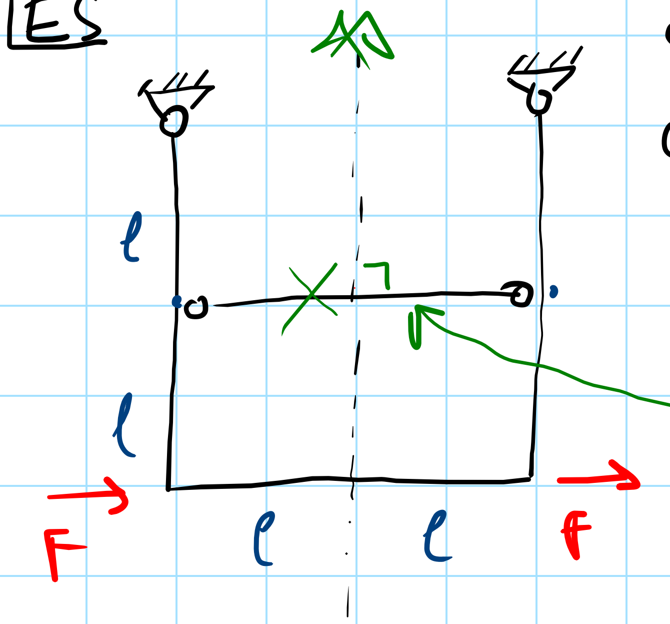
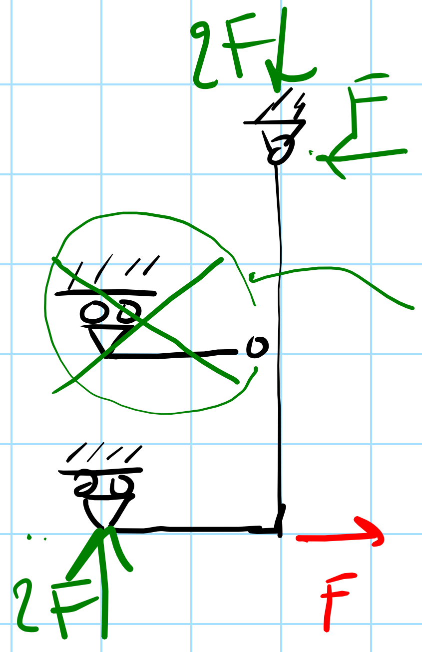
LES

18/11/25



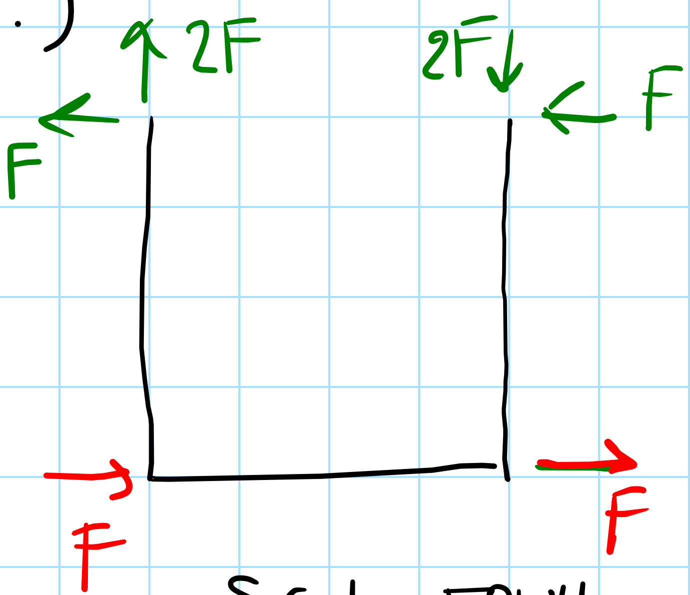
STR. SYM
CARICO ANTISYM

BIELLA
SCARICA

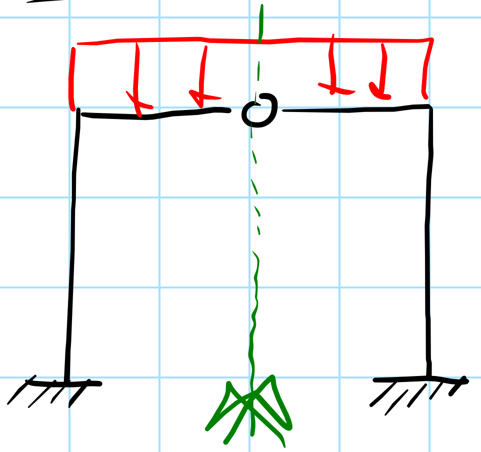


STR
RIDOTTA (ISOST.)

ASTA SCARICA

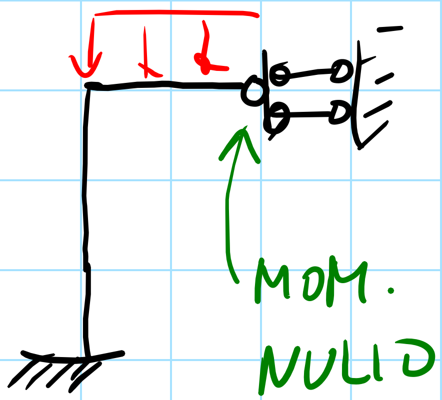


S.C.L. EQUIL.

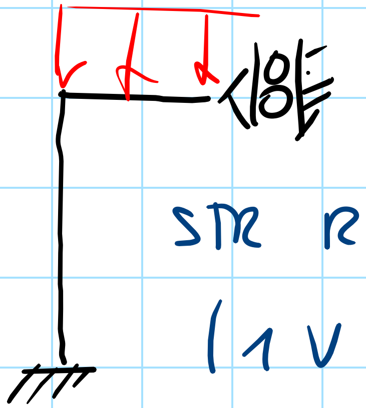


STR SYM
CAR SYM
(CERN. SU
ASSE DI SIMM.)

