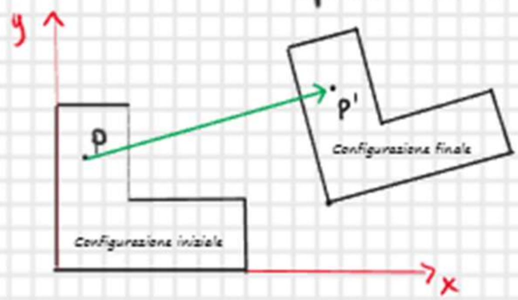
Titolo:

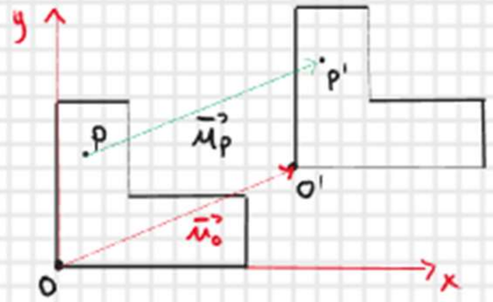
Calcolo reazioni vincolari tramite PLV

Spostamento $\vec{u} = u_x \vec{i} + u_y \vec{j} + u_w \vec{k}$



$\vec{u}_P = P' - P$ Spostamento del punto P

Traslazione rigida

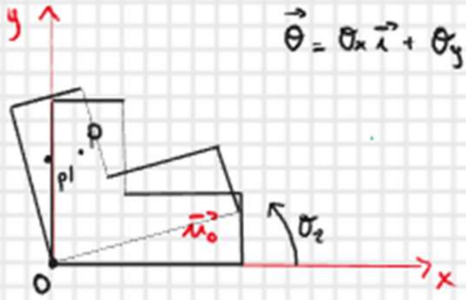


$\vec{u}_P = \vec{u}_o$

Tutti i punti del corpo compiono spostamenti uguali in direzione, verso e intensità

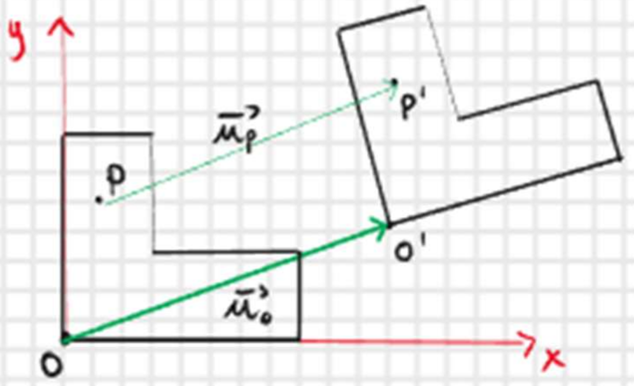
Rotazione rigida attorno ad un asse

$\vec{\theta} = \theta_x \vec{i} + \theta_y \vec{j} + \theta_z \vec{k}$

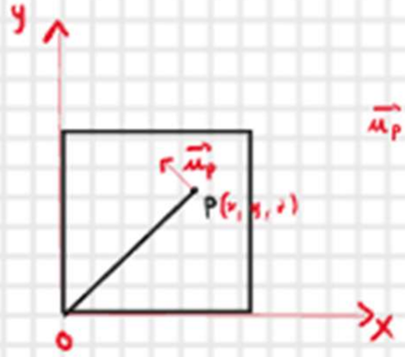


z: asse di rotazione

Roto-traslazione = Rotazione + traslazione



Introduciamo l'ipotesi di piccoli spostamenti



$$\vec{u}_P = \vec{\theta} \times \vec{OP} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \theta_x & \theta_y & \theta_z \\ x & y & z \end{vmatrix} = (\theta_y z - \theta_z y)\vec{i} + (\theta_z x - \theta_x z)\vec{j} + (\theta_x y - \theta_y x)\vec{k}$$

Nel caso piano $\theta_z = \theta$

spostamento di un punto P

$$(1) \quad \boxed{\begin{cases} u_x = u_{x0} - \theta_z y \\ u_y = u_{y0} + \theta_z x \end{cases}}$$

Piccoli spostamenti (piano)

Cerchiamo un punto nel piano che non subisce spostamenti

$\downarrow$ $u_x = 0 \quad u_y = 0$

$y_c = \dfrac{u_{x0}}{\theta} \qquad x_c = -\dfrac{u_{y0}}{\theta}$ Coordinate del Centro d'istantanea rotazione C.I.R.

