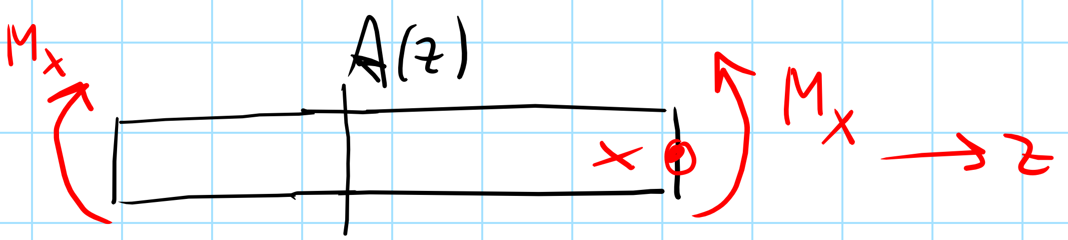
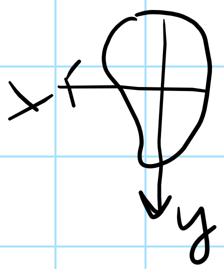
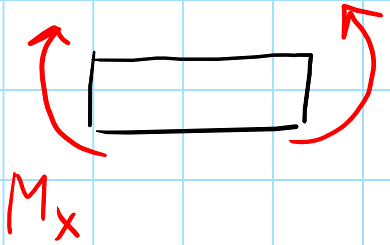
2) FLESSIONE RETTA (o SEMPLICE)

19/11/25







$M_x(z) = M_x$

STATO TENS : $\tau_{xz} = \tau_{yz} = 0$; $\sigma_{zz} \neq 0$

IPOTESI : $\sigma_{zz} = a x + b y$ (a, b COST INCOGNITE)

CDS

$$N(z) = \boxed{0 \overset{?}{=} \int_{A(z)} \sigma_{zz}\, dA} \quad , \quad 0 \overset{?}{=} a \overbrace{\int x\, dA}^{S_y} + b \underbrace{\int y\, dA}_{S_x}$$

=0 perché x, y BARICENTR.

OK

$$M_x(z) = \boxed{M_x \overset{?}{=} \int \sigma_{zz}\, y\, dA} \quad , \quad M_x = a \underbrace{\cancel{\int xy\, dA}}_{J_{xy}=0} + b \overbrace{\int y^2 dA}^{J_x \neq 0} \Rightarrow M_x = b J_x \Rightarrow \boxed{b = \frac{M_x}{J_x}}$$

(x, y : PRINCIPALI)

