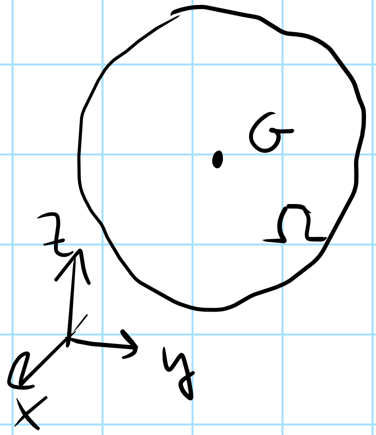
INTEGR 1 CFU (ultima lezione) 25/9/25

GEOMETRIA DELLE MASSE E MOM D' INERZIA



ρ ; $\frac{M}{L^3}$

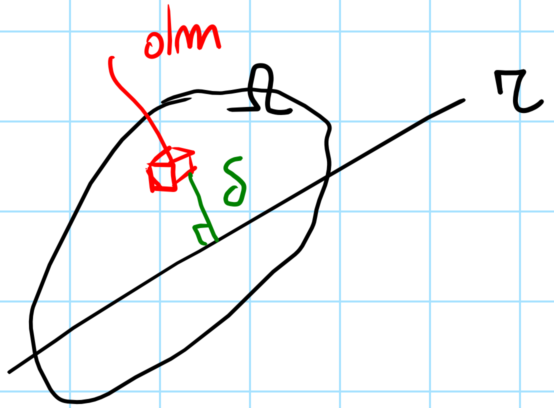
$$x_G = \frac{1}{M} \int_\Omega \rho x \, dV \quad ; \quad y_G = \frac{1}{M} \int_\Omega \rho y \, dV$$

$$M = \int_\Omega \rho \, dV$$

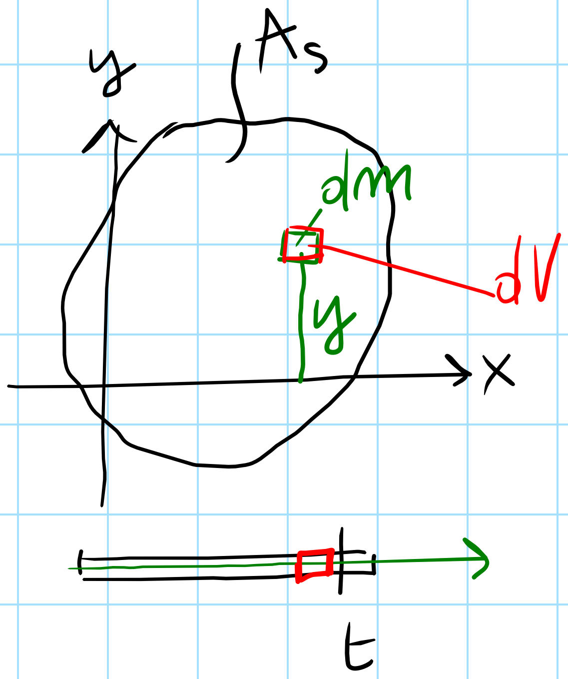
SE IL CORPO È OMOGENEO : $x_G = \frac{1}{\cancel{\rho} \int_\Omega dV} \cancel{\rho} \int_\Omega x \, dV = \frac{\int_\Omega x \, dV}{V}$

$\rho = \text{cost}$

MOM D' INERZIA ASSIALE



$$I_r = \int_\Omega \delta^2 dm = \int_\Omega \delta^2 \rho \, dV$$



$$I_x = \int_\Omega y^2\, dm = \int_\Omega y^2 \rho\, dV = \int_{A_s} y^2 \underbrace{\rho t}_{\sigma}\, dA = \int_{A_s} y^2 \sigma(x,y)\, dA$$

