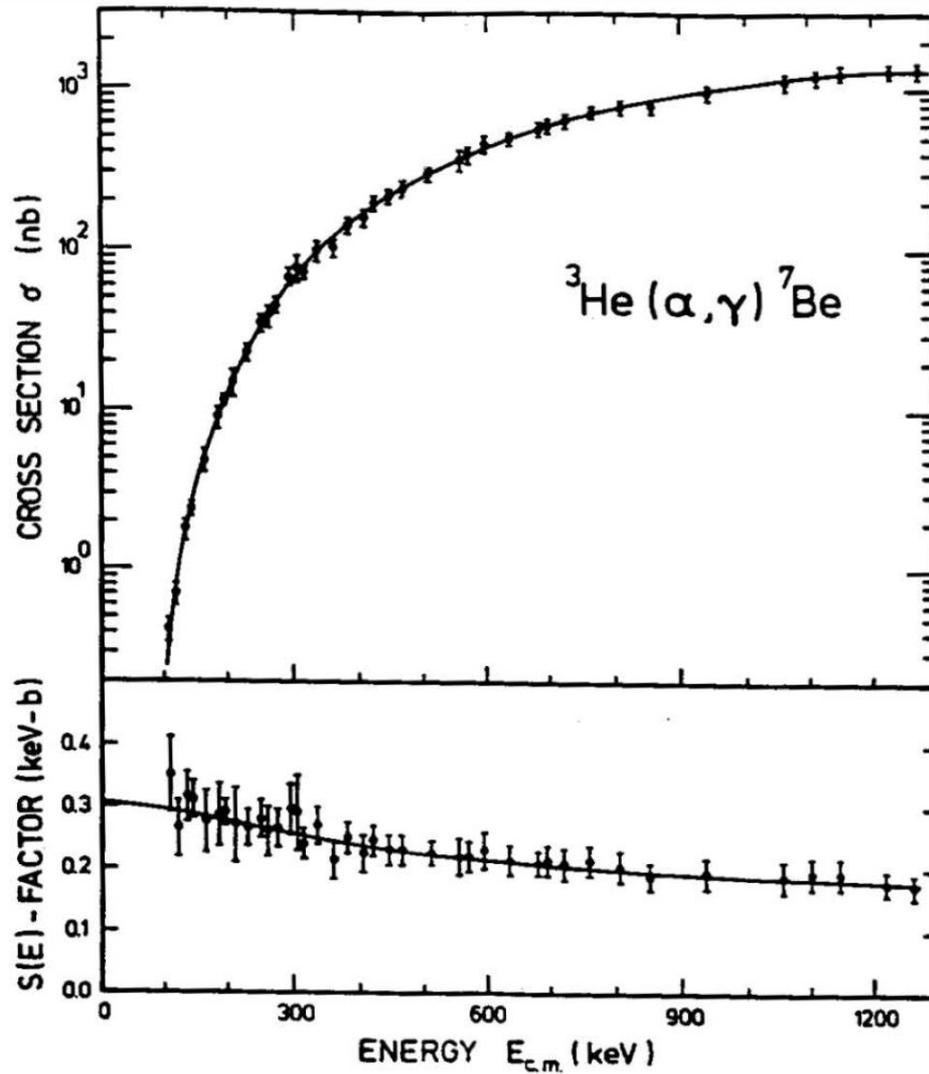


Figure 2.3: Energy dependence of the measured cross section (top) of the  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  reaction. An extrapolation to  $E < 50$  keV, which is relevant in astrophysical environments, appears treacherous. However, the  $S$ -factor (bottom) is only weakly dependent on energy, and therefore much easier to extrapolate (solid line). Figure from Rolfs & Rodney (1988).

# Hydrogen and helium burning

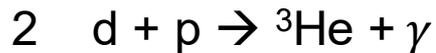
The most important series of fusion reactions are those converting H to He (H-burning). As we shall see this dominates ~90% of lifetime of nearly all stars.

- Fusion of 4 protons to give one  ${}^4\text{He}$  is completely negligible
- Reaction proceeds through steps – involving close encounter of 2 particles
- We will consider the main ones: the **PP-chain** and the **CNO cycle**

## The PP Chain

The PP chain has three main branches called the PPI, PPII and PPIII chains.

### PPI Chain



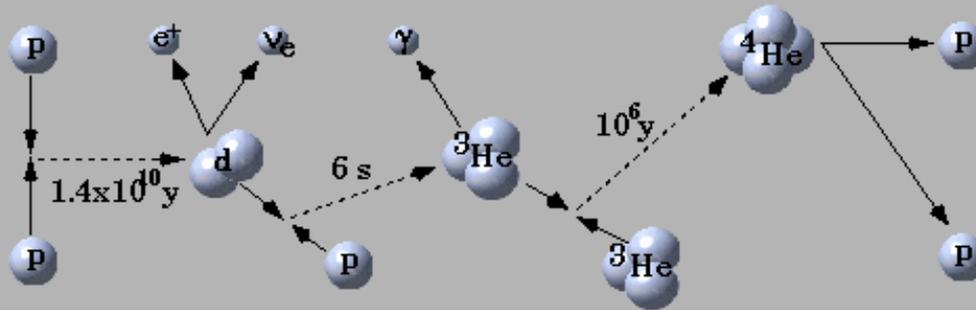
### PPII Chain



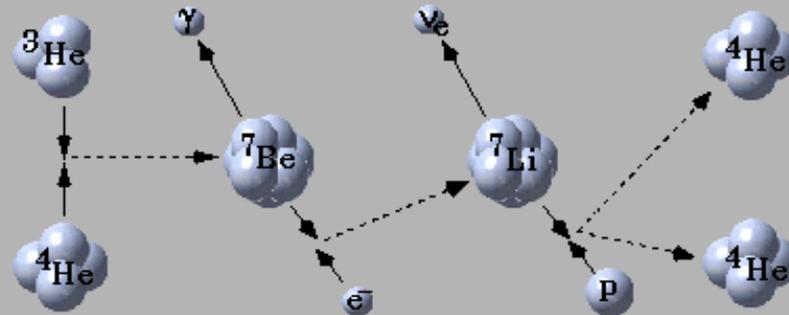
### PPIII Chain



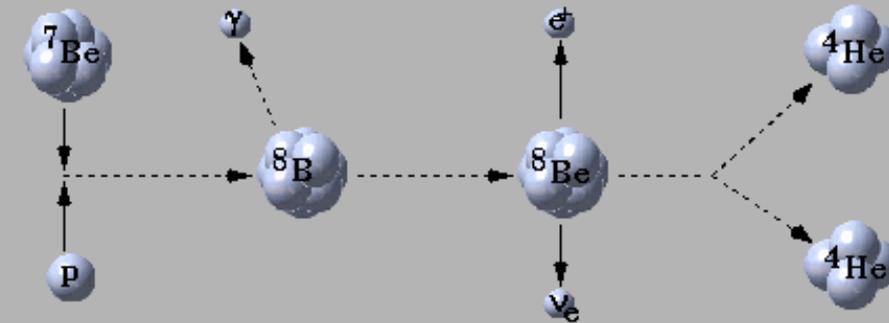
**PPI**



**PPII**

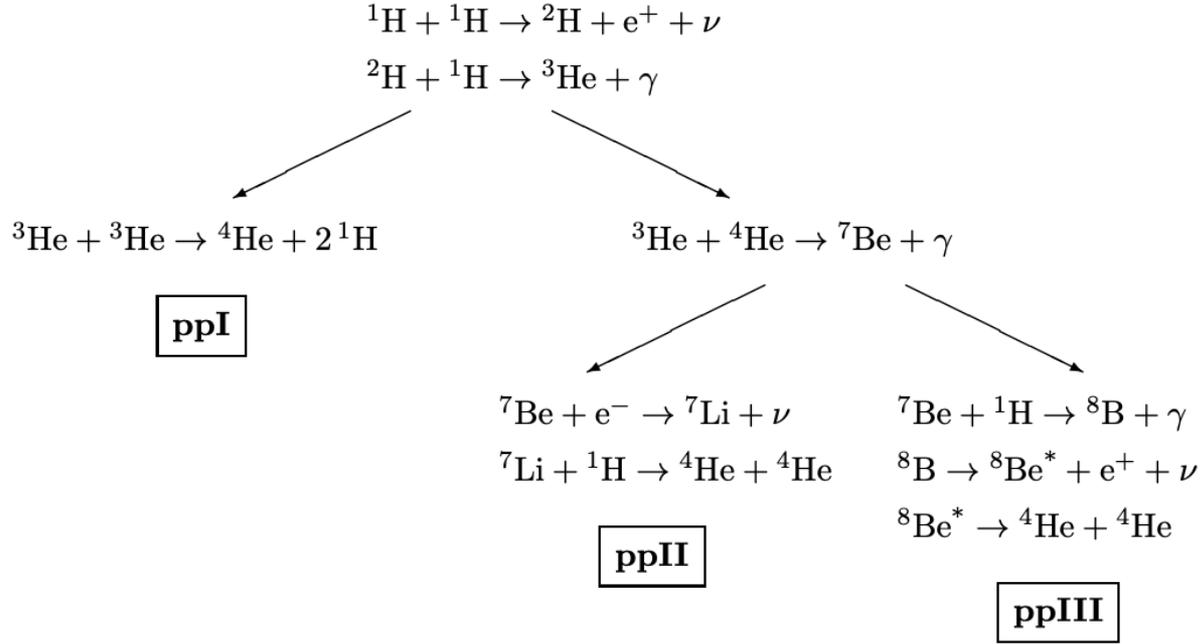


**PPIII**



Relative importance of PPI and PPII chains (*branching ratios*) depend on conditions of H-burning ( $T$ ,  $\rho$ , abundances). The transition from PPI to PPII occurs at temperatures in excess of  $1.3 \cdot 10^7 \text{ K}$ .

Above  $3 \cdot 10^7 \text{ K}$  the PPIII chain dominates over the other two, but another process takes over in this case.



reaction	$Q$ (MeV)	$\langle E_\nu \rangle$ (MeV)	$S(0)$ (keV barn)	$dS/dE$ (barn)	$\tau$ (yr)
${}^1\text{H}(p, e^+ \nu){}^2\text{H}$	1.442	0.265	$3.94 \times 10^{-22}$	$4.61 \times 10^{-24}$	$10^{10}$
${}^2\text{H}(p, \gamma){}^3\text{He}$	5.493		$2.5 \times 10^{-4}$	$7.9 \times 10^{-6}$	$10^{-8}$
${}^3\text{He}({}^3\text{He}, 2p){}^4\text{He}$	12.860		$5.18 \times 10^3$	$-1.1 \times 10^1$	$10^5$
${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$	1.587		$5.4 \times 10^{-1}$	$-3.1 \times 10^{-4}$	$10^6$
${}^7\text{Be}(e^-, \nu){}^7\text{Li}$	0.862	0.814			$10^{-1}$
${}^7\text{Li}(p, \alpha){}^4\text{He}$	17.347		$5.2 \times 10^1$	0	$10^{-5}$
${}^7\text{Be}(p, \gamma){}^8\text{B}$	0.137		$2.4 \times 10^{-2}$	$-3 \times 10^{-5}$	$10^2$
${}^8\text{B}(e^+ \nu){}^8\text{Be}^*(\alpha){}^4\text{He}$	18.071	6.710			$10^{-8}$

### ppI summary:

- D is *destroyed* (in fact already on the pre-main sequence)
- ${}^3\text{He}$  is destroyed in the center, enriched above the core of the Sun
- the pp-reaction is the slowest, and (when  ${}^3\text{He}$  is in equilibrium) sets the pace of the overall rate of the ppI chain

### Comparing ppI, ppII and ppIII

- ppII, III start with  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ , but the  $\alpha$ -particle is used only as a catalyst: it is liberated in the end, by  ${}^7\text{Li}(\text{p}, \alpha){}^4\text{He}$  or by  ${}^8\text{B}(\alpha){}^4\text{He}$
- ppII, III need the (slow) pp-reaction only *once* to produce one  ${}^4\text{He}$  nucleus, whereas ppI needs *two* pp-reactions per  ${}^4\text{He}$  nucleus  $\Rightarrow$  at high temperature: ppII, III may dominate

# Energy production and neutrino emission

Energy released in the formation of an alpha particle by fusion of four protons. Is essentially given by the difference of the mass excesses of four protons and one  $\alpha$  particle.

$$Q_{p-p} = \left[ 4\Delta M(^1H) - \Delta M(^4He) \right] c^2 = 26.7 \text{ MeV}$$

Since any reaction branch that completes this must turn 2 protons in 2 neutrons, two neutrinos are also emitted, which carry energy away from the reaction site.

It is these neutrinos that *directly* confirm the occurrence of nuclear reactions in the interior of the Sun. No other *direct* observational test of nuclear reactions is possible.

The mean neutrino energy flux is  $\sim 0.26 \text{ MeV}$  for d creation (PPI/II) and  $\sim 7.2 \text{ MeV}$  for B decay (PPIII). But as PPIII is negligible, the energy released for each He nucleus assembled is  $\sim 26 \text{ MeV}$  (or  $6 \cdot 10^{14} \text{ JKg}^{-1}$ )

## **Exercise**

Estimate the total number of neutrinos generated in the Sun per second, and the flux of solar neutrinos on the Earth (i.e., the number of neutrinos per unit area and per unit time).

Assume that the cross section for reaction between a neutrino and a nucleus is  $10^{46}$  cm<sup>2</sup>.

Estimate the total number of neutrino reactions per year in the body of a typical student.

*(EXTRA NEUTRINOS)*

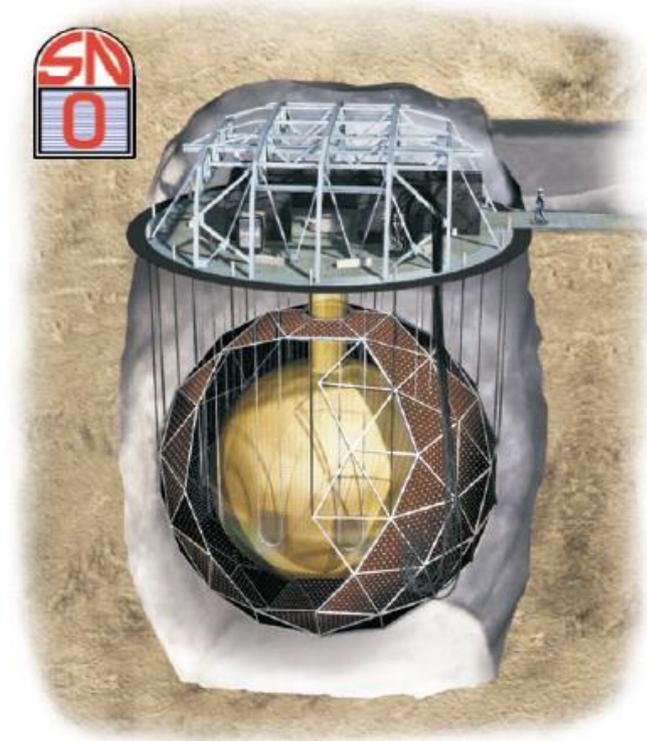
## SNO

*The detector consisted of 1000 tonnes of ultrapure heavy water (99.917%  $2H$  by mass) contained within a 12-m-diameter, 5.6-cm thick transparent acrylic vessel.*

*The heavy water was viewed with the aid of 9438 sensitive Photomultiplier Tubes on an 18-m diameter support structure.*

*Highly enriched heavy water was required because the neutron capture cross section on  $1H$  is 640 times that on  $2H$ .*

*The entire cavity outside the acrylic vessel was filled with ultrapure light water.*



**Figure 5**

A schematic drawing of the SNO detector, which is located in a 34-m-high by 22-m-diameter barrel-shaped cavity 2039 m below the surface in an active nickel mine owned by Vale-INCO near Sudbury, Ontario, Canada.

