

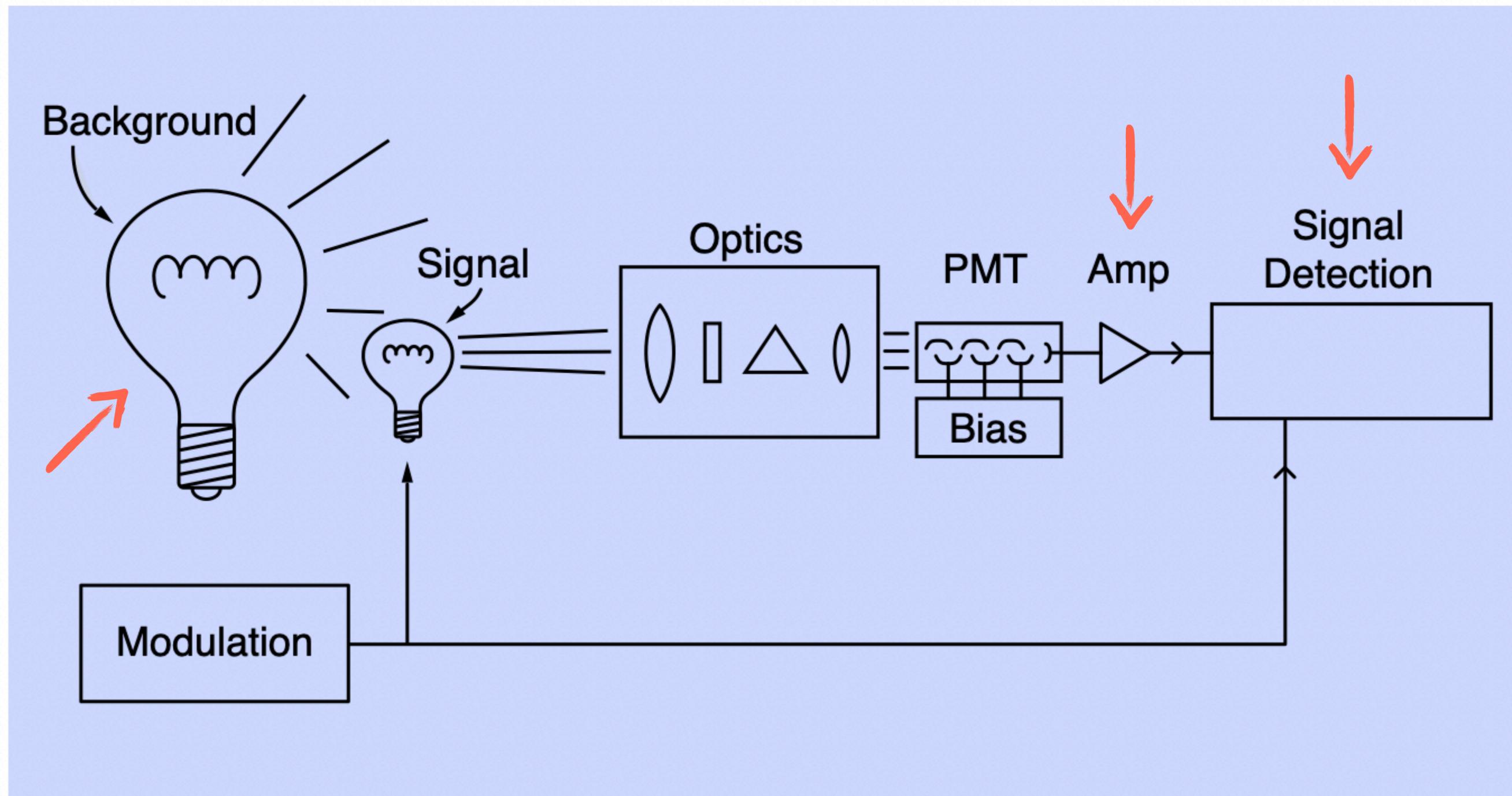
# **Recovering signals out of noise: the lock-in amplifier**

**Laboratorio di Fisica della Materia Condensata, a.a. 2025/26**

# Noise is ubiquitous in experiments!

- \* Almost all measurements are done using electronic equipment. This means that experimental techniques rely on the **quantitative measurement of electrons** (voltages, currents, charge etc.)
  - E.g. measurement of a DC anode current by an ampèremeter, charge counting in the pixels of a charge-coupled device (CCD) camera chip, ...
- \* Any electronic signal always shows random, uncorrelated fluctuations: **noise**
- \* The signal of interest may be **obscured by noise**! The noise may be fundamental to the process: e.g. discrete charges (as well as discrete light quanta, i.e. photons) are governed by Poisson statistics which gives rise to shot noise.

# Prototype experiment



# Two types of noise

- \* Sometimes noise is **extrinsic** and “**non-essential**”: it can be minimised by good laboratory practice
- \* Often noise sources are **intrinsic**, related to the physics (and the statistics) of the system and the probe used in the measurements: this cannot be acted upon

Intrinsic sources of noise:

- Shot noise
  - Johnson-Nyquist noise
  - Flicker noise (aka  $1/f$  noise)
- \* Understanding the noise sources in a measurement is critical to achieving a satisfactory signal-to-noise performance! → The quality of a measurement may be substantially degraded by a trivial error...

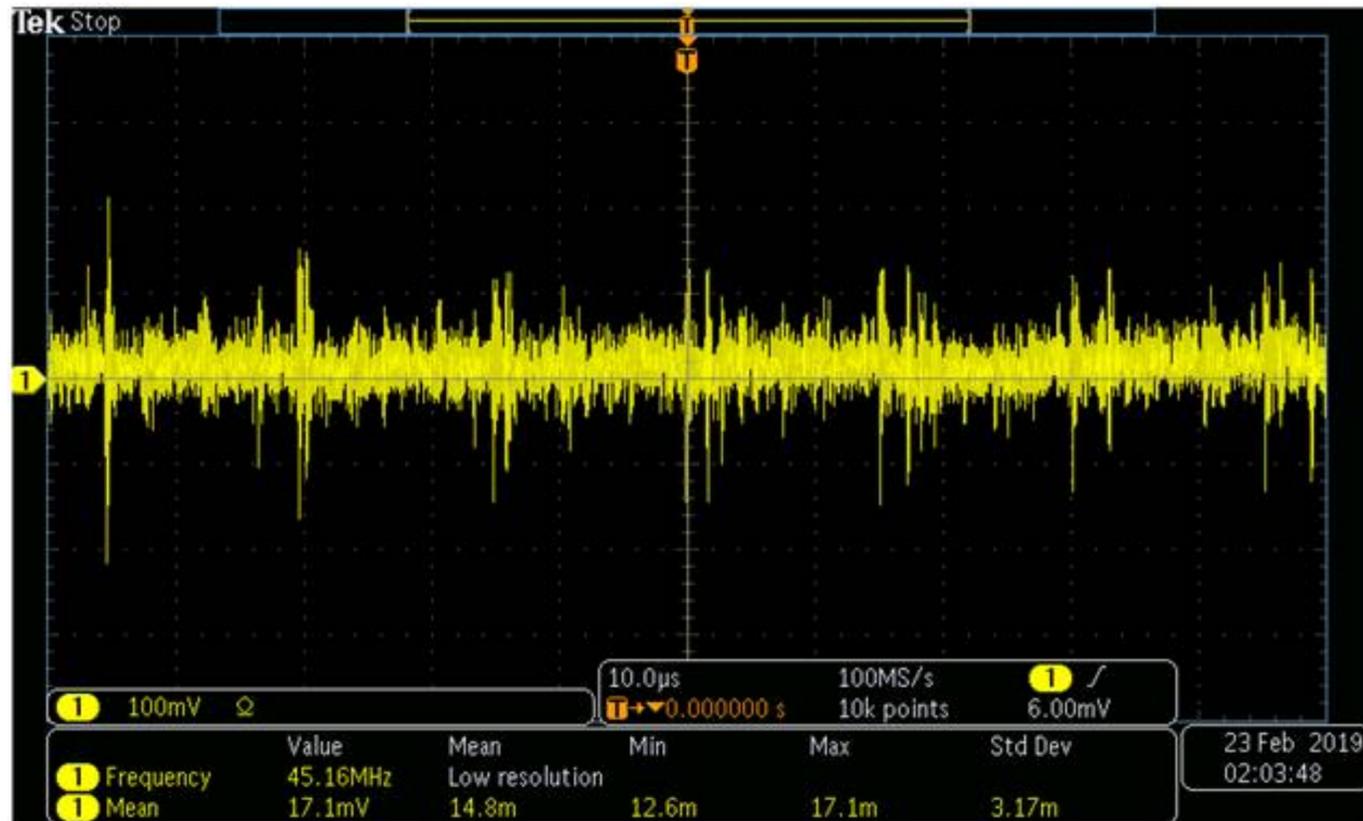
# Spectral noise density

- \* Noise characteristics of a system are often represented by the noise spectral density (PSD)
- \* Let's consider a quantity of interest  $X(t)$ : the PSD  $S_X(f)$  is defined as the squared modulus of the Fourier transform of  $X(t)$