Sostituiamo x_c e y_c nelle (1)

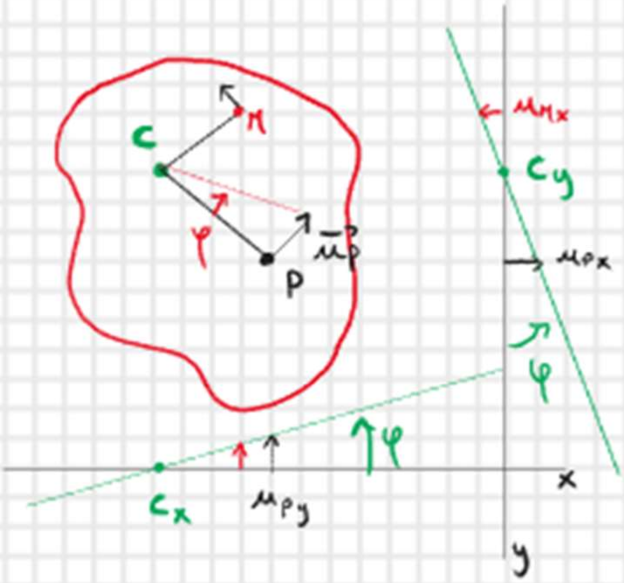
$$\boxed{\begin{aligned} u_x &= \theta(y_c - y) \\ u_y &= \theta(x - x_c) \end{aligned}}$$

Ogni spostamento rigido, piano è riconducibile ad una rotazione

Nel caso della traslazione rigida

$$\theta = 0 \qquad x_c \to \infty \qquad y_c \to \infty$$

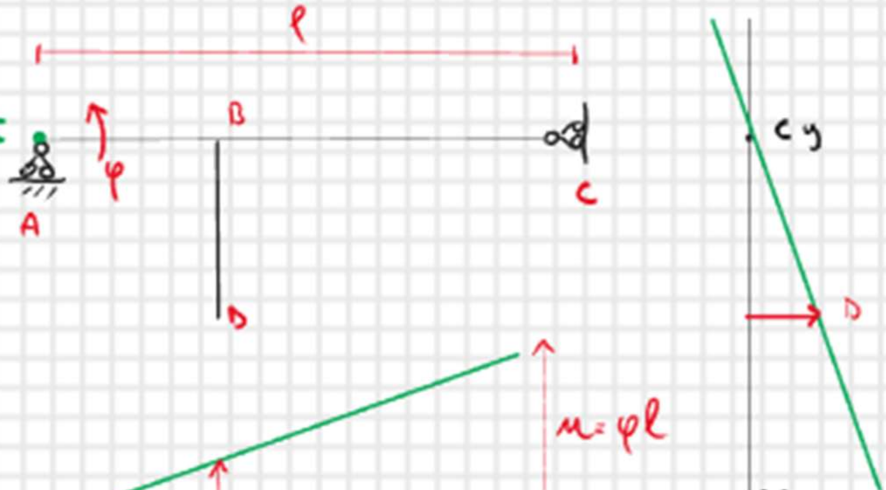
Traslazione come una rotazione rispetto ad un punto improprio (infinito)



Assegnato lo spostamento di due punti (P e M) possiamo determinare in CIR (C), intersezione con le normali allo spostamento.