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The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2015 to

**Takaaki Kajita**

**Super-Kamiokande Collaboration**

University of Tokyo, Kashiwa, Japan

and

**Arthur B. McDonald**

**Sudbury Neutrino Observatory Collaboration**

Queen's University, Kingston, Canada

*“for the discovery of neutrino oscillations, which shows that neutrinos have mass”*

# The CNO Cycle

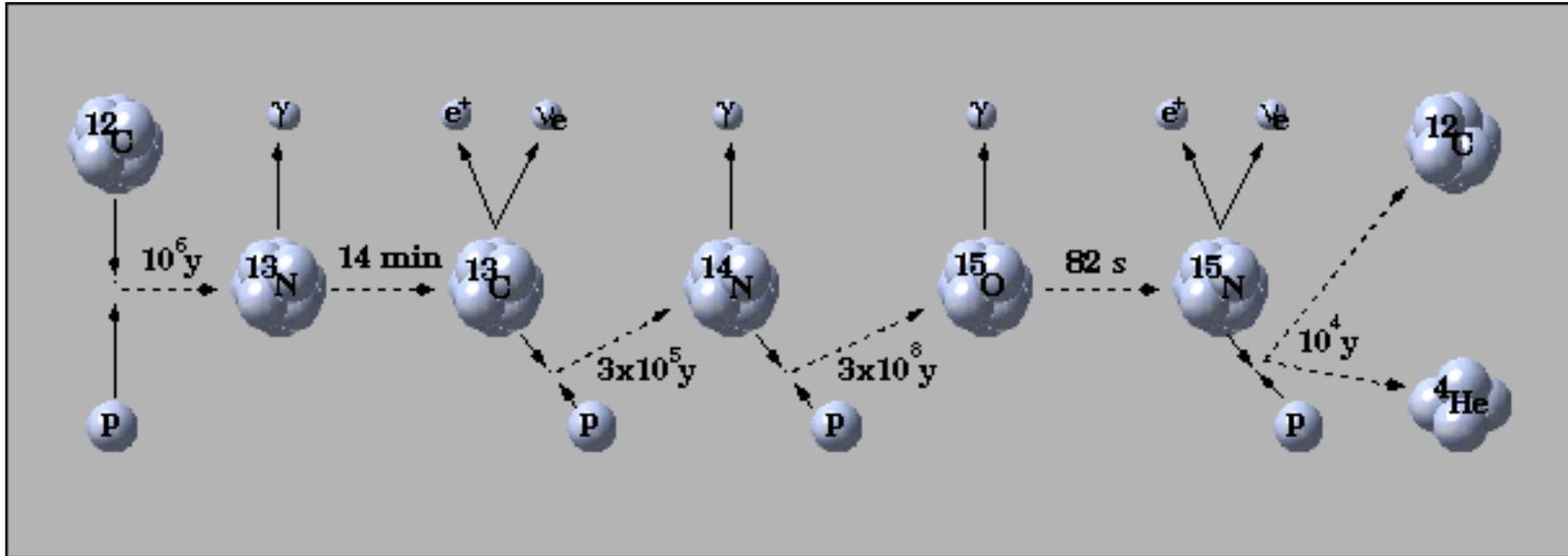
At birth stars contain a small (2%) mix of heavy elements, some of the most abundant of which are carbon, oxygen and nitrogen (CNO). These nuclei may induce a chain of H-burning reactions in which they act as catalysts.

The process is known as the CNO Cycle. There are alternative names that you may come across :

- The CNO bi-cycle
- The CNOF cycle
- The CN and NO cycles
- The CN and NO bi-cycles

In this course we will just refer to it all as the CNO cycle – and discuss the branches.

# The main branch



- 1  $^{12}\text{C} + \text{p} \rightarrow ^{13}\text{N} + \gamma$
- 2  $^{13}\text{N} \rightarrow ^{13}\text{C} + \text{e}^+ + \nu_e$
- 3  $^{13}\text{C} + \text{p} \rightarrow ^{14}\text{N} + \gamma$
- 4  $^{14}\text{N} + \text{p} \rightarrow ^{15}\text{O} + \text{g}$
- 5  $^{15}\text{O} \rightarrow ^{15}\text{N} + \text{e}^+ + \nu_e$
- 6  $^{15}\text{N} + \text{p} \rightarrow ^{12}\text{C} + ^4\text{He}$

In the steady state case, the abundances of isotopes must take values such that the isotopes which react more slowly have higher abundance. The slowest reaction is p capture by  $^{14}\text{N}$ .

Hence most of  $^{12}\text{C}$  is converted to  $^{14}\text{N}$ .

## CN nucleosynthesis

Lifetimes:  $^{15}\text{N} : ^{13}\text{C} : ^{12}\text{C} : ^{14}\text{N} \simeq 1 : 45 : 190 : 26\,000$

$\Rightarrow$  CN-equilibrium abundance distribution:

$$^{15}\text{N} : ^{13}\text{C} : ^{12}\text{C} : ^{14}\text{N} \simeq \frac{1}{26\,000} : \frac{1}{600} : \frac{1}{140} : 1$$

*independent* of the initial composition

Comparison with solar system composition:

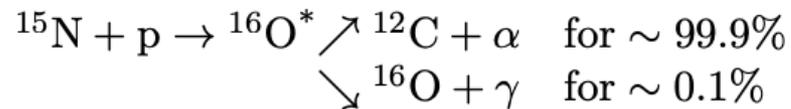
$$^{15}\text{N} : ^{13}\text{C} : ^{12}\text{C} : ^{14}\text{N} = \frac{1}{1136} : \frac{1}{114} : \frac{4}{5} : \frac{1}{5}$$

$\Rightarrow$   $^{15}\text{N}$ ,  $^{13}\text{C}$  and  $^{12}\text{C} \downarrow$      $^{14}\text{N} \uparrow$

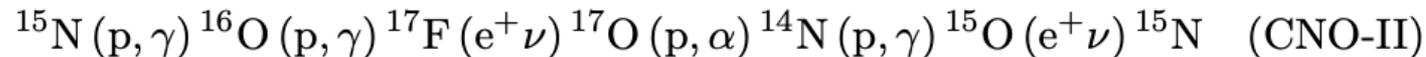
$\Rightarrow$   $^{14}\text{N}$  is the major nucleosynthesis product of the CN-cycle (except for  $^4\text{He}$ ).

## The CNO bi-cycle

Not all proton captures on  $^{15}\text{N}$  lead to  $^{12}\text{C} + \alpha$ :



This initiates a second cycle (Fig. 4.10):



ratio of 1:1000  $\Rightarrow$

- second cycle *not* relevant for energy generation
- but: very relevant for **nucleosynthesis**:  
it makes the  $^{16}\text{O}$ -reservoir available to flow into the CN-cycle.

Table 4.4: Reactions of the CNO bi-cycle. Cross section factors are taken from Bahcall (1989). The last column gives the lifetime of the CNO nuclei for  $T_6 = 20$ .

reaction	$Q$ (MeV)	$\langle E_\nu \rangle$ (MeV)	$S(0)$ (MeV barn)	$dS/dE$ (barn)	$\tau$ (yr)
$^{12}\text{C}(p, \gamma)^{13}\text{N}$	1.944		$1.45 \times 10^{-3}$	$2.45 \times 10^{-3}$	$6.6 \times 10^3$
$^{13}\text{N}(e^+ \nu)^{13}\text{C}$	2.220	0.707			863 s
$^{13}\text{C}(p, \gamma)^{14}\text{N}$	7.551		$5.50 \times 10^{-3}$	$1.34 \times 10^{-2}$	$1.6 \times 10^3$
$^{14}\text{N}(p, \gamma)^{15}\text{O}$	7.297		$3.32 \times 10^{-3}$	$-5.91 \times 10^{-3}$	$9.3 \times 10^5$
$^{15}\text{O}(e^+ \nu)^{15}\text{N}$	2.754	0.997			176 s
$^{15}\text{N}(p, \alpha)^{12}\text{C}$	4.965		$7.80 \times 10^1$	$3.51 \times 10^2$	$3.5 \times 10^1$
$^{15}\text{N}(p, \gamma)^{16}\text{O}$	12.127		$6.4 \times 10^{-2}$	$3 \times 10^{-2}$	$3.9 \times 10^4$
$^{16}\text{O}(p, \gamma)^{17}\text{F}$	0.600		$9.4 \times 10^{-3}$	$-2.3 \times 10^{-2}$	$7.1 \times 10^7$
$^{17}\text{F}(e^+ \nu)^{17}\text{O}$	2.761	0.999			93 s
$^{17}\text{O}(p, \alpha)^{14}\text{N}$	1.192		resonant reaction		$1.9 \times 10^7$

# Temperature dependence of PP chain and CNO Cycle

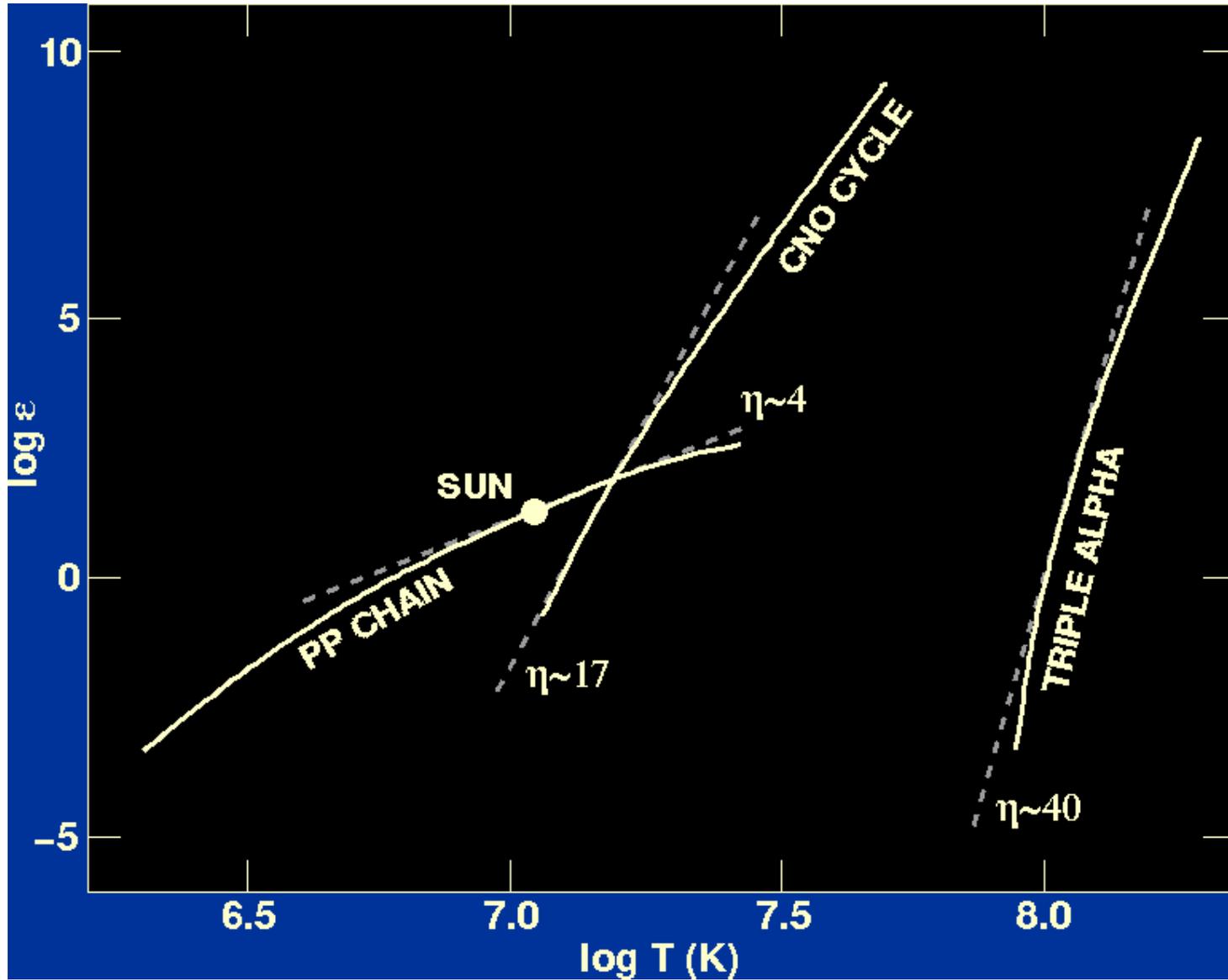
The two processes have very different temperature dependences. The rate of energy production in each (see pages 84-87 Pols notes):

$$\epsilon_{PP} = \epsilon_0 \rho X_H^2 \left( \frac{T}{T_0} \right)^{4.6} \quad \epsilon_{CNO} = \epsilon_0 \rho X_H X_{CNO} f_N \left( \frac{T}{25 \times 10^6} \right)^{16.7}$$

Equating this two gives the  $T$  at which they produce the same rate of energy production:

$$T \approx 1.7 \times 10^7 \left( \frac{X_H}{50 X_{CN}} \right)^{\frac{1}{12.1}} \text{ K}$$

Below this temperature the PP chain is most important, and above it the CNO Cycle dominates. This occurs in stars slightly more massive than the sun e.g.  $1.2-1.5M_{\odot}$ .