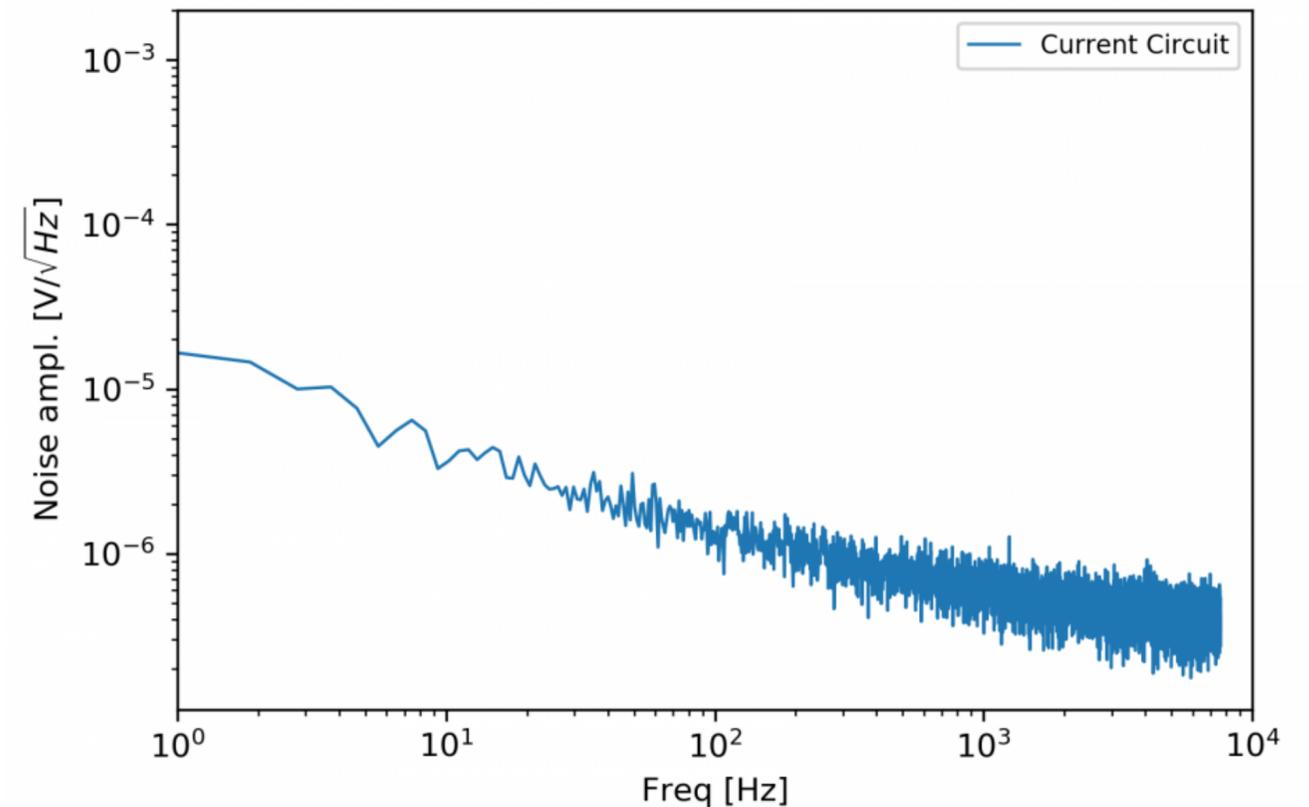
$$S_X(f) = \lim_{T \rightarrow \infty} \frac{1}{T} \left\langle \left| \int_{-T/2}^{+T/2} X(t) e^{+i2\pi ft} dt \right|^2 \right\rangle$$

- \* PSDs are statistical measures: they can be estimated from real data by averaging over many measurements → Taking a single measurement trace gives only a rough estimate of the PSD

