INITIAL CONDITIONS

$$M_{gas}(0) = M_{TOT}$$

$$Z(0) = 0$$

EVOLUZIONE DEL GAS

$$\frac{dM_{gas}}{dt} = -\psi + E(t)$$

RATE AT WHICH DYING STARS RESTORE BOTH ENRICHED AND UNENRICHED MATERIAL IN ISM

$$E(t) = \int_{m(t)}^{\infty} (m - M_{REM})\,\psi(t - \tau_m)\,\varphi(m)\,dm$$

$m(t)$ ← MASS OF THE STAR BORN AT $t=0$ and DIE $t=t$

IRA $m \leq 1\,M_\odot$ LIVE FOREVER

$> 1\,M_\odot$ DIE INSTANTANEOUSLY

$\Rightarrow$ IRE

$$E(t) = \psi R$$

$$\frac{dM_{gas}}{dt} = -\psi + E = -(1-\psi)R$$

EQS. FOR THE METALS

$$\frac{d(M_Z)}{dt} = -Z\psi + E_Z(t)$$

$$E_Z(t) = \int_{m(t)}^{8} \Bigg[\underbrace{(m - M_{REM})\, Z(t-\tau_m)}_{\text{MASS OF PRISTINE METALS RESTORED IN THE ISM WITHOUT SUFFERING ANY NUCLEAR PROCESSING}} + \overbrace{m\, p_{Z_m}}^{\text{NEWLY FORMED AND EJECTED METALS}} \Bigg] \varphi(m)\, \psi(t-\tau_m)\, dm$$

IRA

$$E_Z(t) = \psi(1-R)\, y_Z + Z R \psi$$

$$\Downarrow$$

$$\frac{dM_{TOT}}{dt} = 0$$

$$\frac{dM_{gas}}{dt} = -(1-R)\psi$$

$$\frac{d(M_Z)}{dt} = \frac{d(M_{gas} Z)}{dt} = -\psi Z + \psi(1-R)\, y_Z + Z R \psi$$

$$\frac{dM_z}{dt} = -\psi z + R\psi z + (1-R)\, y_z \psi$$

$$= (-z + y_z)(1-R)\,\psi(t)$$

$$\frac{dM_{gas}}{dt} z + M_{gas}\frac{dz}{dt} = (-z + y_z)(1-R)\,\psi(t)$$

$$\cancel{-(1-R)\psi z} + M_{gas}\frac{dz}{dt} = (\cancel{-z} + y_z)(1-R)\,\psi(t)$$

$$\frac{M_{gas}\, dz}{dM_{gas}} = \frac{y_z \cancel{(1-R)}\psi}{-1\cancel{(1-R)}\psi}$$

$$dZ = \frac{-y_z\, dM_{gas}}{M_{gas}} \Rightarrow \int_0^Z dZ' = -y_z \ln\left(\frac{M_{gas}(t)}{M_{gas}(0)}\right)$$

$$M_{gas}(0) = M_{tot}(t)$$

$$Z = y_z \ln\left(\frac{M_{tot}(t)}{M_{gas}(t)}\right) \Rightarrow Z = y_z \ln \mu^{-1}$$

MODEL ONLY OUTFLOW

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$$W(t) = \lambda(1-R)\psi$$ WIND PROP. TO SFR

$$\frac{dM_{TOT}}{dt} = -\lambda(1-R)\psi$$

$$\frac{dM_{gas}}{dt} = -\psi + E(t) - \lambda(1-R)\psi = -(1-R)\psi - \lambda(1-R)\psi$$

$$= -(1+\lambda)(1-R)\psi$$

$$\frac{dM_Z}{dt} = -\psi Z + E_Z(t) - \lambda Z(1-R)\psi$$

$$= -\psi Z + \psi Z R + (1-R)y_Z\psi - \lambda Z(1-R)\psi$$

$$\cancel{\frac{dM_{gas}}{dt}} Z + \frac{dZ}{dt}M_{gas} = -\cancel{\psi Z} + \cancel{\psi} Z R + (1-R)y_Z\cancel{\psi} - \lambda Z\cancel{(1-R)}\psi$$