Oppure noto il CIR (C) e la rotazione del corpo possiamo determinare lo spostamento dei punti del corpo

Esempio con delle travi



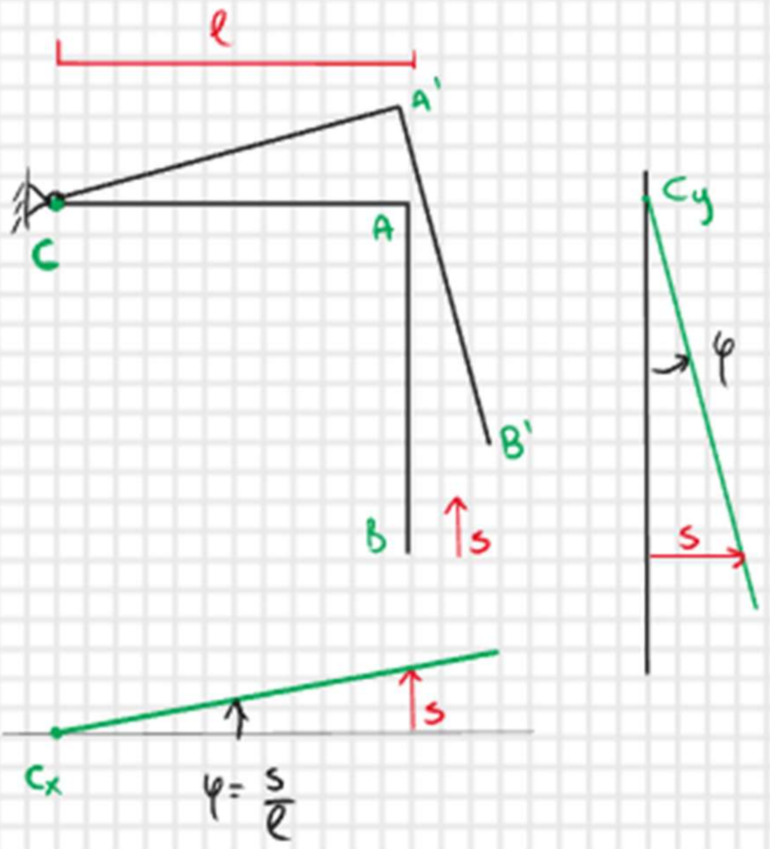
Retta x e y, riferimento dal quale misurare lo spostamento verticale e orizzontale del corpo