# Noise PSD in practice...



FT  
→

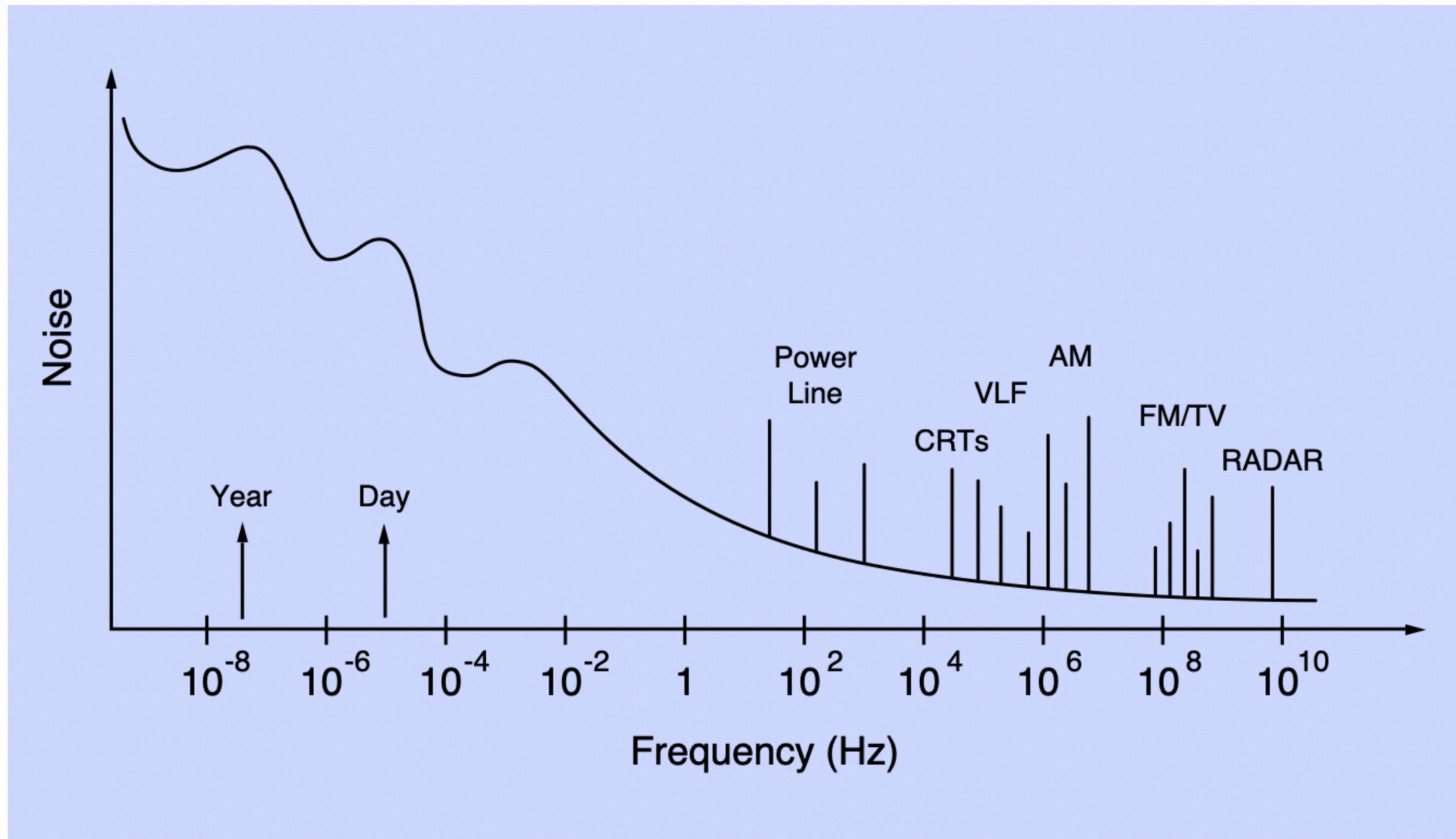


Remove signal mean,  
only fluctuations around the mean

Units of  $V/\sqrt{\text{Hz}}$

An RMS is obtained by integrating the PSD over a chosen frequency window

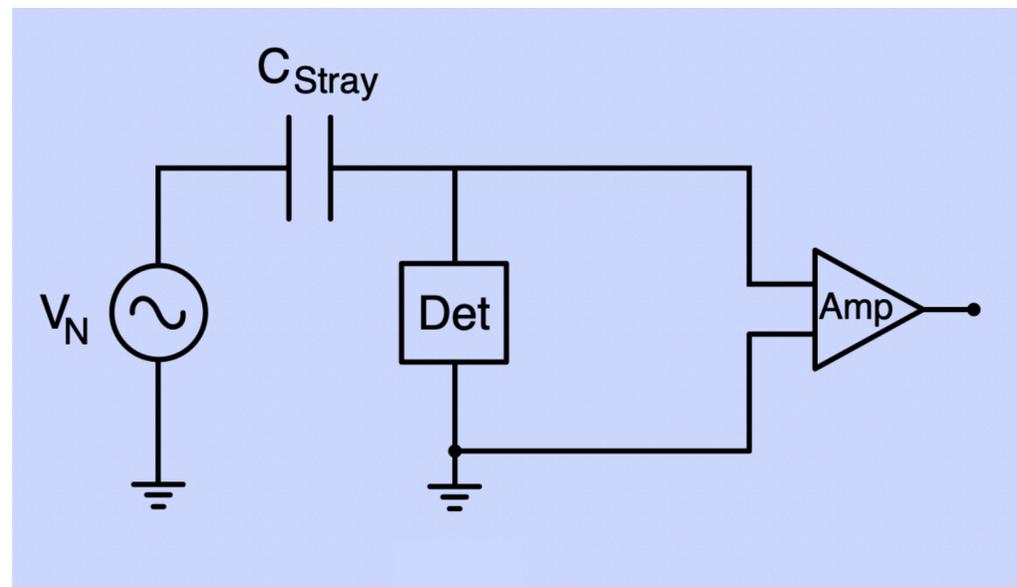
# Examples of PSD and noise sources



# Examples of extrinsic noise sources

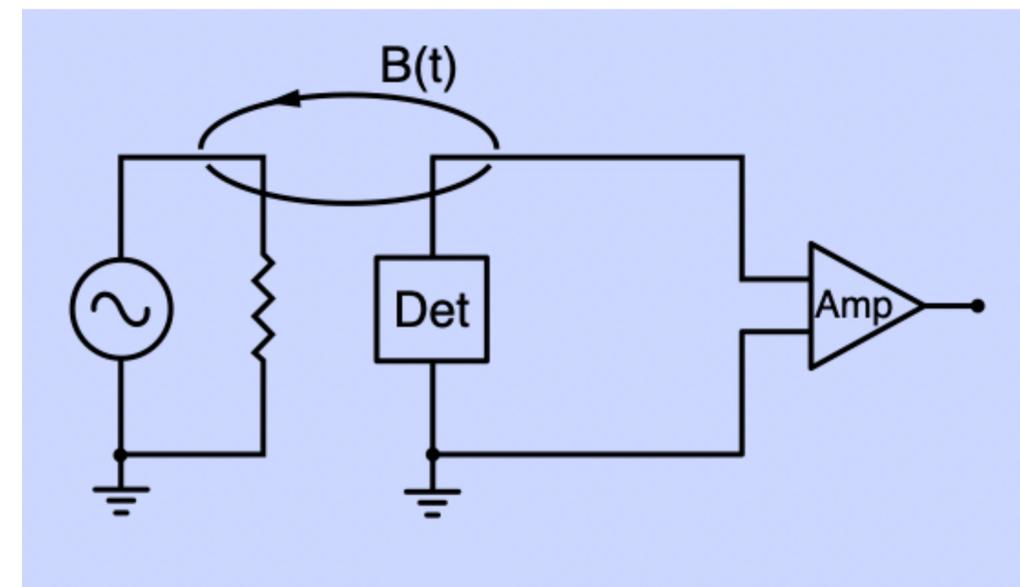
## Capacitive and inductive couplings

- \* Noise can be picked up through the **capacitive coupling** with a nearby apparatus with varying voltage



**Cure:** shielding the detector

- \* Noise can be picked up through the **inductive coupling** to a time-varying magnetic field, which induces a e.m.f. in the detection circuit



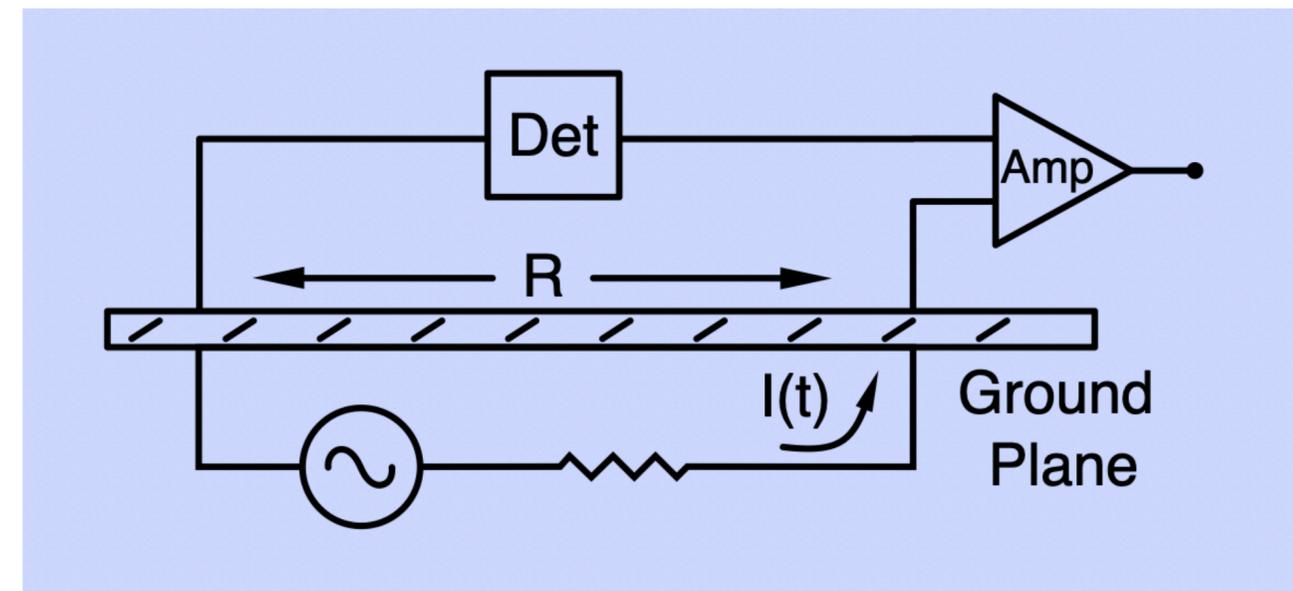
**Cure:** use twisted pairs or coaxial cables

# Examples of extrinsic noise sources

## Resistive couplings: ground loops

- \* Currents through common connections can give rise to noisy voltages. E.g. the detector can be contaminated by the noise on the ground bus.

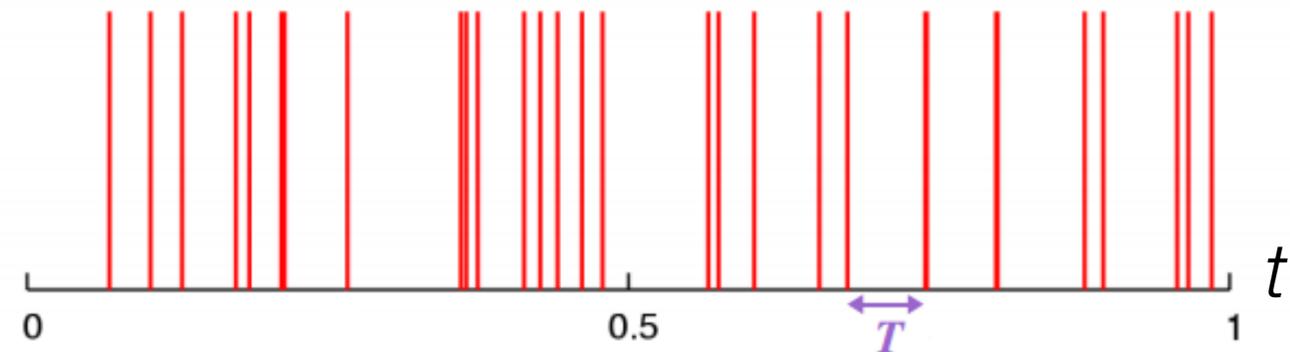
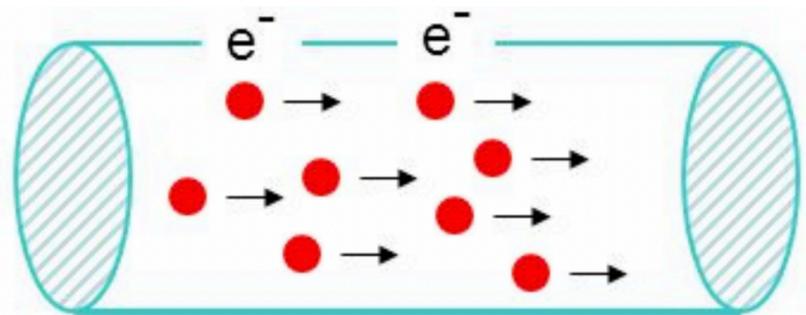
**Cure:** ground all instruments to the same point, remove sources of large currents from ground wires used for small signals!