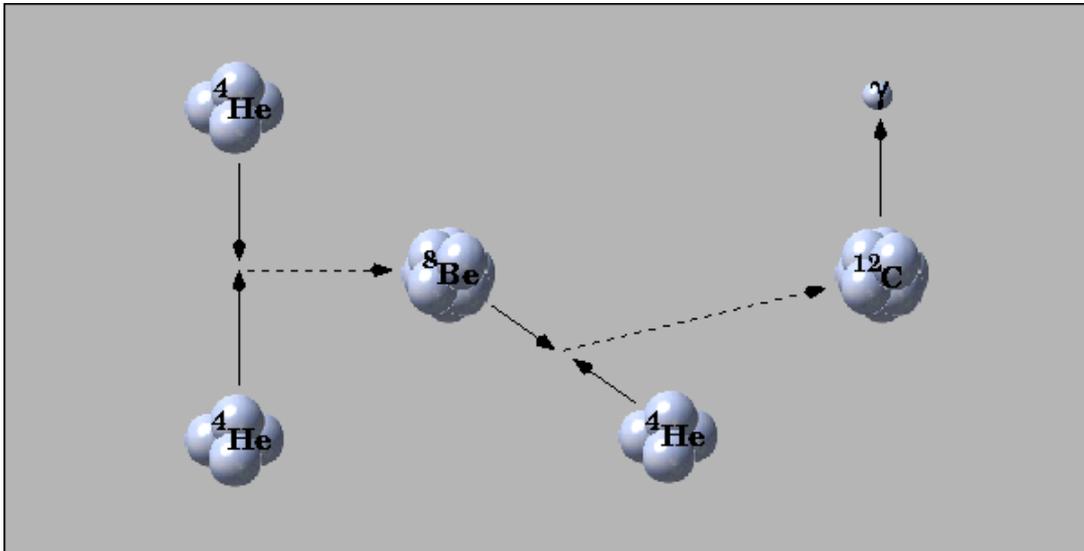


# Helium Burning: the triple- $\alpha$ reaction.

Simplest reaction in a helium gas should be the fusion of two helium nuclei.  
There is no stable configuration with  $A=8$ . For example, the beryllium isotope  ${}^8\text{Be}$  has a lifetime of only  $2.6 \cdot 10^{-16}$  s



But a third helium nucleus can be added to  ${}^8\text{Be}$  before decay, forming  ${}^{12}\text{C}$  by the “triple-alpha” reaction



# Helium Burning: the triple- $\alpha$ reaction.

Fred Hoyle (1952-54) suggested this small probability of alpha-capture by short lived  $^8\text{Be}$  would be greatly enhanced if the C nucleus had an energy level close to the combined energies of the reacting  $^8\text{Be}$  and  $^4\text{He}$  nuclei. The reaction would be a faster “resonant” reaction.

This resonant energy level of  $^{12}\text{C}$  was not experimentally known at the time. Hoyle’s prediction led to nuclear experiment at Caltech, and resonant level discovered.

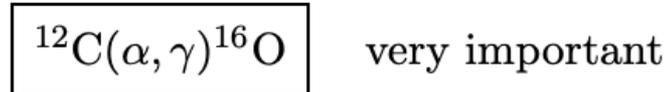
Thus helium burning proceeds in a 2-stage reaction, and energy released is

$$Q_{3\alpha} = \left[ 3\Delta M(^4\text{He}) - \Delta M(^{12}\text{C}) \right] c^2 = 7.275\text{MeV}$$

In terms of energy generated per unit mass =  $5.8 \cdot 10^{13} \text{ J Kg}^{-1}$  (i.e. 1/10 of energy generated by H-burning). But the  $T$  dependence is astounding:

$$\epsilon_{3\alpha} \propto \rho^2 T^{40}$$

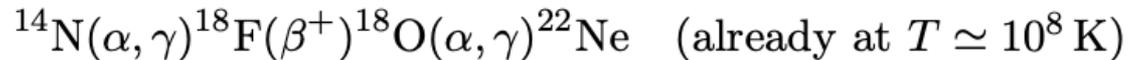
### Further reactions:



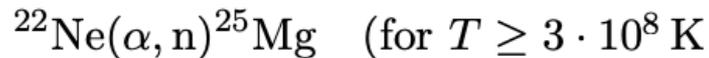
Remark: the  ${}^{12}\text{C}(\alpha, \gamma)$ -rate is not very well known today.

$\Rightarrow$  unclear whether main product of He-burning is carbon or oxygen!

### Secondary nucleosynthesis during helium burning:



$\Rightarrow$  production of  ${}^{18}\text{O}$  and  ${}^{22}\text{Ne}$



$\Rightarrow$  n-production !!  $\Rightarrow$  s-process

# Carbon burning

Carbon burning (fusion of 2 C nuclei) requires temperatures above  $5 \cdot 10^8$  K

Interactions of C and O nuclei are negligible – as at the intermediate temperatures required by the coulomb barrier the C nuclei are quickly destroyed by interacting with themselves

Starting composition: ashes of helium burning, mainly  $^{12}\text{C}$ ,  $^{16}\text{O}$   
no light particles (p, n,  $\alpha$ ) available initially!

**Main reaction of carbon burning:**  $^{12}\text{C} + ^{12}\text{C}$

There are two main exit channels:



$\Rightarrow$  production of p and  $\alpha$  particles; those are immediately captured  $\Rightarrow$

**many side reactions**, e.g.:  $^{23}\text{Na}(\text{p}, \alpha)^{20}\text{Ne}$ ,  $^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg}$  ...  
also  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ ,  $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$  ...

Composition after carbon burning:  $^{16}\text{O}$ ,  $^{20}\text{Ne}$ ,  $^{24}\text{Mg}$  (together: 95%)

# Ne burning

First expectation: oxygen burning should follow carbon burning, as  $^{16}\text{O}$  is the lightest remaining nucleus, **but:**

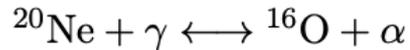
$^{16}\text{O}$  is doubly-magic nucleus (magic n and magic p number!)

$\Rightarrow$  extremely stable:  $\alpha$ -separation energy: 7.2 MeV

$^{20}\text{Ne}$  is much less stable:  $\alpha$ -separation energy: 4.7 MeV

**Consequence:**  $^{20}\text{Ne}(\gamma, \alpha)^{16}\text{O}$  occurs at lower temperatures than  $^{16}\text{O} + ^{16}\text{O}$

**Main reactions of neon burning:**



Effectively:  $2^{20}\text{Ne} \rightarrow ^{16}\text{O} + ^{24}\text{Mg} \Rightarrow$  effective energy generation  $> 0!$

Composition after neon burning:

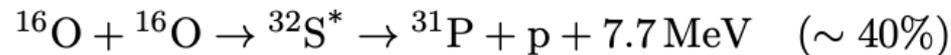
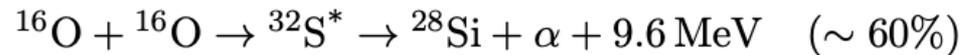
$^{16}\text{O}, ^{24}\text{Mg}$ (together: 95%)
---

# O burning

Starting composition: ashes of neon burning, mainly  $^{16}\text{O}$ ,  $^{24}\text{Mg}$   
no light particles (p, n,  $\alpha$ ) available initially!

**Main reaction of oxygen burning:  $^{16}\text{O} + ^{16}\text{O}$**

There are two main exit channels:



$\Rightarrow$  production of p and  $\alpha$  particles (like in carbon burning) which are immediately captured  $\Rightarrow$

**many side reactions**, e.g.:  $^{31}\text{P}(\text{p}, \alpha)^{28}\text{Si}$ ,  $^{28}\text{Si}(\alpha, \gamma)^{32}\text{S}$  ...  
also  $^{24}\text{Mg}(\alpha, \gamma)^{28}\text{Si}$ , ... but  $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$  is blocked!

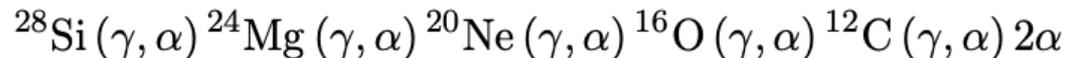
Composition after oxygen burning:  $^{28}\text{Si}$ ,  $^{32}\text{S}$  (together: 90%)

# Silicon burning: nuclear statistical equilibrium

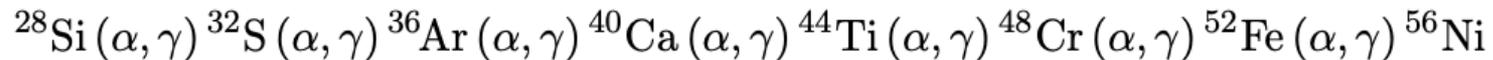
Main ash of oxygen burning, and lightest remaining isotope:  $^{28}\text{Si}$   
**but:**  $^{28}\text{Si} + ^{28}\text{Si}$  is prohibited by the high Coulomb-barrier

instead:  $^{28}\text{Si}(\gamma, \alpha)^{24}\text{Mg}$  occurs for  $T > 3 \cdot 10^9$  K

⇒ Si-burning occurs similar to Ne-burning: through  $(\gamma, \alpha)$  and  $(\alpha, \gamma)$ -reactions:



and



For  $T > 4 \cdot 10^9$  K: almost nuclear statistical equilibrium (NSE) may be reached

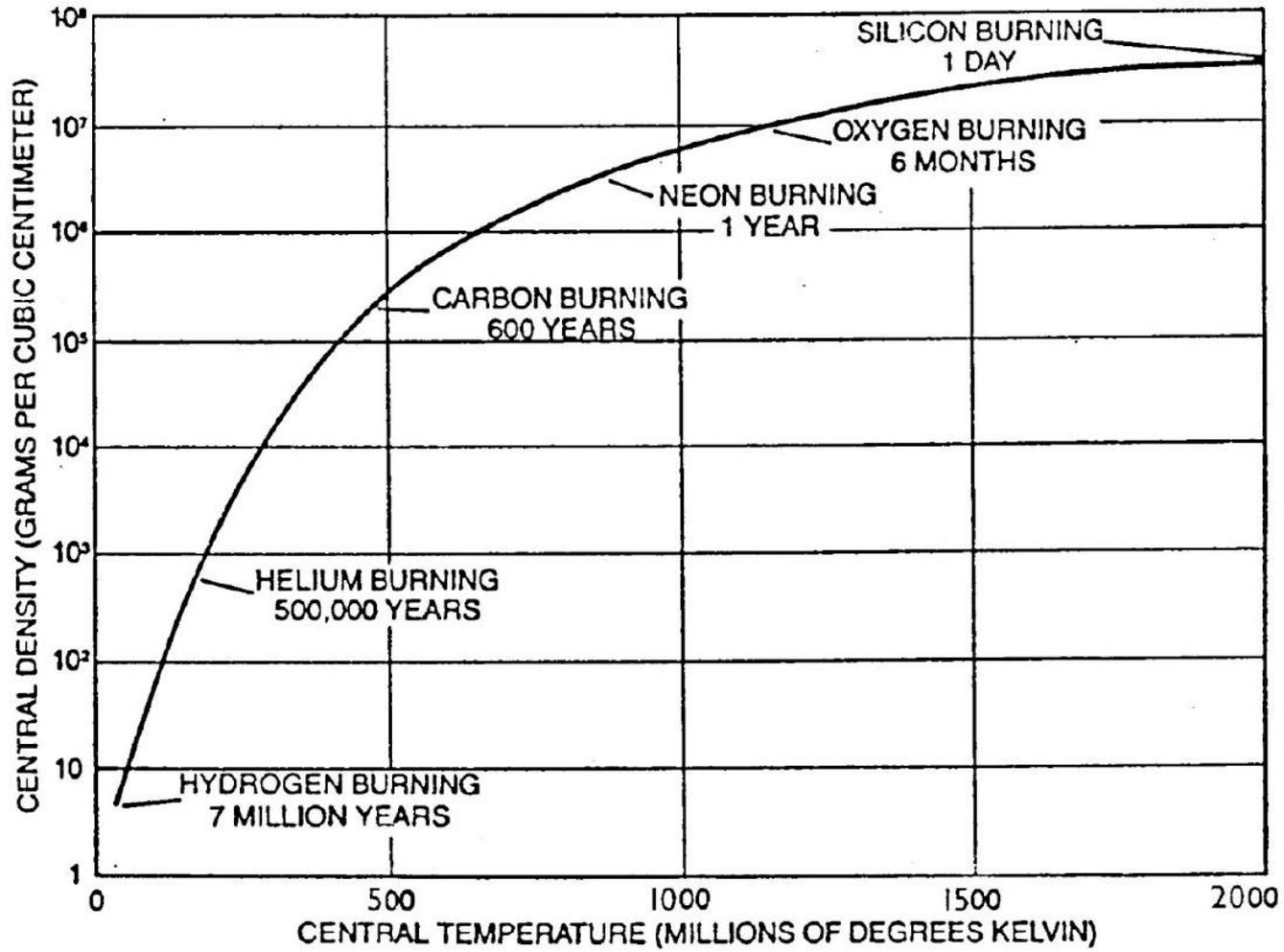
As in the matter  $p/n < 1$  due to  $\beta$ -decays and possibly  $e^-$ -captures: final composition may be mostly  $^{56}\text{Fe}$ .

# Major nuclear burning processes

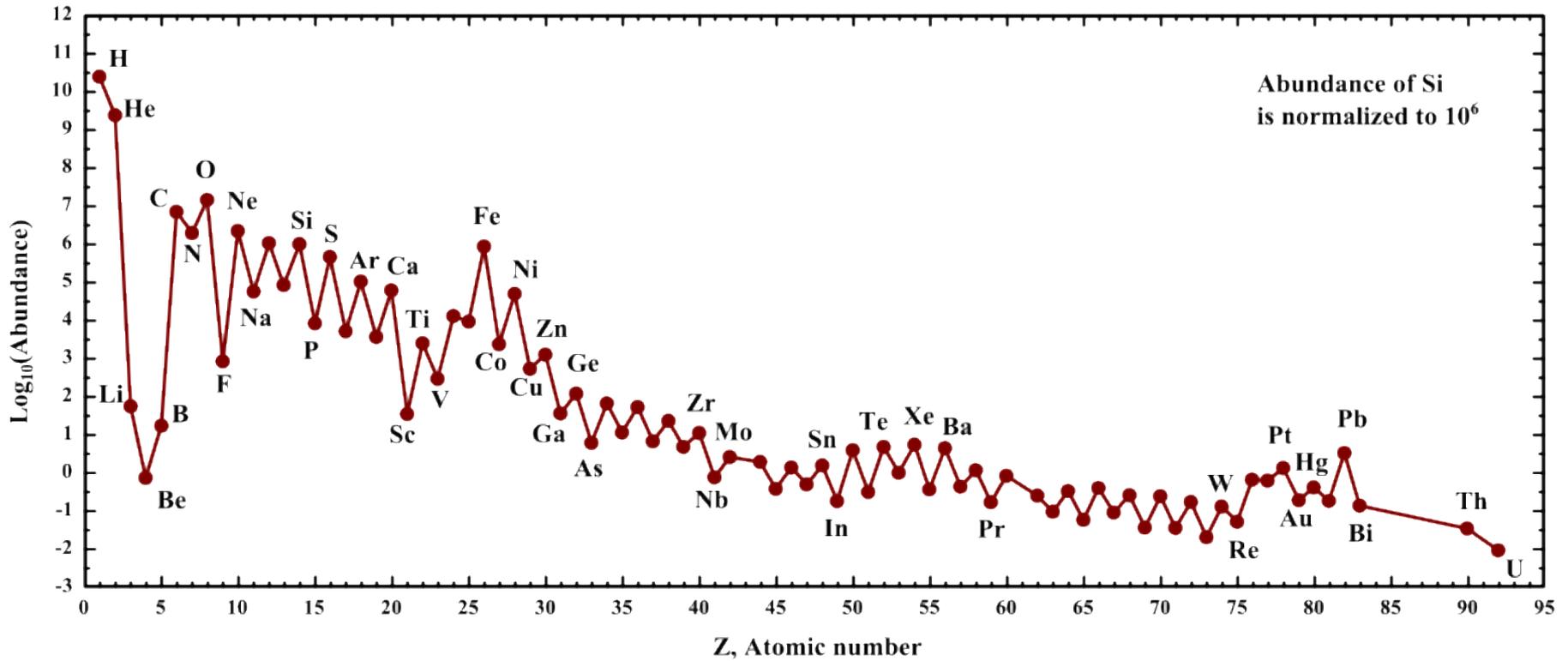
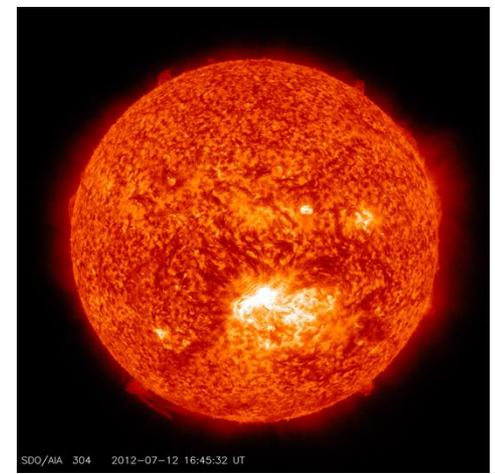
Common feature is release of energy by consumption of nuclear fuel. Rates of energy release vary enormously. Nuclear processes can also absorb energy from radiation field, we shall see consequences can be catastrophic.