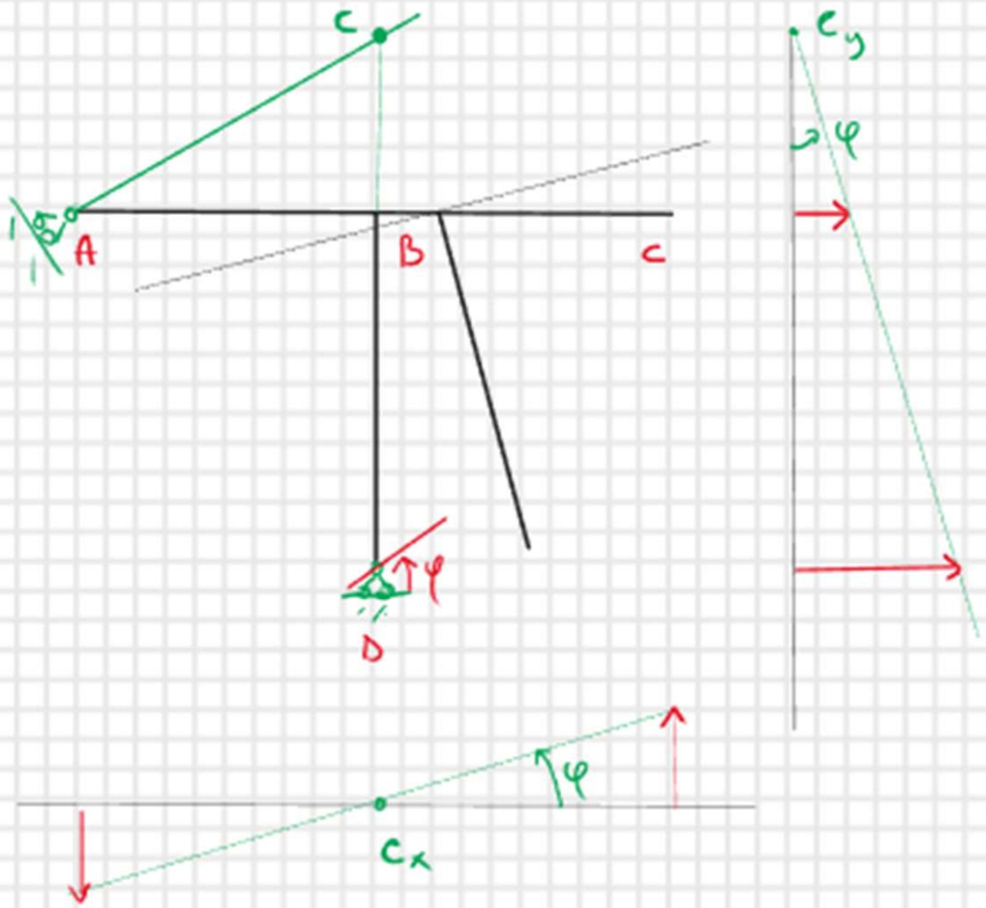
C: centro d'istantanea rotazione

$C \; \infty \longrightarrow$

C: centro d'istantanea rotazione, punto improprio

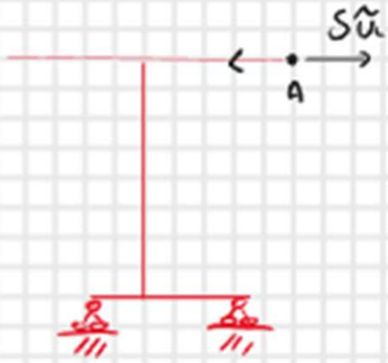
Imponiamo lo spostamento di un punto del corpo e determiniamone la configurazione finale





Lavoro virtuale di un sistema di forze

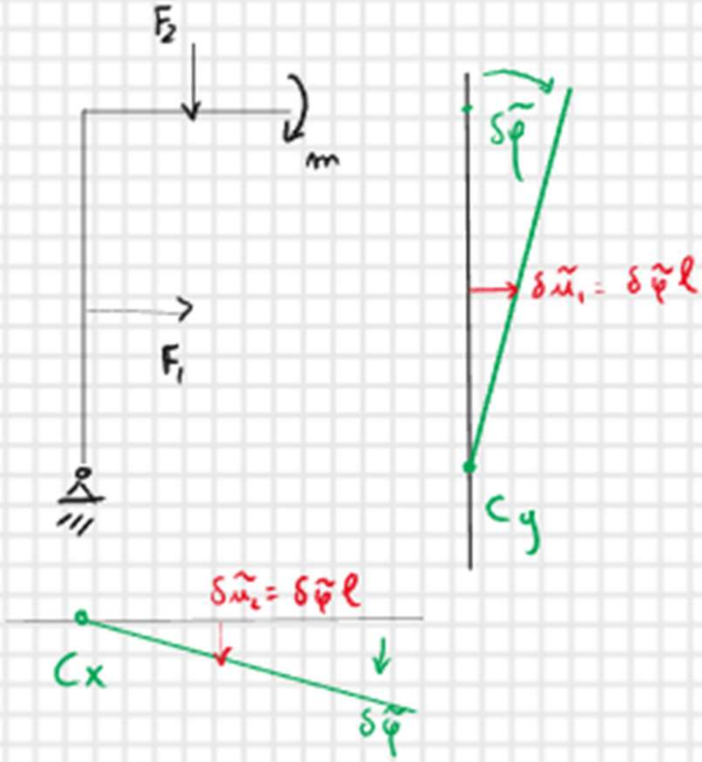
spostamento virtuale : classe di spostamenti infinitesimi compatibili con i vincoli del sistema