# Examples of intrinsic noise sources

## Shot noise

- \* Light and electrical charge are quantized: so the number of photons or electrons which pass a point during a period of time are subject to Poissonian statistical fluctuations.



→ if the signal mean is  $M$  photons, the standard deviation (noise) will be  $\sqrt{M}$ , hence the  $S/N = M/\sqrt{M} = \sqrt{M}$ .

- \*  $M$  may be increased by increasing the photon rate (laser power) or increasing the integration time.

# Examples of intrinsic noise sources

## Johnson-Nyquist noise (or thermal noise)

- \* In a conductor there are a large number of moving electrons. Point-by-point their density shows statistical fluctuations at finite temperature as a function of time, like the local density of air in a given point of a room. These density fluctuations give rise to voltage fluctuations:

$$V_{\text{JN,rms}} = 4k_B R T \Delta f$$

where  $R$  is resistance of the conductor,  $k_B$  is Boltzmann's constant,  $T$  is the temperature, and  $\Delta f$  is the bandwidth over which the noise is measured.

- \* This is a *white* noise, since its PSD does not depend on the frequency, i.e. this noise contains Fourier components at any frequency. Example: for  $1\text{M}\Omega$  resistor the J-N noise is  $V_{\text{JN,rms}} \simeq 100\mu\text{V}$  between 0 and 100 kHz

# Examples of intrinsic noise sources

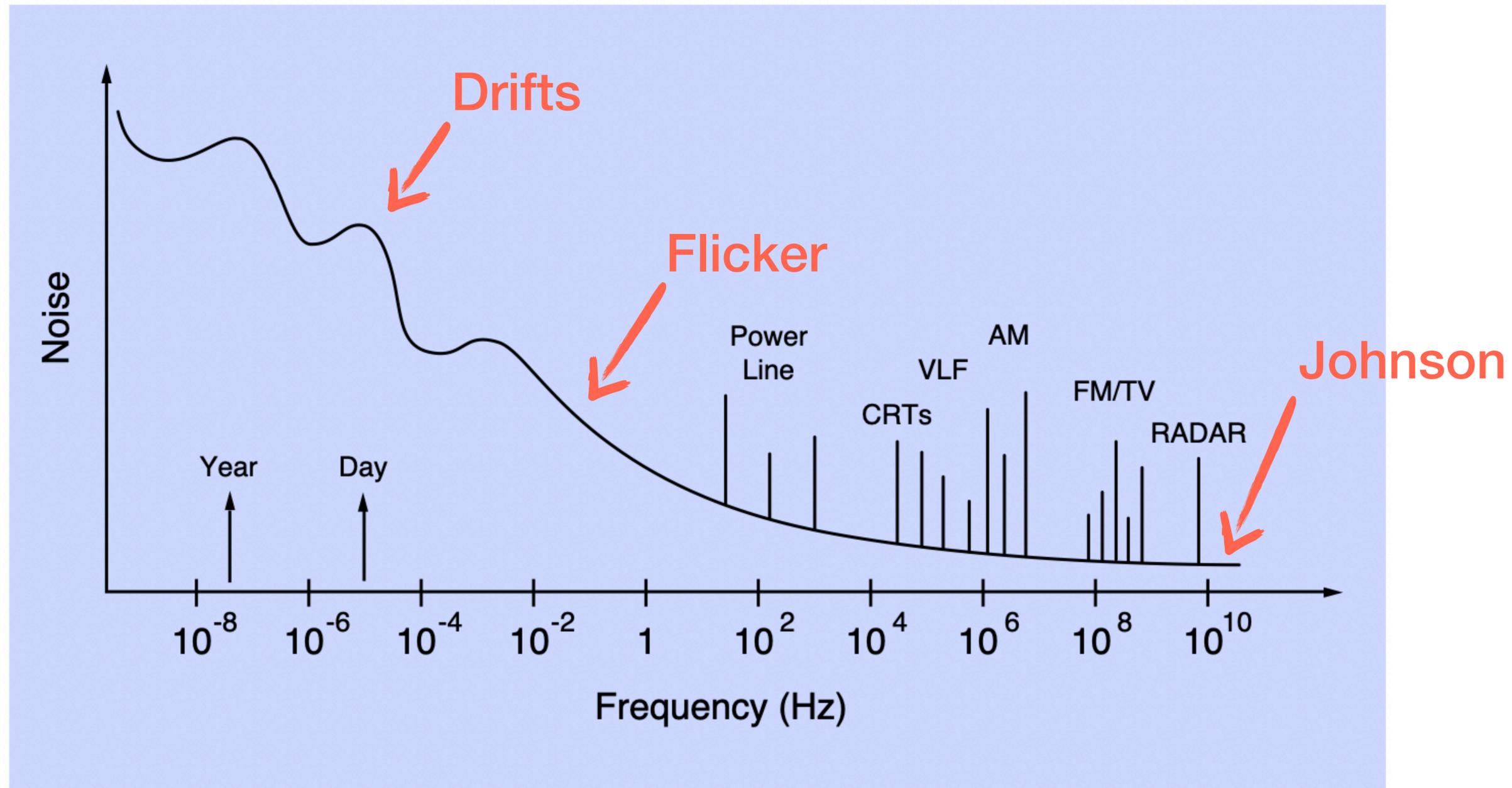
## Flicker noise (or 1/f noise)

- \* The voltage across a resistor carrying a constant current will fluctuate because the resistance of the material used in the resistor fluctuates, giving rise to a frequency-dependent noise. However, there is no general accepted theory that explains it in all the cases where the 1/f noise is present. It occurs in almost all electronic devices:

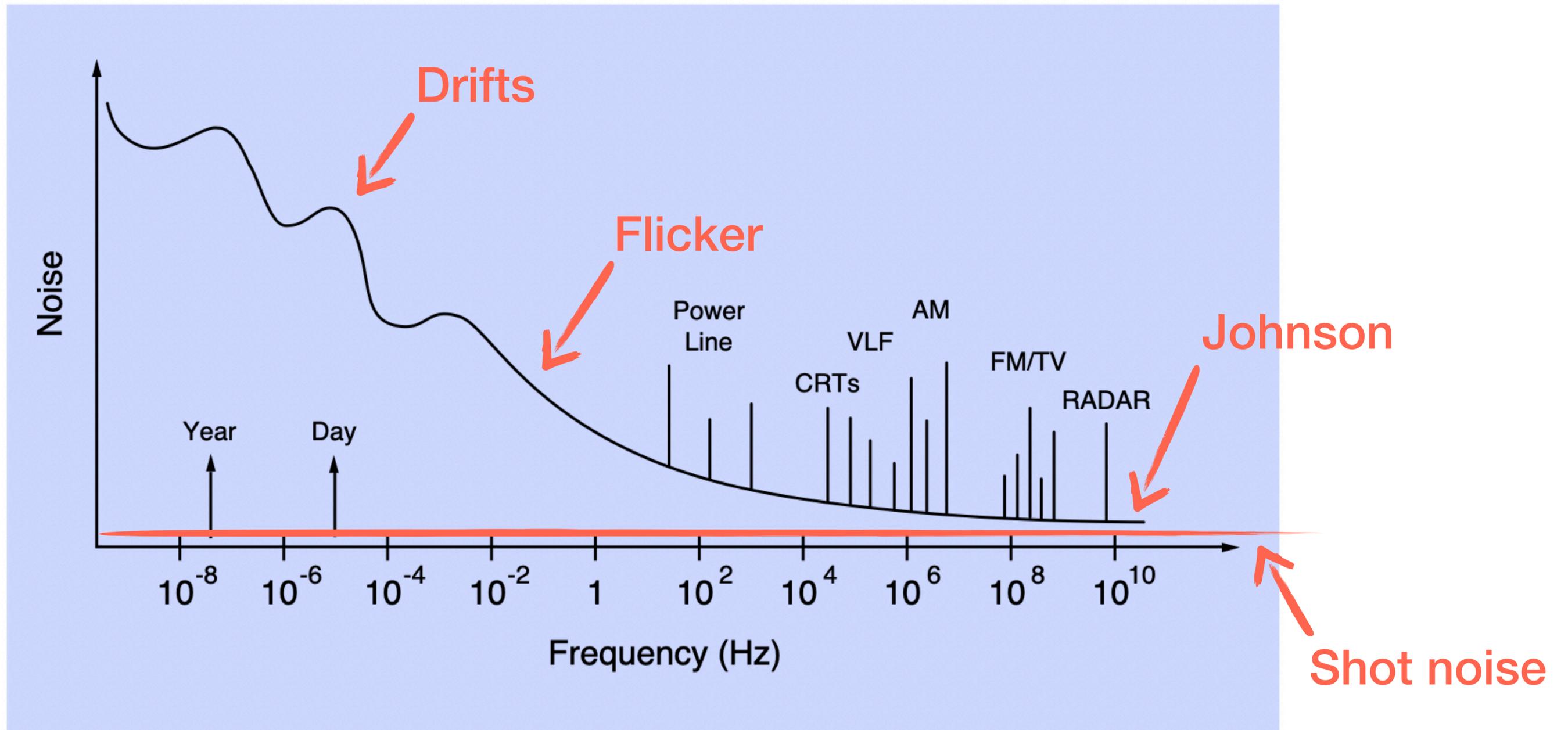
$$V_{\text{pink,rms}}^2 \propto \Delta f / f$$