Evolutionary Time Scales for a  $15 M_{\text{sun}}$  Star

Fused	Products	Time	Temperature
H	$^4\text{He}$	$10^7$ yrs.	$4 \times 10^6$ K
$^4\text{He}$	$^{12}\text{C}$	Few $\times 10^6$ yrs	$1 \times 10^8$ K
$^{12}\text{C}$	$^{16}\text{O}$ , $^{20}\text{Ne}$ , $^{24}\text{Mg}$ , $^4\text{He}$	1000 yrs.	$6 \times 10^8$ K
$^{20}\text{Ne} +$	$^{16}\text{O}$ , $^{24}\text{Mg}$	Few yrs.	$1 \times 10^9$ K
$^{16}\text{O}$	$^{28}\text{Si}$ , $^{32}\text{S}$	One year	$2 \times 10^9$ K
$^{28}\text{Si} +$	$^{56}\text{Fe}$	Days	$3 \times 10^9$ K
$^{56}\text{Fe}$	Neutrons	< 1 second	$3 \times 10^9$ K



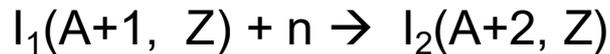
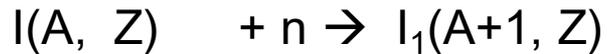
# Chemical composition of the Sun



# The s-process and r-process

Interaction between nuclei and free neutrons (neutron capture) – the neutrons are produced during C, O and Si burning.

Neutrons capture by heavy nuclei is not limited by the Coulomb barrier – so could proceed at relatively low temperatures. The obstacle is the scarcity of free neutrons. If enough neutrons available, chain of reactions possible:



Eventually, a radioactive isotope is formed it will undergo beta-decay, creating a new element.



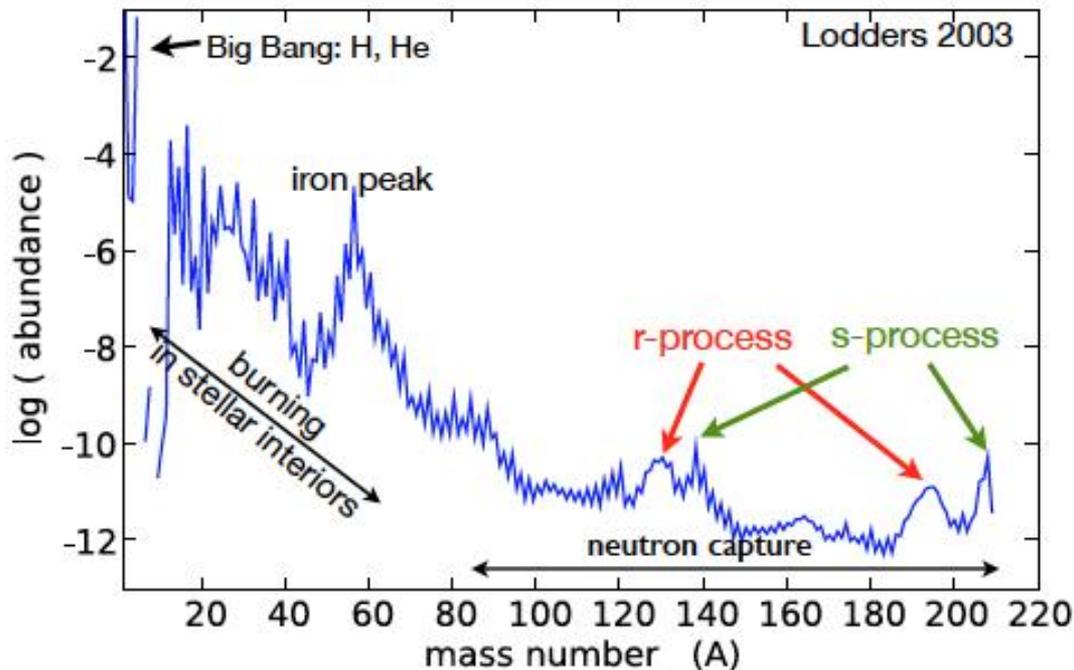
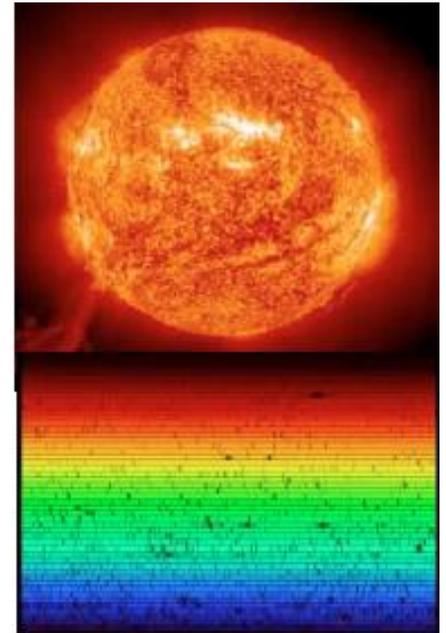
if new element stable, it will resume neutron capture, otherwise my undergo series of beta-decays



# Solar system abundances

Solar photosphere and meteorites:  
chemical signature of gas cloud where the Sun formed

Contribution of all nucleosynthesis processes



s-process:  
slow neutron capture  
r-process:  
rapid neutron capture

abundance = mass fraction / mass number

# Neutron capture elements: r-s process

The elements beyond the iron peak ( $A > 60$ ) are mainly formed through neutron capture on seed nuclei (iron and silicon).

Two cases:

**s-process**

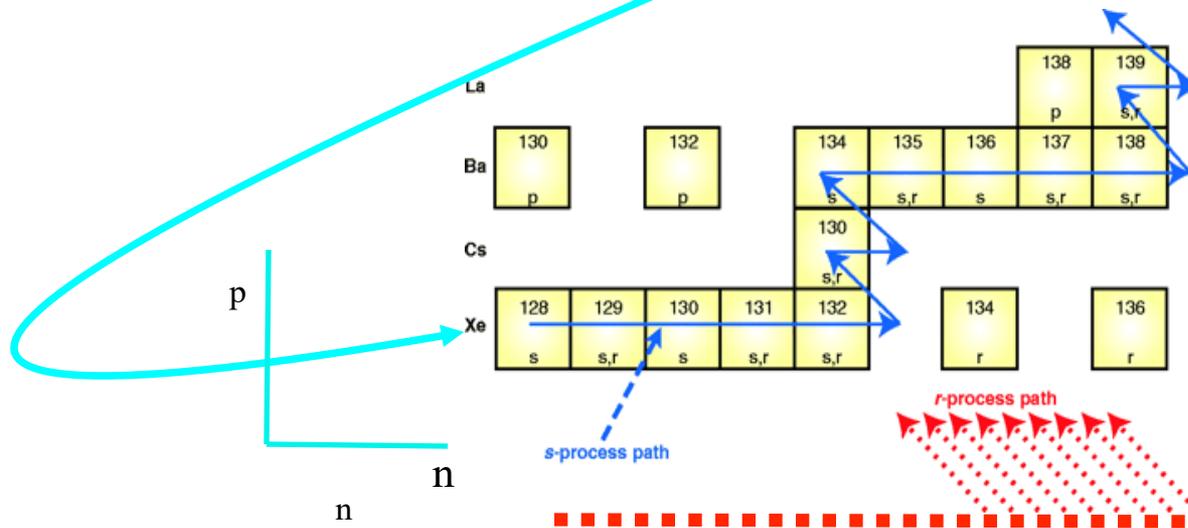
Different Timescale of the neutron capture

**r-process**

$\tau_{\beta} \ll \tau_c$

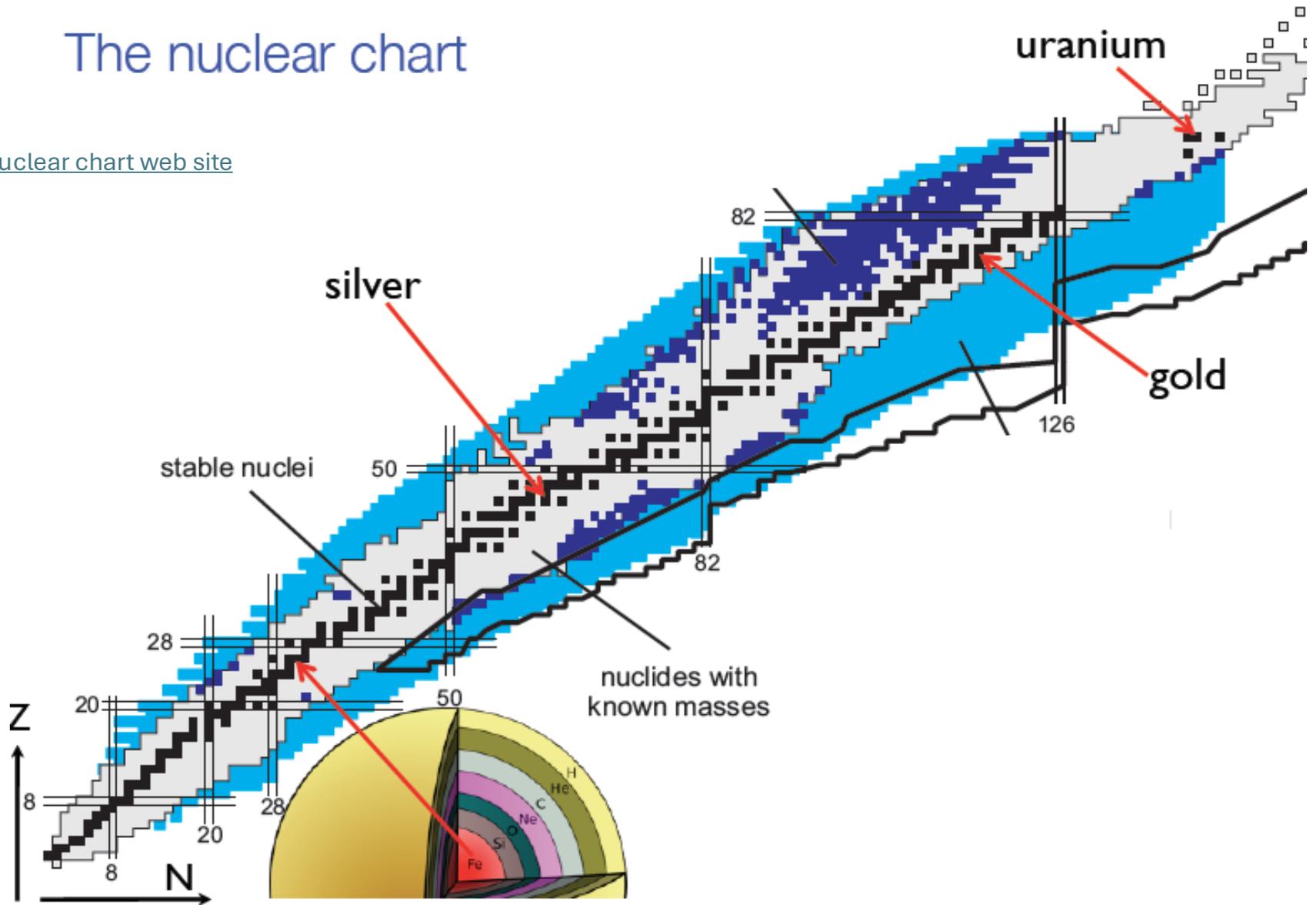
Different process path

$\tau_{\beta} \gg \tau_c$

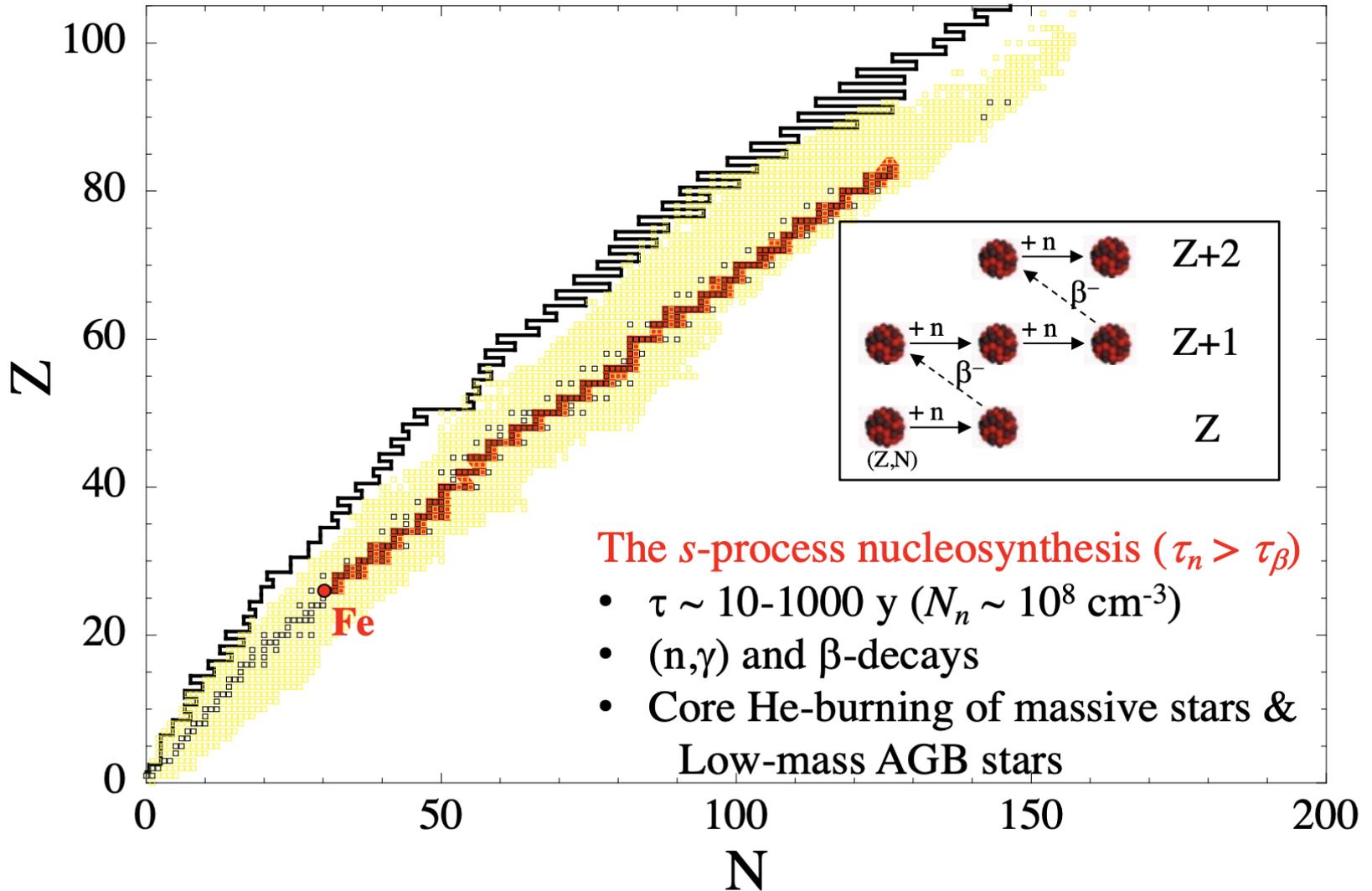


# The nuclear chart

[nuclear chart web site](#)



