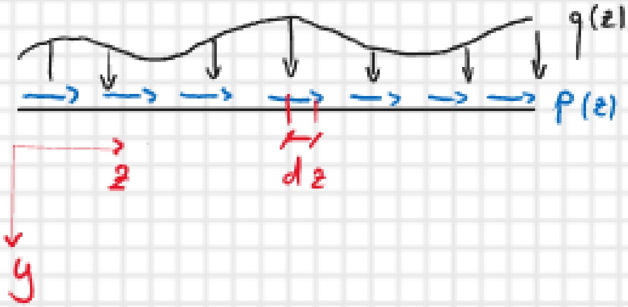
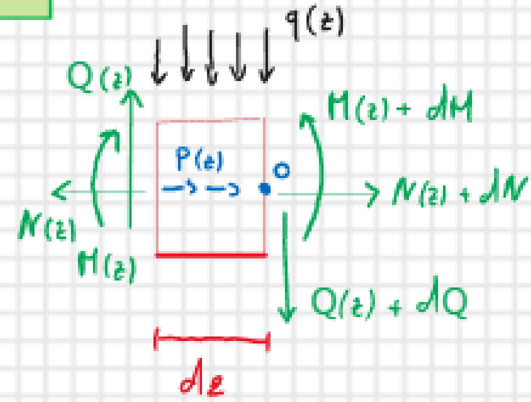
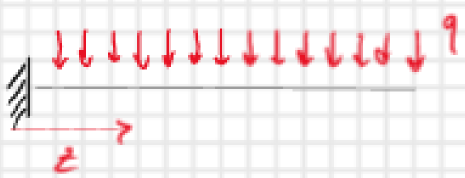
Titolo: Equazioni indefinite di equilibrio

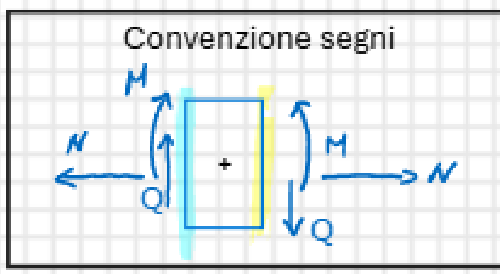


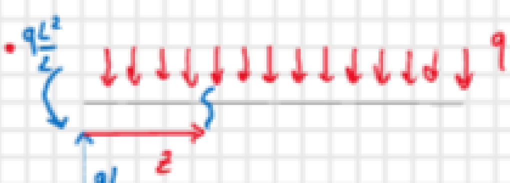


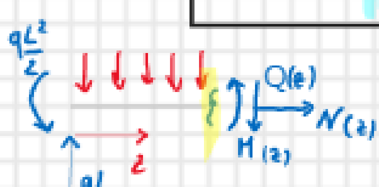
Imponiamo l'eq. del concio

- $-N(z) + N(z) + dN + p(z)\,dz = 0 \longrightarrow \frac{dN(z)}{dz} = -p(z)$
- $Q(z) + dQ - Q(z) + q(z)\,dz = 0 \longrightarrow \frac{dQ(z)}{dz} = -q(z)$
- $M(z) + dM - M(z) - T(z)\,dz + \underbrace{q(z)\,dz\left(\frac{dz}{2}\right)}_{\text{Trascurabile}} = 0 \longrightarrow \frac{dM(z)}{dz} = Q(z)$









Equilibrio della parte di struttura delimitata dalla sezione (considero la faccia gialla)

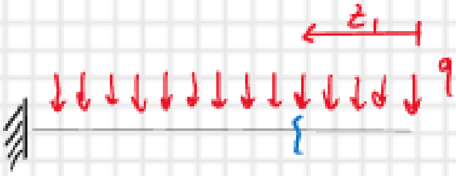
- $N(z) = 0$
- $qL - qz - Q(z) = 0 \qquad Q(z) = qL - qz$