$$\frac{dZ\, M_{gas}}{dM_{gas}} = \frac{-y_Z}{(1+\lambda)}$$

$$dz = \frac{dM_{gs}}{M_{gs}}\left(\frac{-g_t}{1+\lambda}\right)$$

$$\boxed{M_{gs}(0) \neq M_{TOT}(t)}$$

IMPORTANT

$$z(t) = \int_{M_{gs}(0)}^{M_{gs}(t)} dM_{gs}$$

$$= \frac{g_t}{1+\lambda} \ln\left(\frac{M_{gs}(0)}{M_{gs}(t)}\right)$$

$$\boxed{M_{\cancel{gs}} \cancel{(0)} \; M_{gs}(0)}$$

$$M_{TOT}(0) = M_{gs}(0)$$

$$\frac{dM_{TOT}}{dM_{gs}} = \frac{\lambda}{1+\lambda}$$

$$M_{TOT}(t) - M_{TOT}(0) = \left(M_{gs}(t) - M_{gs}(0)\right)\left(\frac{\lambda}{1+\lambda}\right)$$

$$M_{gs}(0)\left(1 - \frac{\lambda}{1+\lambda}\right) = M_{TOT}(t) - \frac{\lambda}{1+\lambda} M_{gs}(t)$$

$$M_{gs}(0) = (1+\lambda) M_{TOT}(t) - \lambda M_{gs}(t)$$

$$\boxed{z = \frac{g_t}{(1+\lambda)} \ln\left((1+\lambda)\mu^{-1} - \lambda\right)}$$

ONLY INFALL

$M_{gas}(0) = M_0$ (12)

SMALL MASS

$$\frac{dM_{TOT}}{dt} = \Lambda(1-R)\psi \qquad A(t) = \Lambda(1-R)\psi$$

$$\frac{dM_{gas}}{dt} = -(1-R)\psi + \Lambda(1-R)\psi = (\Lambda-1)(1-R)\psi$$

CONSTANT

$$\frac{dM_Z}{dt} = -\psi Z + R\psi Z + (1-R)y_Z\psi + \Lambda Z_A(1-R)\psi$$

$$= (-Z + y_Z + \Lambda Z_A)(1-R)\psi$$

$$\cancel{\frac{dM_{gas}}{dt}} Z + M_{gas}\frac{dZ}{dt} = $$

$$(Z_A\Lambda + y_Z - Z\Lambda)(1-R)\psi$$

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$$\frac{M_{gs}\,dZ}{dM_{gs}} = \frac{(Z_A\Omega + y_t - Z\Omega)}{\Omega - 1}$$

$x = -Z\Omega$

$$\int_{M_0}^{M_{gs}} \frac{dM_{gs}}{M_{gs}} = \int_0^{Z} \frac{(\Omega - 1)\;dZ}{(Z_A\Omega + y_t - Z\Omega}$$

$$\ln\left(\frac{M_{gs}}{M_0}\right) = -\frac{(\Omega - 1)}{\Omega}\ln\left(\frac{Z_A\Omega - Z\Omega + y_t}{Z_A\Omega + y_t}\right)$$

6 I WANT ~~VALU~~ QUANTITIES COMPUTED AT TIME t!!!

$$\frac{dM_{gs}}{dM_{TOT}} = \frac{\Omega - 1}{\Omega}$$

M_{gs0}

$\equiv$

$$M_{gs}(t) - M_{gs}(0) = \left(M_{TOT}(t) - M_{TOT}(0)\right)\frac{\Omega - 1}{\Omega}$$

$$M_{gs}(0)\left(1 - \frac{\Omega - 1}{\Omega}\right) = M_{gs}(t) - M_{TOT}(t)\,\frac{\Omega - 1}{\Omega}$$

$$M_{gs}(0)\left(\frac{1}{\Omega}\right) = M_{gs}(t) - \frac{(\Omega - 1)}{\Omega}M_{TOT}(t)$$