- \* This is a *pink* noise, and it will impact mostly the low-frequency part of the noise spectral density

# Examples of noise sources



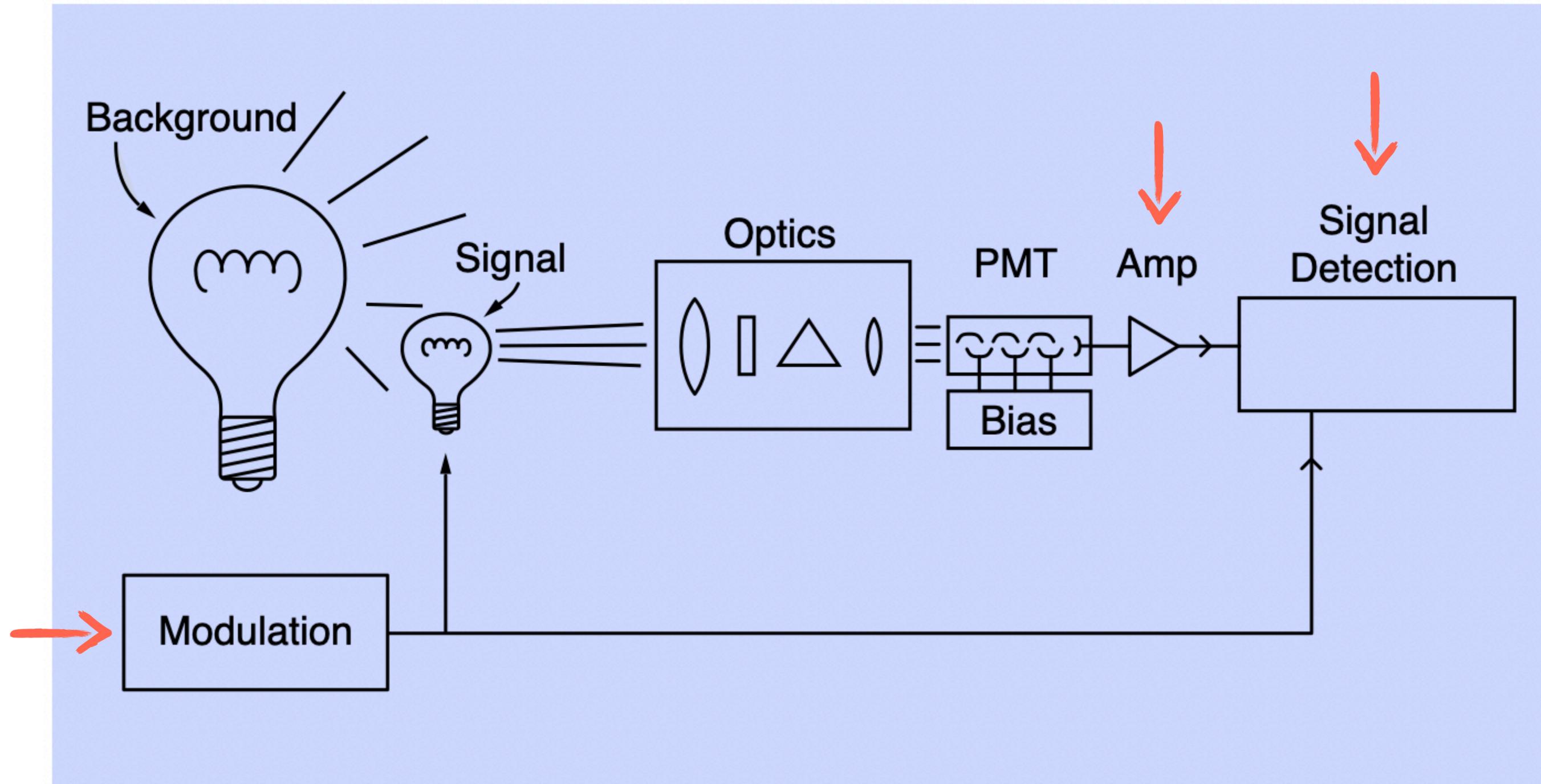
# Examples of noise sources



# Measuring small signals: amplifiers

- \* Several considerations are involved in choosing the correct amplifier for a particular application: bandwidth, gain, impedance, noise characteristics...
- \* General technique: perform AC measurements to avoid noise close to DC frequencies → Modulate the source and analyse at the modulation frequency
- \* When the source is modulated, one may choose from gated integrators, boxcar averagers, transient digitizers, lock-in amplifiers, spectrum analyzers... We will only use the **lock-in amplifier**
  - Lock-in amplifiers are used to detect and measure very small AC signals... all the way down to a few nV!