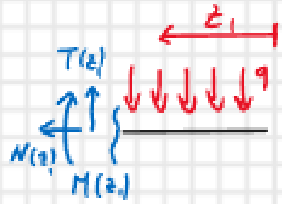
$Q(0) = qL \qquad Q(L) = 0$

- $q\frac{L^2}{2} - qLz + q\frac{z^2}{2} + M(z) = 0 \qquad M(z) = -q\frac{L^2}{2} + qLz - q\frac{z^2}{2}$

$M(0) = -q\frac{L^2}{2} \qquad M(L) = 0$

Valuto le CDS partendo dall'estremo destro (uso z1)





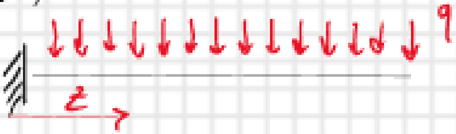
$N(z_1) = 0$

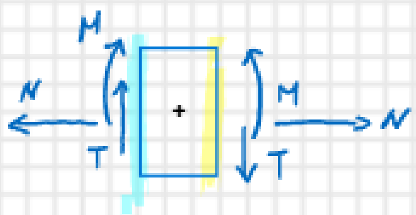
$T(z_1) = q z_1$ $\quad T(0) = 0$ $\quad T(L) = qL$

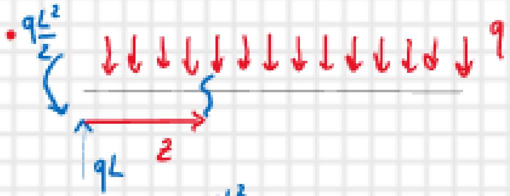
$\boxed{z_1 = L - z}$

$M(z_1) = -q\frac{z_1^2}{2}$ $\quad M(0) = 0$ $\quad M(L) = -qL^2$

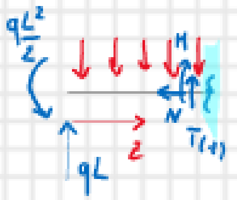
Equivalenza delle forze sulla parte di struttura delimitata dalla sezione (considero la faccia celeste)







Le forze nella parte di struttura delimitata dalla sezione sono equivalenti alle caratteristiche di sollecitazione sulla faccia celeste



- $N(z) = 0$
- $T(z) = qL - qz$
- $M(z) = -q\frac{L^2}{2} + qLz - q\frac{z^2}{2}$

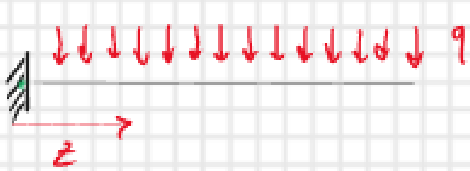
Calcolo delle CDS con eq. indefinite di equilibrio

$\frac{dN(z)}{dz} = -p(z)$ $\longrightarrow$ Problema assiale

2) $\frac{dQ(z)}{dz} = -q(z)$

3) $\frac{dM(z)}{dz} = Q(z)$

$\Big\}\longrightarrow$ Problema flessionale

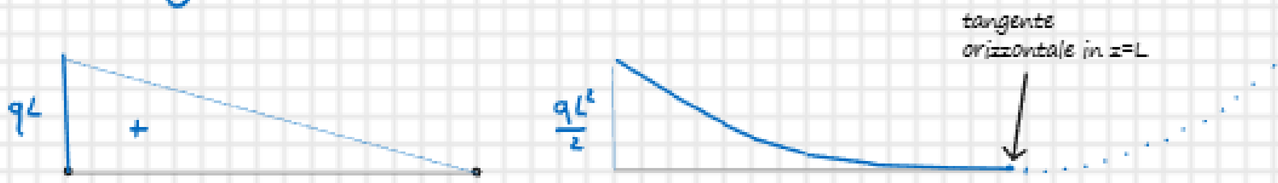


$q(z) = q$ carico costante

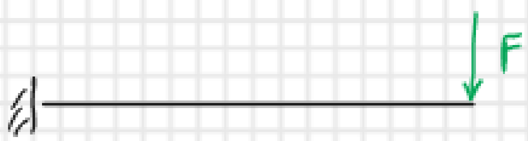
2) $\frac{dQ(z)}{dz} = -q$ $\quad Q(z) = -qz + C$ $\quad Q(0) = C = qL \longrightarrow Q(z) = qL - qz$