2 V IPERST.

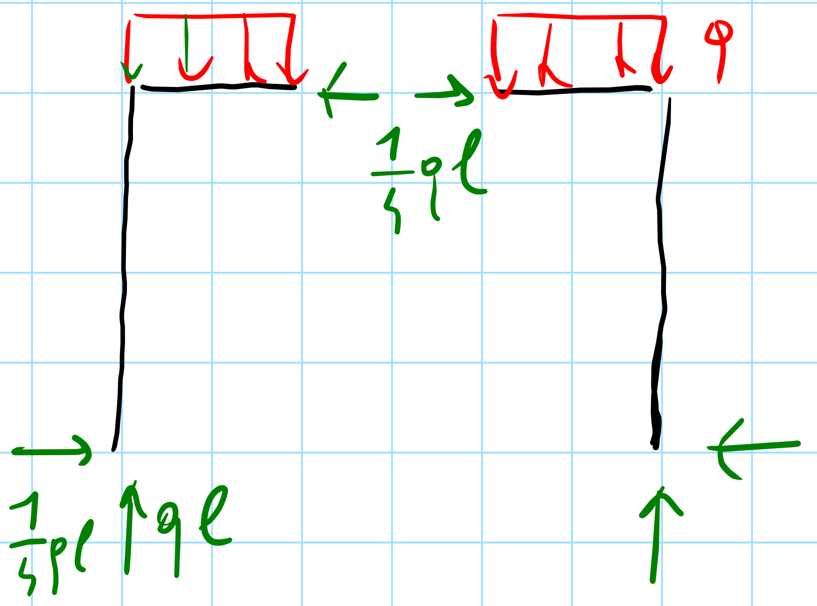


MOM.
NULLO

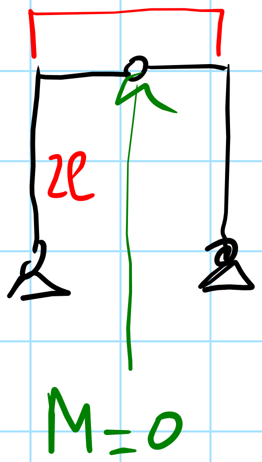


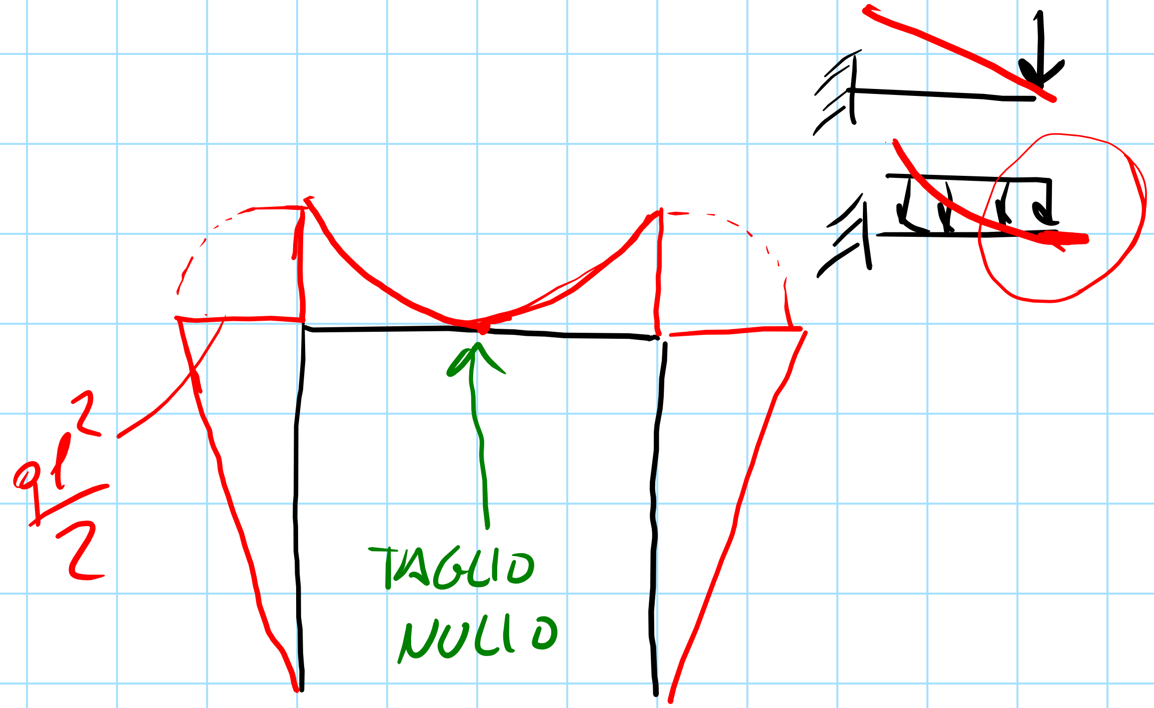
STR RIDOTTA
(1 V IPERST)

ES IN SOSPESO (ARCO A 3 CERNIERE)



STR SYM
CARICO SYM





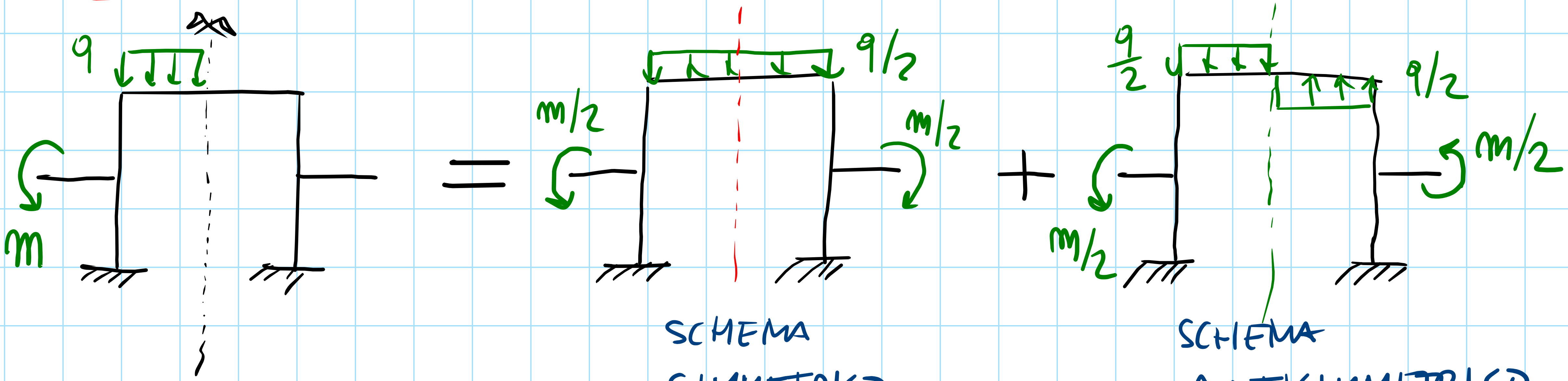
(M) SIMMETRICO

(Q) ANTYSIMMETRICO

STR SYM
CARICO ANTI SYMM.

(M) ANTISYM
(Q) SYM

STR SIMMETRICHE CARICATE IN MODO GENERICO



REAZ. VINC.