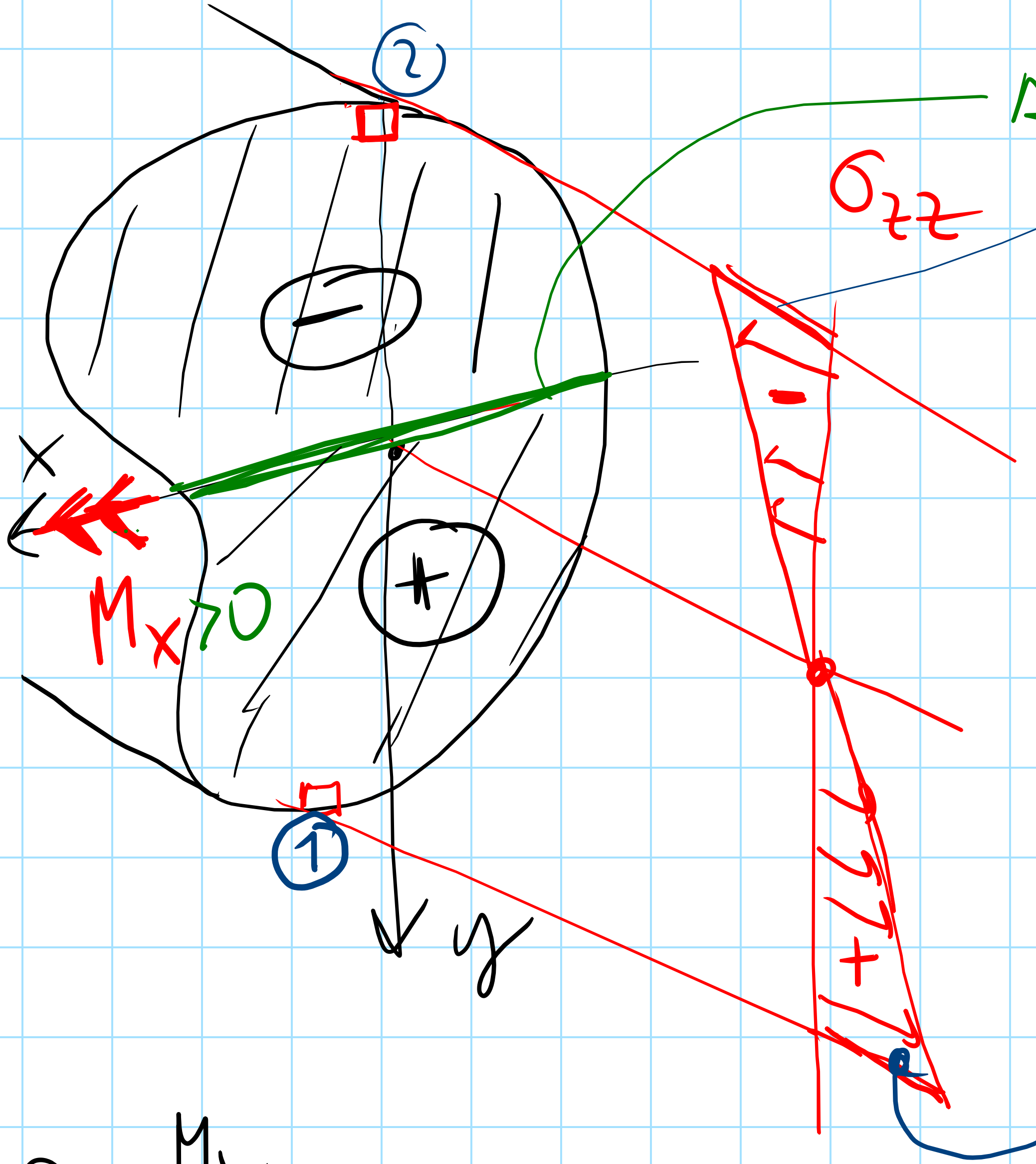
$$\boxed{M_y \overset{?}{=} -\int \sigma_{zz}\, x\, dA = 0}$$

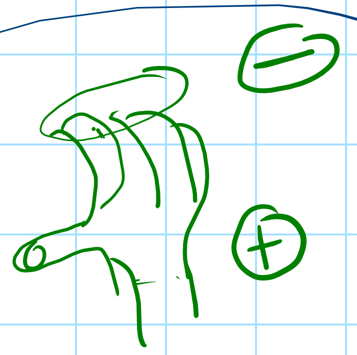
$$-a \underbrace{\int x^2 dA}_{J_y} - b \underbrace{\cancel{\int xy\, dA}}_{J_{xy}} = 0$$

$$\Rightarrow a \cancel{J_y}^{\neq 0} = 0 \; ; \Rightarrow \boxed{a = 0}$$

$$\boxed{\sigma_{zz} = \frac{M_x}{J_x}\, y}$$

FORMULA DI NAVIER





MANO DESTRA

$$\sigma_{zz}^{MIN} = \frac{M_x}{J_x} y_{MIN} \quad (2)$$

$$\sigma_{zz}^{MAX} = \frac{M_x}{J_x} y_{MAX} \quad (1)$$

$$[\underset{\sim}{\sigma}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{M_x}{J_x} y \end{bmatrix}$$

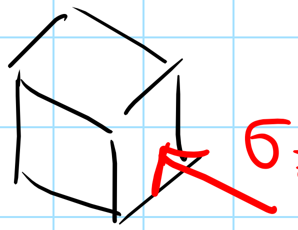
TENS DI CAUCHY PER TUTTO IL SOLIDO DI S-V.

