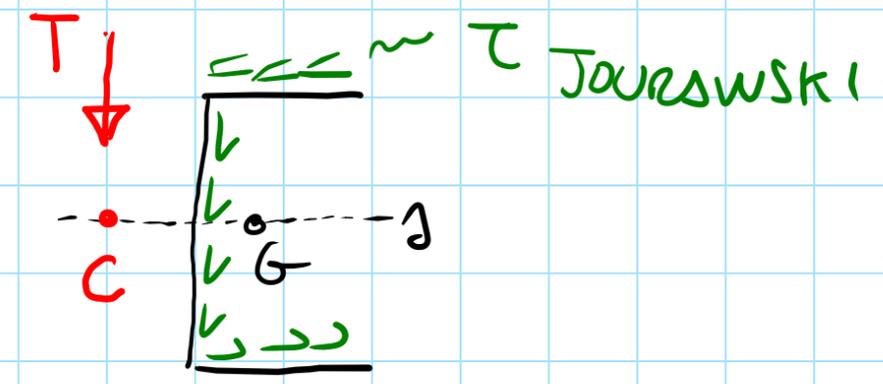
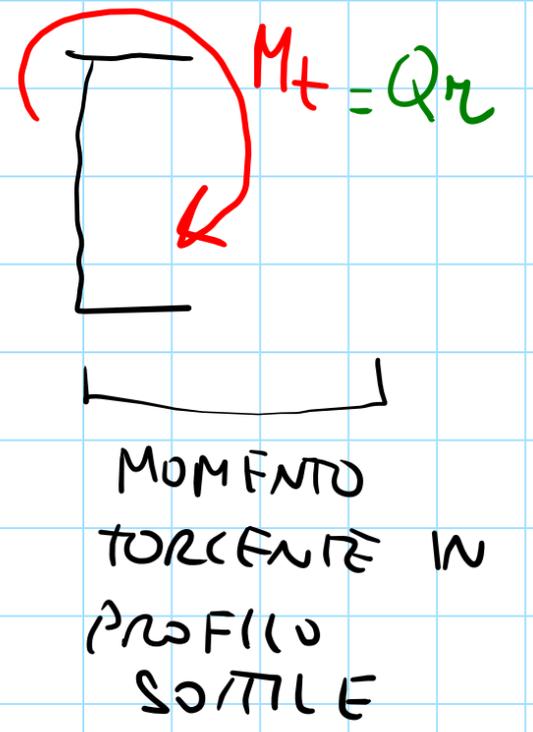
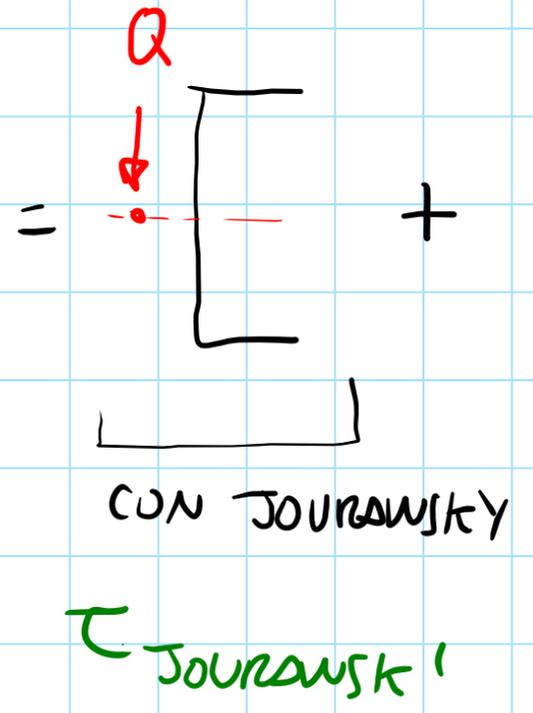
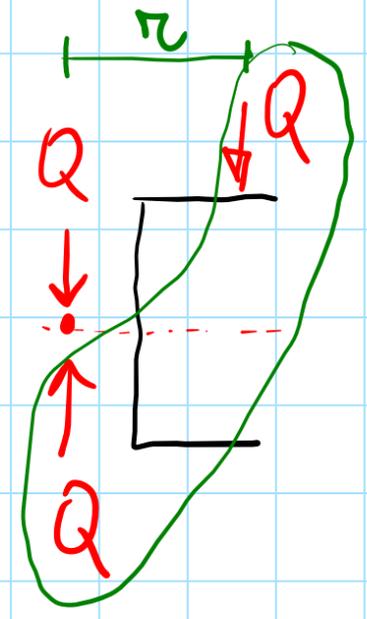
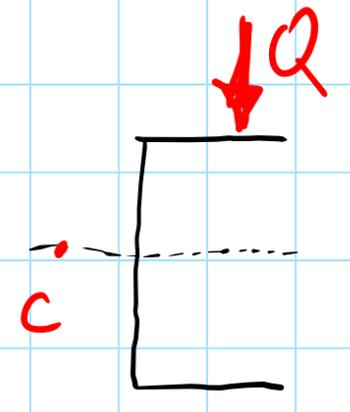


... TAGLIO - FLESSIONE

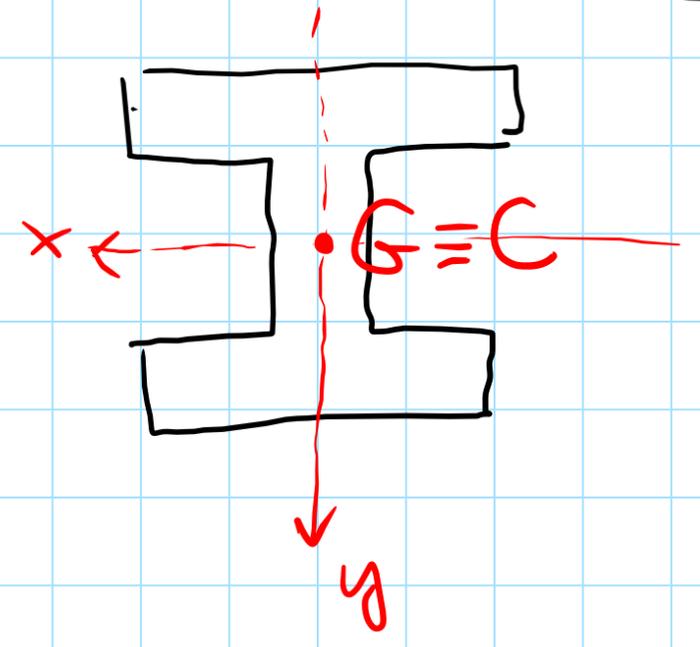


I DUE SISTEMI SONO EQUIVALENTI

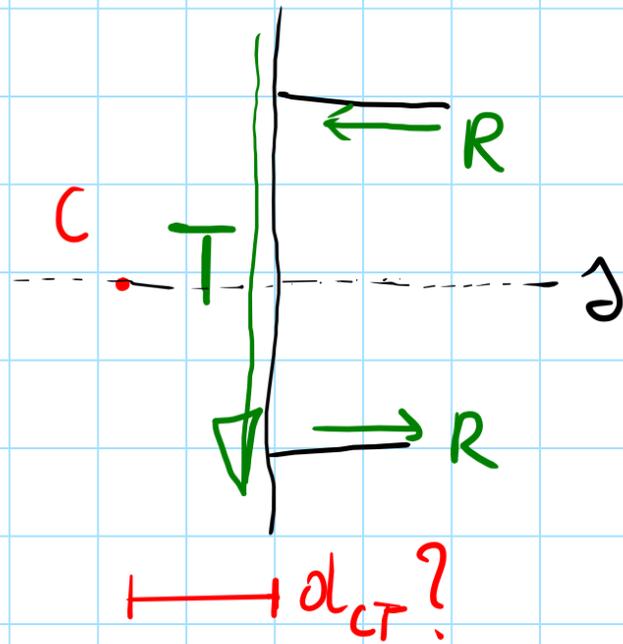
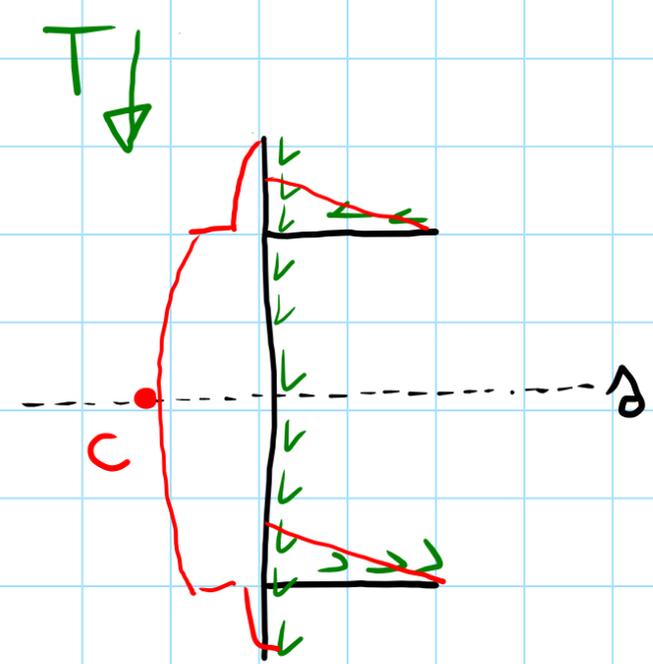


SEZ. MA DUE ASSI DI SIMM. $C \equiv G$

SEZ CON UN ASSE DI SIMMETRIA (Δ): $C \in \Delta$

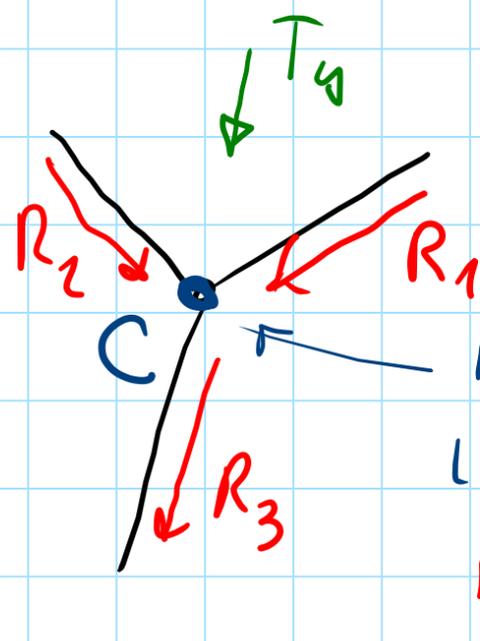
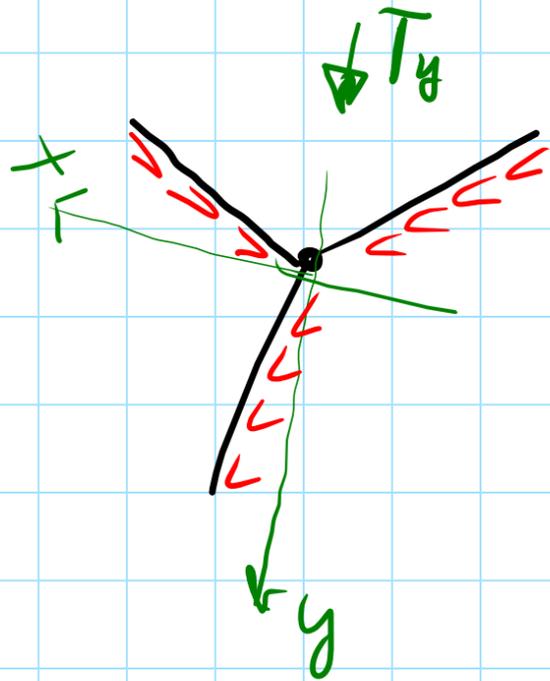