# The slow neutron-capture process (or s-process)



The s-process is responsible for about half of the elements heavier than iron in the Universe

## Slow and steady, the s-process

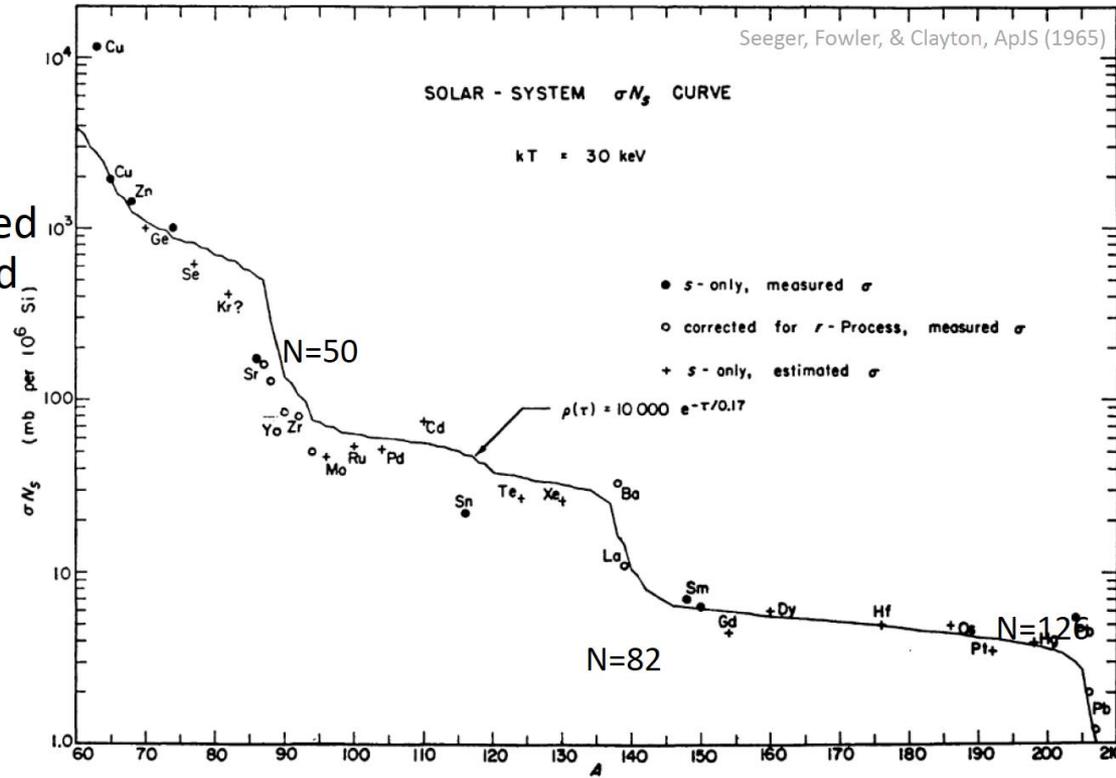
- For the s-process, we need an environment where nuclei can bathe for a long time in a moderate neutron density ( $n_n \sim 10^7-10^8 \text{ cm}^{-3}$ ). Let's consider the reaction flow for this case.
- For isotope (Z,A), the abundance change is the sum of production & destruction mechanisms,  
$$\frac{dN_{Z,A}}{dt}(t) = n_n(t)n_{Z,A-1}(t)\langle\sigma v\rangle_{Z,A-1} - n_n(t)n_{Z,A}(t)\langle\sigma v\rangle_{Z,A} - \lambda_{\beta:Z,A}n_{Z,A}(t)$$
- We'll ignore species with fast  $\beta$ -decays, since they'll just form the more stable isobar more or less instantly, so we can set the last term to zero
- Also, recall  $\langle\sigma v\rangle_{n \text{ cap}} \sim \text{constant}$ , and anyhow we'll assume a constant temperature, so  $\langle\sigma v\rangle_i \rightarrow \sigma_i v_T$ , where  $v_T$  is the thermal velocity for the environment temperature
- As such, our abundance change equation is now  $\frac{dN_{Z,A}}{dt}(t) = n_n(t)v_T(\sigma_{Z,A-1}n_{Z,A-1} - \sigma_{Z,A}n_{Z,A})$
- Note that  $n_n(t)v_T$  is the neutron flux. Integrating this over time gives the neutron irradiation  $\tau = \int_0^t n_n(t)v_T dt = v_T \int_0^t n_n(t) dt$ , which is referred to as the neutron exposure
- Re-casting our abundance change in terms of  $\tau$ ,  $\frac{dN_{Z,A}}{d\tau} = \sigma_{Z,A-1}n_{Z,A-1} - \sigma_{Z,A}n_{Z,A}$
- In equilibrium,  $\frac{dN_{Z,A}}{d\tau} = 0$ , so  $\sigma_{Z,A-1}n_{Z,A-1} = \sigma_{Z,A}n_{Z,A} = \text{constant}$
- So, we can get s-process relative abundances based solely on neutron-capture cross sections!

# The steady flow approximation

- The steady-flow approximation holds roughly in between magic  $N$
- Tests can be performed for elements with multiple isotopes that are shielded from the  $r$ -process and have measured neutron-capture cross sections, e.g.  $^{122,123,124}\text{Te}$

$^{122}\text{Te}$	$^{123}\text{Te}$	$^{124}\text{Te}$	$^{125}\text{Te}$	$^{126}\text{Te}$	$^{127}\text{Te}$
$^{121}\text{Sb}$	$^{122}\text{Sb}$	$^{123}\text{Sb}$	$^{124}\text{Sb}$	$^{125}\text{Sb}$	$^{126}\text{Sb}$
$^{120}\text{Sn}$	$^{121}\text{Sn}$	$^{122}\text{Sn}$	$^{123}\text{Sn}$	$^{124}\text{Sn}$	$^{125}\text{Sn}$
$^{119}\text{In}$	$^{120}\text{In}$	$^{121}\text{In}$	$^{122}\text{In}$	$^{123}\text{In}$	$^{124}\text{In}$

$r$

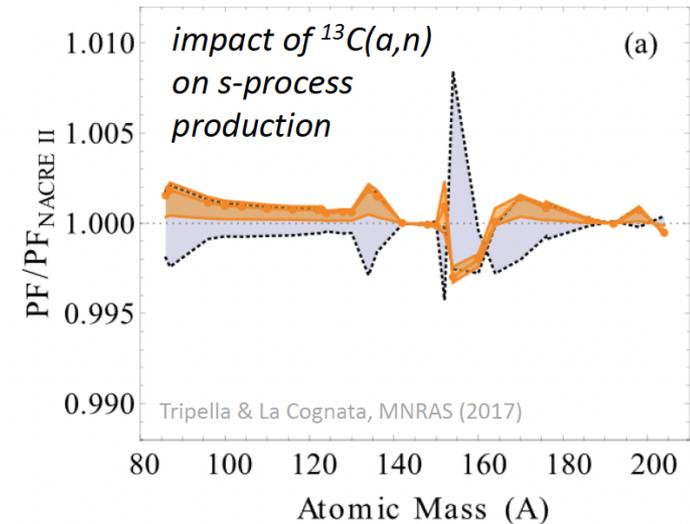
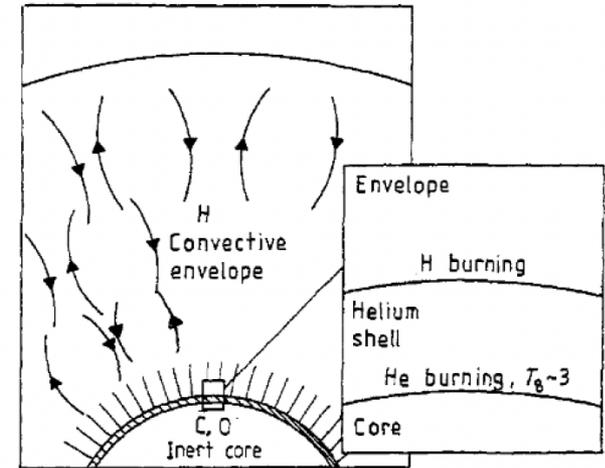


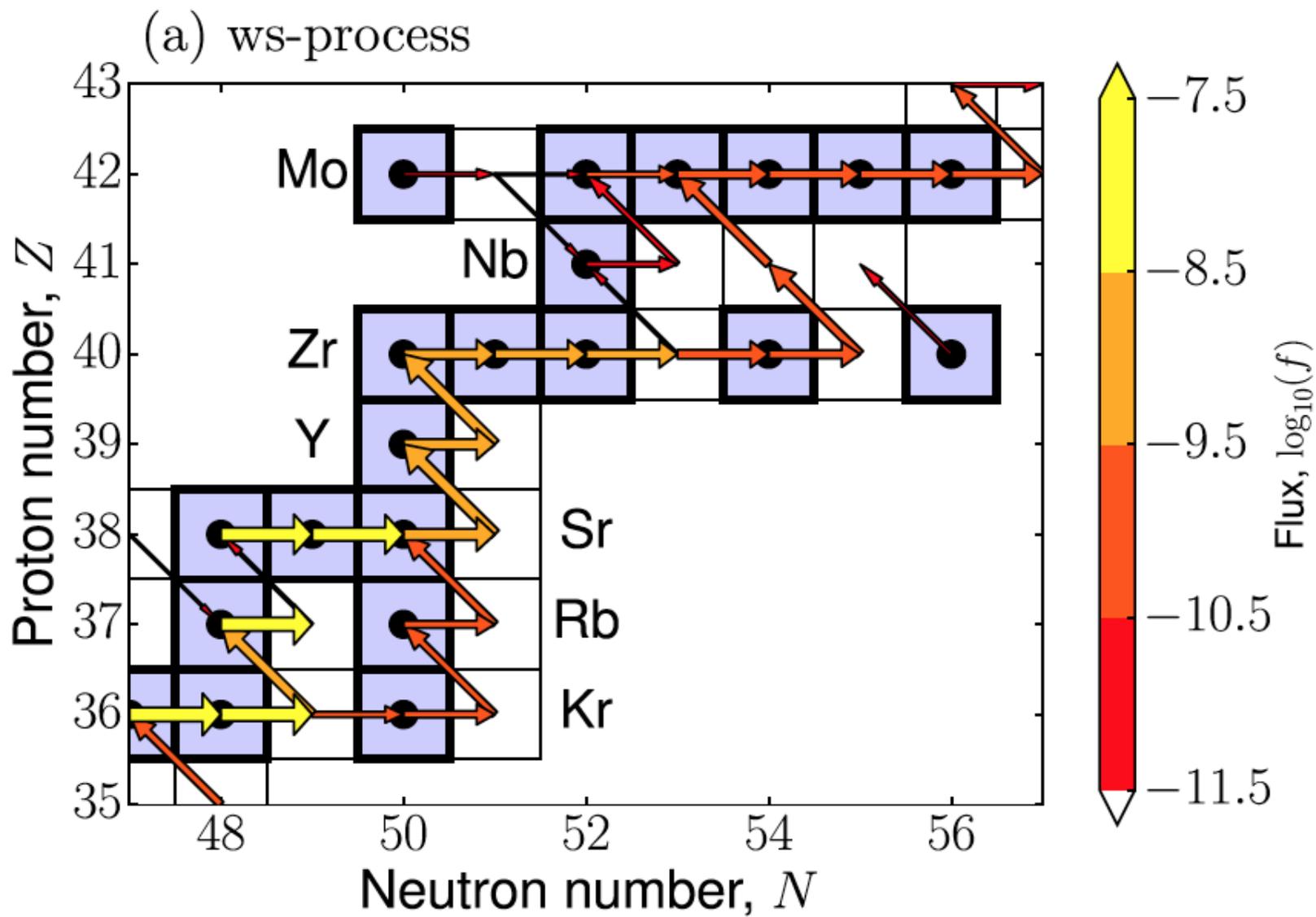
Reactions of special interest are those with half-lives similar to the neutron-capture time (you'll calculate this time in the group activity). They're called branch-points. They not only alter the abundance pattern, but can be used as  $s$ -process thermometers and neutron-density probes, since  $\langle\sigma v\rangle$  isn't exactly constant.