1 + 2

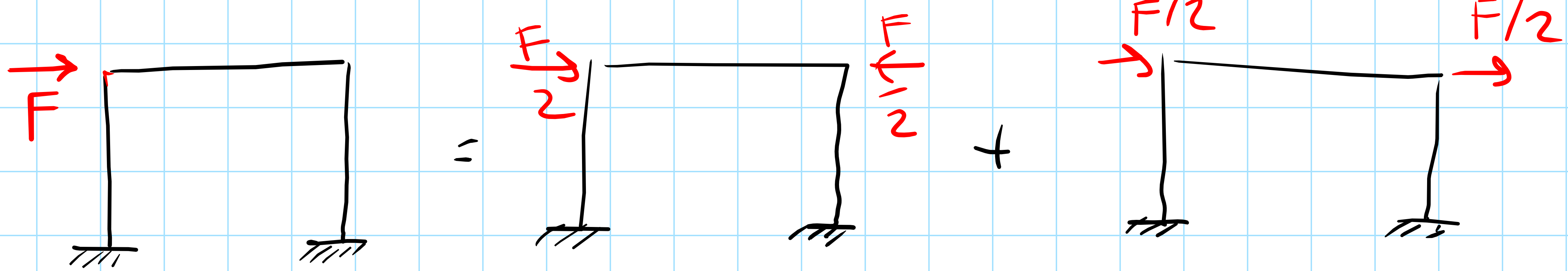
SCHEMA SIMMETRICO

DETERM. LE REAZ. VINC. 1

SCHEMA ANTISIMMETRICO

DETERM. LE REAZ. VINCOLARI 2

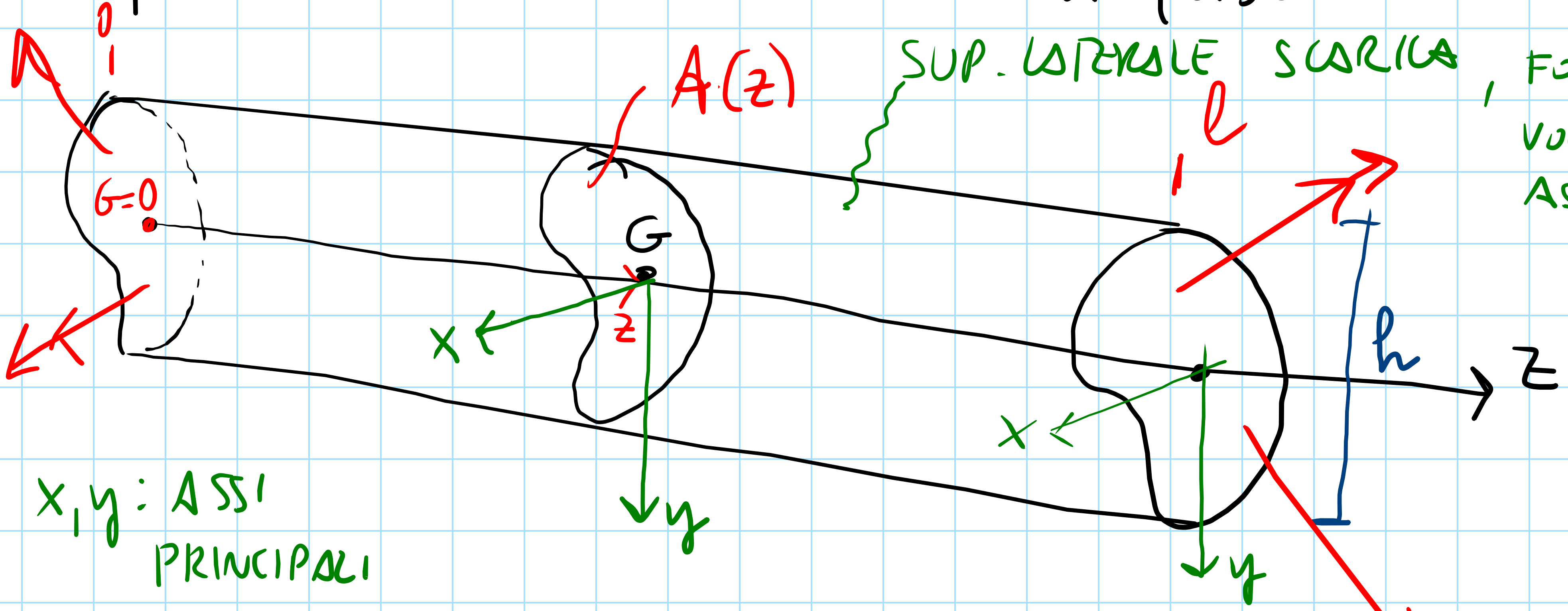
ES



PROBLEMA DI SAINT-VENANT (1855) (ALLA BASE DELLA TEORIA TECNICA DELLA TRAVE)

(ADHEMAR BARRÈ DE S-V)

È UN PROBLEMA ELASTICO LINEARE DOVE IL CORPO Ω è costituito da un cilindro allungato a sezione compatta caricato solo sulle basi (libero da vincoli)



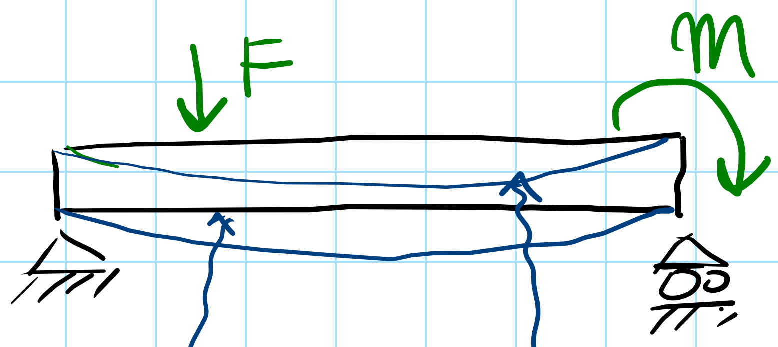
SUP. LATERALE SCARICA, FORZE DI VOLUME ASSENTI

x, y: ASSI PRINCIPALI

CARICHI SULLE BASI (AUTOEQUILIBRATI)

$h \ll \ell$

MATERIALE: ELASTICO LINEARE (E, ν, G)



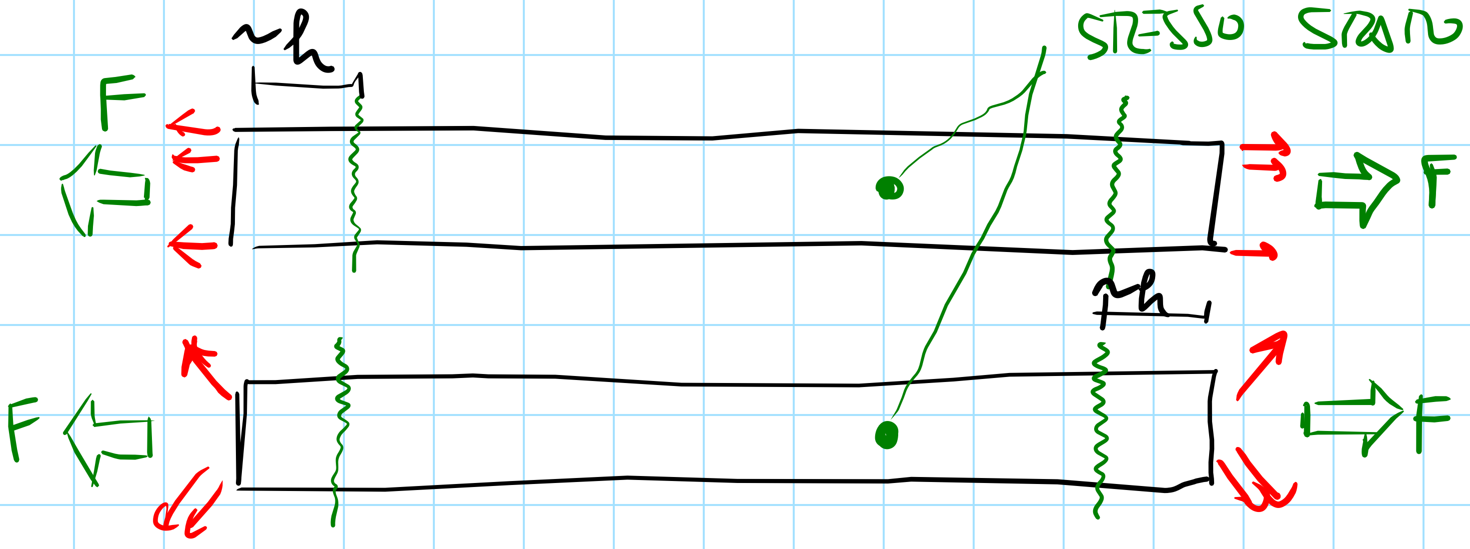
SPOSTAM.