τ_{MONTORC}



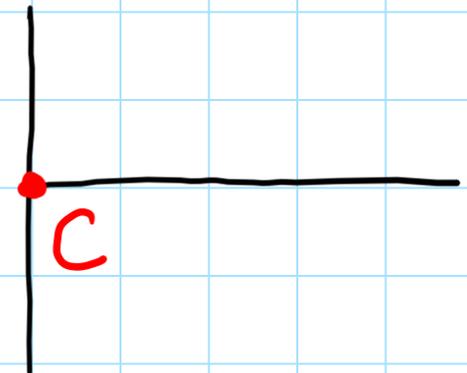
SI FA COME
CON LA I

SEZ. CON PROFILI RETTILINEI CONVERGENTI

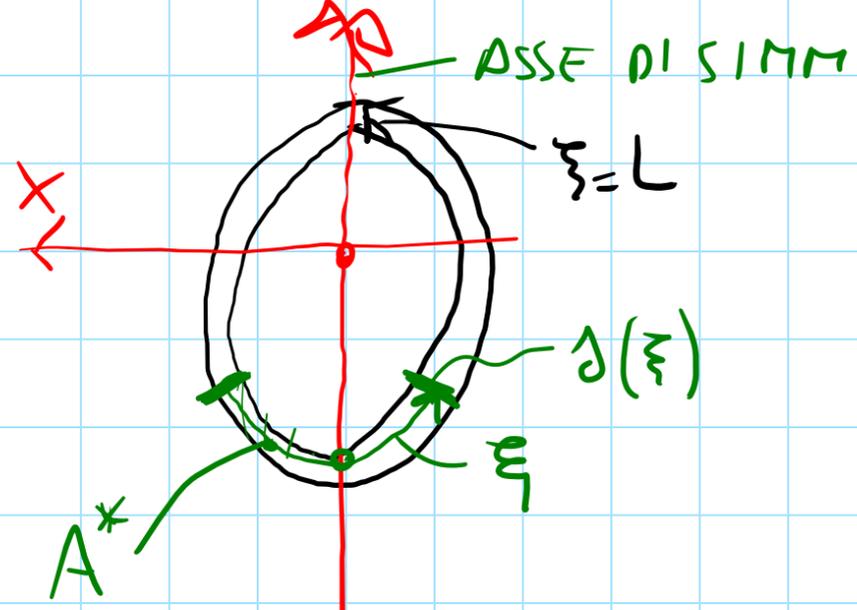


$$\{T_y\} = \{R_1, R_2, R_3\}$$

DA QUI PASSA
LA FORZA RESULTANTE
 R_1, R_2, R_3

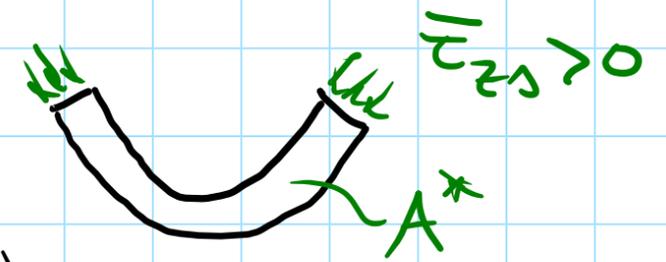


SE2. SOTTILI CHIUSE CON ASSE DI SIMMETRIA



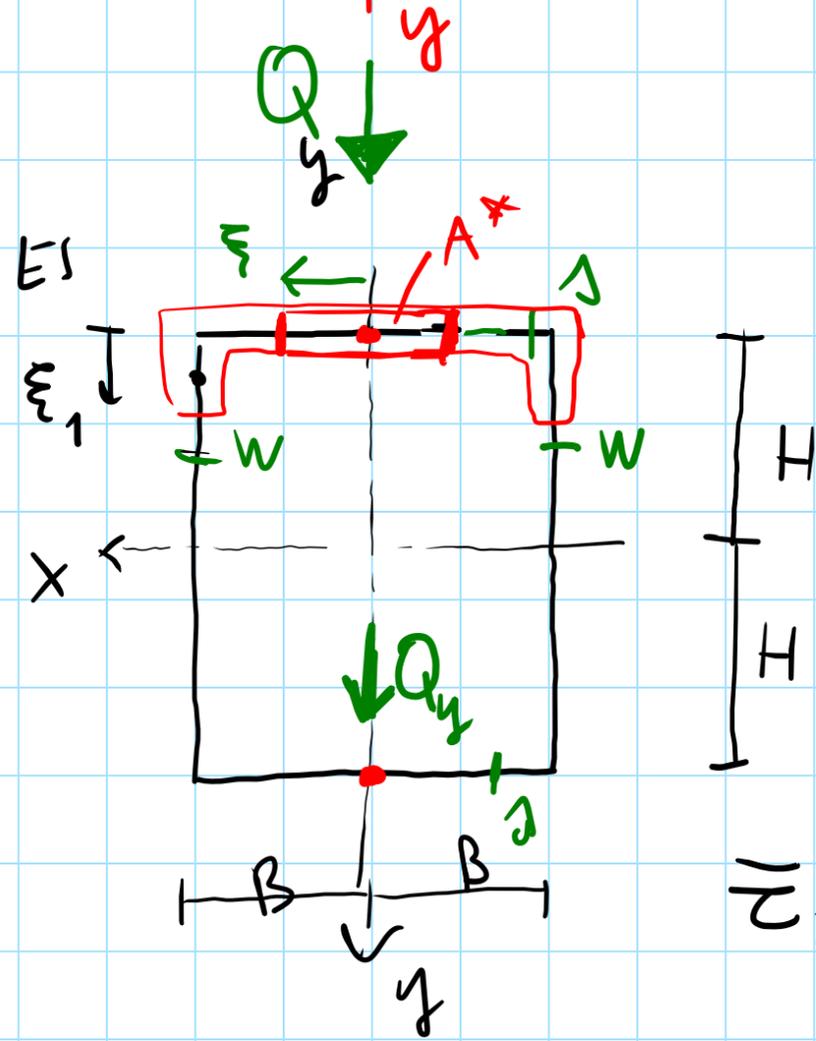
$$\bar{\tau}_{z\alpha} = \frac{Q_y S_x^*}{J_x \underbrace{2\delta(\xi)}_{\text{"CORDA TOTALE"}}$$

$\bar{\tau}_{z\alpha} > 0$ se "ENTRANO" IN A^*

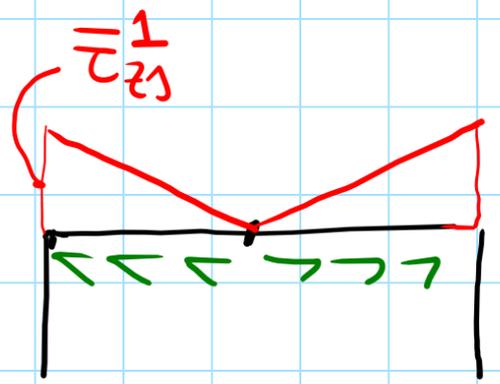


$\xi \rightarrow 0, \bar{\tau}_{z\alpha} \rightarrow 0 \quad (S_x^* \rightarrow 0)$

$\xi \rightarrow L, \bar{\tau}_{z\alpha} \rightarrow 0 \quad (S_x^* \rightarrow 0)$

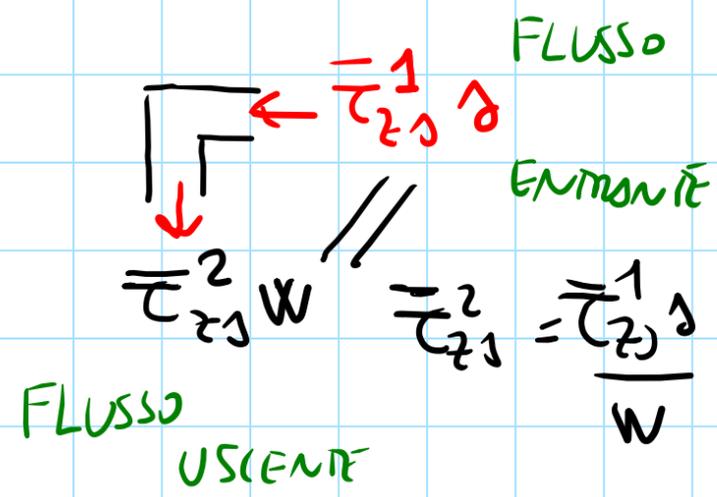


$$\bar{\tau}_{z\alpha}(\xi) = \frac{Q_y}{J_x 2\delta} \underbrace{2\xi\delta(-H)}_{S_x^*} < 0$$

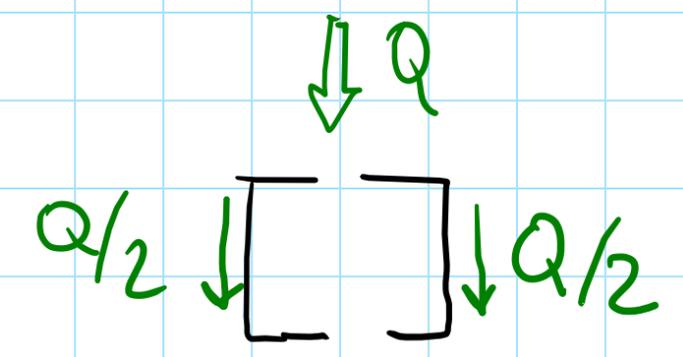
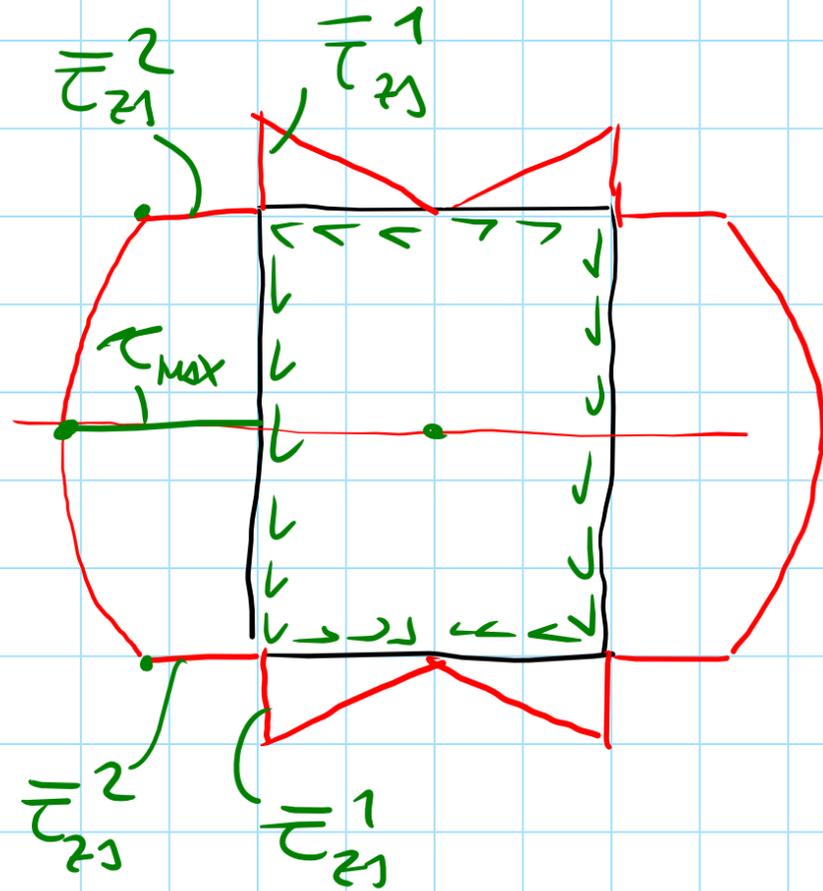


$$\bar{\tau}_{z\alpha}(\epsilon_1) = \frac{Q_y}{J_x 2w} \left[-2B\delta H + 2w\epsilon_1 \left[-\left(H - \frac{\epsilon_1}{2}\right) \right] \right]$$