# Where do the $s$ neutrons come from?

- More careful analyses (e.g. checks with the Te isotopes and branch-points) demonstrate the need for different neutron fluxes for different durations. These are the
  - **Weak:**  $60 < A < 90$ , near He/C shells in massive stars
  - **Main:** Sr-Pb, thermal pluses in AGB stars
  - **Strong:** Pb enhancement for low-metal AGB stars
- components. For each  $kT \sim 30\text{keV}$  or thereabouts.
- No matter, the neutron sources turn-out to be  $^{13}\text{C}(\alpha, n)$  and  $^{22}\text{Ne}(\alpha, n)$ , where the former only operates for the AGB cases and the latter only operates during brief flashes for the AGB cases
- Other important reactions are the neutron “poisons”, light nuclei that swallow-up perfectly good neutrons that would have otherwise been captured by heavier seeds, other exit channels for  $^{22}\text{Ne}$  and  $^{13}\text{C}$   $\alpha$ -capture which fail to produce neutrons, and reactions producing  $^{22}\text{Ne}$  and  $^{13}\text{C}$

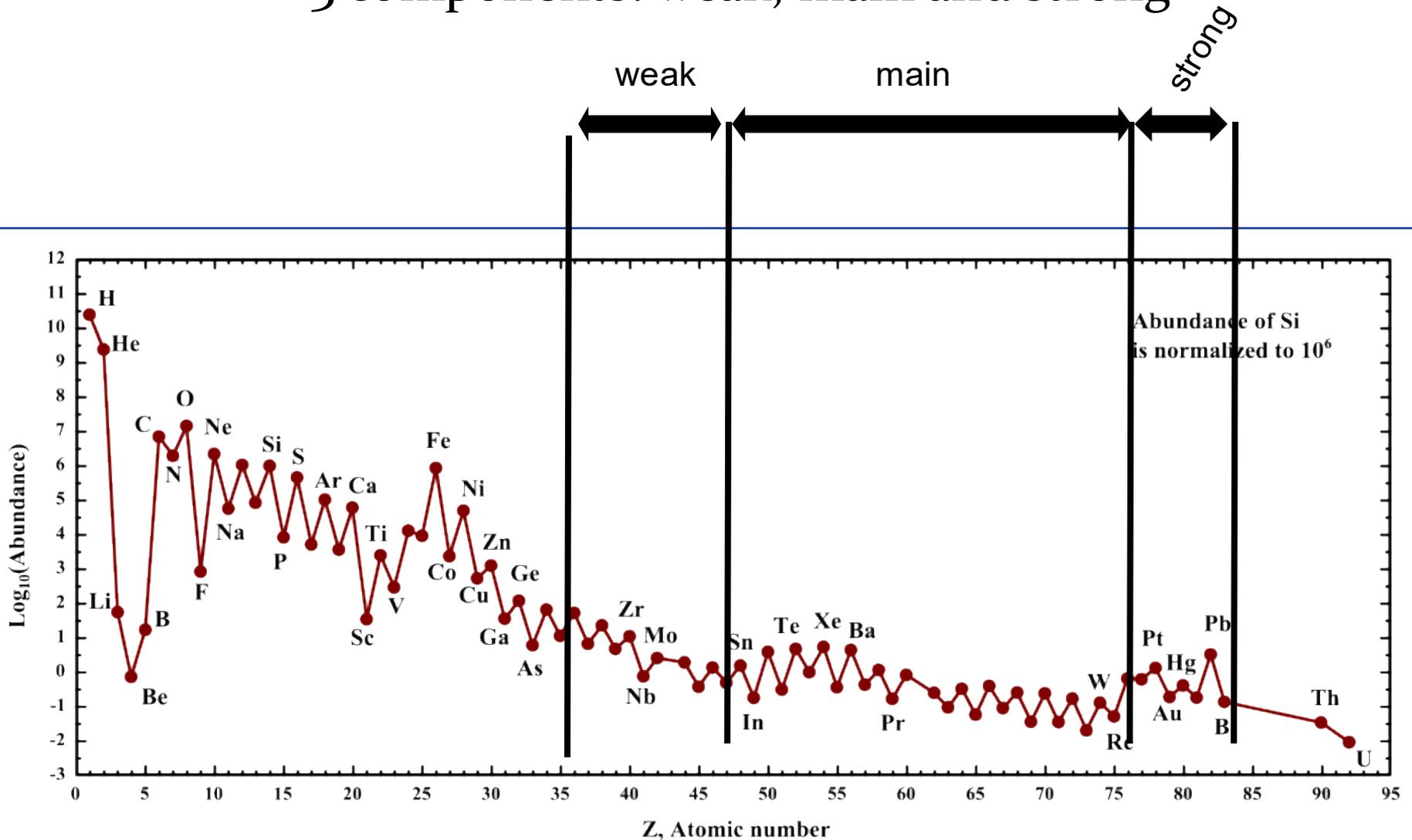
Käppeler, Beer, & Wisshak, Rep.Prog.Phys. (1989)





# On the s-process

3 components: weak, main and strong



## Uncertainties in s-process nucleosynthesis in low-mass stars determined from Monte Carlo variations

G. Cescutti,<sup>1★†</sup> R. Hirschi,<sup>2,3†</sup> N. Nishimura,<sup>4†</sup> J. W. den Hartogh,<sup>2,5†</sup> T. Rauscher,<sup>6,7†</sup>  
A. St. J. Murphy<sup>8†</sup> and S. Cristallo<sup>9,10</sup>

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<sup>4</sup>*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

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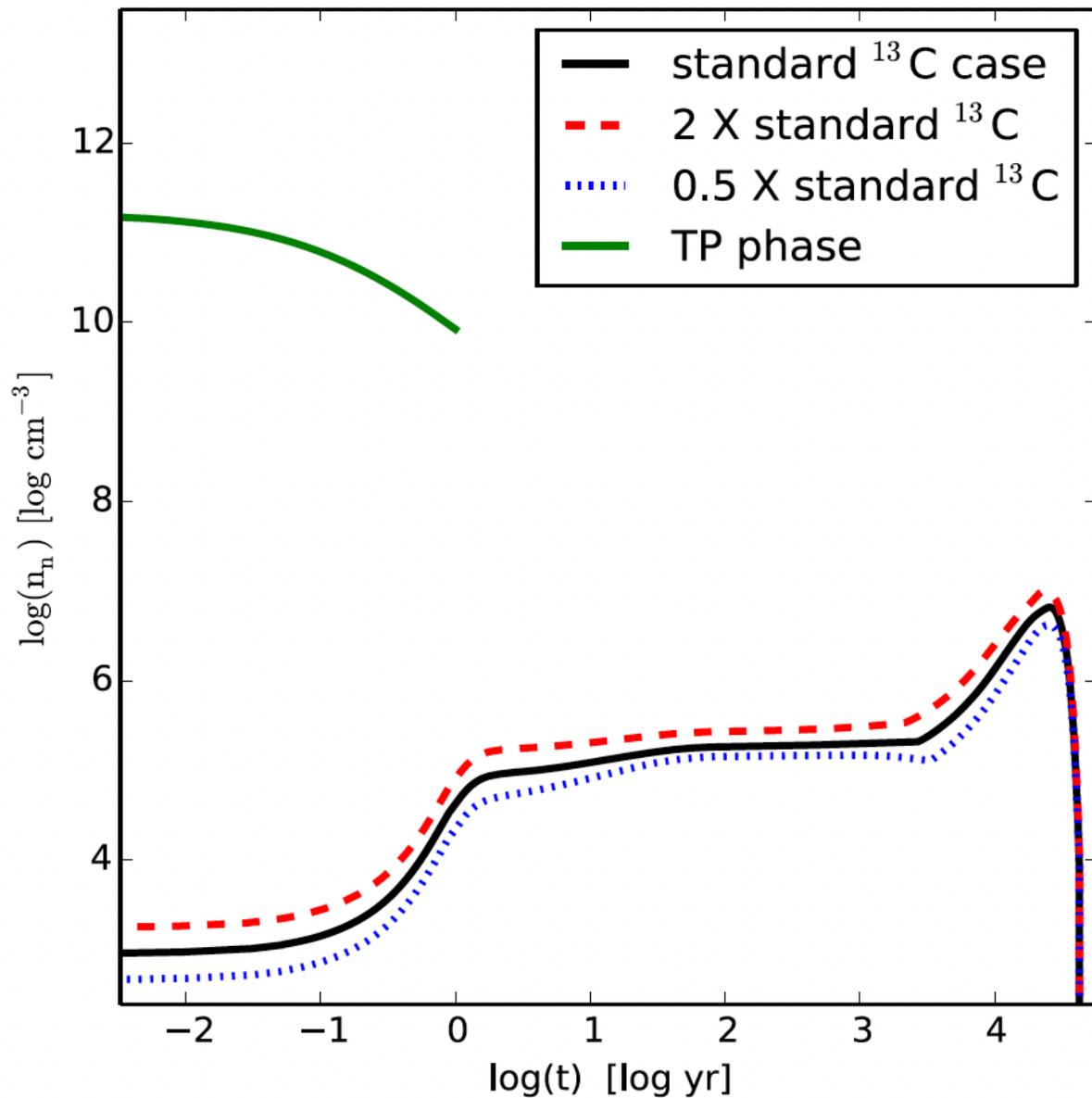
<sup>7</sup>*Centre for Astrophysics Research, University of Hertfordshire, College Lane, Hatfield AL10 9AB, UK*

<sup>8</sup>*SUPA, School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3FD, UK*

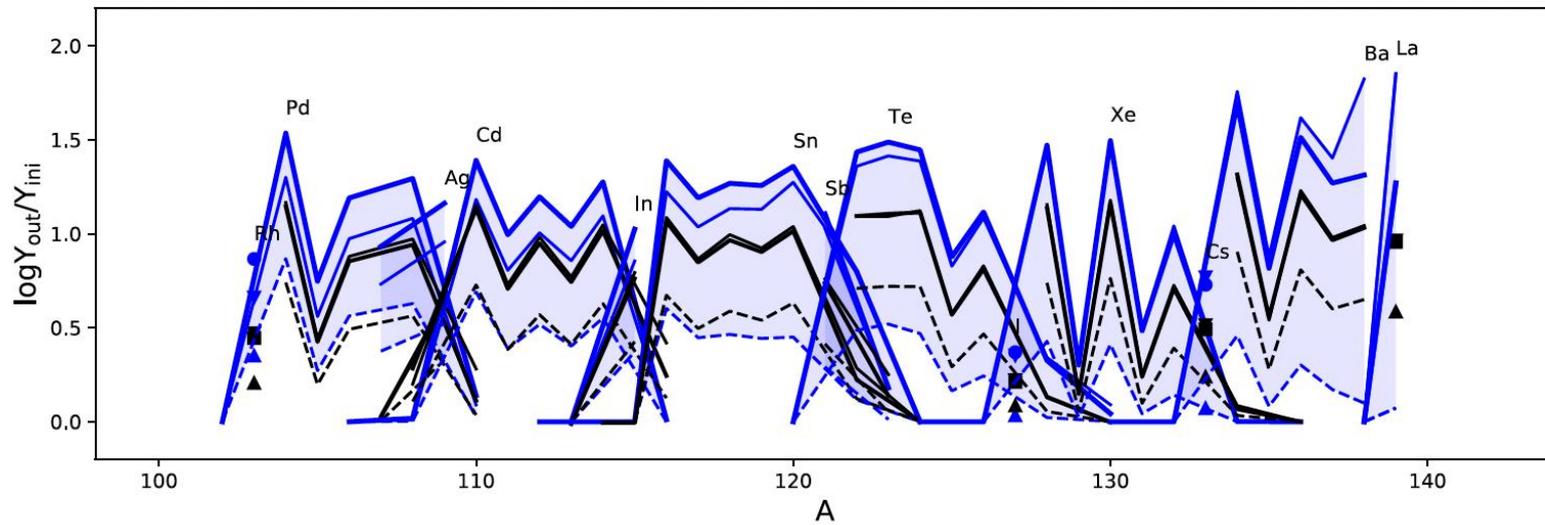
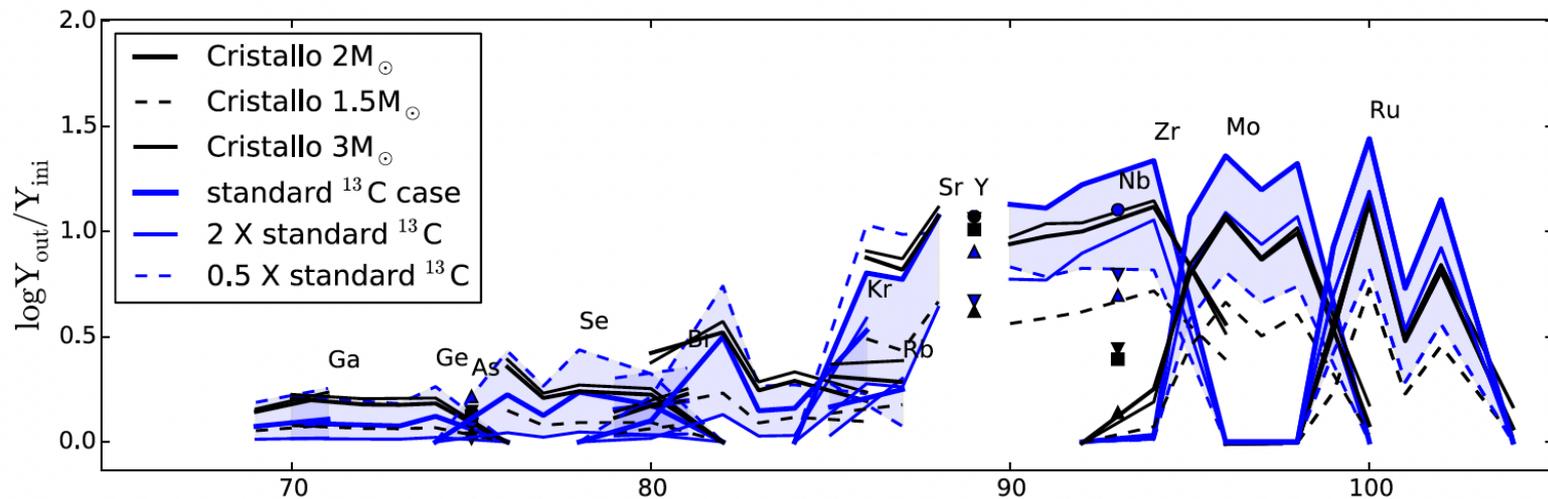
<sup>9</sup>*INAF, Osservatorio Astronomico d'Abruzzo, I-64100 Teramo, Italy*