SOLIDO 3D CHE DEVE RESIST. ALLE SOLLECITAZ. ($N, Q, M \rightarrow \underset{\sim}{\sigma}$ IN OGNI PUNTO e $\Rightarrow$ CRITERIO DI RESIST. SODDISF.)

TEORIA TECNICA: DA CDS A $\underset{\sim}{\sigma}$ IN OGNI PUNTO PER TRAVI ELASTICHE LINEARI

OBIETTIVO: CON LE IPOTESI INDICATE $\Longrightarrow$ DETERMINARE LO STATO TENS. NEL CILINDRO ($\underset{\sim}{\sigma}(x,y,z)$) IN FUNZ. DEI CARICHI APPLICATI ALLE BASI.

ALTRE CONSIDERAZIONI



STESSO STATO TENSIONALE ANCHE SE LA DISTRIB. DELLE FORZE SUPERF. È DIVERSA NEI 2 CASI.

2 CONDIZ. DI CARICO CHE HANNO LA STESSA RISULT.

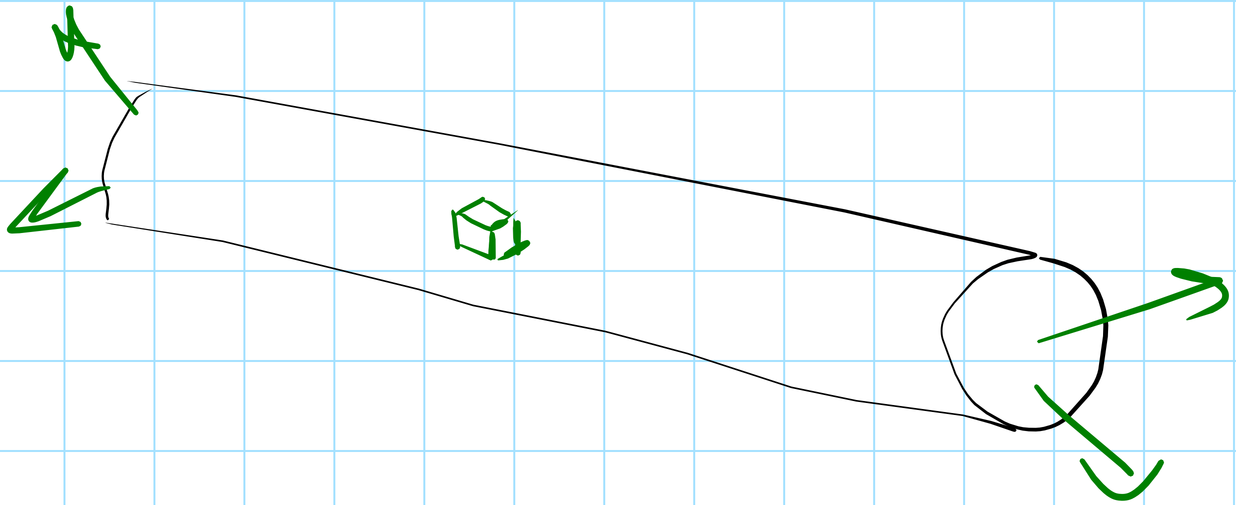
<u>POSTULATO DI S-V.</u>

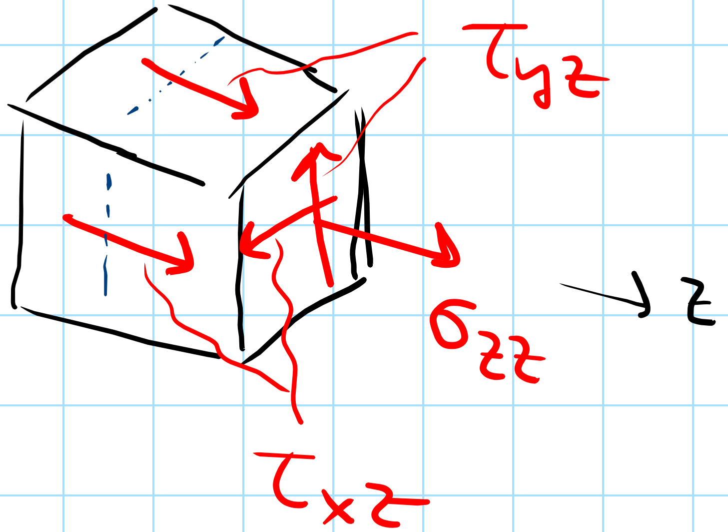
OLTRE UNA DISTANZA PARI A CIRCA h LO STATO TENSIONALE DIPENDE SOLO DALLE RISULTANTI DEI CARICHI SULLE BASI.