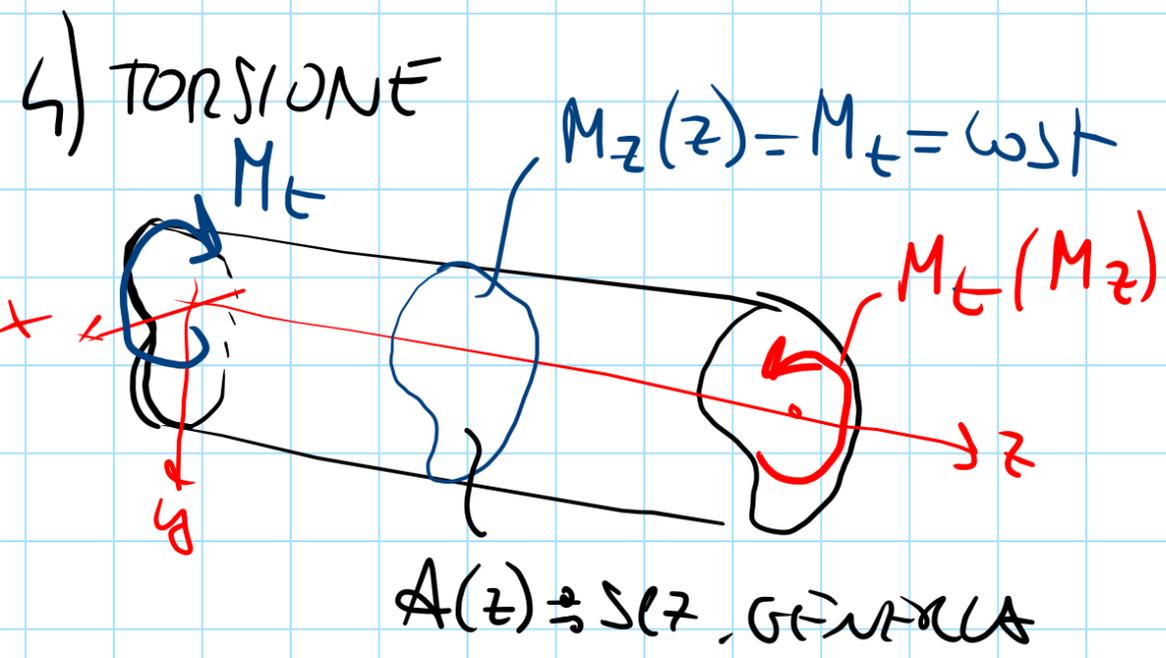
PARABOLICHE < 0



$$\bar{\tau}_{z\alpha}(0) = \frac{Q_y}{J_x 2w} (-2B\delta H) = \bar{\tau}_{z\alpha}^2$$



IL DIAGR. DELLE $\bar{\tau}_{21}$ È
 COME QUELLO DI 2 [AFFIANCATE



$$\tau_{xz}, \tau_{yz} \neq 0$$

$$\sigma_{zz} = 0$$

$$\tau_{xz}(x,y), \tau_{yz}(x,y)$$

IL PROBL DI TORSIONE È

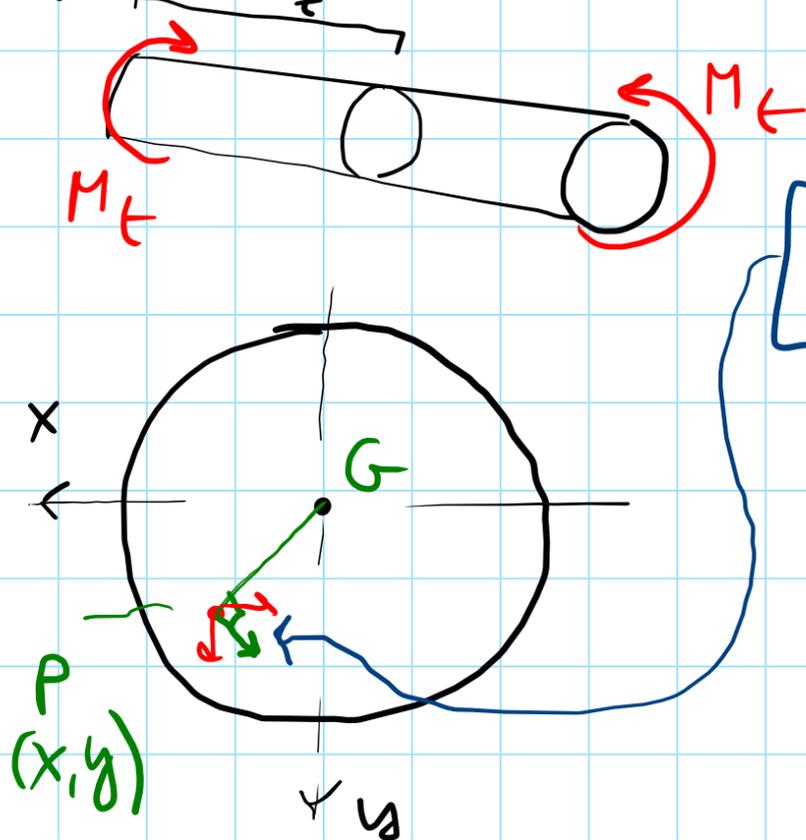
PIU' DI UNO COMPLESSO

E NON LO RISOLVEREMO

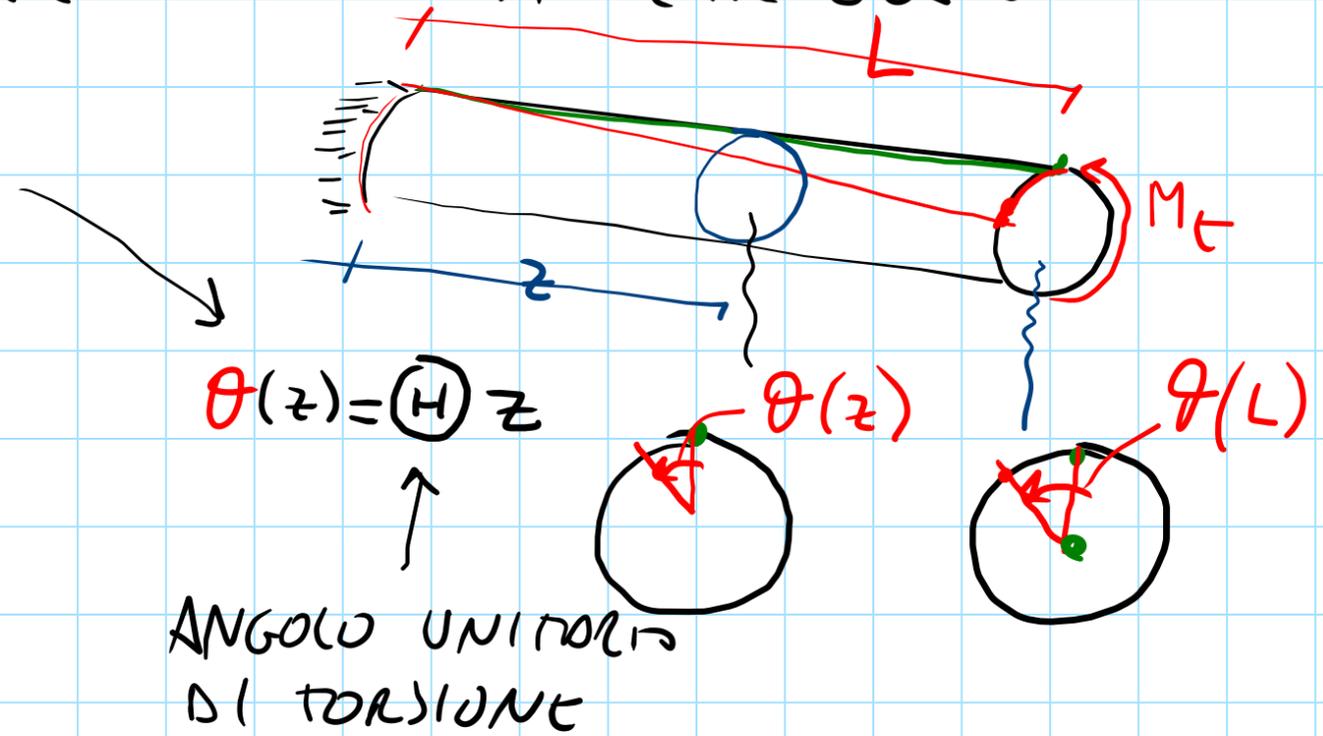
IN FORMA CHIUSA.