3) $\frac{dM(z)}{dz} = Q(z) = qL - qz$ $\quad M(z) = qLz - q\frac{z^2}{2} + D$ $\quad M(0) = D = -q\frac{L^2}{2}$ $\quad M(z) = -q\frac{L^2}{2} + qLz - q\frac{z^2}{2}$

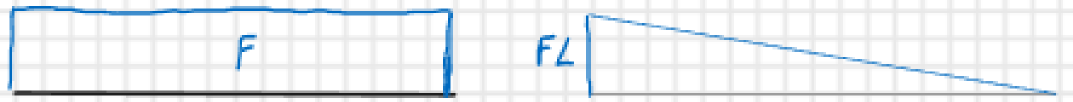
Diagrammi



Q (z) M (z)



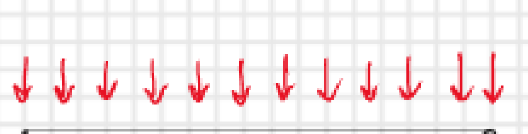
Mensola con carico concentrato



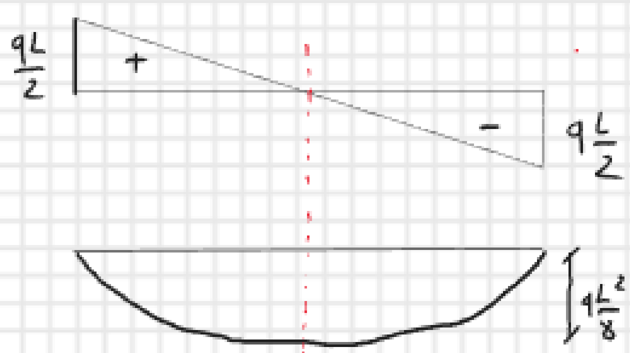
Q(z) M(z)

Considerazioni sull'andamento delle azioni interne servendosi delle equazioni indefinite di equilibrio

- Carico distribuito nullo → Taglio costane → Momento lineare
- Carico distribuito costante → Taglio lineare → Parabolico
- Carico " lineare → Taglio parabolico → Momento cubico



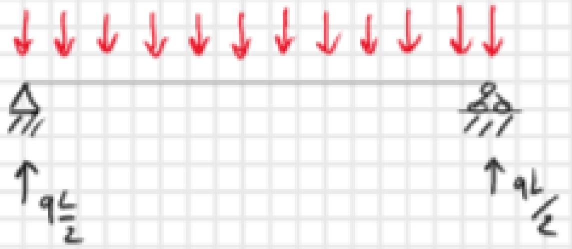
$\frac{dM(z)}{dz} = Q(z) \longrightarrow Q = 0 \longrightarrow M_{max}$



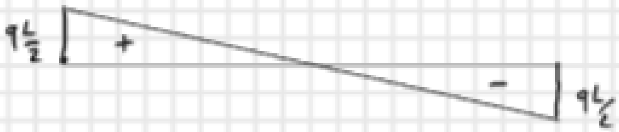
$Q(z) = q\frac{L}{2} - q z$ $\quad Q(z) = 0 \rightarrow z = \frac{L}{2}$