$$dV = dA \cdot t$$

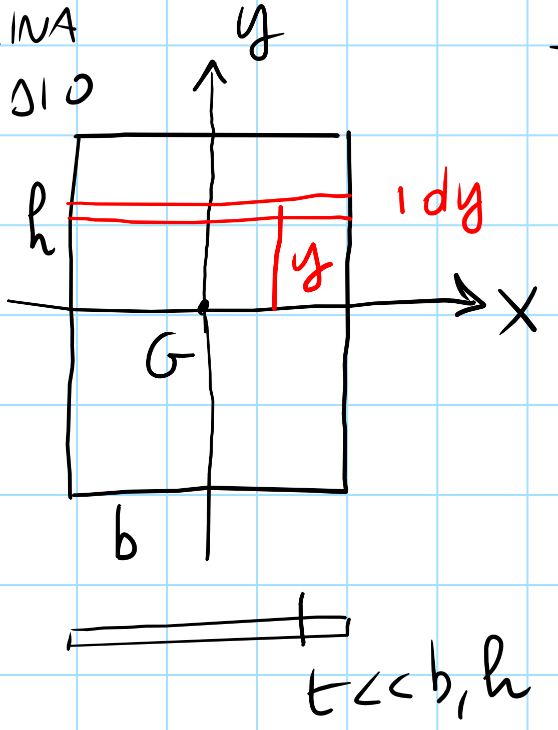
se $\sigma = cost$ $\quad I_x = \sigma \int_{A_s} y^2\, dA$

σ: DENSITA' DI MASSA SUPERFICIALE

$$I_y = \int_{A_s} x^2 \sigma(x,y)\, dA \;;\; \sigma = cost \Rightarrow I_y = \sigma \int_{A_s} x^2\, dA$$

$$\rho t : \left[\frac{M}{L^3} L\right] = \left[\frac{M}{L^2}\right]$$

LAMINA ACCIAIO

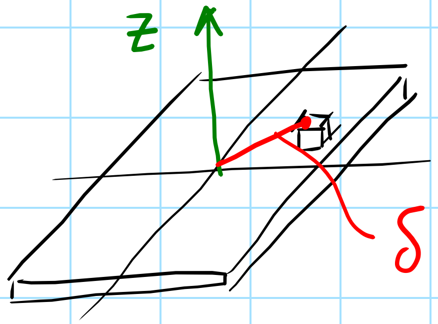


$$I_x = \sigma \int_{A_s} y^2\, dA = \sigma \int_{-h/2}^{+h/2} y^2 b\, dy = \sigma b \left[\frac{y^3}{3}\right]_{-h/2}^{h/2} = \sigma b \left[\frac{h^3}{24} + \frac{h^3}{24}\right] = \sigma \frac{h^3 b}{12}$$

$$\sigma = \rho t$$

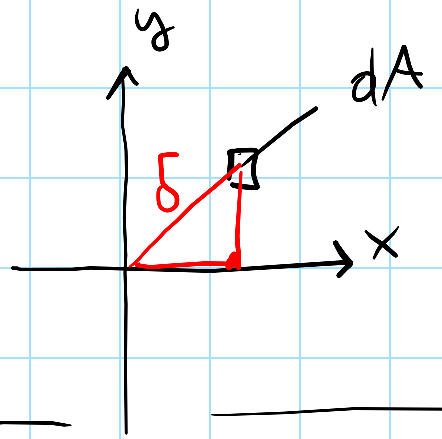
$$M = \sigma b h$$

$$= M \frac{h^2}{12} \;;\quad I_y = \sigma \frac{b^3 h}{12} = \frac{M b^2}{12}$$

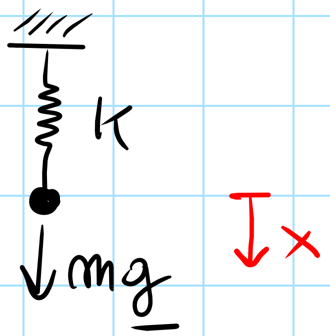


$$I_z = \int_\Omega \delta^2 \rho \, dV = \int_{A_S} (x^2+y^2)\,\sigma\, dA = \underbrace{\int_{A_S} x^2 \sigma\, dA}_{I_y} + \underbrace{\int_{A_S} y^2 \sigma\, dA}_{I_x}$$