CI CONCENTRIAMO SUL PR. DI TORS. CON SEZ. COMPACTA CIRCOLARE (COULOMB 1780)

L'IPOTESI DI COULOMB È DI TIPO CINEMATICO



$$\begin{cases} u_x = -\theta(z)y = -(\oplus)zy \\ u_y = +\theta(z)x = (\oplus)zx \\ u_z = 0 \end{cases}$$



LEG COSTRUTTIVO : $\tau_{xz} = G \gamma_{xz} = G (u_{x,z} + u_{z,x}) = G(-\Theta y)$

$\tau_{yz} = G \gamma_{yz} = G (u_{y,z} + u_{z,y}) = G \Theta x$

J MOM D'INERZIA
 G POTERE

CDS $M_t = \int_A \tau_{yz} x - \tau_{xz} y \, dS = \int_A G \Theta x^2 + G \Theta y^2 \, dS = G \Theta \int_A x^2 + y^2 \, dS$

$M_t = G \Theta J_G$
 NOTO NOTO NOTO

$\Theta = \frac{M_t}{G J_G}$

$J_G = \frac{\pi R^4}{2}$

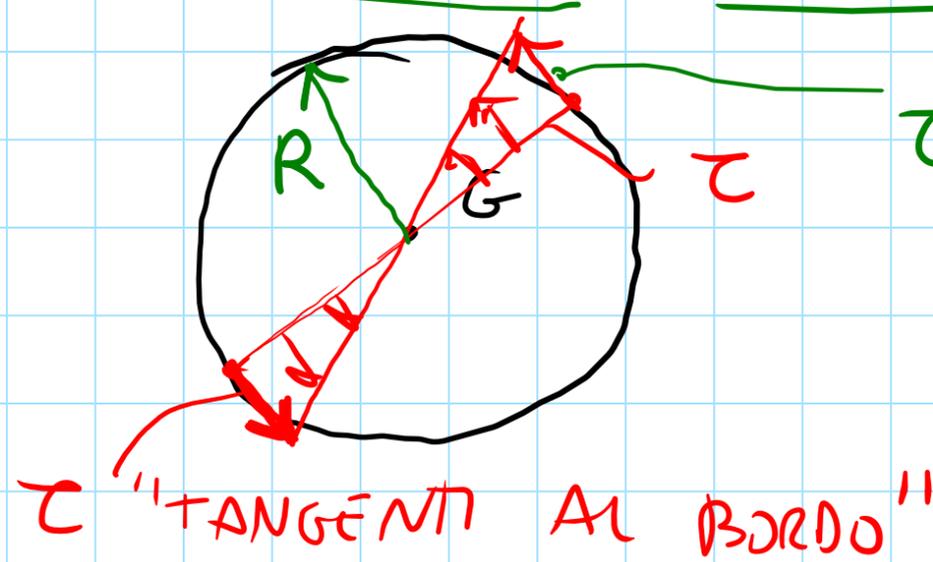
$\tau_{xz} = -\frac{M_t}{J_G} y$

$\tau_{yz} = \frac{M_t}{J_G} x$

VALE PER
 TUTTO IL
 CILINDRO

$|\underline{\tau}| = \sqrt{\tau_{xz}^2 + \tau_{yz}^2} = \frac{M_t}{J_G} r$

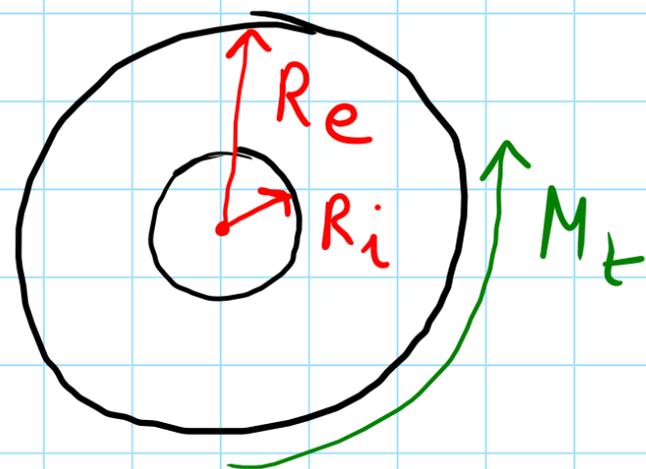
r : DIST |PG|



$\tau_{max} = \tau(R) = \frac{M_t}{J_G} R$

VERIFICHE DI RESIST.
 VON MISES $\sqrt{3} \tau_{max} \leq \sigma_0$
 TRESCA $2 \tau_{max} \leq \sigma_0$

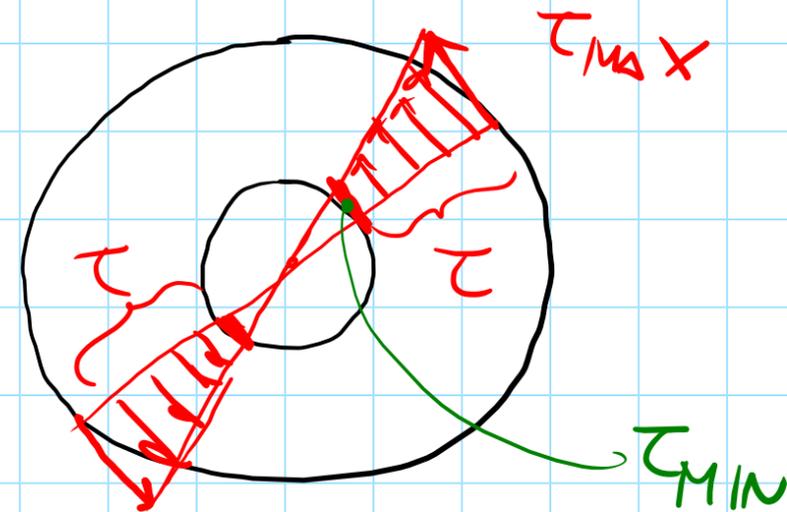
SE AVESSIMO UN CILINDRO CAVO ;



STESSI RISULTATI MA:

$$J_G = \frac{\pi}{2} (R_e^4 - R_i^4)$$

$$\tau(r) = \frac{M_t}{J_G} r \quad \begin{cases} \tau_{max} = \frac{M_t}{J_G} R_e \\ \tau_{min} = \frac{M_t}{J_G} R_i \end{cases}$$



$$\theta = \frac{M_t}{G J_G}$$

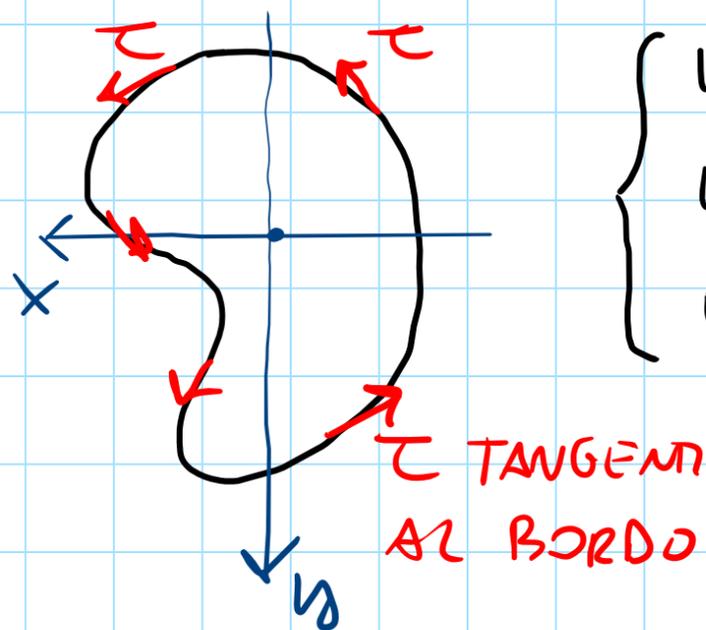
OSS: COSA SUCCEDERE ALL'EQ [CDS]

$$Q_x = 0 = \int_A \tau_{zx} dS \Rightarrow 0 = \int -\frac{M_t}{J_G} y dS$$

$$0 = -\frac{M_t}{J_G} \int_A y dS \quad S_x = 0 \quad \text{OK}$$

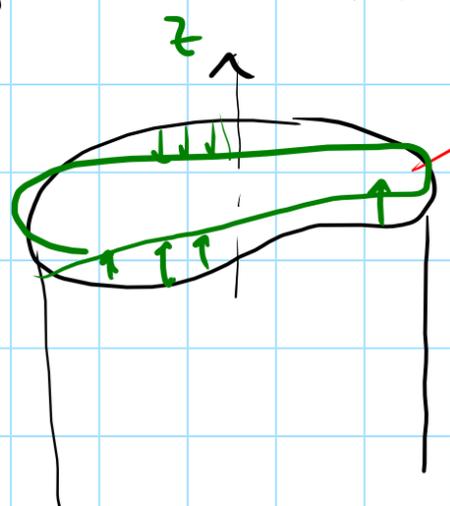
VERIFICARE ANCHE PER Qy

CENNI AL PROBLEMA CON SEZ. COMPATTA GNERICA (SV)



$$\begin{cases} u_x = -\textcircled{H} z y \\ u_y = \textcircled{H} z x \\ u_z = \textcircled{H} \omega(x, y) \end{cases}$$

FUNZ. DI INGORBBAMENTO (INCOGNITA)



$u_z \neq 0$ spost. fuori dal piano

S.-V. HA STUDIATO IL PROBL. ELASTICO \Rightarrow

$$\boxed{\nabla^2 \omega(x, y) = 0} \text{ su } A(z)$$

+ CONDIZ AL BORDO ($\partial A(z)$)

PER LA SEZ CIRC $\Rightarrow \omega(x, y) = 0$

$$\Rightarrow J_t = J_G$$

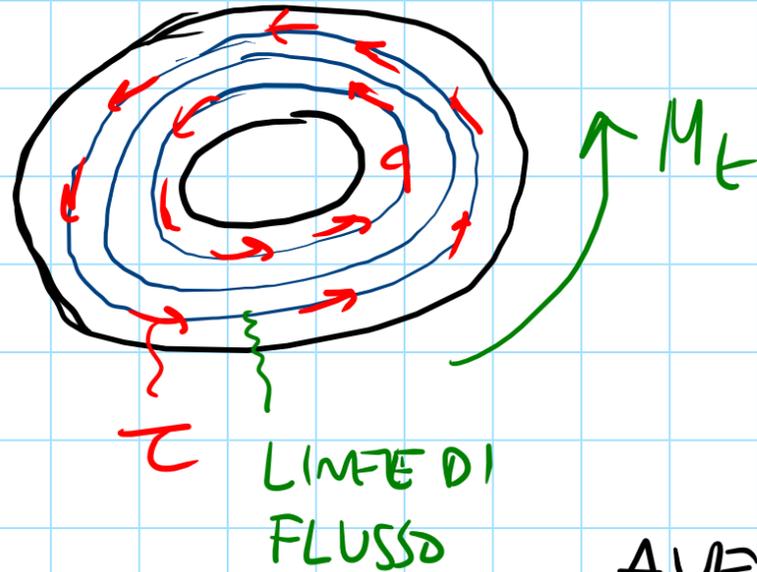
$$\textcircled{H} = \frac{M_t}{G \textcircled{J}_t}$$