$M(z) = q\frac{L}{2} z - q\frac{z^2}{2}$ $\quad M(z = \frac{L}{2}) = qL^2/8$

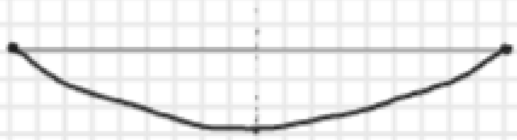
In corrispondenza del punto nel quale si annulla il taglio Q(z)=0 il momento assume il valore massimo



q costante

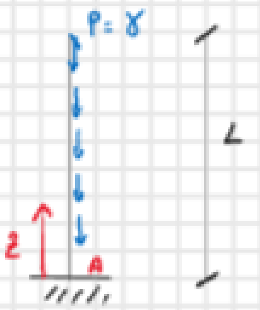


Q(z) lineare



M(z) parabolico

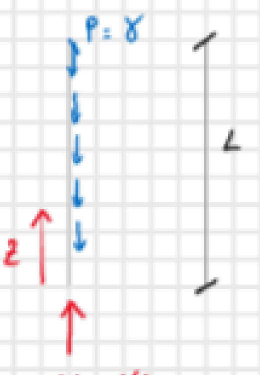
Problema assiale

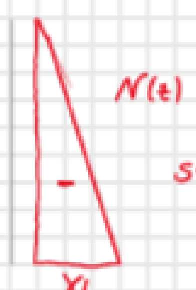


$$\frac{dN(z)}{dz} = -p(z)$$

$$\frac{dN(z)}{dz} = \gamma \qquad N(z) = \gamma z + C \qquad N(L) = \gamma L + C = 0$$

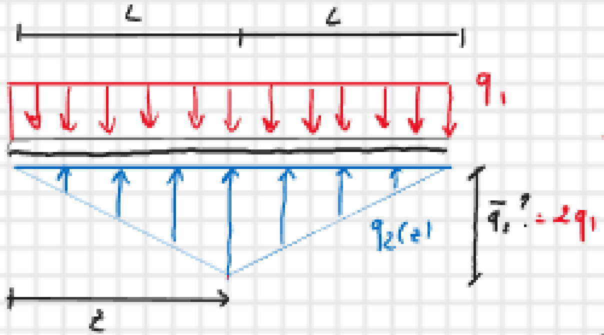
$$C = -\gamma L \qquad N(z) = -\gamma L + \gamma z \qquad N(L) = 0 \qquad N(0) = -\gamma L$$





struttura compressa

Esempio (carico triangolare)



Eq. tr. verticale

$$2q_1 L = 2\,\bar{q}_2 \frac{L}{2} \longrightarrow \bar{q}_2 = 2q_1$$

$$q_2(z) = \bar{q}_2 \frac{z}{L}$$

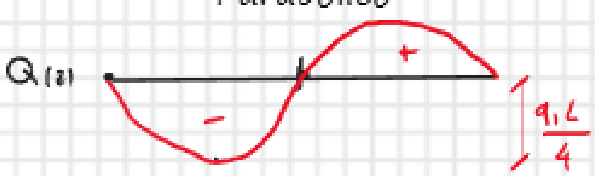
$$q(z) = q_1 - q_2(z) = q_1 - 2q_1\frac{z}{L} \qquad z \in [0, L]$$

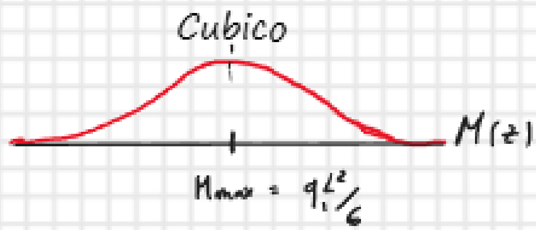
$$\frac{dQ(z)}{dz} = -q(z) = 2q_1\frac{z}{L} - q_1 \qquad Q(z) = \cancel{2}q_1\frac{z^2}{\cancel{2}L} - q_1 z + C \qquad Q(0) = C = 0$$