Costruiamo lavoro virtuale

$$\delta L = \sum_{i=1}^{n} \vec{F}_i \cdot \delta \tilde{u}_i + \sum_{j=1}^{m} m_j \cdot \delta \tilde{\varphi}_j$$

Generale



$$\delta L = F_1 \overset{\delta \tilde{u}_1}{\delta \tilde{\varphi} l} + F_2 \overset{\delta \tilde{u}_2}{\delta \tilde{\varphi} l} + m\, \delta \tilde{\varphi}$$

Teorema dei lavori virtuali

$L_{ve} = L_{vi}$

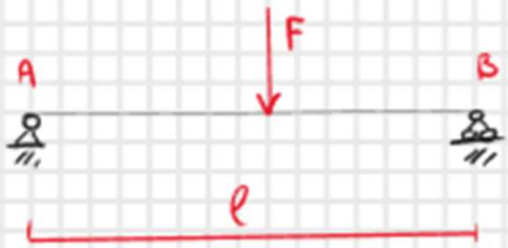
Per corpi rigidi $L_{vi} = 0$ (no deformazioni)

$\downarrow$

$L_{ve} = 0 \longrightarrow \delta L = 0 \quad \forall \delta \hat{u}$

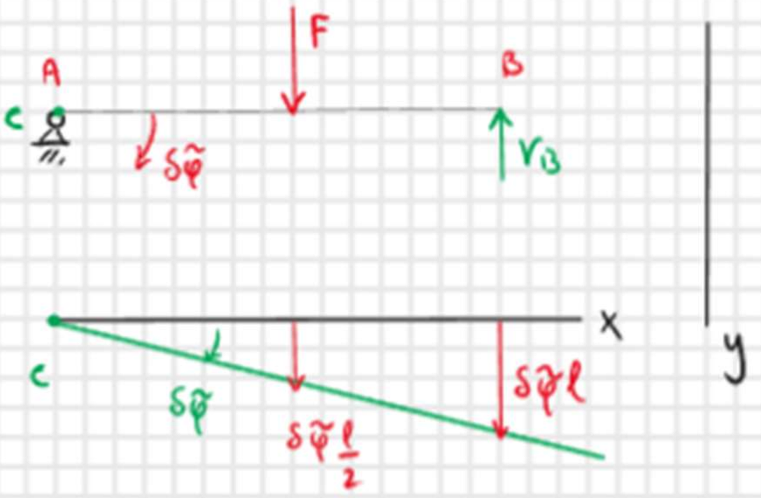
Condizione NS per garantire l'equilibrio del sistema

Calcolare le reazioni vincolari



$\longrightarrow$ Calcolare V_B

Sostituisco al vincolo la reazione che voglio determinare



$\delta L = 0 \longrightarrow F\,\delta\tilde{\varphi}\,\frac{l}{2} - V_B\,\delta\tilde{\varphi}\,l = 0 \longrightarrow V_B = \frac{F}{2}$

Calcolare H_A

$\delta L = 0 : H_A\,\delta\tilde{u} + F \cdot 0 = 0 \quad \forall \delta\tilde{u}$