FAITORE DI RIGIDEZZA TORSIONALE

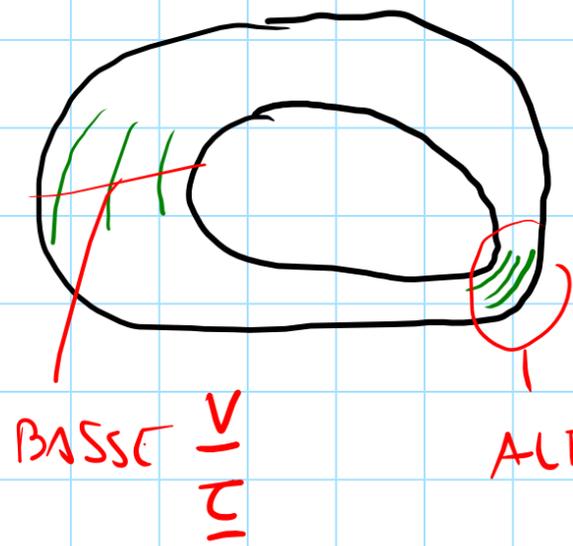
SI PUO' RISOLVERE ANCHE ATTRAVERSO UN'ALTRA FUNZIONE: FUNZIONE DELLE TENSIONI σ DI PRANDTL

$$J_t(\omega(x, y))$$

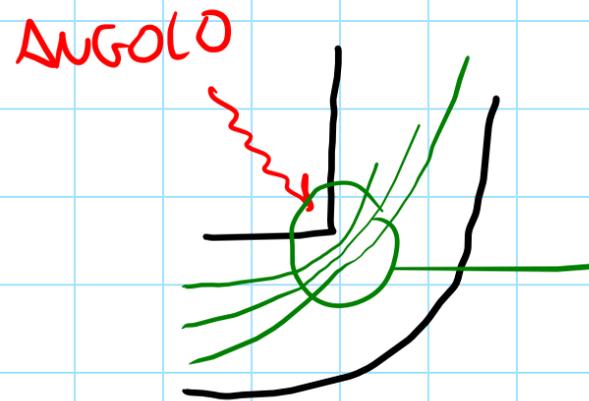
ANALOGIA IDRODINAMICA DELLA TORSIONE



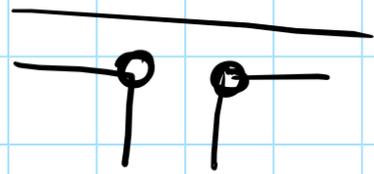
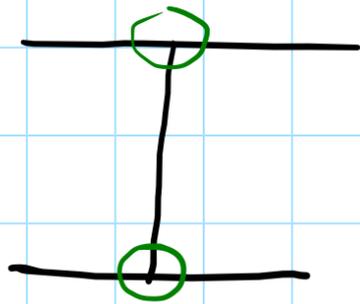
LA DISTRIBUZIONE DEL VETTORE TENS. TANGENZIALE τ COINCIDE CON IL CAMPO DELLE VELOCITA' v DI UN FLUIDO PERFETTO IN MOVIMENTO LAMINARE-STAZIONARIO CONTENUTO IN UN RECIPIENTE AVENTE LA FORMA DELLA SEZ. DEL SOLIDO.



(CONCENTRAZ. DI TENSIONI)



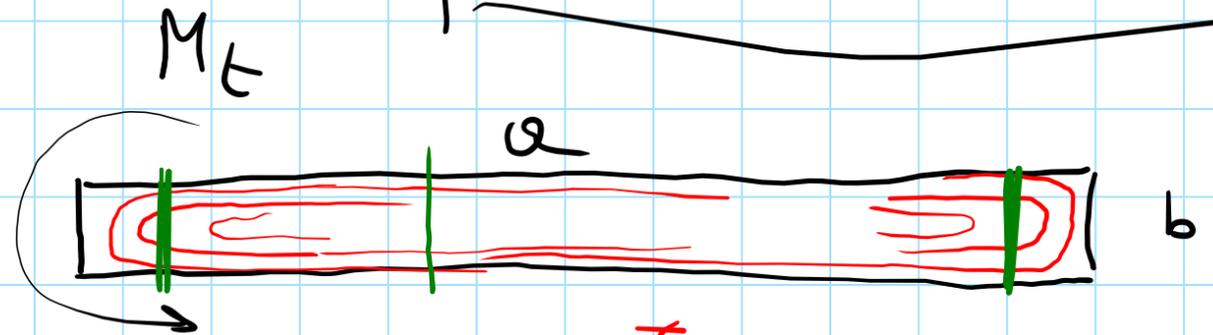
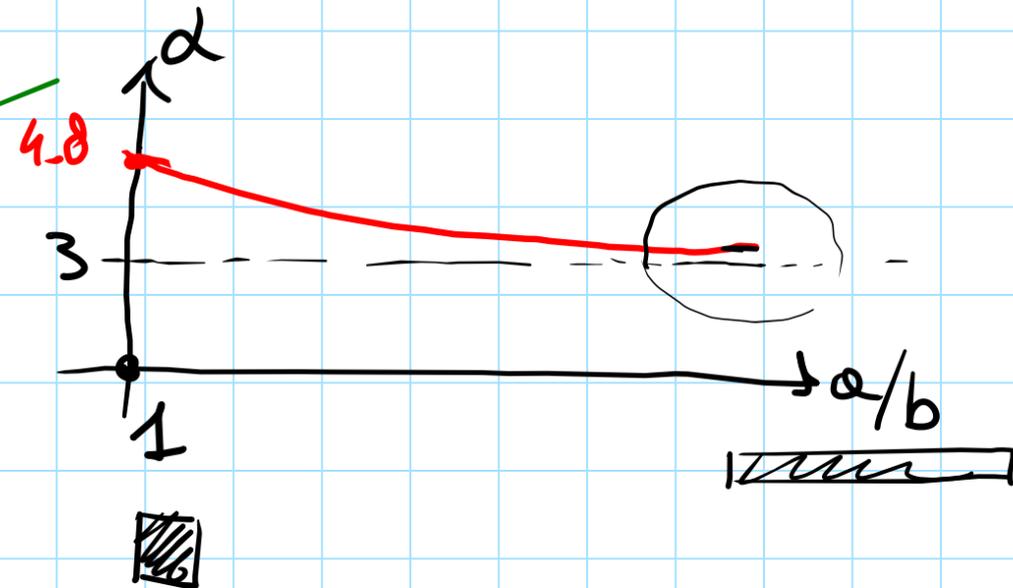
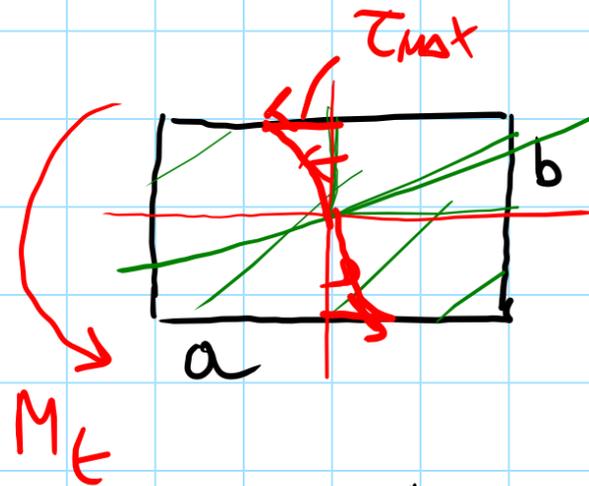
CONCENTR. DI τ (SITUAT. CRITICA) NECESSARI RACCORDI



TORSIONE NELLE SEZ. A PROFILO SOTTILE

INSTABILITA' DEL 'EQUILIBRIO SE LE COMPRESIONI SONO ELEVATE

RETTANGOLO COMPATTO



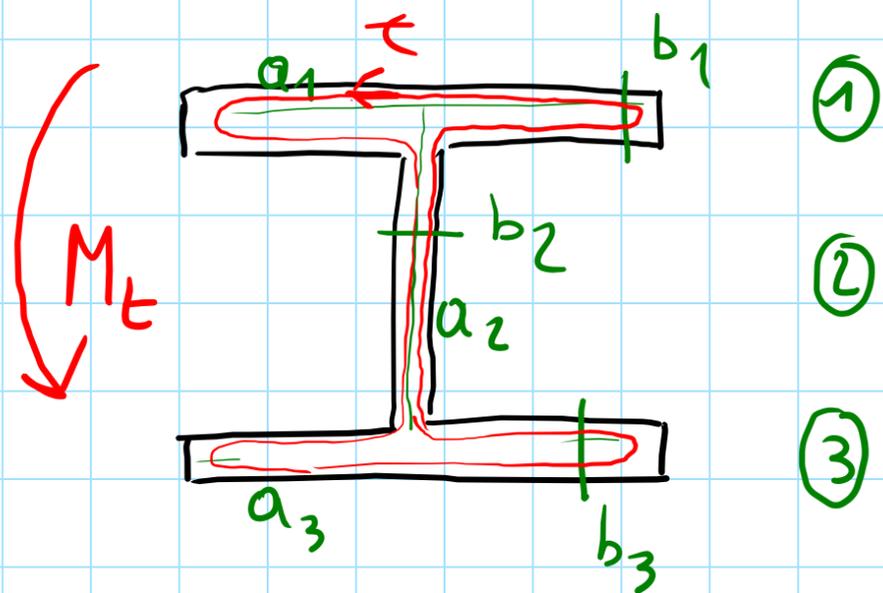
$$\tau_{max} = \alpha \frac{M_t}{ab^2}$$

↑
 $\alpha(a/b)$ COEFF.

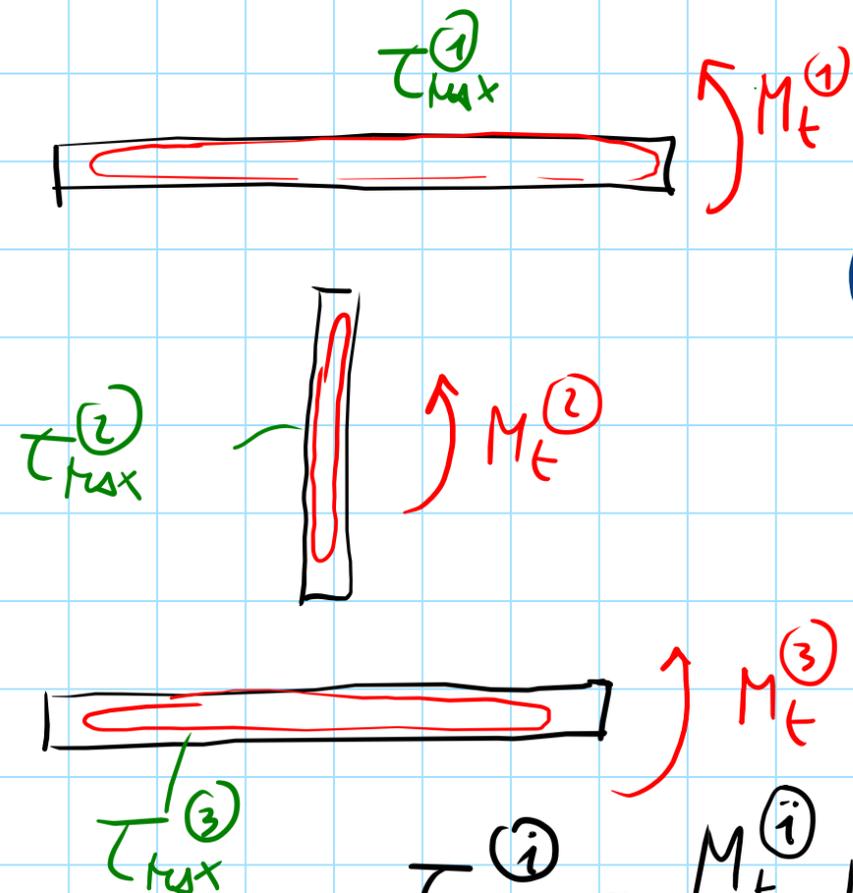
$$\tau_{max} \approx 3 \frac{M_t}{ab^2} = 3 \frac{M_t b}{ab^3} \frac{1}{J_t}$$

$$J_t = \frac{1}{3} ab^3$$

ESERCIZIO ESEMPLIFICATIVO DI STUDIO DI SEZ. CON PROFILI SOTTILI
(SEZ. "APERTA")



?? $\tau_{max}^{(1)}$, $\tau_{max}^{(2)}$, $\tau_{max}^{(3)}$??