$$Q(z) = q_1\frac{z^2}{L} - q_1 z \qquad z \in [0, L]$$

$$\frac{dM(z)}{dz} = Q(z) = q_1\frac{z^2}{L} - q_1 z \qquad M(z) = q_1\frac{z^3}{3L} - q_1\frac{z^2}{2} + D \qquad M(0) = D = 0$$

$$M(z) = q_1\frac{z^3}{3L} - q_1\frac{z^2}{2}$$

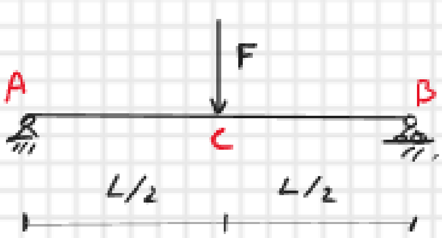


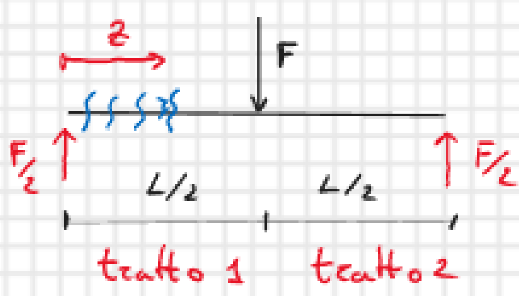


$$Q(z = L/2) = -q_1 L/4$$

Esercizi sulle C.D.S

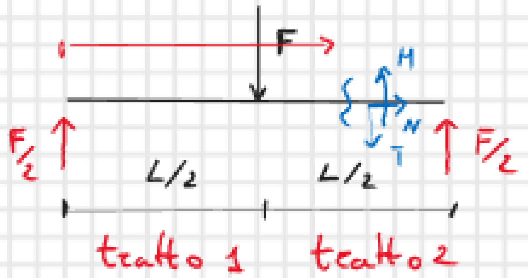
Carico (o momento) concentrato induce una discontinutà nelle CDS)



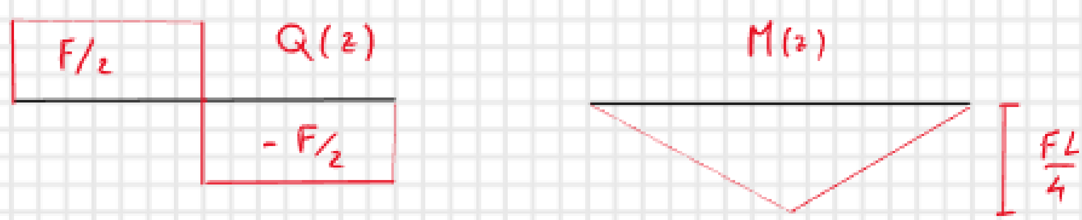


Tratto 1 $\quad z \in [0, L/2] \quad N(z) = 0 \quad T(z) = F/2 \quad M(z) = \frac{F}{2} z$

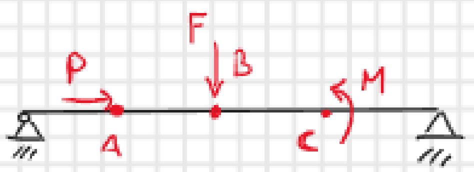
• Tratto 2 $z \in [L/2 , L]$ supero il carico