DIM. (1937) ZANABONI

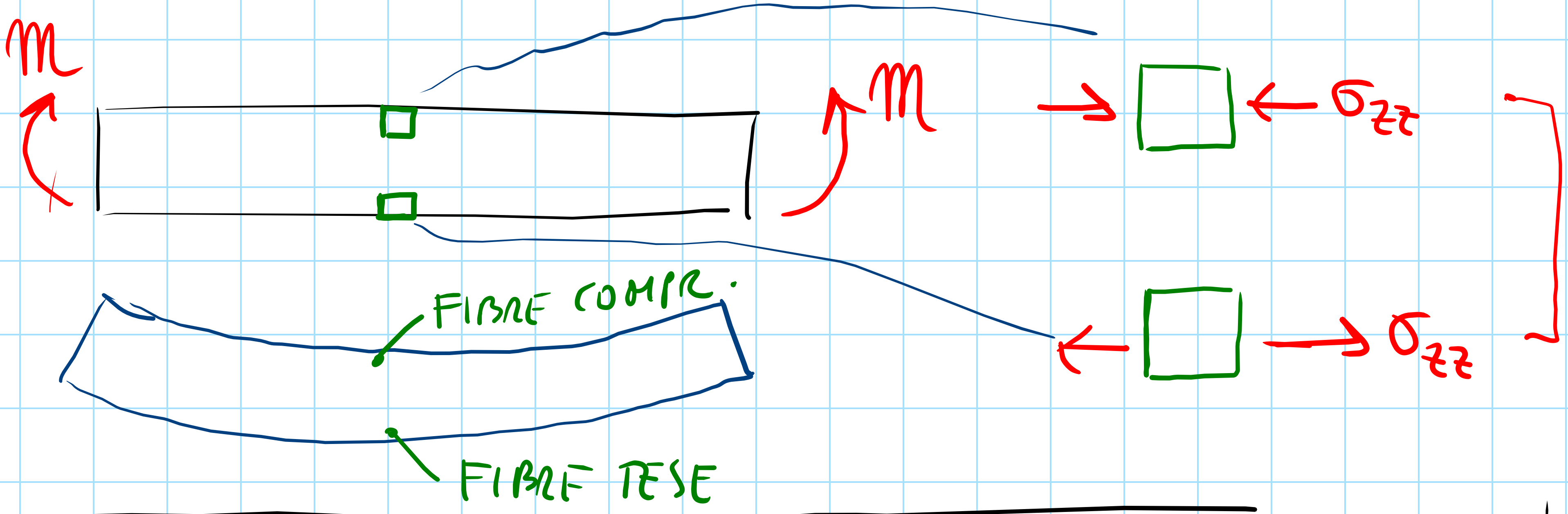
ULTERIORE IPOTESI:

LE UNICHE TENSIONI NON NULLE NEL CILINDRO SONO $\sigma_{zz}, \tau_{xz}, \tau_{yz}$. $(\sigma_{xx} = \sigma_{yy} = \tau_{xy} = 0)$





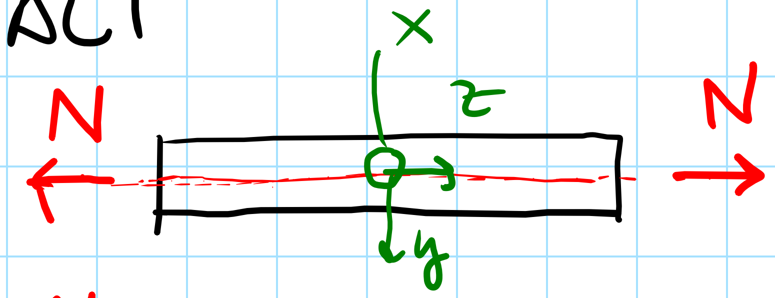
In una soluz. completa σ_{xx}, σ_{yy} sono molto più piccole di σ_{zz}. Questo vale anche per le τ_{xy}



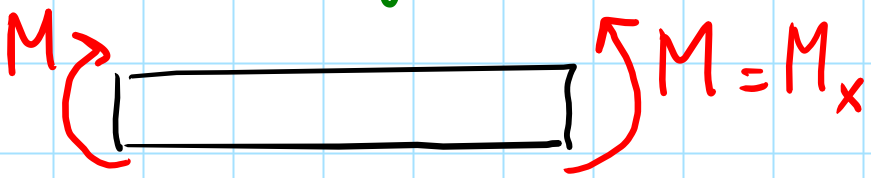
Valori estremi di σ_{zz} ('molto elevati')

4 casi fondamentali

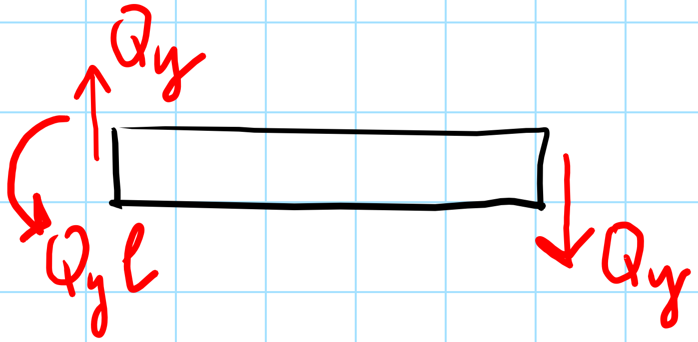
1) Forza normale



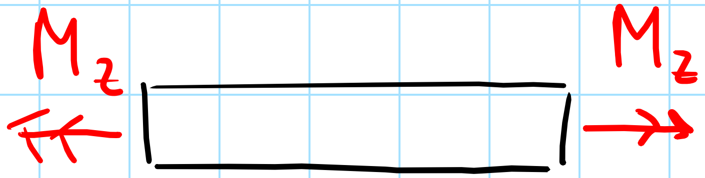
2) Flessione retta (semplice)