EQUIVALENZA:

$$\odot M_t = M_t^{(1)} + M_t^{(2)} + M_t^{(3)}$$

CONGRUENZA:

$$\odot \Theta_1 = \Theta_2 = \Theta_3 = \Theta$$

$$\Theta(z) \quad [\Theta] = [L^{-1}]$$

$$= D \begin{matrix} M_t^1 \\ M_t^2 \\ M_t^3 \end{matrix}$$

← GLOBALG
DELLA
SEZ.

$$\odot \frac{M_t^{(1)}}{G J_t^{(1)}} = \frac{M_t^{(2)}}{G J_t^{(2)}} = \frac{M_t^{(3)}}{G J_t^{(3)}} = \frac{M_t}{G J_t} \Rightarrow M_t^{(1)} = \frac{M_t}{J_t} J_t^{(1)}$$

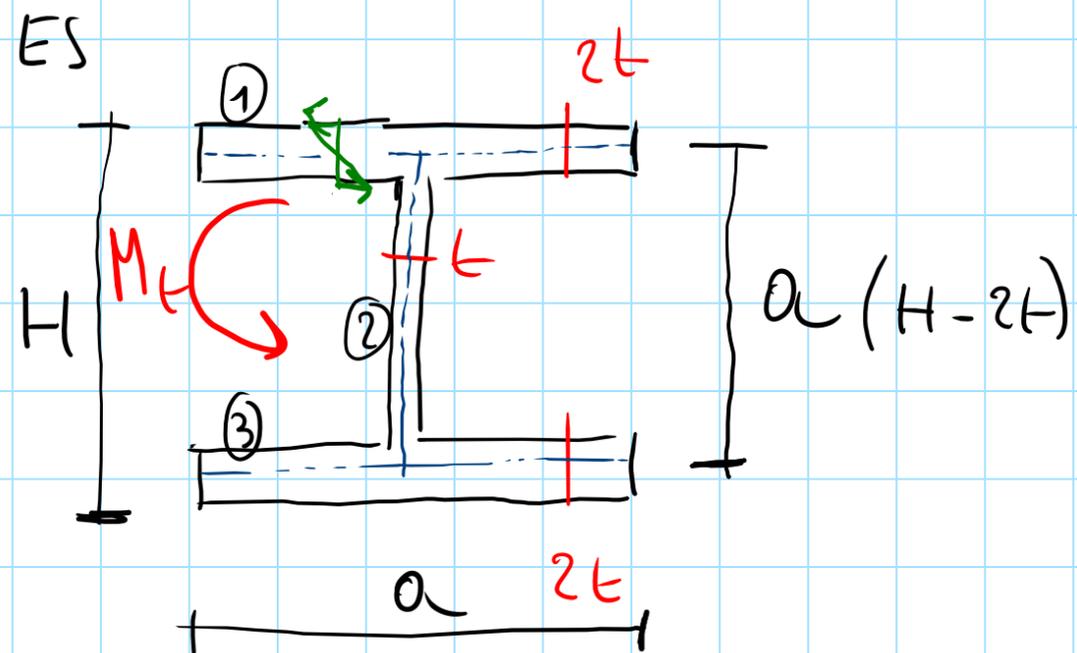
$$\odot M_t = G J_t^{(1)} \Theta + G J_t^{(2)} \Theta + G J_t^{(3)} \Theta = G \Theta \left[\sum_i J_t^{(i)} \right] J_t$$

$$M_t^{(i)} = M_t \frac{J_t^{(i)}}{J_t}$$

$$J_t = \sum_{i=1}^3 J_t^{(i)}$$

3/12/25

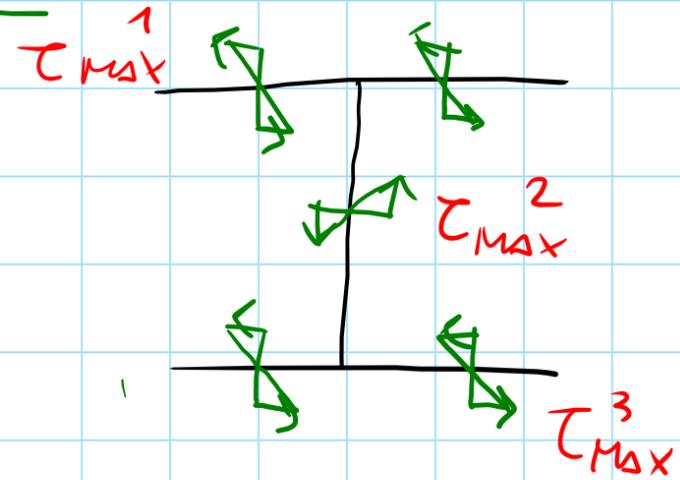
$$\tau_{\max}^{(i)} = \frac{M_t^{(i)}}{J_t^{(i)}} b_i \Rightarrow \boxed{\tau_{\max} = \max \left\{ \tau_{\max}^{(i)} \right\} = \max \left\{ \frac{M_t^{(i)}}{J_t^{(i)}} b_i \right\} = \frac{M_t}{J_t} \max \{ b_i \}}$$



$$J_{t_1} = J_{t_3} = \frac{1}{3} a (2t)^3 = \frac{8}{3} at^3$$

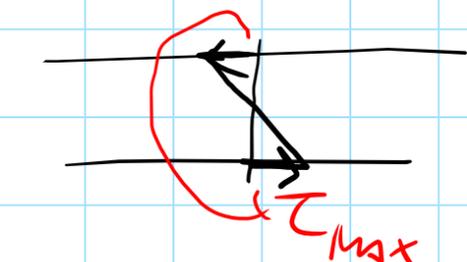
$$J_{t_2} = \frac{1}{3} at^3$$

$$J_t = \sum J_{t_i} = \frac{17}{3} at^3$$



$$\frac{M_t}{J_t}$$

SPESSORE
MAGGIORE

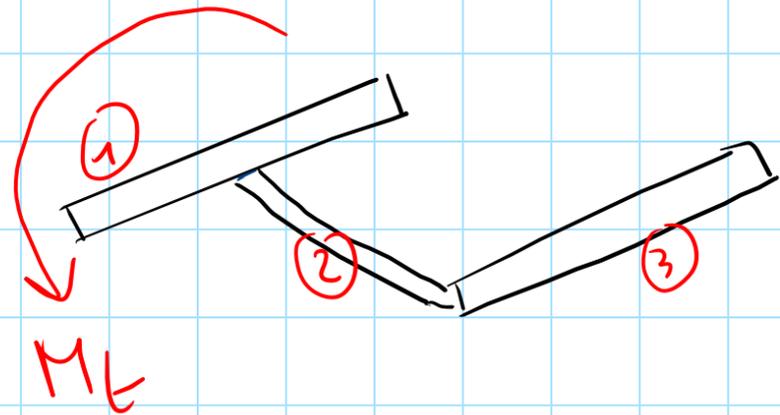


$$M_{t_1} = \frac{M_t}{J_t} J_{t_1} = M_t \frac{8}{3} at^3 \frac{3}{17 at^3} = \frac{8}{17} M_t (= M_{t_3})$$

$$M_{t_2} = M_t \frac{J_{t_2}}{J_t} = M_t \frac{1}{3} at^3 \frac{3}{17 at^3} = \frac{1}{17} M_t$$

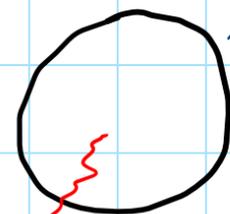
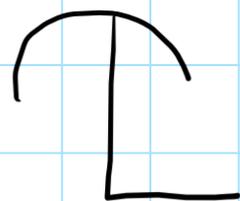
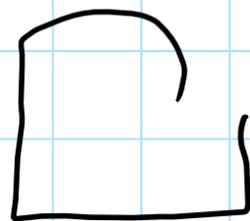
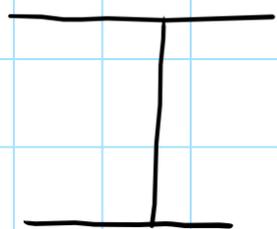
$$\tau_{\max}^{(1)} = \tau_{\max}^{(3)} = \frac{M_{t_1}}{J_{t_1}} 2t = \frac{8}{17} M_t \frac{3}{8 at^3} 2t = \frac{6}{17} \frac{M_t}{at^2} = \tau_{\max}$$

$$\tau_{\max}^{(2)} = \frac{M_{t_2}}{J_{t_2}} t = \frac{1}{17} M_t \frac{3}{at^3} t = \frac{3}{17} \frac{M_t}{at^2} (= \tau_{\max}/2)$$

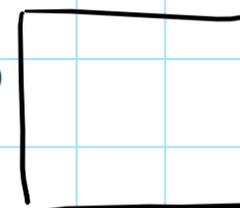


SOLUZ. ESATT. UGUALE
ALLA PRECEDENTE

SEZ. "APERTA" e SEZ. "CHIUSA" (SEZ. FORM. DA PROFILI SOTTILI)



CIRCUITO

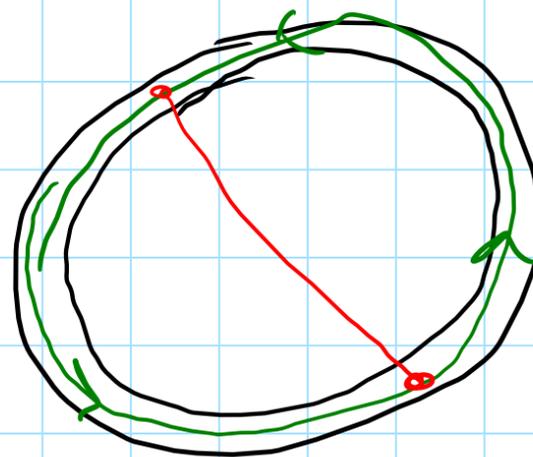
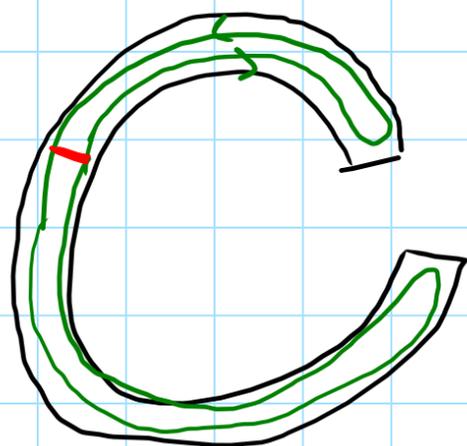