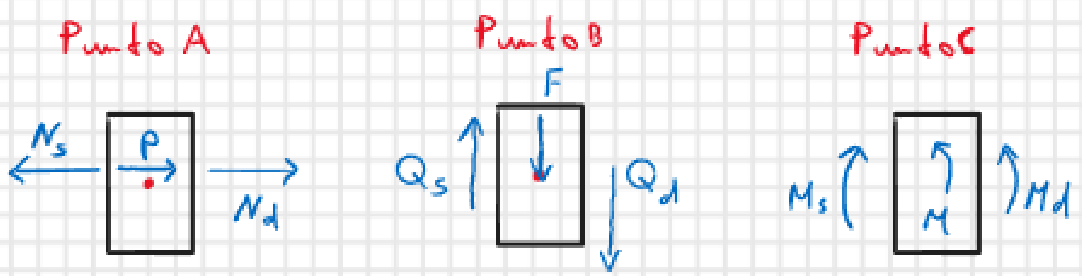


$N(z) = 0$ $Q(z) = \frac{F}{2} - F = -\frac{F}{2}$ $M(z) = \frac{F}{2} z - F\left(z - \frac{L}{2}\right) = \frac{F}{2}(L - z)$

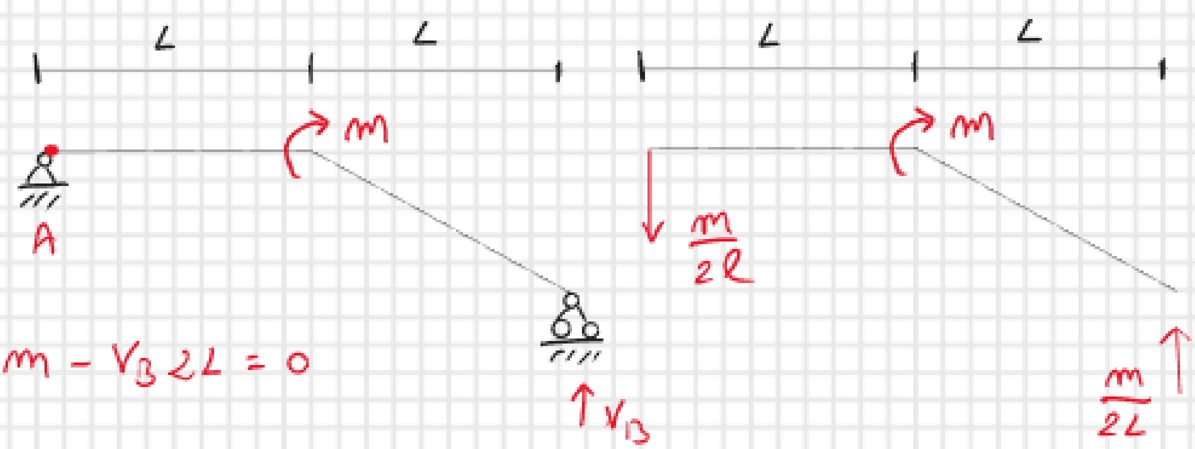


Equazioni di salto delle caratteristiche di sollecitazione

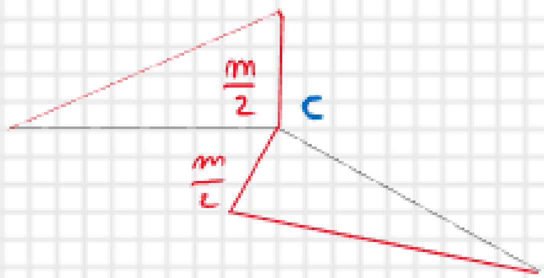


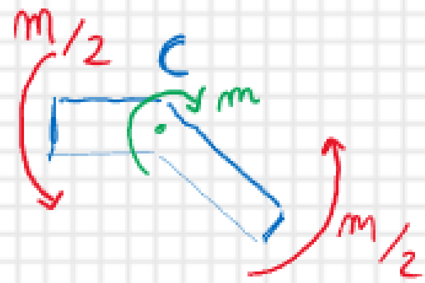


$\Delta N = N_d - N_s = -P$ $\Delta Q = T_d - T_s = -F$ $\Delta M = M_d - M_s = -M$