# Prototype experiment

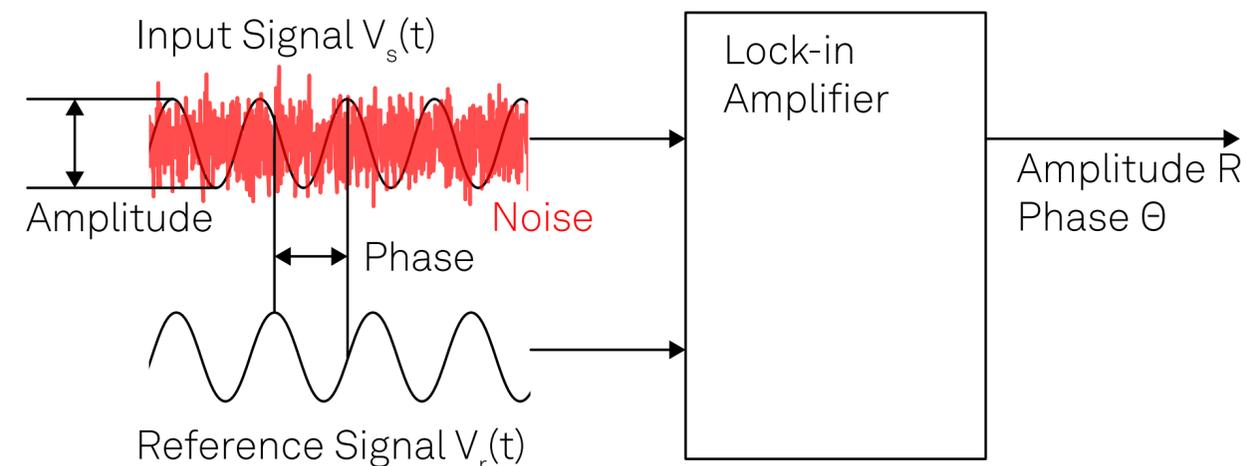


# Let's see an example...

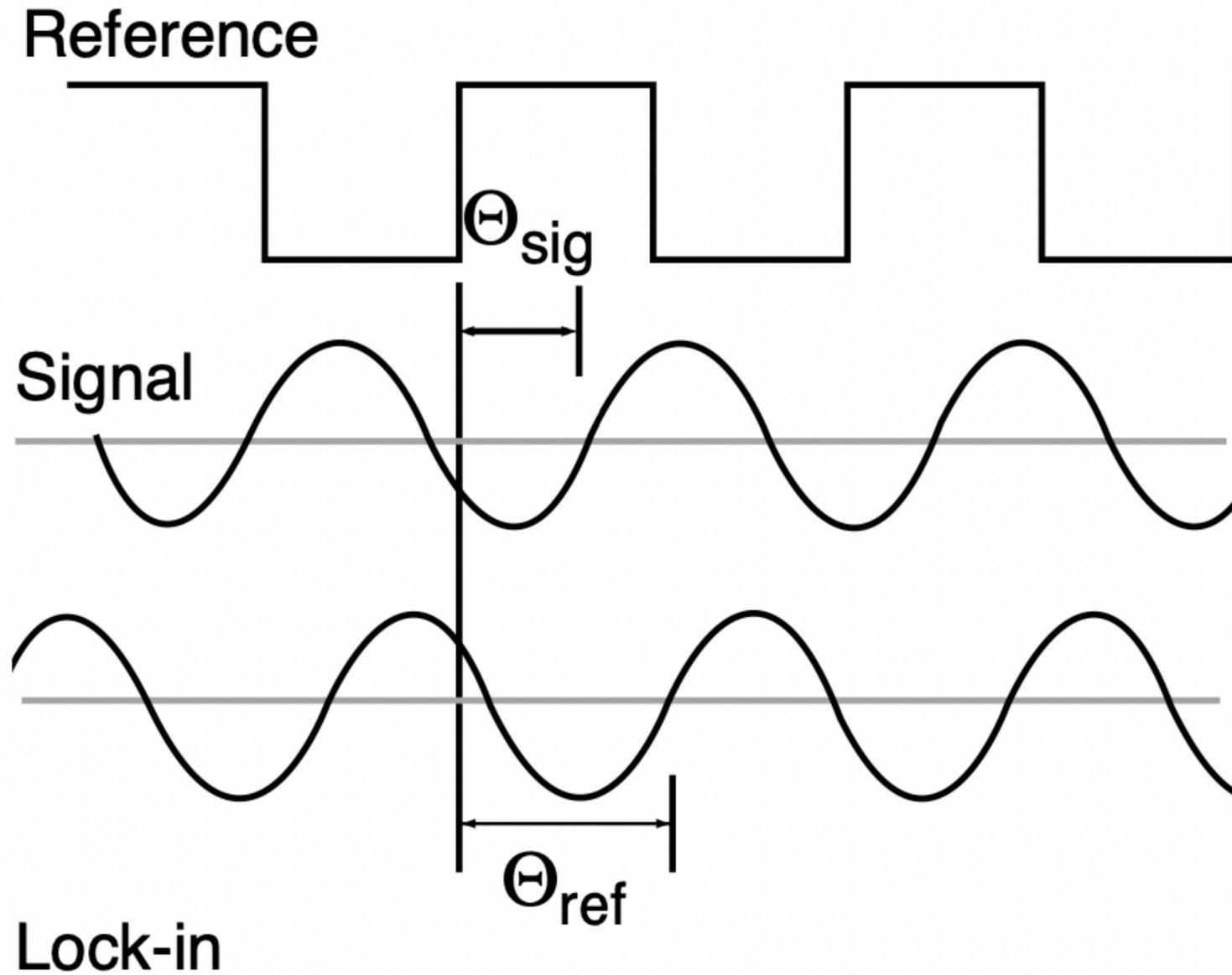
- \* Suppose the signal is a 10 nV sine wave at 10 kHz. Clearly some amplification is required to bring the signal above the noise. A very good low-noise amplifier will have about 5 nV/ $\sqrt{\text{Hz}}$  of input noise.
- \* If the amplifier bandwidth is 100 kHz and the gain is 1000, we can expect our output to be 10  $\mu\text{V}$  of signal (10 nV  $\times$  1000) and 1.6 mV of broadband noise (5 nV/ $\sqrt{\text{Hz}}$   $\times$   $\sqrt{100 \text{ kHz}}$   $\times$  1000) 😞
- \* Supposing we know the frequency of our signal, we follow the amplifier with a very good band-pass filter with a Q=100 centered at 10 kHz. Any signal in a 100 Hz bandwidth will be detected (10 kHz/Q). The noise in the filter pass band will be 50 $\mu\text{V}$  (5 nV/ $\sqrt{\text{Hz}}$   $\times$   $\sqrt{100\text{Hz}}$   $\times$  1000), and the signal will still be 10  $\mu\text{V}$ .
  - The output noise is much greater than the signal! 😞

# Phase-sensitive detection

- \* An amplifier with a phase-sensitive detector can detect the signal at 10 kHz with a bandwidth as narrow as 0.01 Hz: in our previous example, the noise in the detection bandwidth will be  $0.5 \mu\text{V}$  ( $5 \text{ nV}/\sqrt{\text{Hz}} \times \sqrt{0.01 \text{ Hz}} \times 1000$ ), while the signal is still  $10 \mu\text{V}$  😊
- \* How to achieve this? Lock-in measurements require a **frequency reference**. Typically, an experiment is excited at a fixed modulation frequency, and the lock-in detects the response from the experiment only at the reference frequency.



# Phase-sensitive detection



$$V_{\text{sig}} \sin(\omega_r t + \theta_{\text{sig}})$$

$$V_L \sin(\omega_L t + \theta_{\text{ref}})$$

# Phase-sensitive detection

- \* The lock-in amplifies the signal and then multiplies it by the lock-in reference using a phase-sensitive multiplier (a mixer). The output is simply the product of two sine waves:

$$\begin{aligned} V_{\text{out}} &= V_{\text{sig}} V_L \sin(\omega_r t + \theta_{\text{sig}}) \sin(\omega_L t + \theta_{\text{ref}}) \\ &= \frac{1}{2} V_{\text{sig}} V_L \cos([\omega_r - \omega_L]t + \theta_{\text{sig}} - \theta_{\text{ref}}) - \\ &\quad \frac{1}{2} V_{\text{sig}} V_L \cos([\omega_r + \omega_L]t + \theta_{\text{sig}} + \theta_{\text{ref}}) \end{aligned}$$

- \* The output is composed by two AC signals, one at the difference-frequency ( $\omega_r - \omega_L$ ) and the other at the sum-frequency ( $\omega_r + \omega_L$ ). If the output is further passed through a low-pass filter, the high-frequency signal (2nd term above) will be removed.
- \* What will be left? Let's consider the case in which  $\omega_L = \omega_r \dots$

# Lock-in amplification

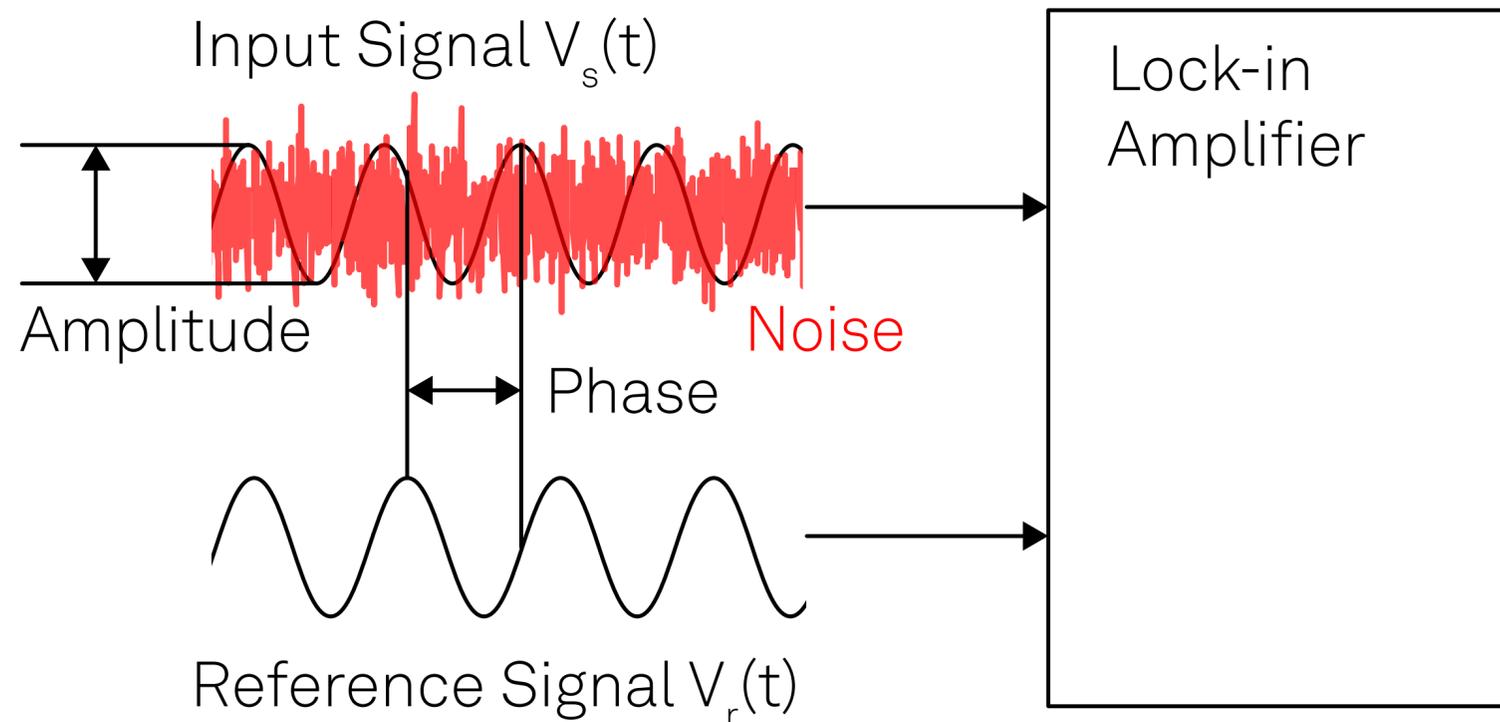
- \* If  $\omega_L$  equals  $\omega_r$ , the difference-frequency component will be a DC signal!  
In this case, the filtered output will be:

$$V_{\text{out}} = \frac{1}{2} V_{\text{sig}} V_L \cos(\theta_{\text{sig}} - \theta_{\text{ref}})$$

- \* This is a very nice output — it is a **DC voltage** proportional to the original sinusoidal signal amplitude  $V_{\text{sig}}$  😊

We have converted the signal at the modulation frequency  $\omega_L$  into a DC signal, while signals at any other frequency are attenuated by the filtering.