A P E R T E

AREA INTERNA CHIUSA

COMPORT. QUALITATIVO

A TORSIONE

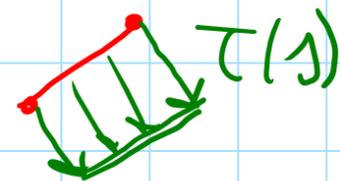
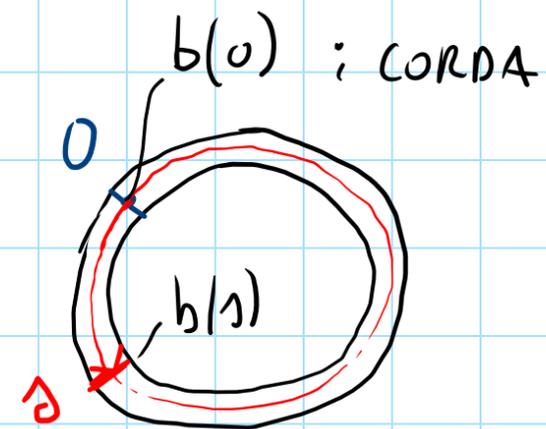
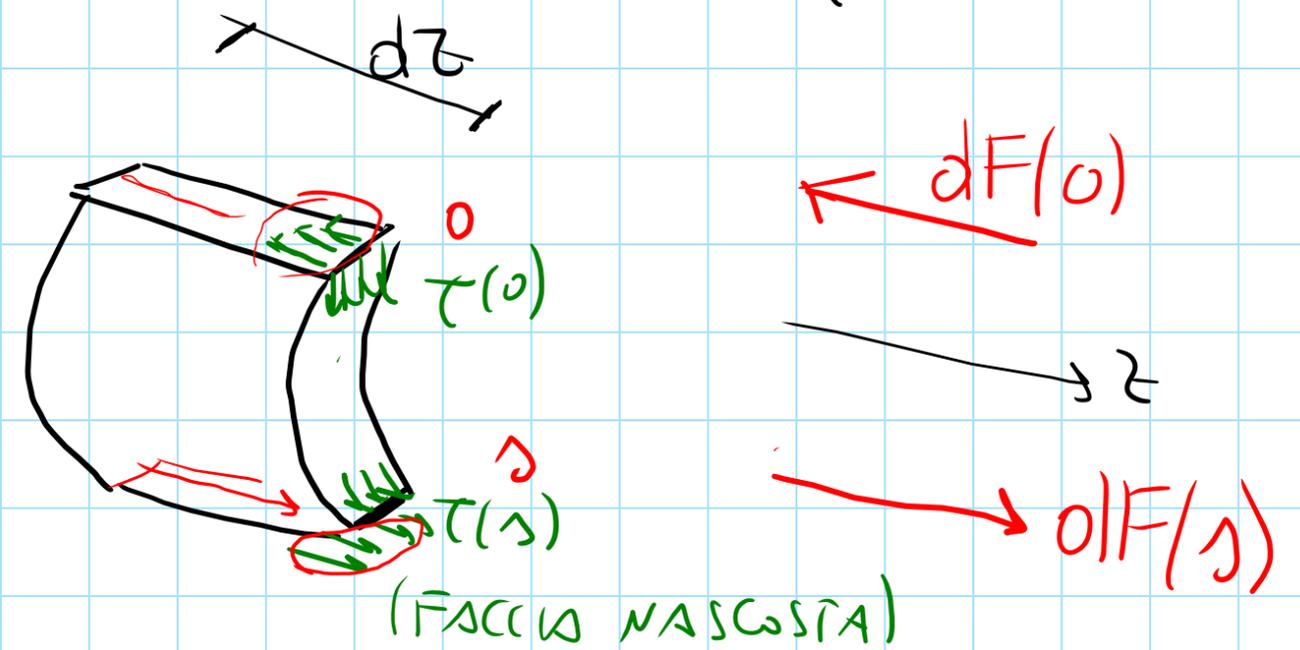
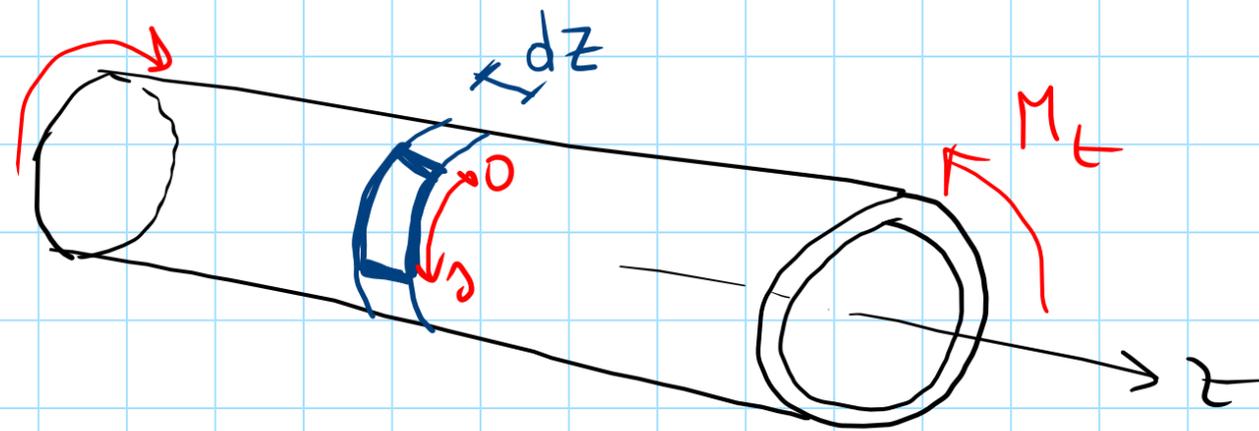


FLUSSO 'T' CON
ANALOGIA
IDRODINAMICA

POCO RESIST A TORSIONE

MOLTO RESISTENTE A TORSIONE

TEORIA DI BREDT DELLE SEZ. SOTTILI CHIUSE SOGGETTE A TORSIONE (TEOR. APPROSS.)



IPOTESI: TENS. TANGENZ.
 COSTANTI SULLA CORDA

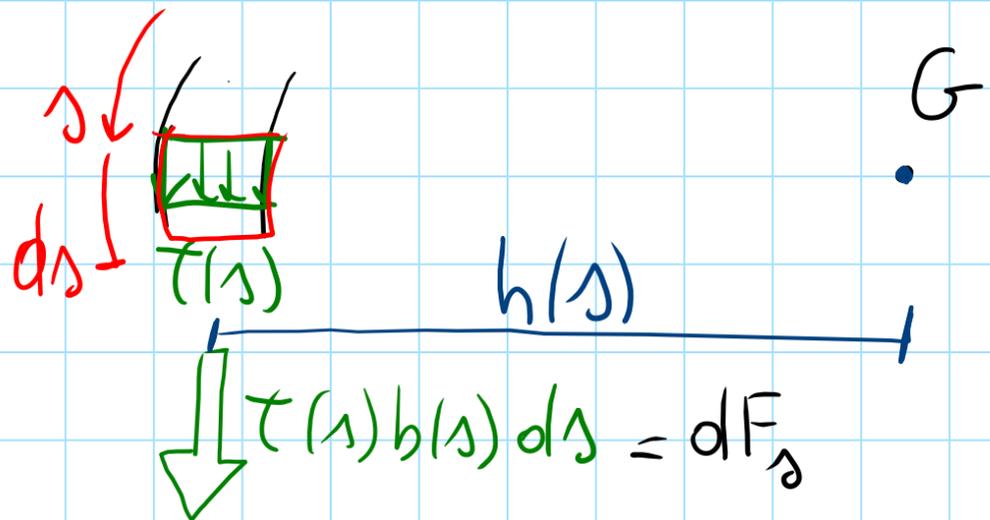
EQUILIBRIO LONGITUDINALE:

$$dF(0) - dF(s) = 0$$

$$\tau(0) b(0) dz - \tau(s) b(s) dz = 0$$

$\tau(s) b(s)$: COSTANTE (NON DIPENDE DA s)

LEGGEME TMA $\tau(s)$ e M_t ?



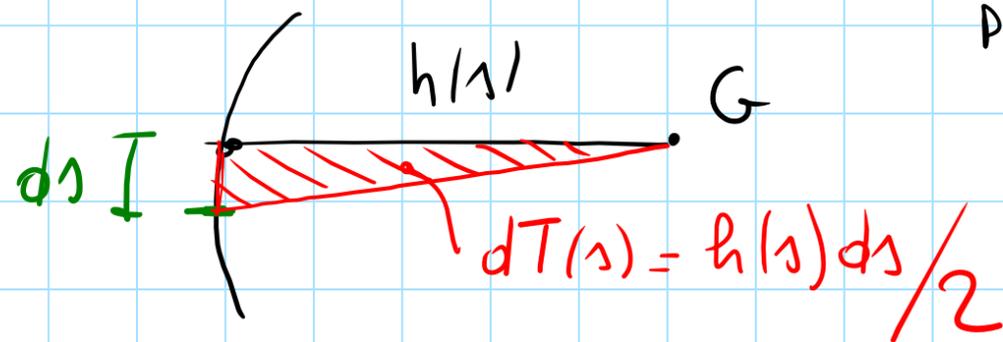
$$dM_s = dF_s h(s) = \tau(s) b(s) ds h(s)$$

$$M_t = \int dM_s = \int \tau(s) b(s) h(s) ds$$

EQUILIBRIO

$$M_t = \tau(s) b(s) \int h(s) ds$$

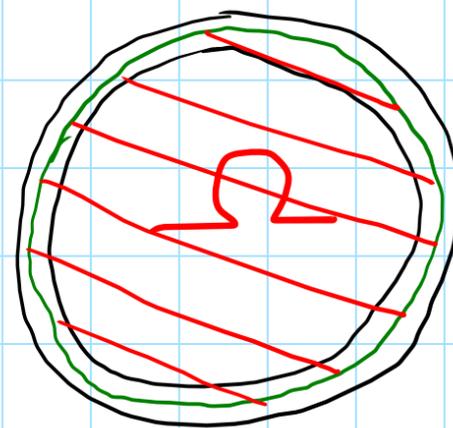
?