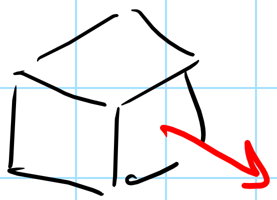
IN OGNI PUNTO DEL CILINDRO LO STATO TENS. È MONOASSIALE

$$\sigma_{zz} = \frac{M_x}{J_x} y$$

$$\sigma_{zz} \overset{?}{=} 0 \Rightarrow y = 0 \quad (\text{ASSE } x)$$

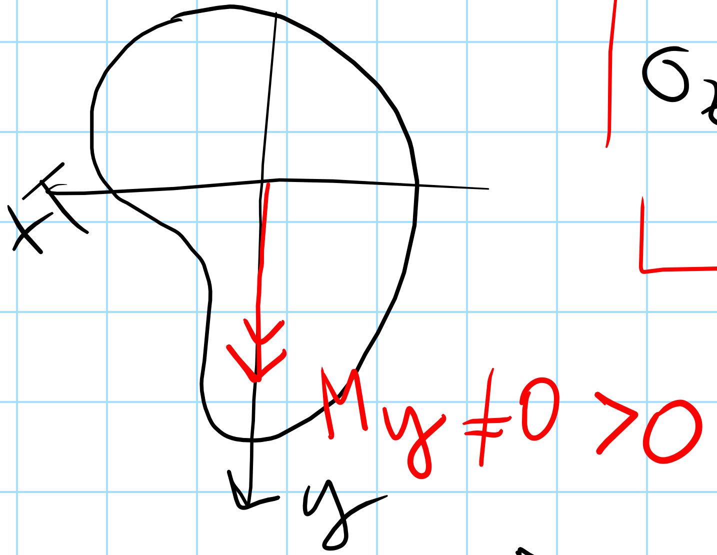
ASSE NEUTRO

(2)  σ_{zz}^{MIN} $\Big)\, y < 0$

(1)  σ_{zz}^{MAX} $\Big)\, y > 0$

VERIF. DI RESIST. (σ_0)

$$\text{MAX}\left\{ \sigma_{zz}^{MAX}, \left|\sigma_{zz}^{MIN}\right| \right\} \le \sigma_0$$

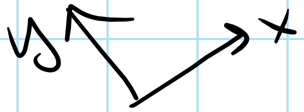


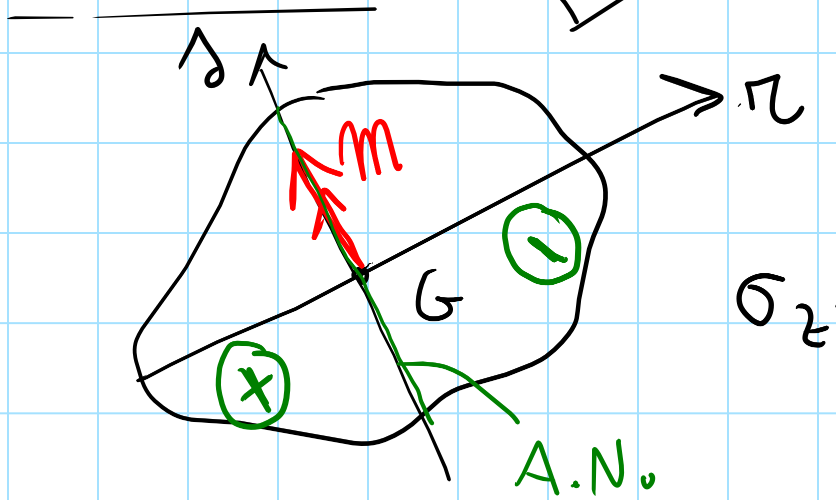
$$\boxed{\sigma_{zz} = - \frac{M_y}{J_y} x}$$