# Lock-in amplification



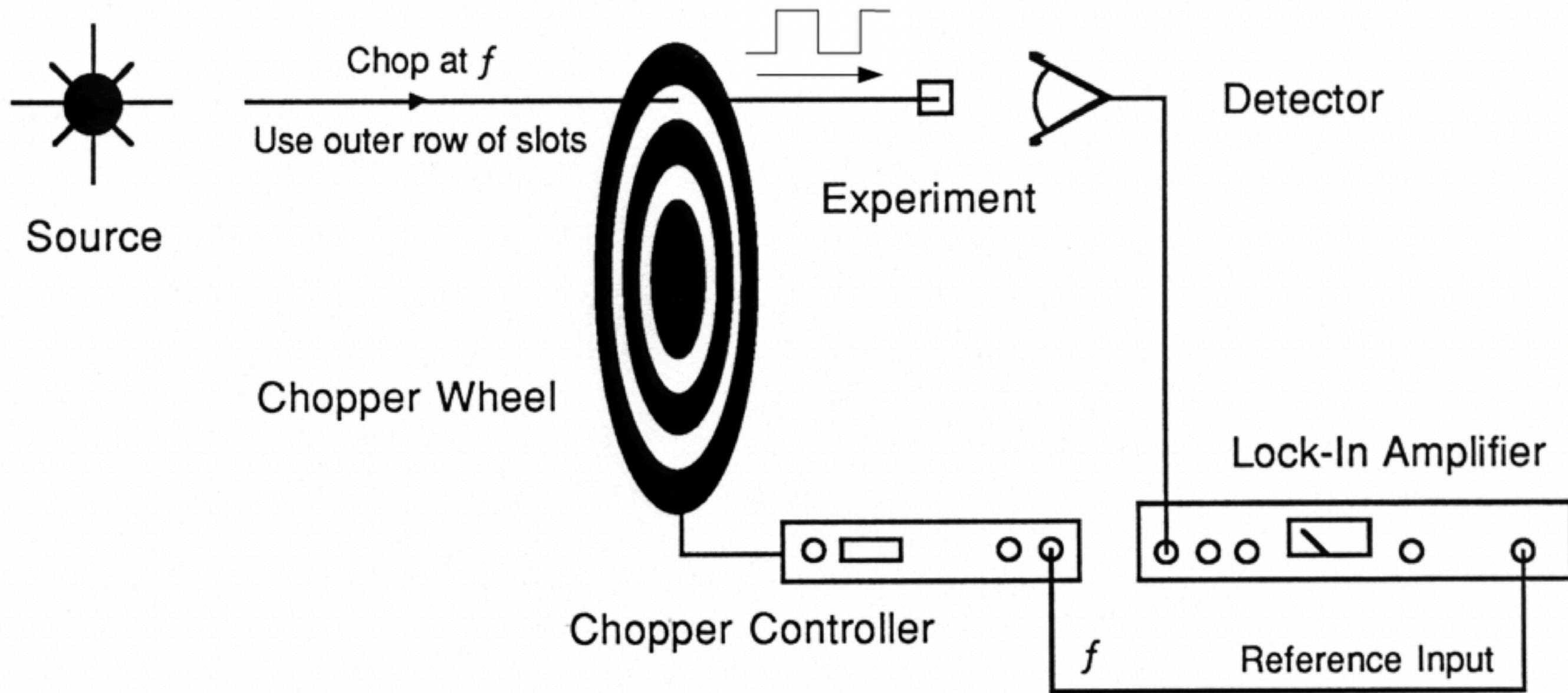
- \* A narrower filter will remove noise sources very close to the reference frequency; a wider bandwidth allows some signals to pass → The low-pass filter bandwidth determines the remaining noise, at the expenses of longer integration

- \* Let's now take the input signal as composed of signal + noise
- \* The lock-in and the low-pass filter only detect signals whose frequencies are very close to the lock-in reference frequency
- \* Noise signals, at frequencies far from the reference, are attenuated by the low pass filter, since  $\omega_{\text{noise}} - \omega_r$  and  $\omega_{\text{noise}} + \omega_r$  are not close to 0.

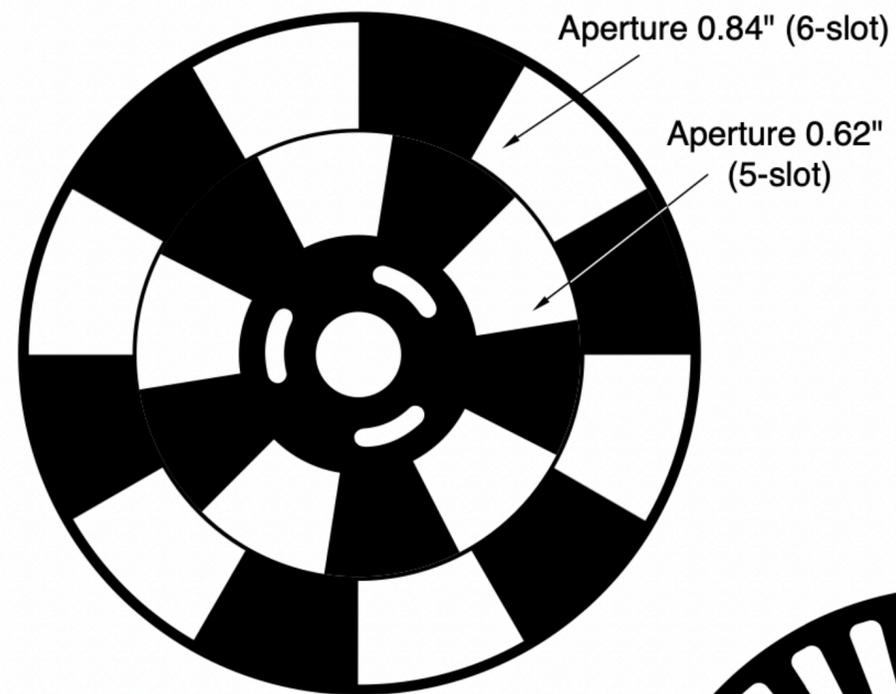
# The phase

- \* We need to make the lock-in reference the same as the signal frequency, i.e.  $\omega_L = \omega_r$ . Not only the frequencies need to be the same, also **the phase between the signals can not change over time**. Otherwise,  $\cos(\theta_{\text{sig}} - \theta_{\text{ref}})$  will change and the output will not be a DC signal.
  - In other words, the lock-in reference needs to be phase-locked to the signal reference.
- \* The lock-in amplifier generates a signal internally, in phase with the frequency reference wave. Let's call  $\theta$  the phase difference between the signal and the lock-in reference oscillator. By adjusting  $\theta_{\text{ref}}$  we can have  $\theta = 0$ , in which case we can measure  $V_{\text{sig}}$ , since  $\cos \theta = 1$ . Conversely, if  $\theta$  is  $90^\circ$ , there will be no output at all.
  - This fact can be used in practice to tune the lock-in phase.