PROPRIETA'

$$\int dT(s) = \Omega$$

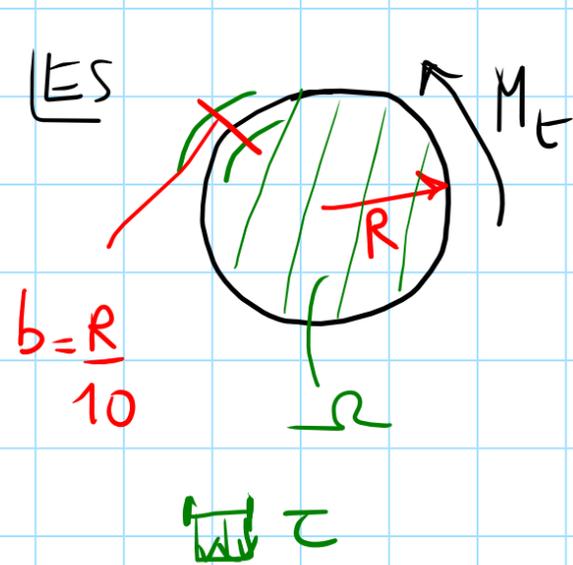
AREA RACCHIUSA
DALLA LINEA MEDIA



$$M_t = \tau(s) b(s) 2\Omega$$

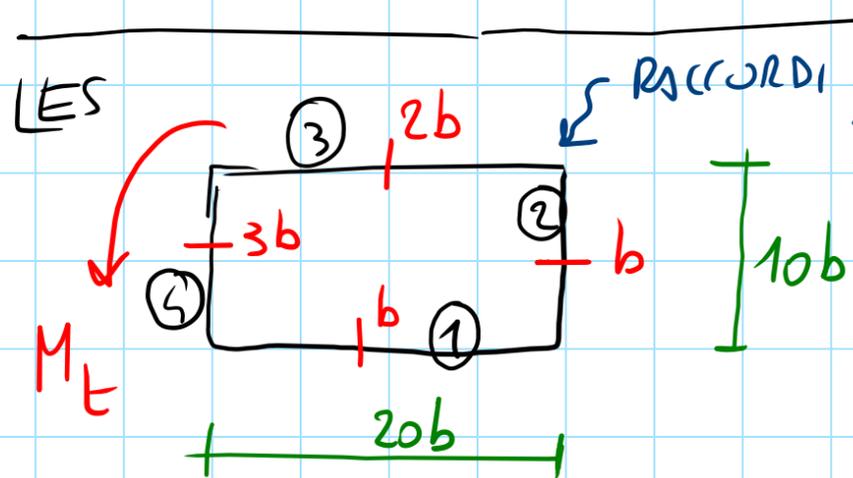
$$\tau(s) = \frac{M_t}{2\Omega b(s)}$$

1° FORMULA DI
BREDT



$$\Omega = \pi R^2$$

$$\tau(\rho) = \tau = \frac{M_t}{2\Omega b} = \frac{M_t}{2\pi R^2 \frac{R}{10}} = 5 \frac{M_t}{\pi R^3}$$



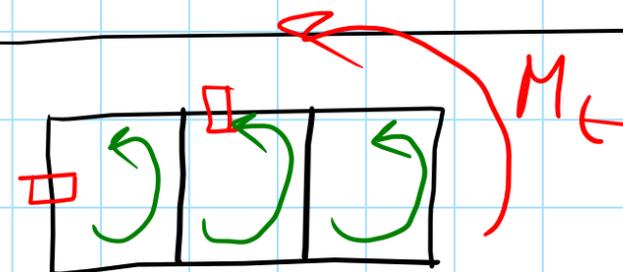
τ_{max} ? (DOVE LO SPESSORE È MINIMO) : $\tau_1 = \tau_2 = \tau_{max}$

$$\tau_1 = \tau_2 = \frac{M_t}{2(200b^2)b} = \tau_{max}$$

$$\tau_4 = \tau_1/3$$

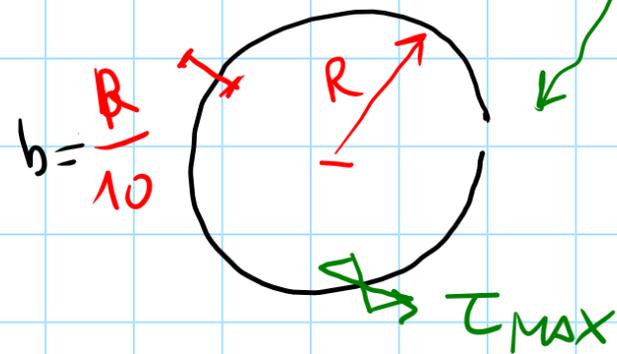
NOTA: 2^a FORMULA DI BRENT

$$J_t = \frac{4\Omega^2}{\oint \frac{ds}{b(s)}}$$



TEORIA DELLE SEZ.
PLURICELLULARI

ES



$$J_t = \frac{1}{3} a b^3 = \frac{1}{3} (2\pi R) b^3$$

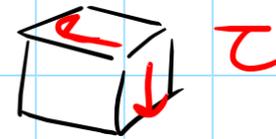
$$\tau_{max} = \frac{M_t}{J_t} b = \frac{M_t}{\frac{2}{3} \pi R b^3} b = \frac{3}{2} \frac{M_t}{\pi R \frac{R^2}{100}} = 150 \frac{M_t}{\pi R^3} = \tau_{max}^{APERTA}$$

CONFRONTO QUESTA τ_{max} CON QUELLA NEL PROFILO CHIUSO

$$\frac{\tau_{max}^{APERTA}}{\tau_{max}^{CHIUSO}} = \frac{150}{5} = 30$$

SEZ. CHIUSA 30 VOLTE PIU' EFFICIENTE DI QUELLA APERTA

TORSIONE (\Rightarrow TAGLIO PURO)

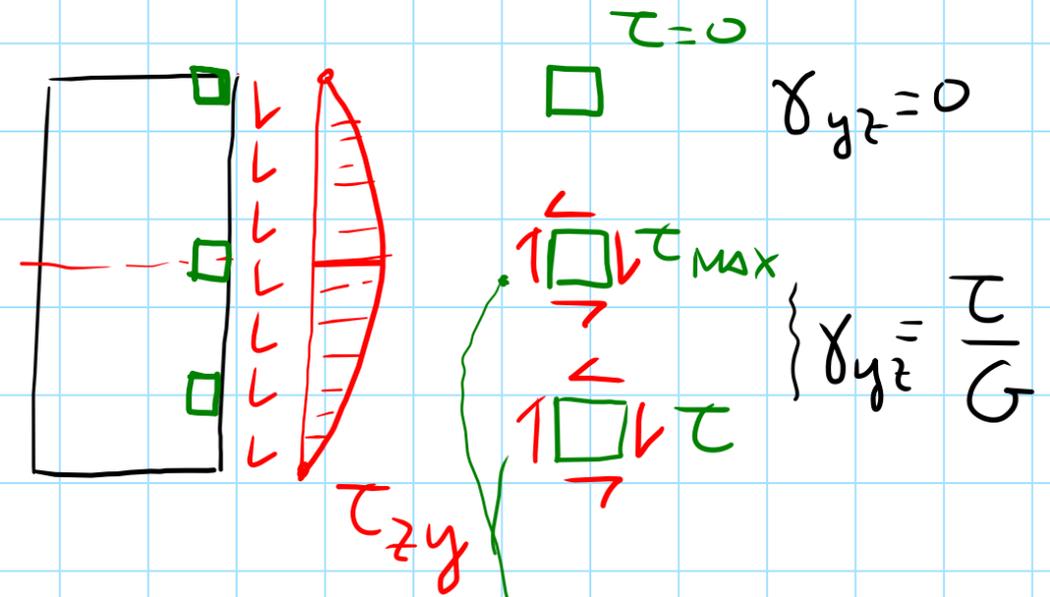
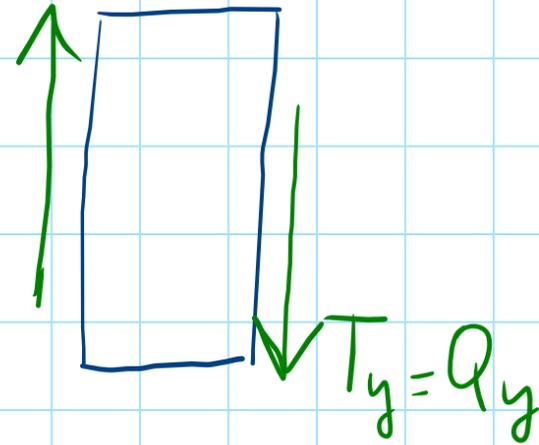
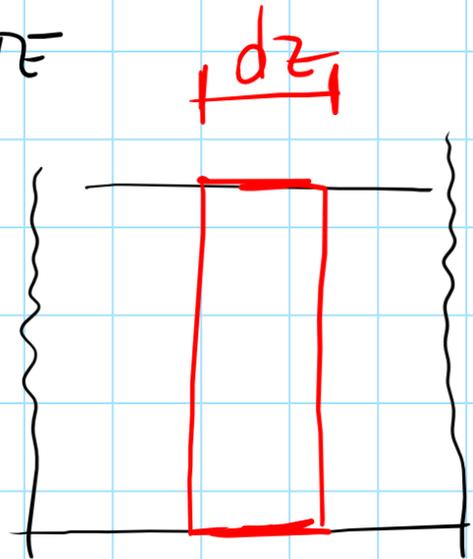
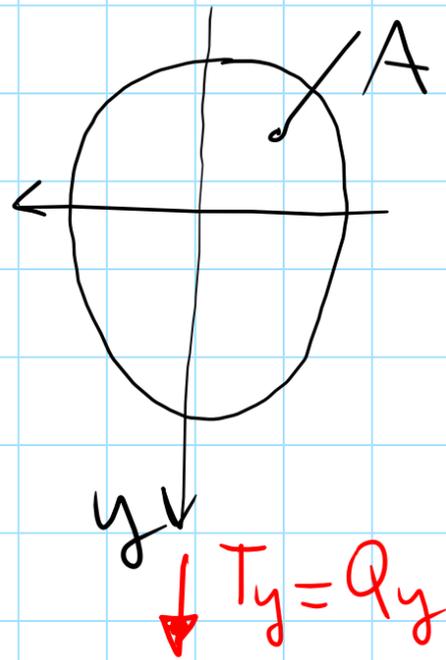


VERIFICA DI RESIST.

VON MISES : $\tau \leq \sigma_0 / \sqrt{3}$

TRESCA : $\tau \leq \sigma_0 / 2$

OSSERVAZIONI SULLA CINEMATICA DEL PROBLEMA TAGLIO-FLESSIONE IN SEZ. COMPASTE



$\gamma_{yz}(y)$; NON COSTANTE

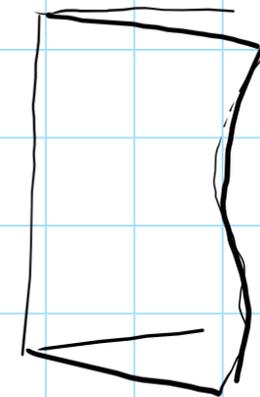
$$\bar{\gamma} = \frac{K T_y}{GA}$$

AREA DELLA SEZIONE

$$\bar{\gamma} = f(T_y)$$



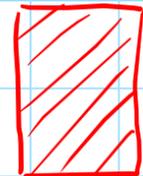
SCORRIM. MEDIO



DEFORM. DELLA PORZIONE DI CILINDRO

FAITORE DI TAGLIO: SI OTTENE CON METODI ENERGI.

$$K > 1$$



$$K = \frac{6}{5}$$