EQ. CDS

$$M_y = - \int_{A(z)} \sigma_{zz} \, x \, dA = - \int - \frac{M_y}{J_y} x^2 dA = \frac{M_y}{J_y} \underbrace{\int x^2 dA}_{J_y}$$

$$M_y = M_y \quad \text{OK}$$





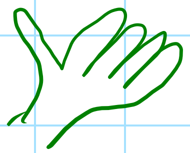
$$\sigma_{zz} = - \frac{m}{J_s} r$$

$r > 0$, $\sigma_{zz} < 0$

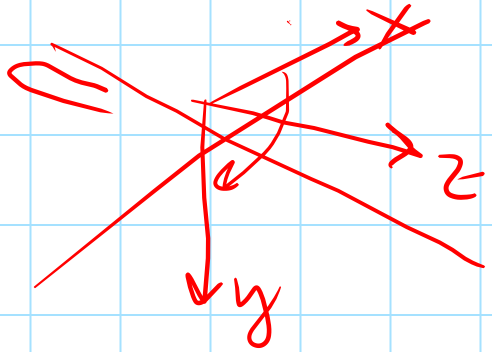
$r < 0$, $\sigma_{zz} > 0$

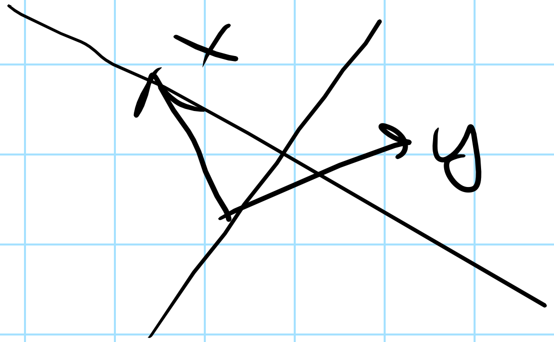
OK

r, s : PRINCIPALI

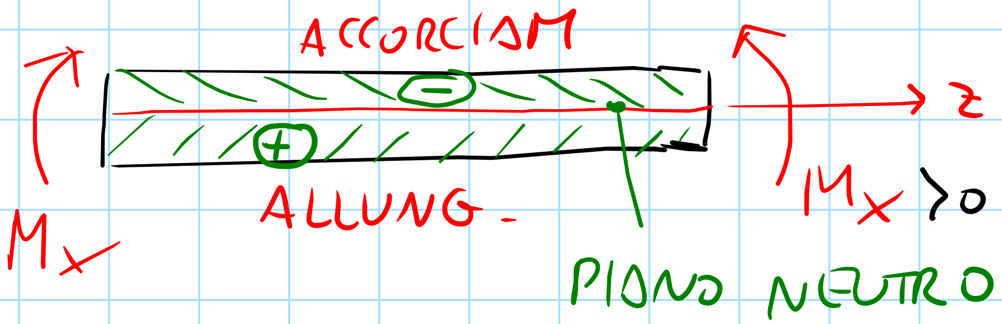


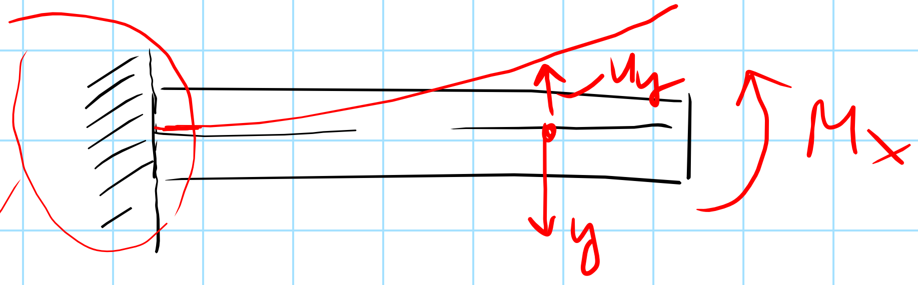
COINCIDENTI





DEFORMATA DELL'ASSE DEL CILINDRO INFLESSO





$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E} = \frac{M_x}{EJ_x} y$$

$$\varepsilon_{yy} = \varepsilon_{xx} = -\nu \frac{M_x}{EJ_x} y$$

LE EQ. DI PARTENZA PER DET. GLI SPOSTAMENTI SONO