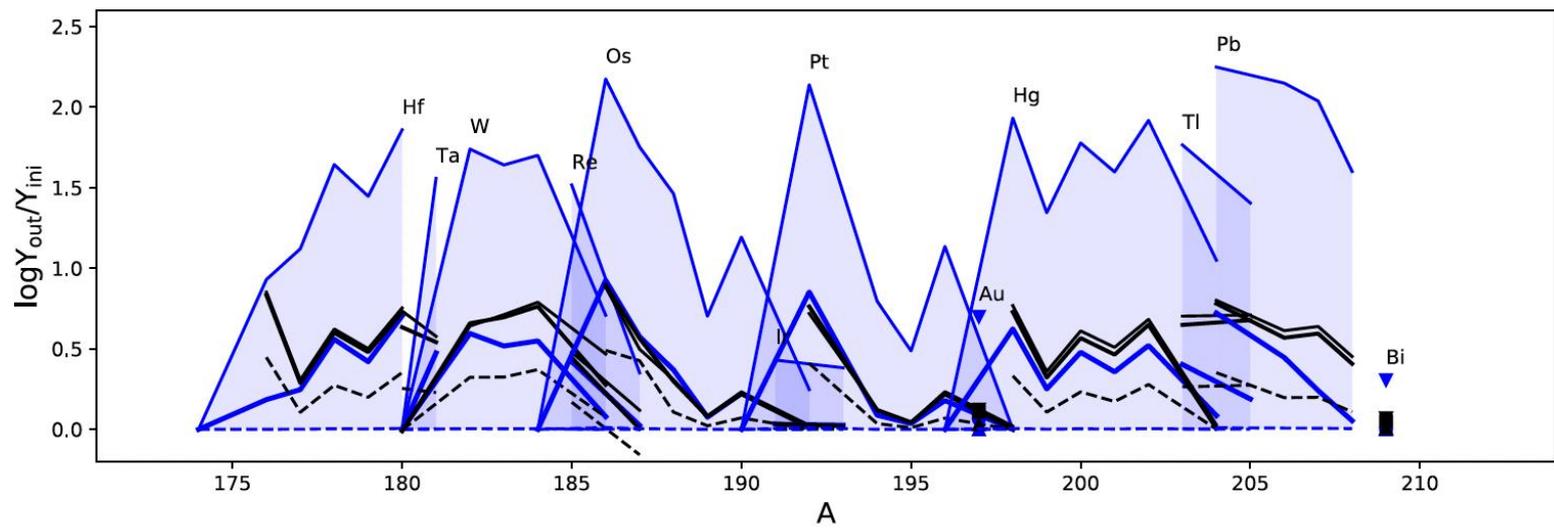
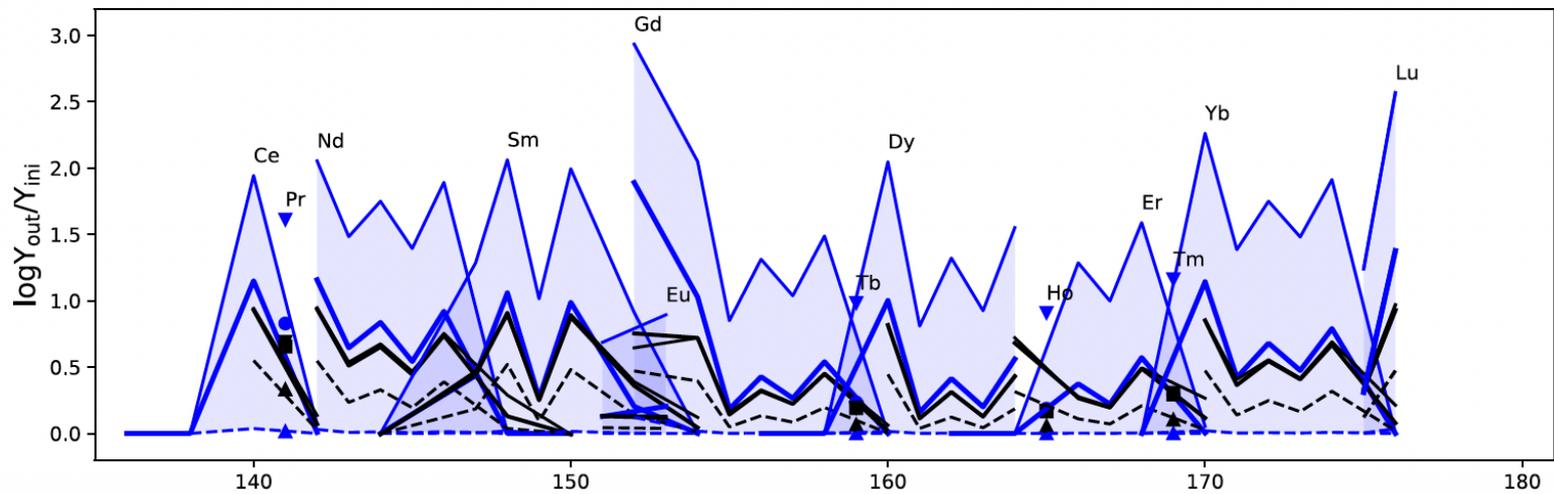
<sup>10</sup>*INFN-Sezione di Perugia, I-06123 Perugia, Italy*

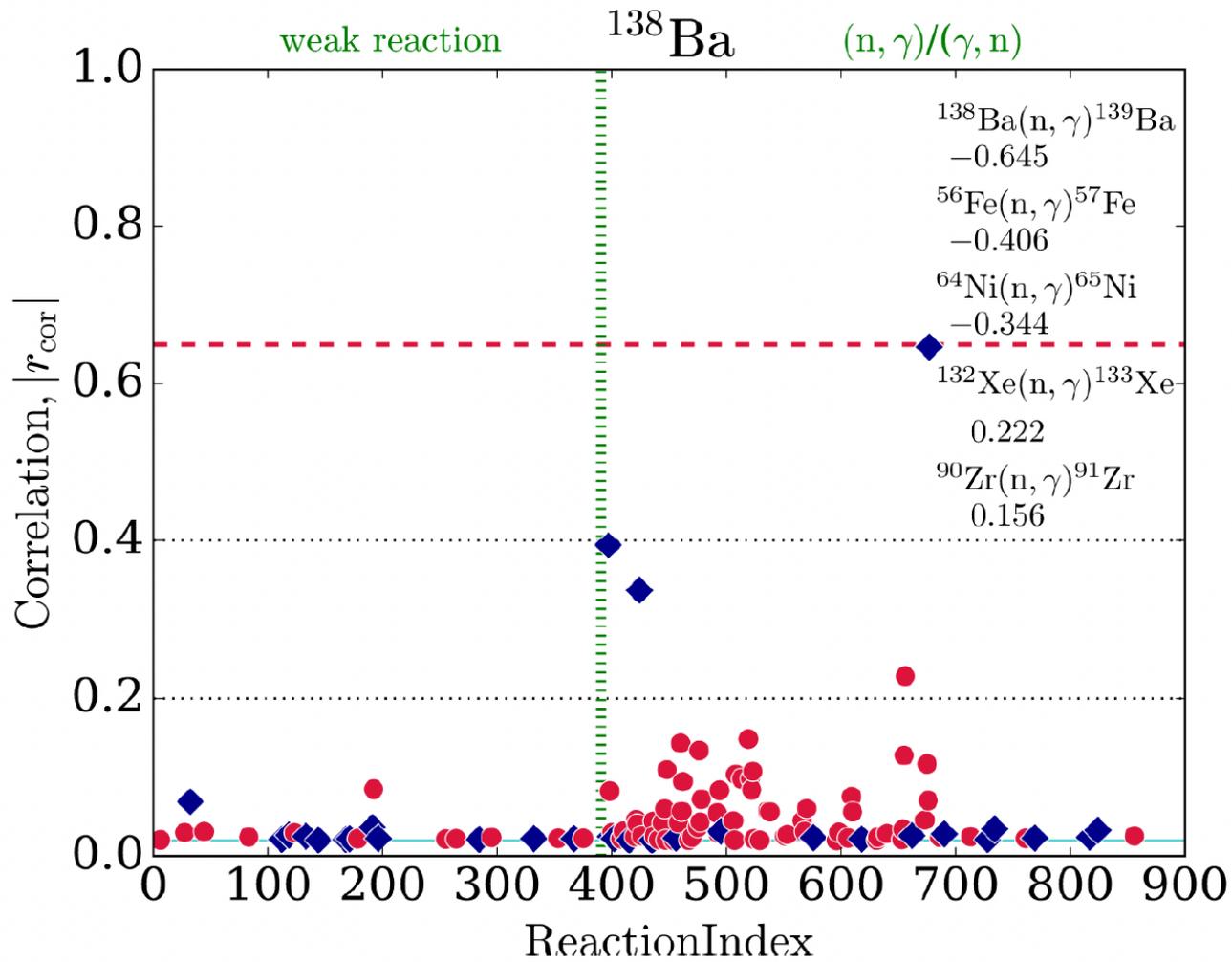
Accepted 2018 May 3. Received 2018 May 3; in original form 2017 November 8

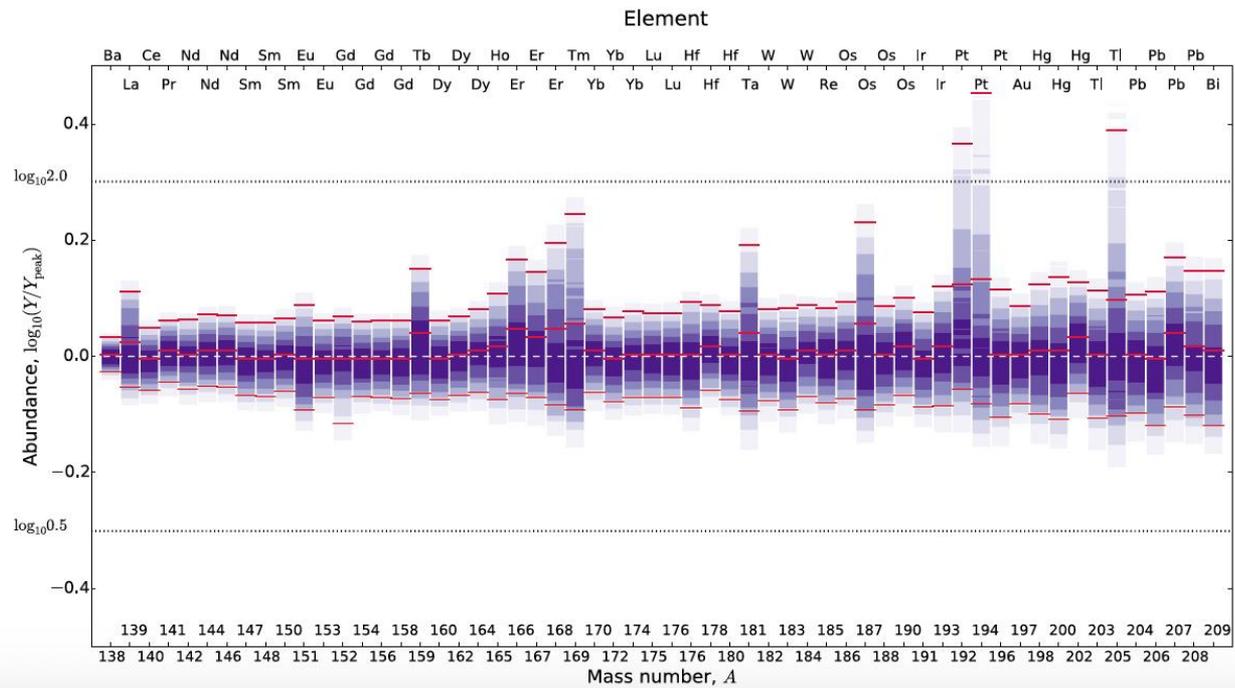
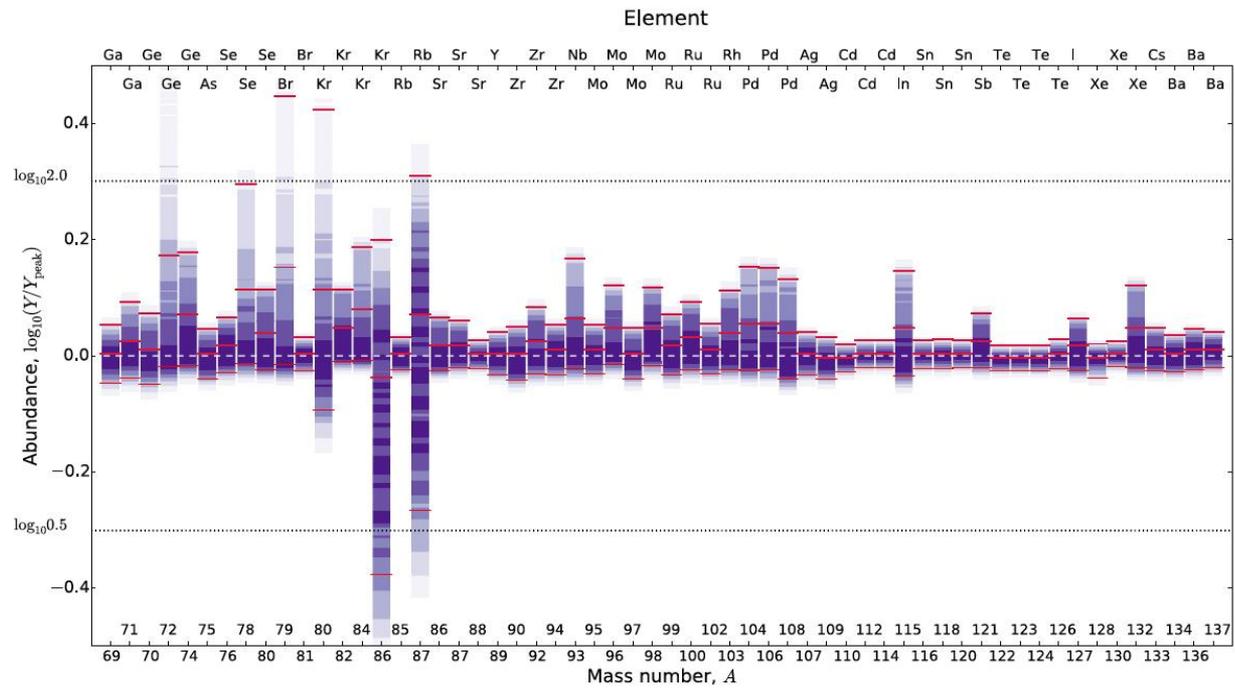


**Figure 2.** Time evolution of the neutron density ( $\text{cm}^{-3}$ ) for the three initial  $^{13}\text{C}$  abundances considered in the  $^{13}\text{C}$ -pocket and the TP phase.