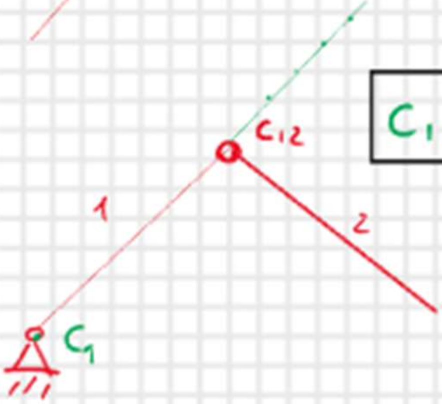
$H_A = 0$

Sistemi con $n > 1$ corpi rigidi

$u_i^x(C_{ij}) = u_j^x(C_{ij})$

$u_i^y(C_{ij}) = u_j^y(C_{ij})$

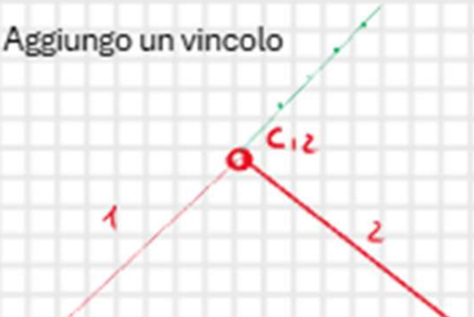
C_{ij} centro rotazione relativo



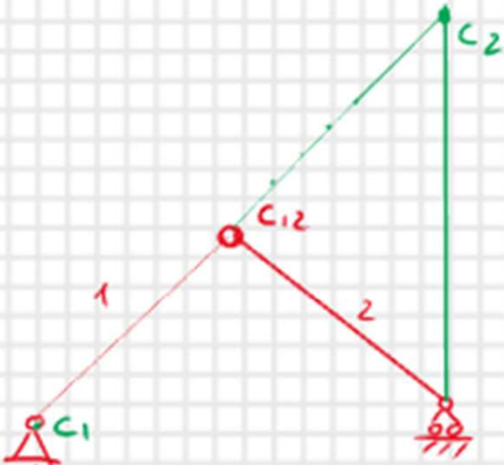