$$= I_x + I_y$$

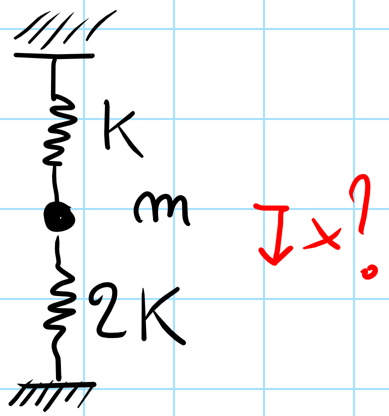


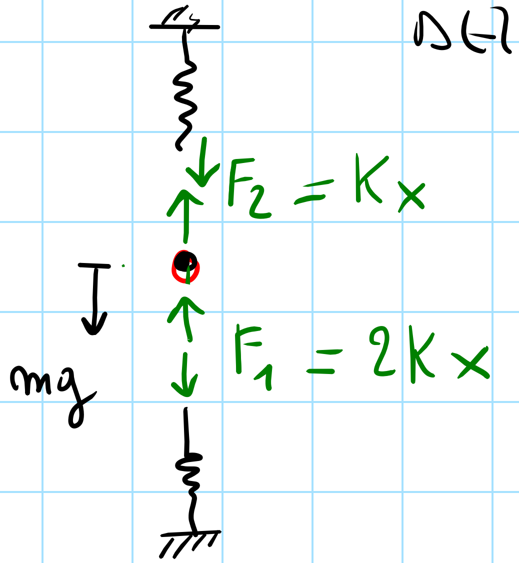
UTILIZZO DELL' EN. POTENZIALE TOTALE PER LO STUDIO DELL'EQUILIBRIO DEI SISTEMI MECCANICI



$$mg - Kx = 0$$

$$x = \frac{mg}{K}$$





$+\downarrow : \; mg - 2Kx - Kx = 0$

INCOGNITA

$$mg = 3Kx$$

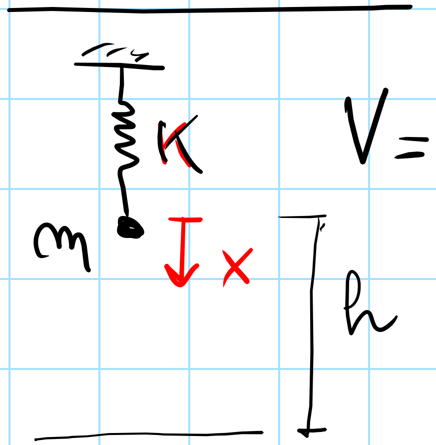
$$\boxed{x = \frac{mg}{3K}}$$

V: EN POT TOTALE IN SIST. MECCANICO = Em elastica delle molle + Em potenziale "dei carichi"

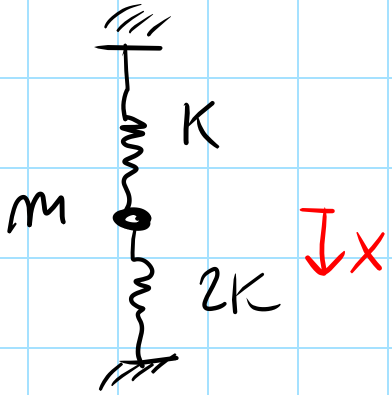
−U: POT. TOTALE

POSIZ DI EQUILIBRIO : $V(q_1, q_2, q_3 \ldots) \Rightarrow \begin{cases} \dfrac{\partial V}{\partial q_1} = 0 \\ \dfrac{\partial V}{\partial q_2} = 0 \\ \dfrac{\partial V}{\partial q_3} = 0 \end{cases}$

q_1, q_2, q_3: PARAM. DI SPOSTAMENTO
= GRADI DI LIBERTA'



$V = \frac{1}{2} K x^2 + mg(h - x)$; $V'(x) = 0$; $Kx - mg = 0$; $x = \frac{mg}{K}$



$V = \frac{1}{2} K x^2 + \frac{1}{2} 2K x^2 - mgx$; $V'(x) = 0$; $3Kx - mg = 0$

$\frac{3}{2} K x^2 - mgx$

$x = \frac{mg}{3K}$