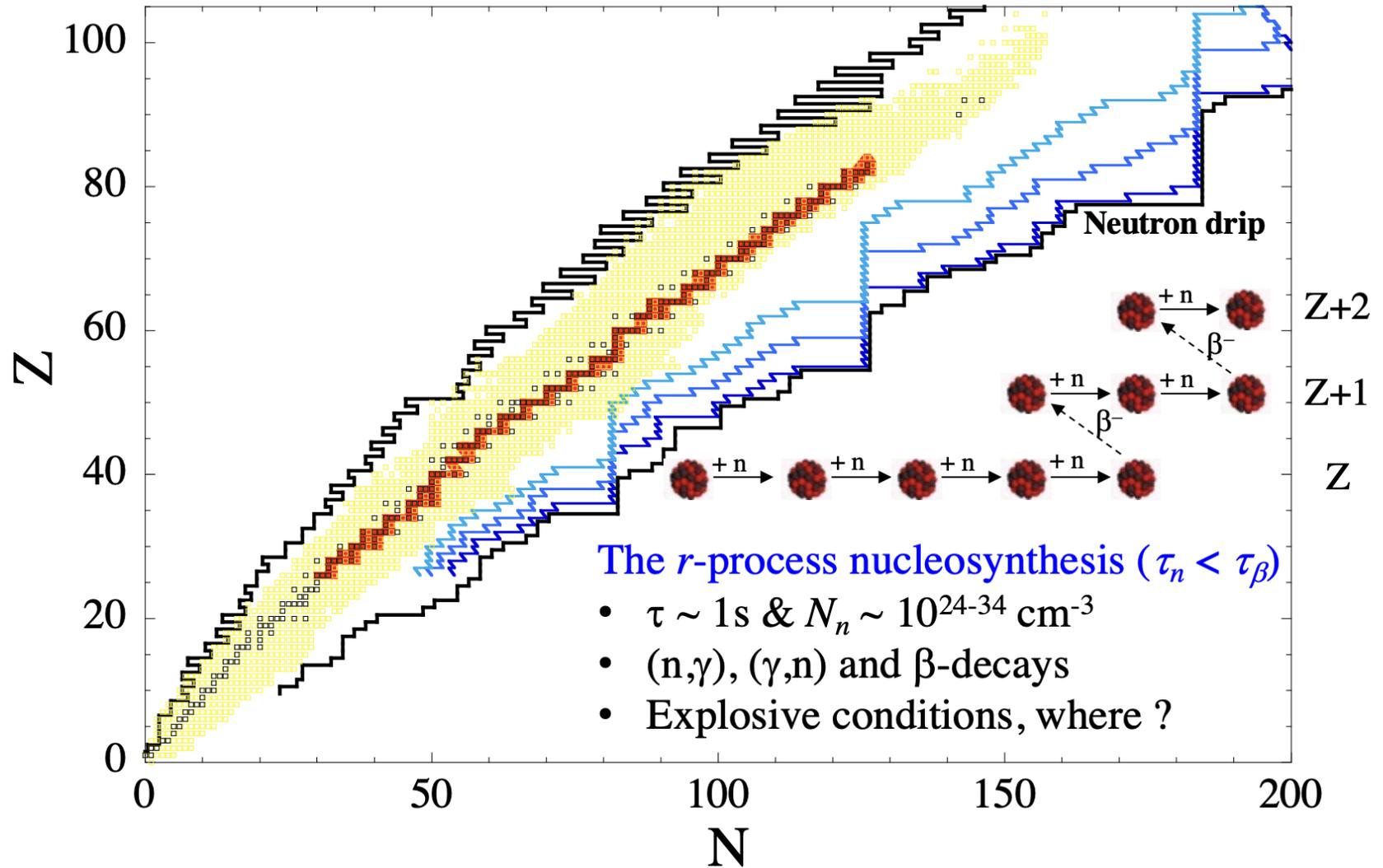






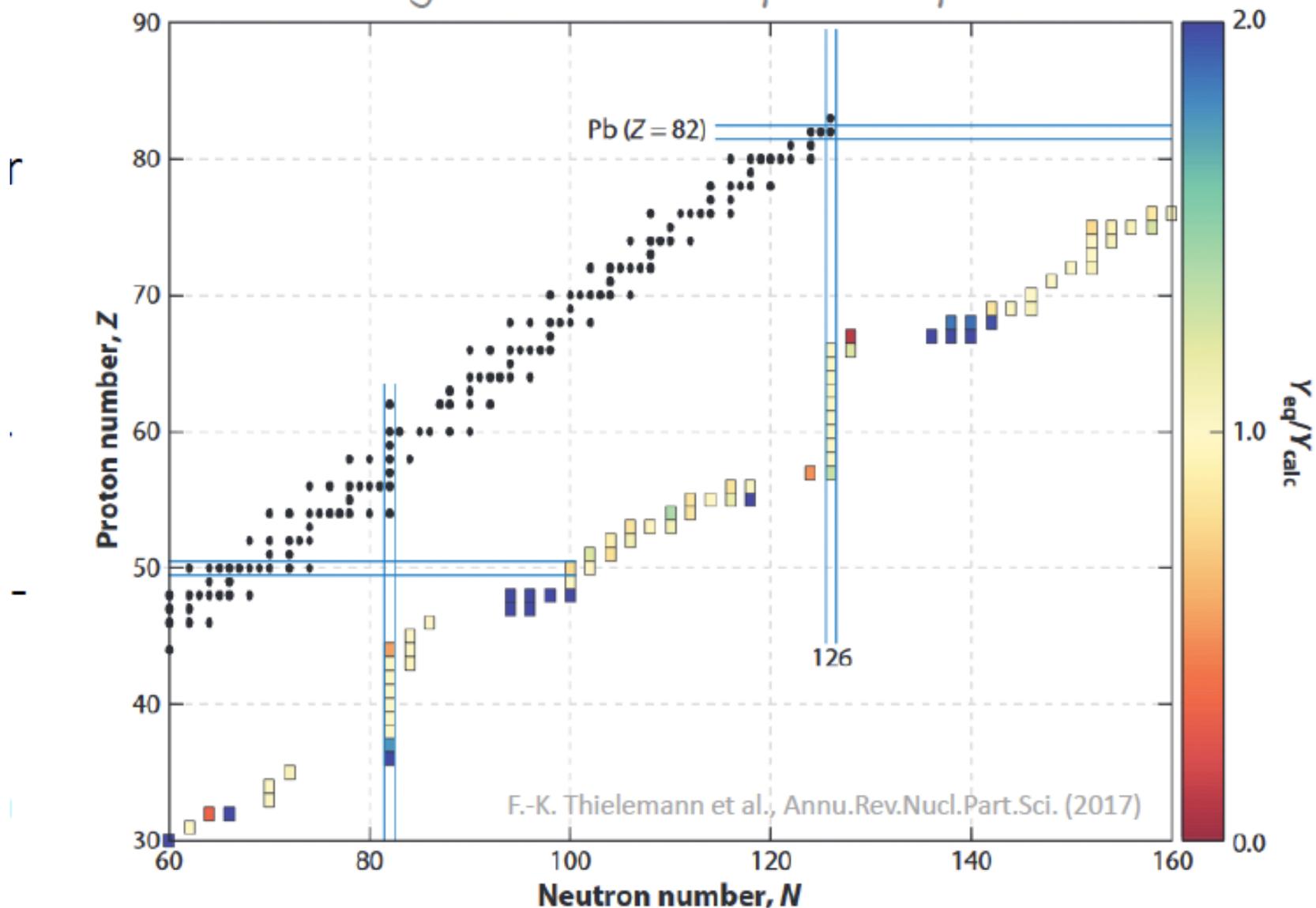


# The rapid neutron-capture process (or r-process)

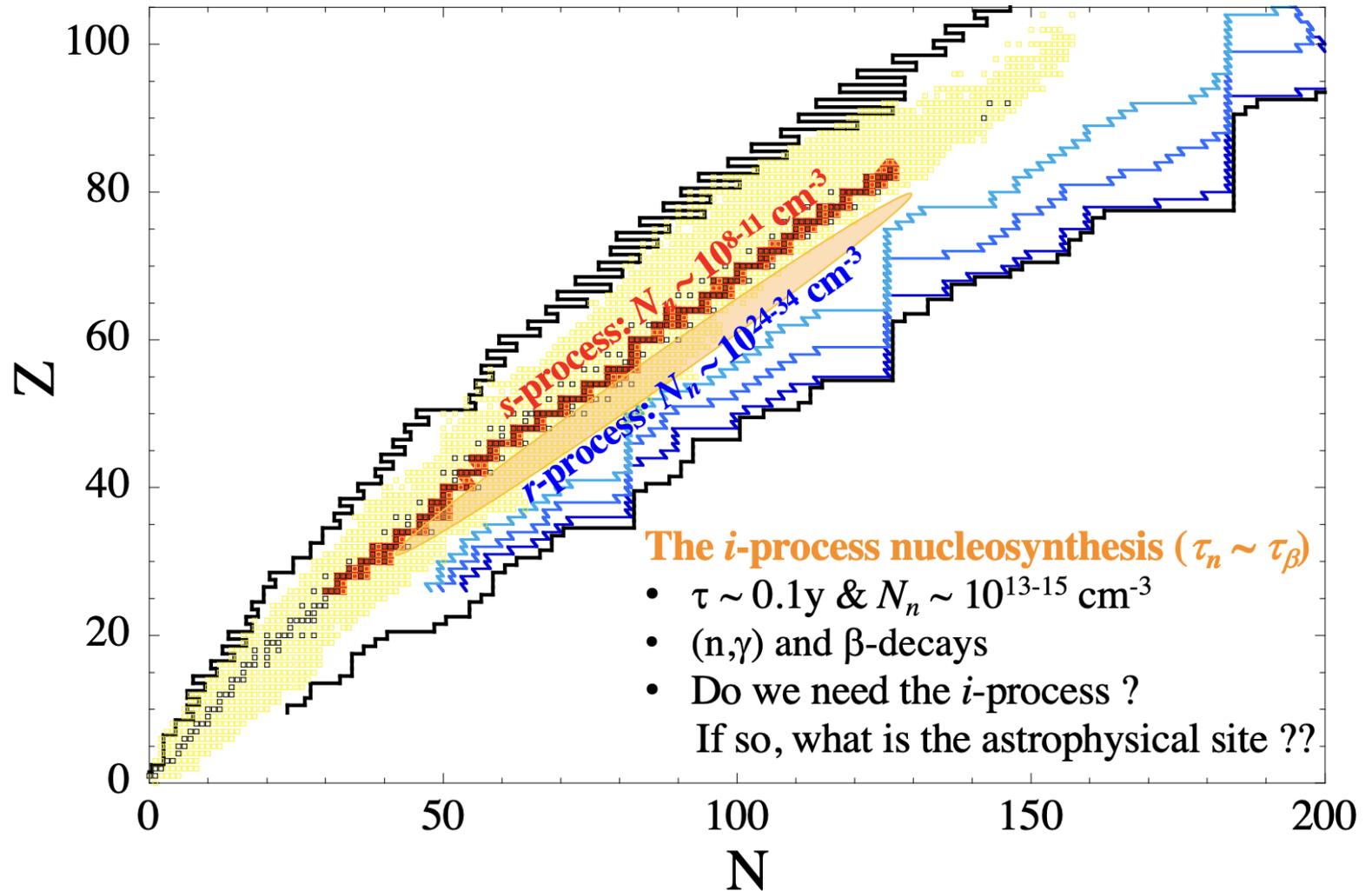


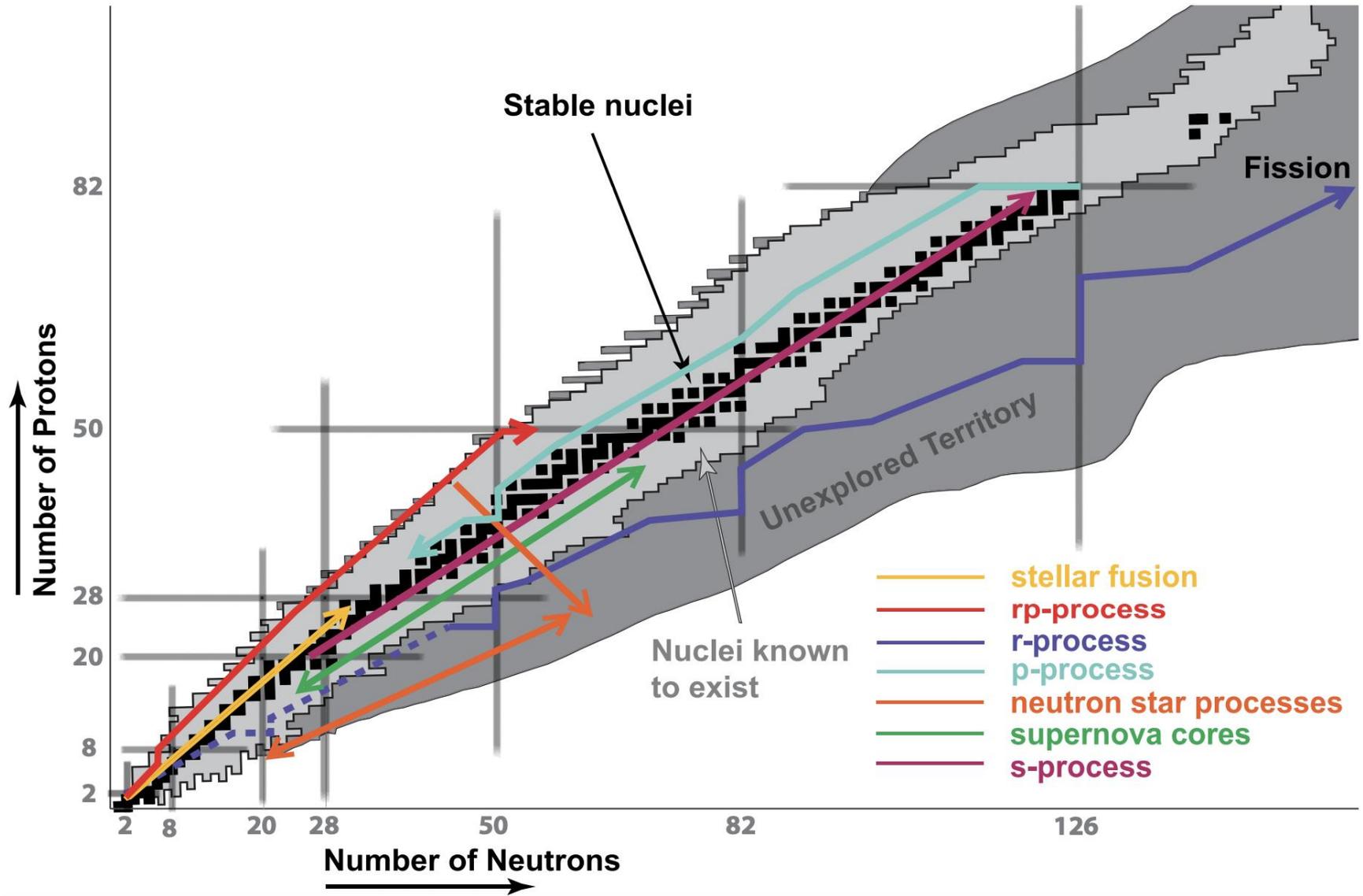
The r-process is responsible for the other half of the elements heavier than iron in the Universe

However,  $(n, \gamma) \leftrightarrow (\gamma, n)$  equilibrium is a pretty decent first guess for the *r*-process path location



## What about an intermediate neutron-capture process ?





# Summary and conclusions

Recalling that we wanted to determine the physics governing  $\square$  will be defined by the nuclear energy source in the interiors. We needed to develop a theory and understanding of nuclear physics and reactions

- We have covered the basic principles of energy production by fusion
- The PP chain and CNO cycle have been described,
- He burning by the triple-alpha reaction was introduced
- Later burning stages of the heavier elements (C,O, Si) discussed
- The r- and s-processes – origin of the elements heavier than Fe