$$u_{z,z} = \frac{M_x}{EJ_x} y \qquad \left.\begin{matrix} u_{y,y} \\ u_{x,x} \end{matrix}\right\rangle = -\nu \frac{M_x}{EJ_x} y \;\Rightarrow\; \begin{matrix} u_x \\ u_y \\ u_z \end{matrix} (x,y,z)$$

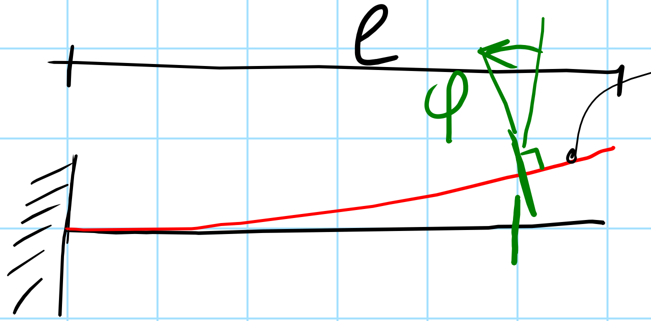
INTEGRANDO QUESTE EQ. ALLE DERIVATE PARZIALI E UTILIZZ. LE CONDIZ. DI VINCOLO IPOTIZZATE

L'ASSE z SUBISCE GLI SPOSTAMENTI:

$$\boxed{u_y(\underbrace{0,0,z}_{\text{ASSE}}) = -\frac{M_x}{2EJ_x} z^2}$$

$u_x = 0$
$u_z = 0$

OBIETTIVO: DETERMINARE LO SPOSTAMENTO SUBITO DALL'ASSE z DEL SOLIDO VINCOLATO NELLA SEZ. $z=0$



DEFORMATA ESAGERATA

M_x If

$$K(z) = \frac{d^2}{dz^2} u_y(0,0,z) = -\boxed{\frac{M_x}{EJ_x}}$$

CURVATURA COSTANTE

EJ_x: COEFFICIENTE DI RIGIDEZZA FLESSIONALE

$$\boxed{\frac{d\varphi}{dz} = \frac{M}{EJ_x}}$$

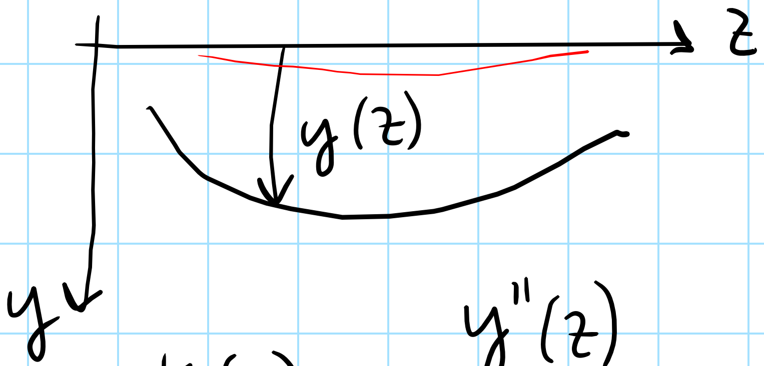
(ELIMINO IL SEGNO)