$m - V_B 2L = 0$

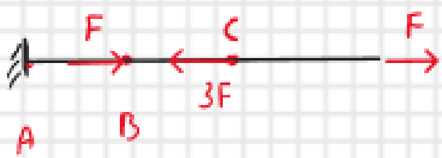


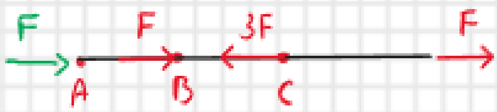


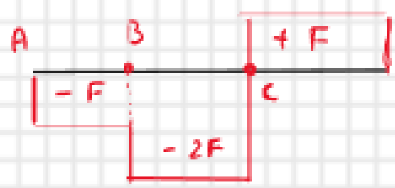
Nodo C è in equilibrio $\Sigma M = 0$

Sommatoria dei momenti interni e di quelli applicati

Esempio di discontinuità dovuta a carichi assiali

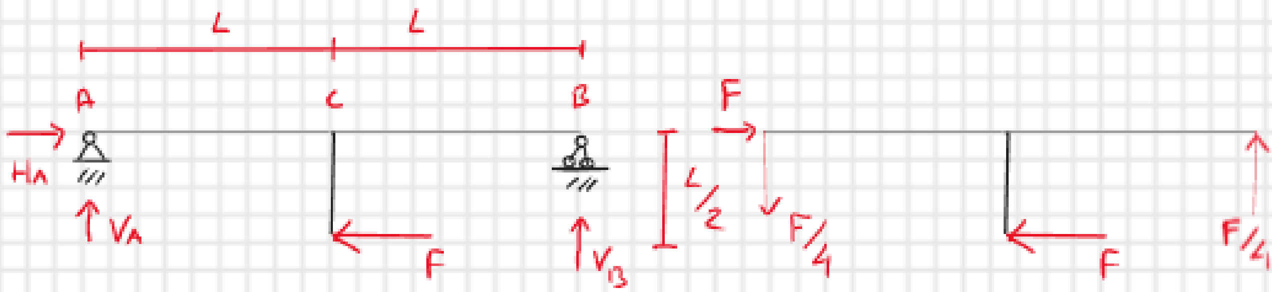






Azione assiale N(z)

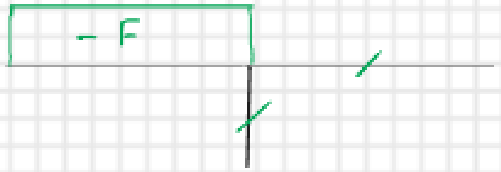
Esempio



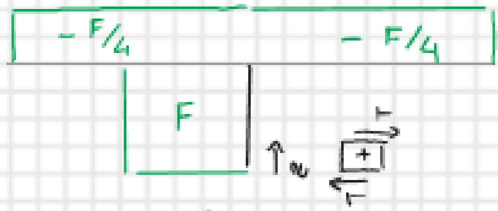
in A)
$$\begin{cases} V_B \cdot 2L - F\frac{L}{2} = 0 \rightarrow V_B = \frac{F}{4} \\ V_A + V_B = 0 \qquad V_A = -F/4 \\ H_A - F = 0 \end{cases}$$