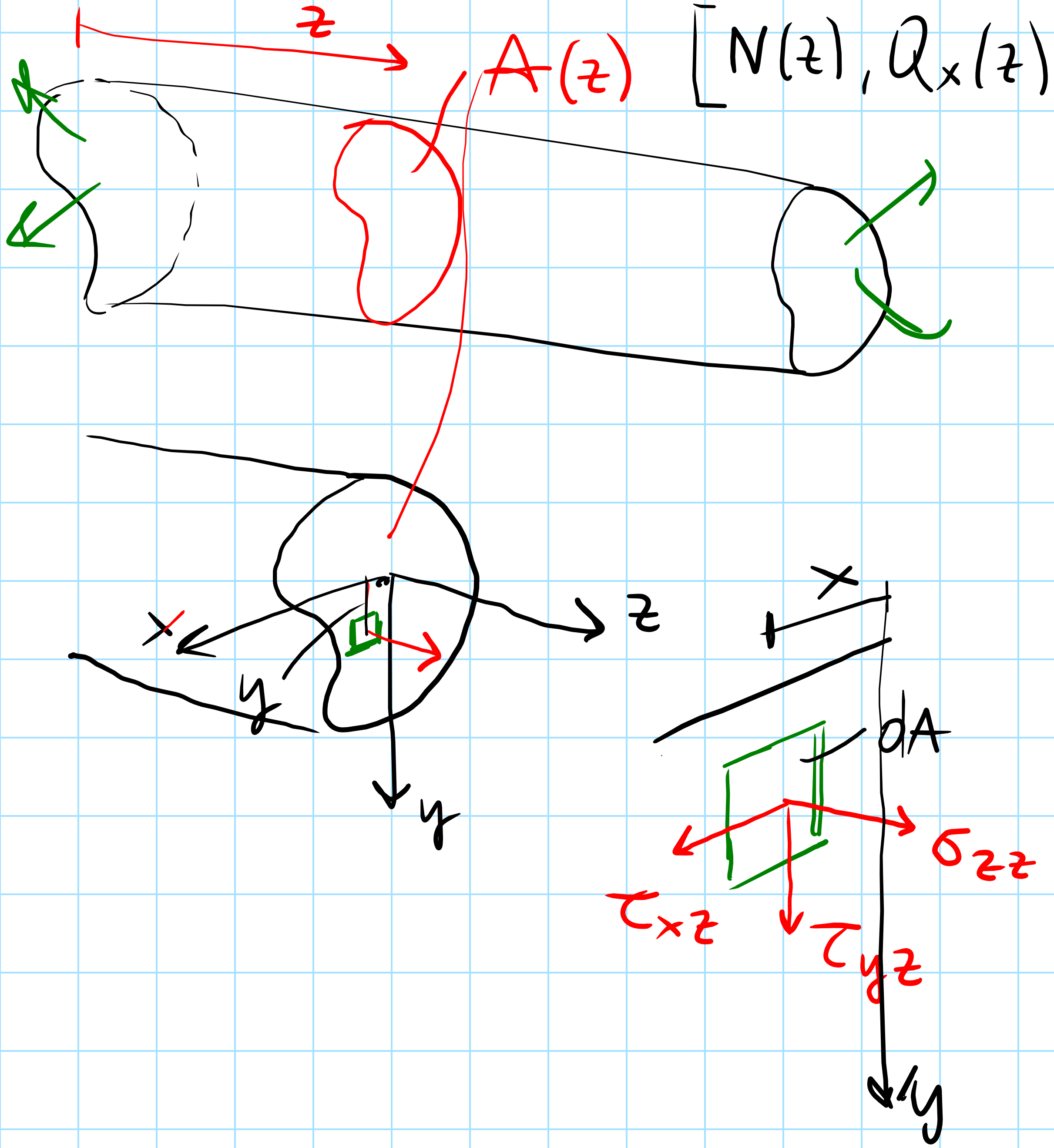


3) Taglio-flessione



4) Torsione





$$[N(z), Q_x(z), Q_y(z), M_x(z), M_y(z), M_z(z)]$$

$$N(z) = \int_{A(z)} \sigma_{zz}\, dA \; ; \; Q_y(z) = \int_{A(z)} \tau_{yz}\, dA , \; Q_x(z) = \int_{A(z)} \tau_{xz}\, dA$$

$$M_x(z) = \int_{A(z)} \underbrace{\sigma_{zz}\, y\, dA}_{dF}$$

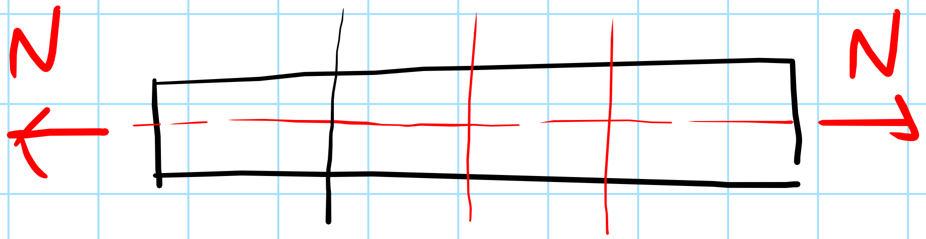
$$M_y(z) = -\int_{A(z)} \sigma_{zz}\, x\, dA$$

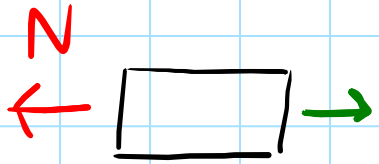
$$M_z(z) = \int_{A(z)} \tau_{yz}\, x - \tau_{xz}\, y\, dA$$

$\boxed{\text{CDS}}$

LE 6 EQUAZ SONO EQUAZIONI DI EQUIVALENZA TRA LE TENSIONI CHE SI SVILUPPANO NELLA SEZIONE $A(z)$ E LE RISPETTIVE C.D.S.

1) FORZA NORMALE (SFORZO NORMALE)





$N(z) = N$

$$\left(\begin{array}{l} M_x(z)=0 \\ Q_y(z)=0 \\ Q_x(z)=0 \\ M_y(z)=0 \\ M_z(z)=0 \end{array}\right)$$

$\sigma_{zz}(x,y,z) = C \quad ; \quad \tau_{xz} = \tau_{yz} = 0$] IPOTESI DI SOLUZIONE

PER DET. LA C UTILIZZO L'EQ $\boxed{\text{CDS}}$:

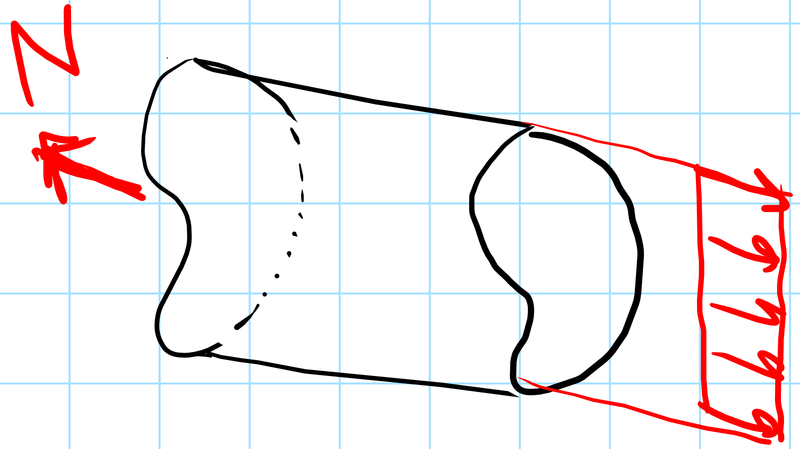
$$N(z) = \int_{A(z)} \sigma_{zz}\, dA = C \underbrace{\int_{A(z)} dA}_{\text{AREA } A} = \boxed{CA = N}$$

$$C = \frac{N}{A}$$

VERIFICO CHE LA SOLUZ. $\sigma_{zz} = N/A$ SODDISFA LA $\boxed{\text{CDS}}$ DI $M_x(z)$ $(=0)$

$$M_x(z) = \boxed{\int_{A(z)} \sigma_{zz}\, y\, dA \overset{?}{=} 0} \Rightarrow \int \frac{N}{A} y\, dA \overset{?}{=} 0, \quad \frac{N}{A}\int y\, dA \overset{?}{=} 0, \quad \frac{N}{A} S_x \overset{?}{=} 0$$

ma $S_x = 0$ (x è BARICENTRICO) QUINDI L'EQ è SODDISFATTA.