EQUAZ CHE, INTEGRATA, FORNISCE LE ROTAZIONI $\varphi(z)$

ASSE: $u_y = -\frac{M_x}{2EJ_x} z^2$

$$f = |u_y(0,0,\ell)| = +\frac{M_x}{2EJ_x}\ell^2$$

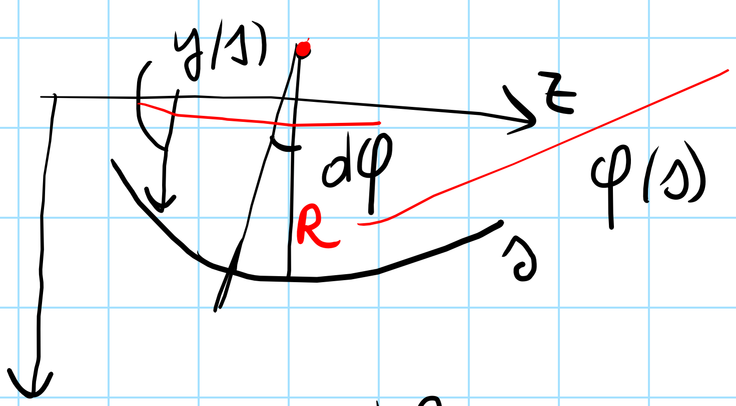
NOTE SULLA CURVATURA (K) DI UNA FUNZ.



$$K(z) = \frac{y''(z)}{\left(1+y'(z)\right)^{3/2}}$$

QUANDO $y(z)$ è PICCOLO ($\left(1+y'(z)\right)^{3/2} \to 1$)

$$\boxed{K(z) \simeq y''(z)}$$



RAGGIO LOCALE (DEL CERCHIO OSCULATORE)

$$K(s) = \frac{d\varphi}{ds}$$

QUANDO $y(s)$ è PICCOLO

$$\Rightarrow K(s) \simeq K(z) = \frac{d\varphi}{dz}$$

$$K(s) = \frac{1}{R}, \quad [K] = [L^{-1}]$$

verifica che $\left[\frac{M_x}{EJ_x}\right] = [L^{-1}]$ $\Longrightarrow$ $\left[\frac{M_x}{EJ_x}\right] = \left[\cancel{F}\cancel{L}\,\frac{\cancel{L^2}}{\cancel{F}}\,\frac{1}{L^{\cancel{4}}}\right] = \left[\frac{1}{L}\right]$

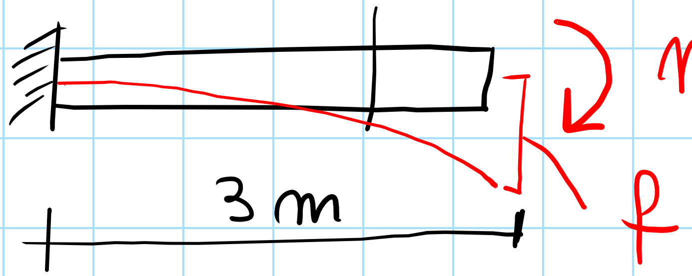
$[M_x] = [FL]$

$[E] = [FL^{-2}]$

$[J_x] = [L^4]$

LES

10 cm × 20 cm ACCIAIO: $E = 210 \cdot 10^9 \ \frac{N}{m^2}$



$m = -10 \ KNm$

$$f = \frac{m\,\ell^2}{2EJ_x} = \frac{10000 \cdot 3^2}{2 \cdot 210 \cdot 10^9 \cdot \frac{1}{15000}} = 0.0032 \ m$$

$$J_x = \frac{0{,}1 \cdot 0{,}2^3}{12} = \frac{1}{15000} \ m^4$$

$$k = \frac{m}{EJ_x} = \frac{10000}{210 \cdot 10^9 \ \frac{1}{15000}} = \frac{1}{1400} \ m^{-1}$$

$R = 1400 \ m$