Equazioni cardinali della statica

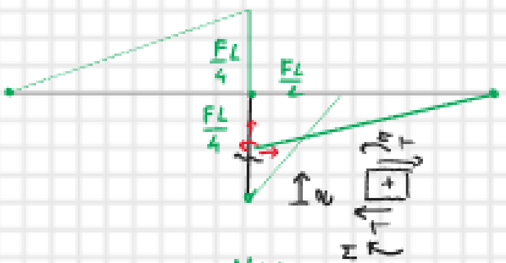
Diagrammi



$N(z)$

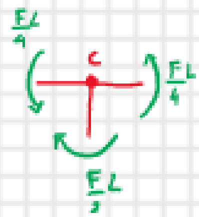


$Q(z)$



$M(z)$

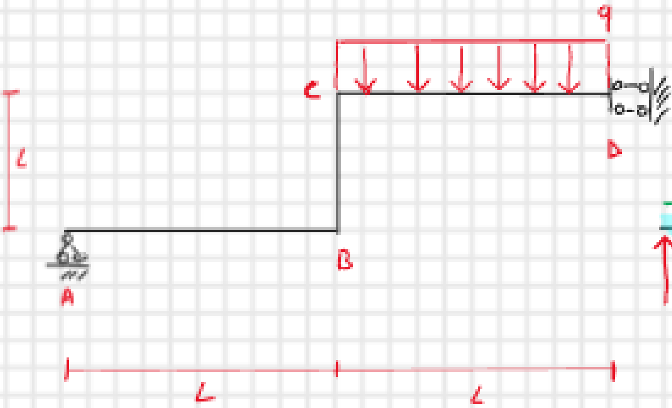
Nodo centrale

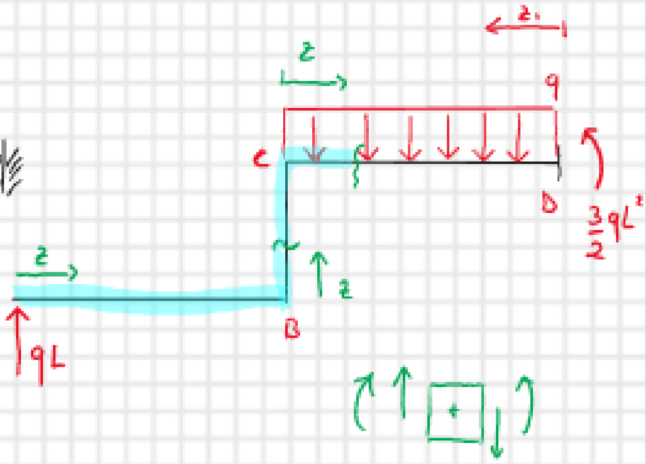


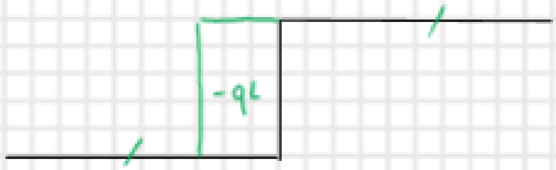
$\Sigma M = 0$

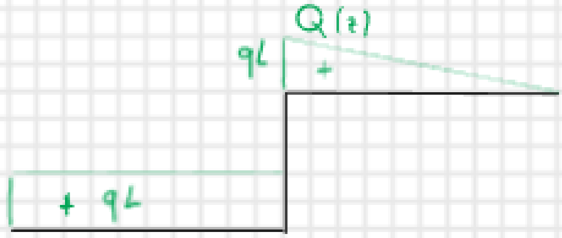
Sommatoria di tutti i momenti delle aste che convergono nel nodo C

Esempio









$N(z) = 0$ in AB e CD

$N(z) = -qL$ in BC

$Q(z) = qL$ in AB

$Q(z) = 0$ in BC

$Q(z) = qL - qz$ in CD

alternativa con z_1 da destra

$Q(z_1) = qz_1$ in CD

$M(z)$

$M(z) = qLz$ in AB

$M(z) = qL^2$ in BC

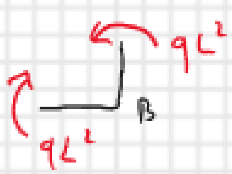
$M(z) = qL(L+z) - q\frac{z^2}{2}$ $\quad M(0) = qL^2$ $\quad M(L) = \frac{3}{2}qL^2$

Alternativa partendo da destra z_1

$M(z_1) = \frac{3}{2}qL^2 - q\frac{z_1^2}{2}$ $\quad M(0) = \frac{3}{2}qL^2$ $\quad M(L) = qL^2$

Verifica dei nodi

nodo B



equilibrati

nodo C

In un nodo formato da due sole aste, senza un momento applicato, il momento flettente nelle due aste è lo stesso (ribaltamento nei grafici)