$$M_{gs}(t) = M_{gs}(t)\,\Omega - (\Omega - 1)\,M_{Tot}(t)$$

$$\ln\left(\frac{\Omega M_{gs}(t) - (\Omega - 1) M_{Tot}(t)}{M_{gs}(1)}\right) = \frac{\Omega - 1}{\Omega} \ln\left(\frac{z_A \Omega - z\Omega + y_t}{z_A \Omega + y_t}\right)$$

$$\left(\Omega - (\Omega - 1)\bar{\mu}^{-1}\right)^{\Omega/\Omega-1} = \frac{z_A \Omega - z\Omega + y_t}{z_A \Omega + y_t}$$

$$(z_A \Omega + y_t)\left(\Omega - (\Omega - 1)\bar{\mu}^{-1}\right)^{\Omega/(\Omega-1)} = z_A \Omega - z\Omega + y_t$$

$$z\Omega = (z_A \Omega + y_t)\left(1 - \left[\Omega - (\Omega - 1)\bar{\mu}^{-1}\right]^{\Omega/\Omega-1}\right)$$

$$z = \frac{\ldots}{\Omega}$$

PRIMARY AND SECONDARY ELEMENTS

SECONDARY ELEMENT S

PRIMARY SEED Z

p_{Sm} YIELD WITH INITIAL METALLICITY $Z_\odot$

FOR A GENERIC METALLICITY $p_{Sm}\left(\frac{Z}{Z_\odot}\right)$

NEWLY PRODUCED

$$E_{S1} = \int_{m(t)}^{\infty} m\, p_{Sm} \frac{Z(t-\tau_m)}{Z_\odot} \psi(t-\tau_m)\, \varphi(m)\, dm$$

$$\overset{IRA}{=} y_S (1-R) \frac{Z}{Z_\odot} \psi$$

ALREADY PRESENT IN THE STAR

$$E_{S2} = \int_{m(t)}^{\infty} (m - m_{REM})\, X_S(t-\tau_m)\, \psi(t-\tau_m)\, \varphi(m)\, dm \overset{IRA}{=} X_S R \psi$$

$$E_{S1} + E_{S2} \overset{IRA}{=} X_S R \psi + y_S (1-R) \frac{Z}{Z_\odot} \psi$$

(2)

$$E_s = E_{s1} + E_{s2}$$

MODELLO CHIUSO

a) $$\frac{dM_{gas}}{dt} = -(1-R)\psi$$

$$\frac{d(M_{gas}X_s)}{dt} = -X_s\psi + RX_s\psi + y_s\frac{Z}{Z_0}(1-R)\psi$$

~~Molte~~

$$M_{gas}\frac{dX_s}{dt} = y_s\frac{Z}{Z_0}(1-R)\psi$$

DIVIDO PER a)

$$\frac{M_{gas}}{dM_{gas}}dX_s = -y_s\frac{Z}{Z_0}$$

Ro

RICORDIAMO DAL MODELLO CHIUSO CHE (3)

$$\frac{dM_{gas}}{dt} = -(1-R)\psi$$

$$\frac{d(M_{gas} Z)}{dt} = -\psi Z + R\psi Z + y_Z(1-R)\psi$$

$$M_{gas}\frac{dZ}{dt} = (1-R)\psi\, y_Z$$

$$M_{gas}\frac{dZ}{dM_{gas}} = -y_Z \qquad \frac{M_{gas}}{dM_{gas}} = -\frac{y_Z}{dZ}$$

AVEVAMO

$$M_{gas}\frac{dX_s}{dM_{gas}} = -y_s\frac{Z}{Z_0}$$

$$-y_Z\frac{dX_s}{dZ} = -y_s\frac{Z}{Z_0}$$

$$dX_s = \frac{y_s}{y_Z}\frac{Z}{Z_0}dZ \iff X_s = \frac{1}{2}\frac{y_s}{y_Z}\frac{Z^2}{Z_0}$$