$\sigma_{zz} = \dfrac{N}{A}$

$N > 0$; $\sigma_{zz} > 0$ TRAZIONE

$N < 0$; $\sigma_{zz} < 0$ COMPRESSIONE

VERIFICO CHE LE EQ. PUNTUALI DI EQUIL. $\left(\text{div}\, \underset{\sim}{\sigma} = \underline{0}\right)$ SIANO SODDISF.:

$$\left.\begin{aligned} &\tau_{xz,z} = 0 \\ &\tau_{yz,z} = 0 \\ &\tau_{xz,x} + \tau_{yz,y} + \sigma_{zz,z} = 0 \end{aligned}\right\} \quad \begin{aligned} &\text{FORZA} \\ &\text{NORMALE} \\ &\tau_{xz} = \tau_{yz} = 0 \\ &\sigma_{zz} = N/A \end{aligned} \qquad \left.\begin{aligned} &0 = 0 \\ &0 = 0 \\ &\underbrace{\left(\frac{N}{A}\right)_{,z}}_{0} = 0 \end{aligned}\right\} \ \text{OK}$$

$$[\underset{\sim}{\sigma}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & N/A \end{bmatrix}$$

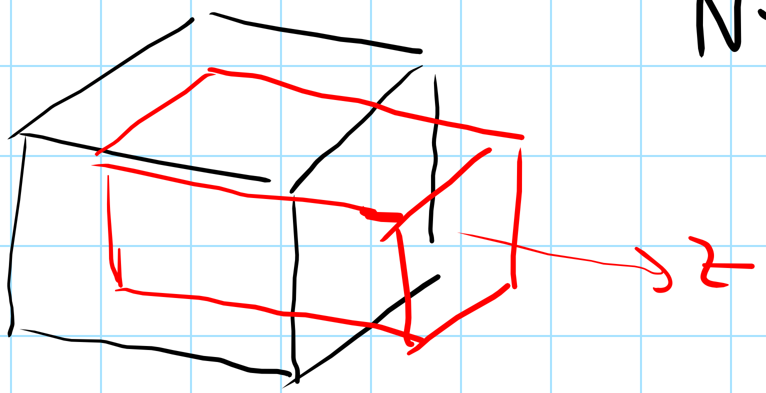
IN OGNI PUNTO C'È UNO STATO TENS. DI TIPO MONOASSIALE legame costitutivo $(\nu > 0)$

$$\left[\varepsilon_{xx} = \frac{1}{E}\left(-\nu\, \sigma_{zz}\right) = \boxed{-\nu \frac{N}{EA}} \;;\quad \varepsilon_{yy} = \boxed{-\nu \frac{N}{EA}} \;;\quad \varepsilon_{zz} = \frac{1}{E}\left(\sigma_{zz}\right) = \boxed{\frac{N}{EA}}\right.$$

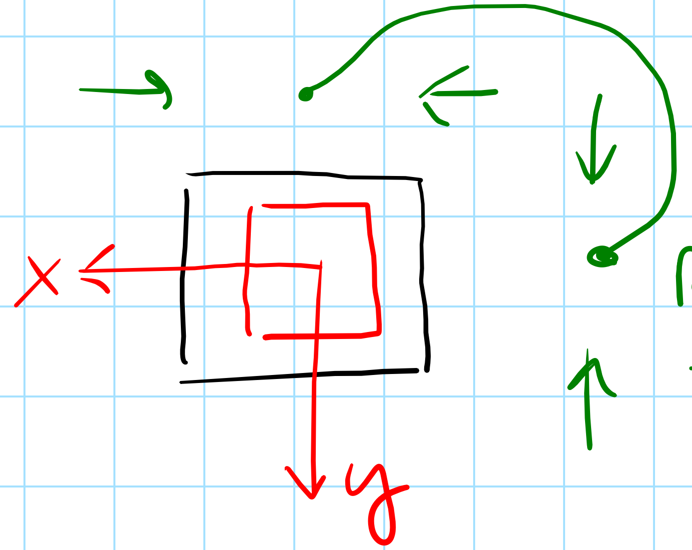
$$\boxed{\underset{\sim}{\varepsilon} = \text{COST.}}$$

$$\varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0 \qquad \text{(NO SCORRIMENTI)}$$

$N > 0$ (TRAZIONE, $\sigma_{zz} = \frac{N}{A} > 0$)

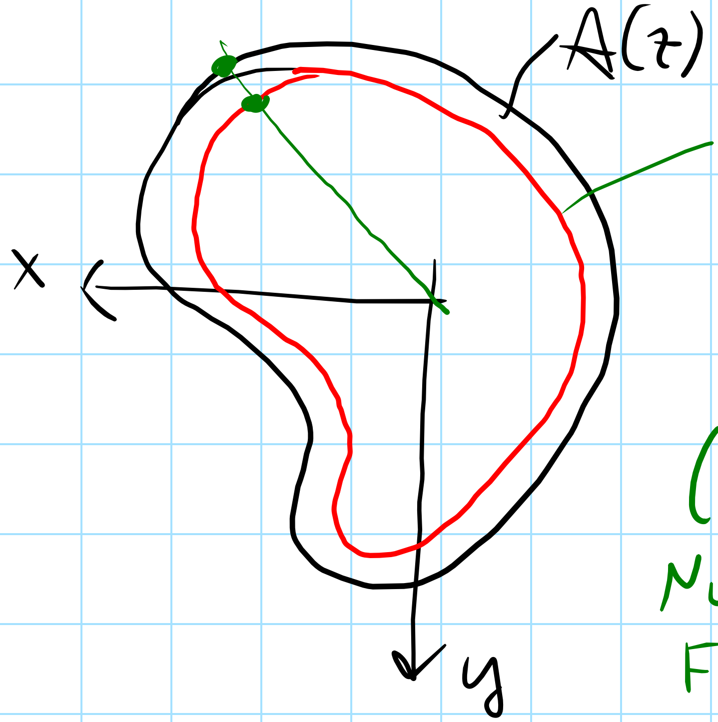


ALLUNG. LONGITUD.



RESTRINGIM. TRASVERSALE

(se $N < 0$; LE DEFORMAZ SONO OPPOSTE.)



RESTRINGIM. DELLA SEZ. TRASVERSALE

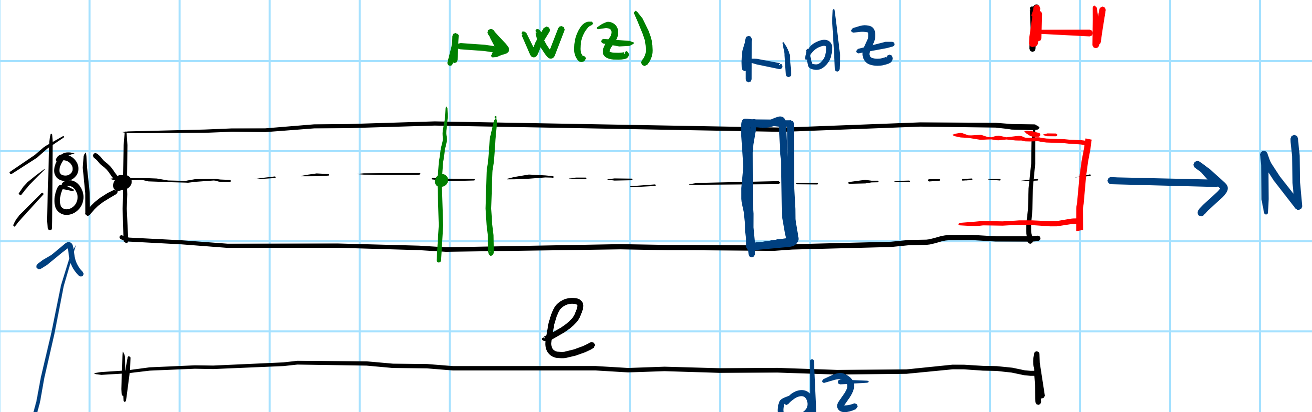
(OMOTETICO)