$\boxed{C_1 \text{ e } C_2 \text{ CIR assoluti del corpo 1 e 2}}$

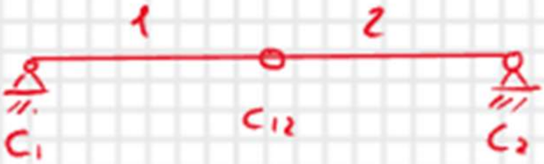
Il C_2 si trova sulla retta che congiunge C_1 e C_{12}

Aggiungo un vincolo



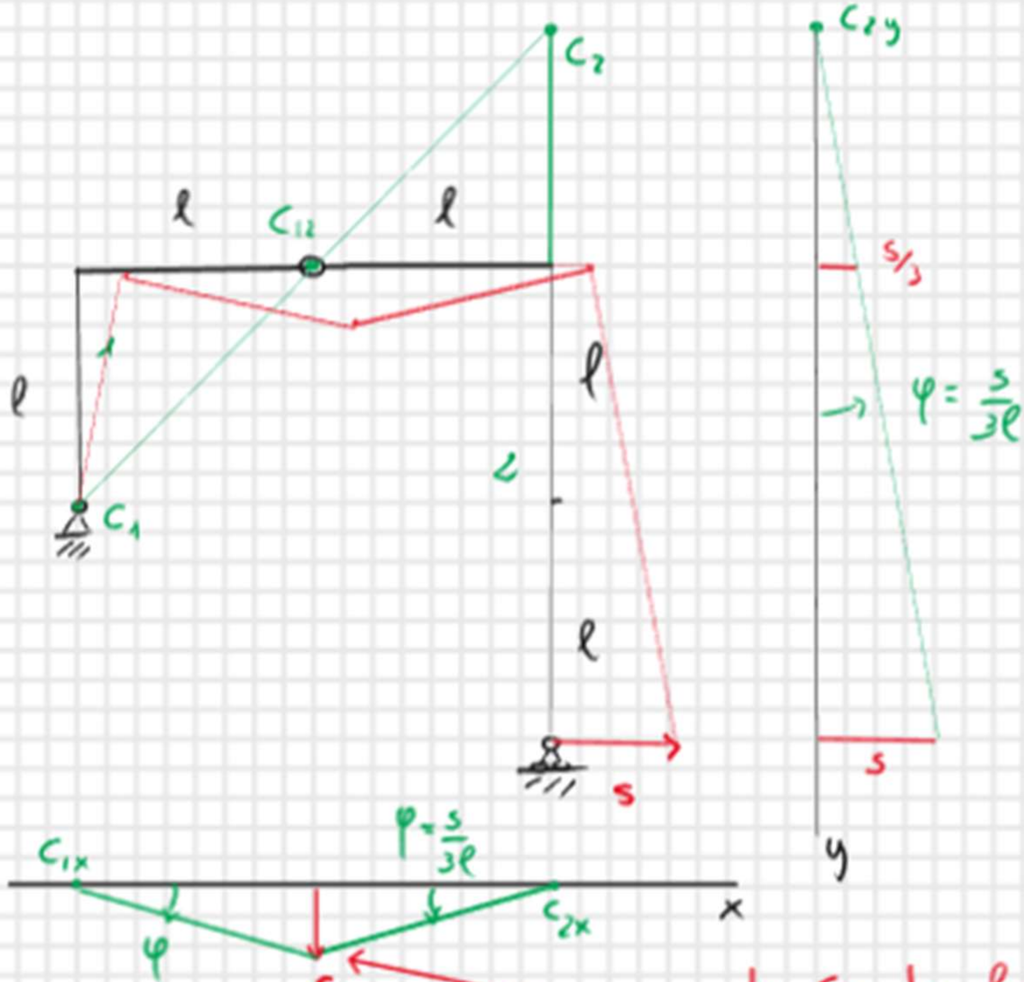
Struttura non labile, non ci sono centri di rotazione