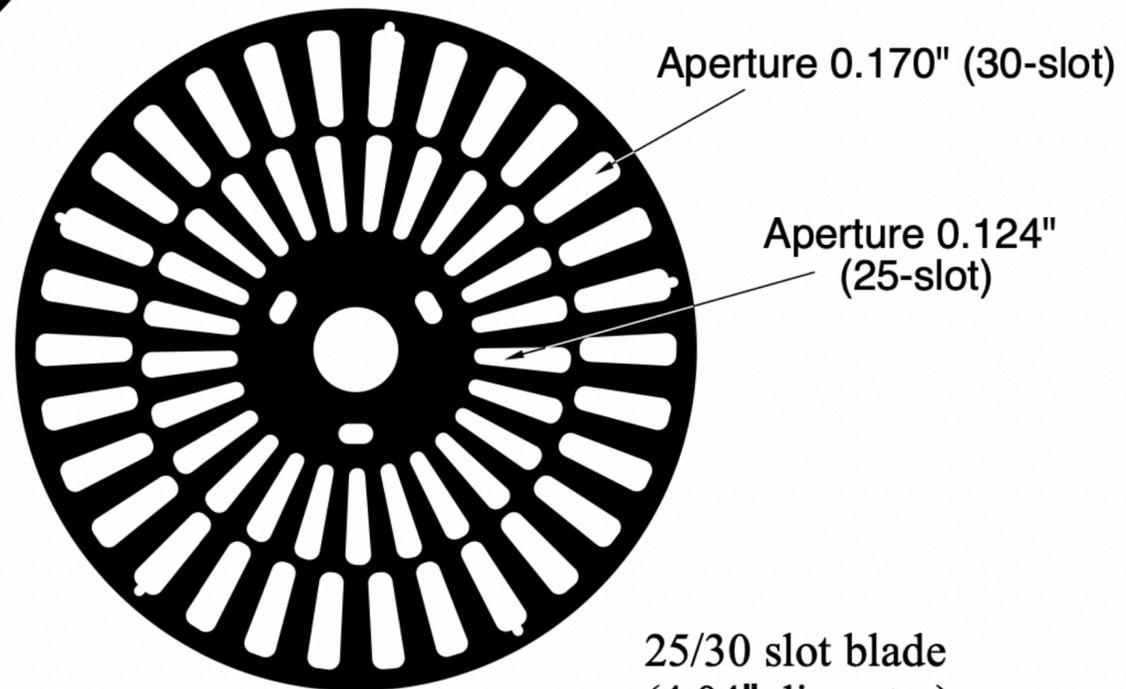
# Experimental scheme



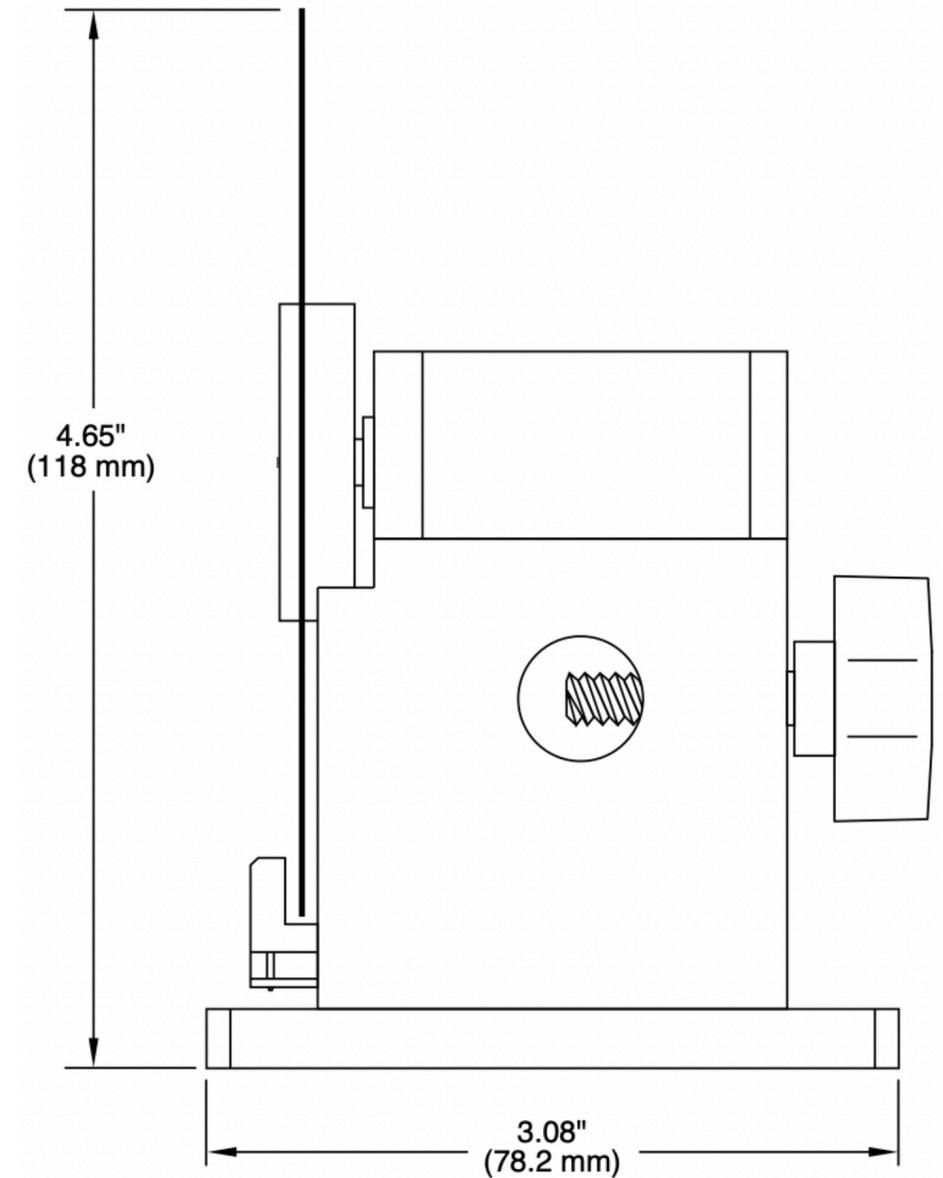
# Optical choppers



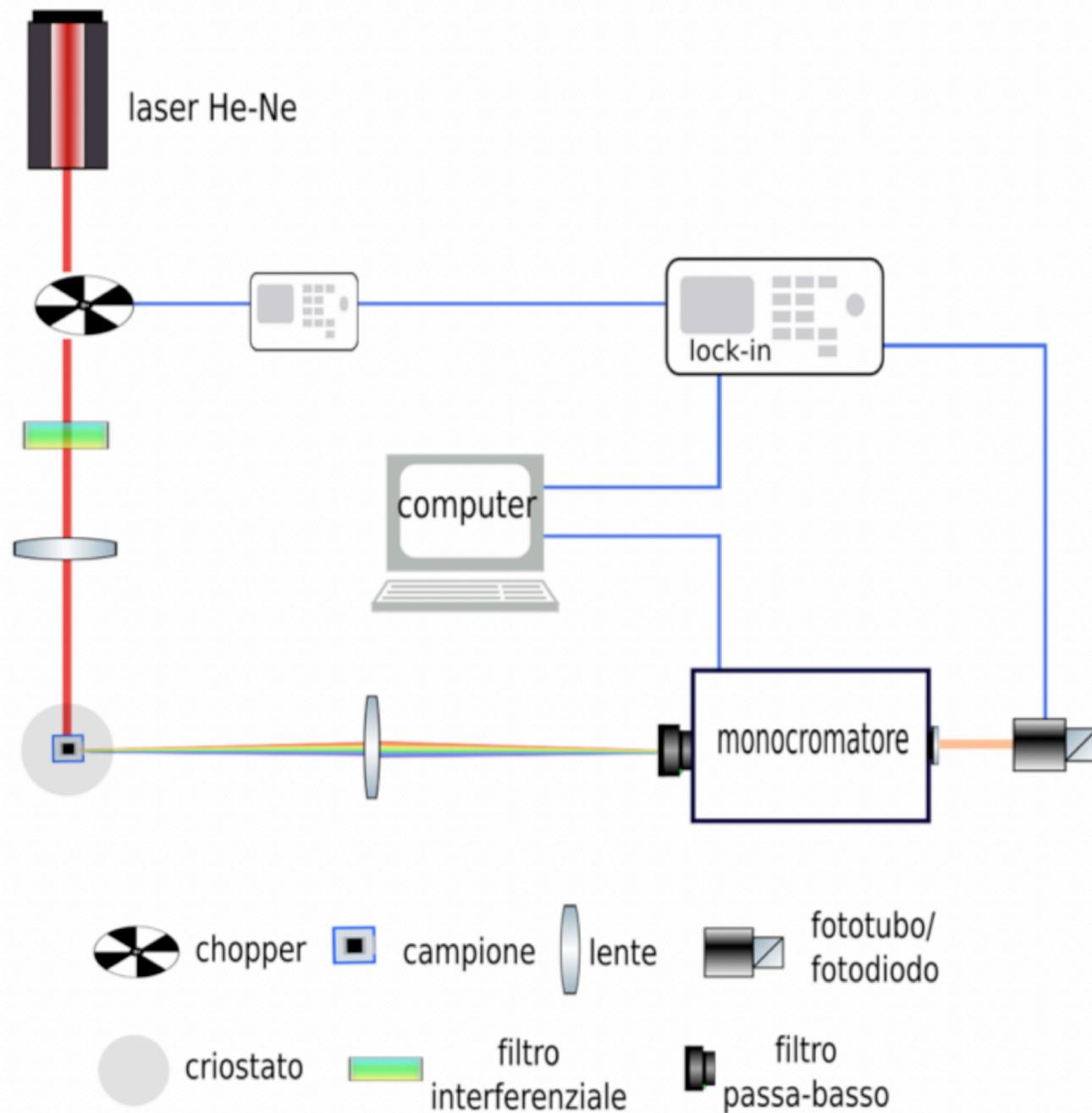
5/6 slot blade  
(4.04" diameter)



25/30 slot blade  
(4.04" diameter)



# Experimental apparatus



- \* Excitation laser (He-Ne)
- \* Chopper for laser modulation
- \* Filter + Focusing lens
- \* Sample
- \* Lens for photoluminescence signal collection (modulated)
- \* Monochromator
- \* Detector, i.e .photomultiplier tube
- \* Lock-in amplifier
- \* Computer acquisition