NO CAMBIO DI FORMA

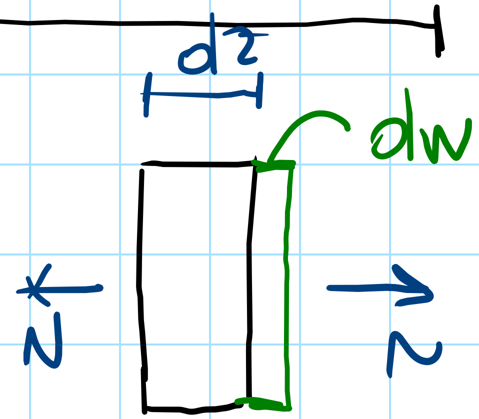
ALLUNG. DEL CILINDRO

$\Delta\ell$: ALLUNG. DEL CILINDRO

$w(z)$: SPOST. LONGIT. DELLA SEZ. GENERICA. $\quad (w(0)=0,\ w(\ell)=\Delta\ell)$



BLOCCA GLI SPOST. @ $z=0$



dw : DIFF DEGLI SPOST. TRA LE DUE FACCE

$$\varepsilon_{zz} = \frac{dw}{dz} \qquad \varepsilon_{zz} = \frac{\overset{\text{cost.}}{N}}{EA}$$

L'EQ DIFF CHE GOVERNA $w(z)$:

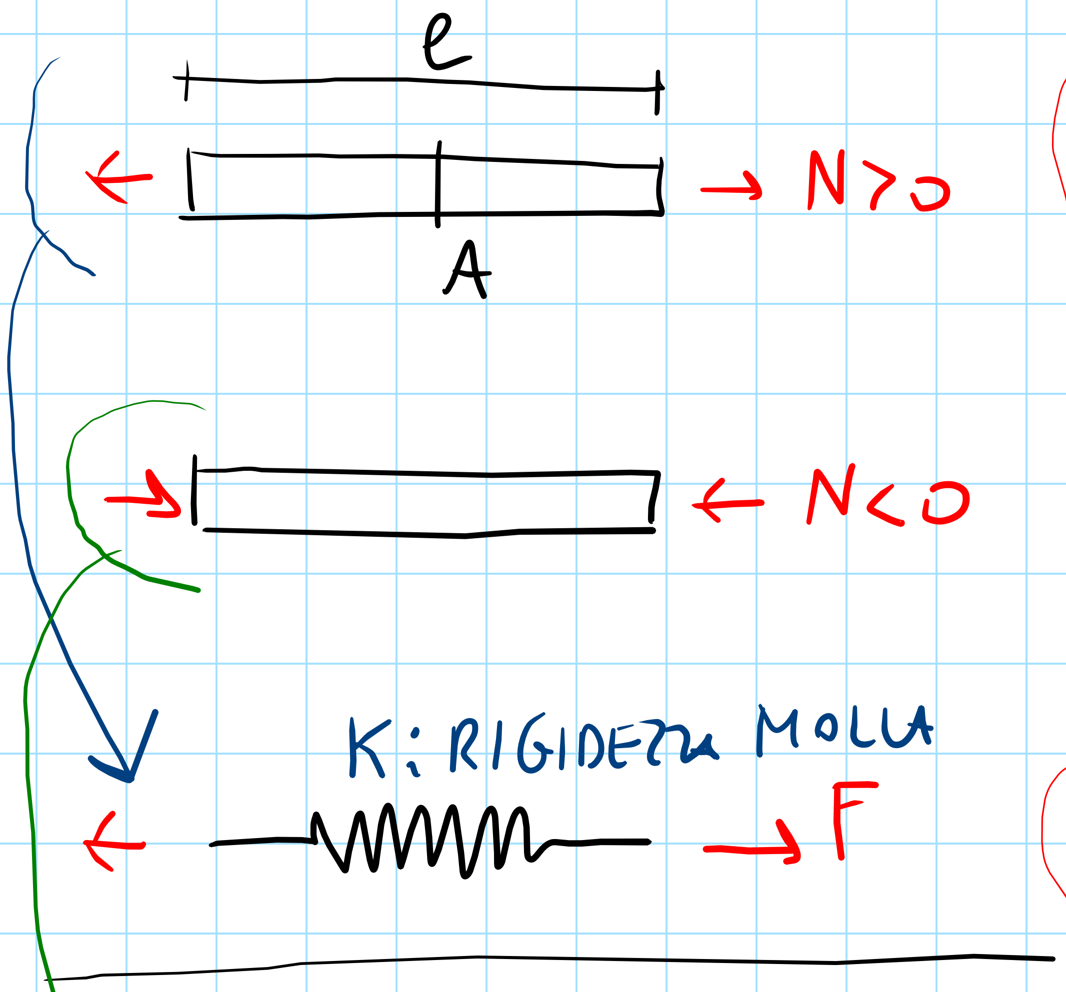
$$\boxed{\frac{dw(z)}{dz} = \frac{N}{EA}} \quad \boxed{w(0)=0}$$

CONDIZ AL BORDO

$$\left.\begin{array}{l} w(z) = \dfrac{N}{EA} z + D \\ D=0 \end{array}\right\} \Rightarrow \boxed{w(z) = \frac{N}{EA} z}$$

FUNZ. LINEARE $\Rightarrow w(\ell) = \boxed{\Delta\ell = \frac{N\ell}{EA}}$

ALLUNG. DOVUTO ALLA TRAZIONE N



$\Delta l = \frac{Nl}{EA} > 0$: ALLUNG.

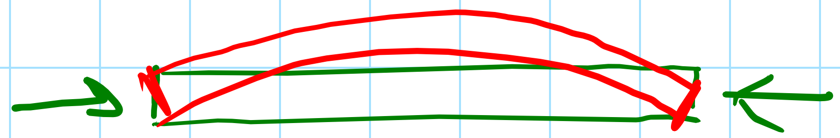
$\Delta l = \frac{Nl}{EA} < 0$: ACCORCIAM. $\left[w(l) = -\frac{Nl}{EA} \right]$

$\Delta l = \frac{F}{K}$: QUANDO VALE K NEL CASO DEL CILINDRO TESA?

$\boxed{K = \frac{EA}{l}}$

EA: COEFFICIENTE DI RIGIDEZZA ASSIALE

NOTA: ATTENZIONE AI SOLIDI SNELLI COMPRESSI PERCHÈ QUI NASCE IL PROBLEMA DELL'INSTABILITÀ.



(PROBLEMA NON CONTEMPL. DALLA SOLUZ DI S-V.)