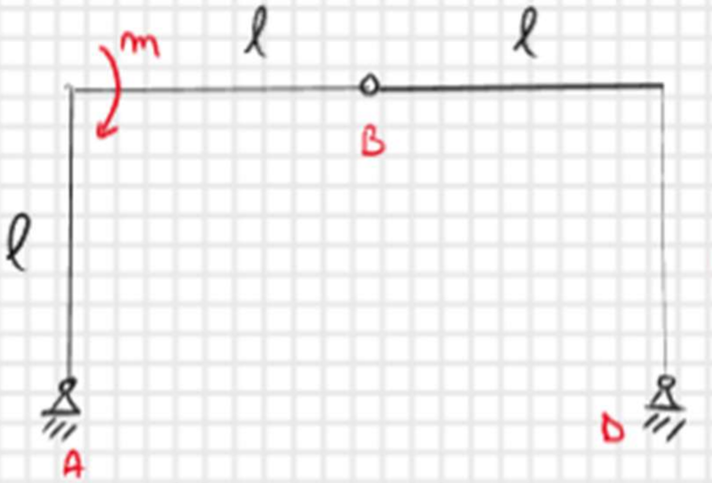
Struttura è labile se C_1, C_2 e C_{12} sono allineati

C_1, C_2 e C_{12} sono allineati



In C_{12} ho la continuità dello spostamento

Calcolo reazioni vincolari

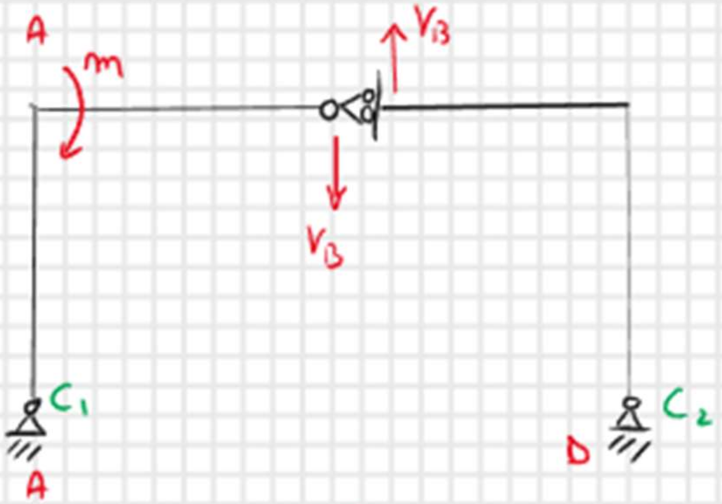


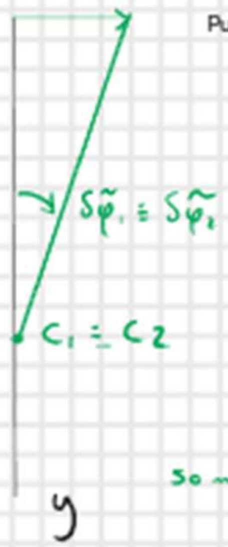
Calcolare V_b

Declasso la cerniera centrale

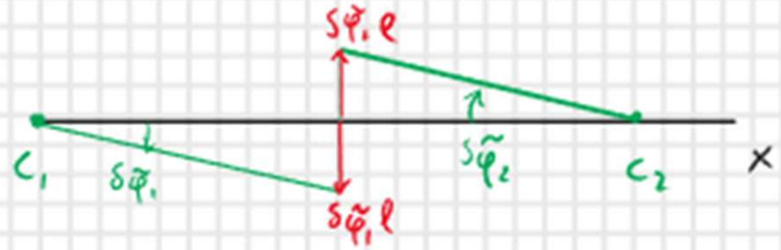
$c_{12} \to \infty$

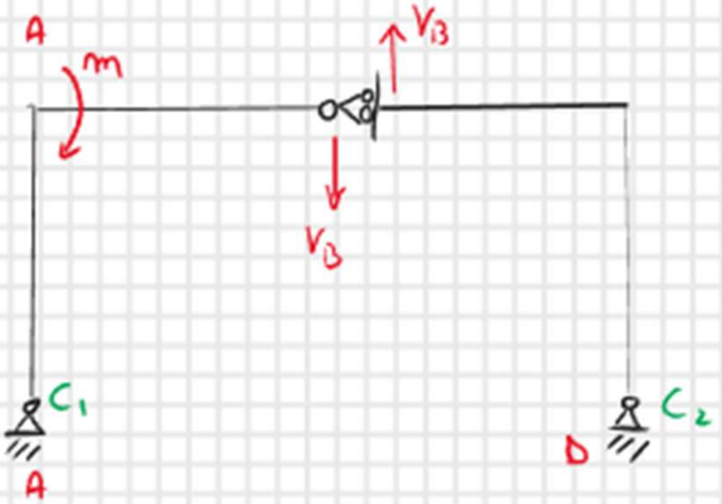
Punto improprio orizzontale

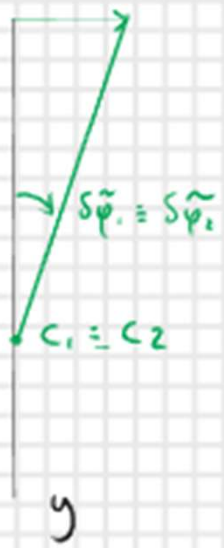


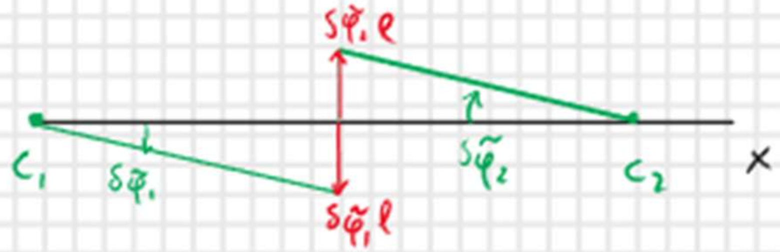


c_1, c_2 e c_{12} sono allineati



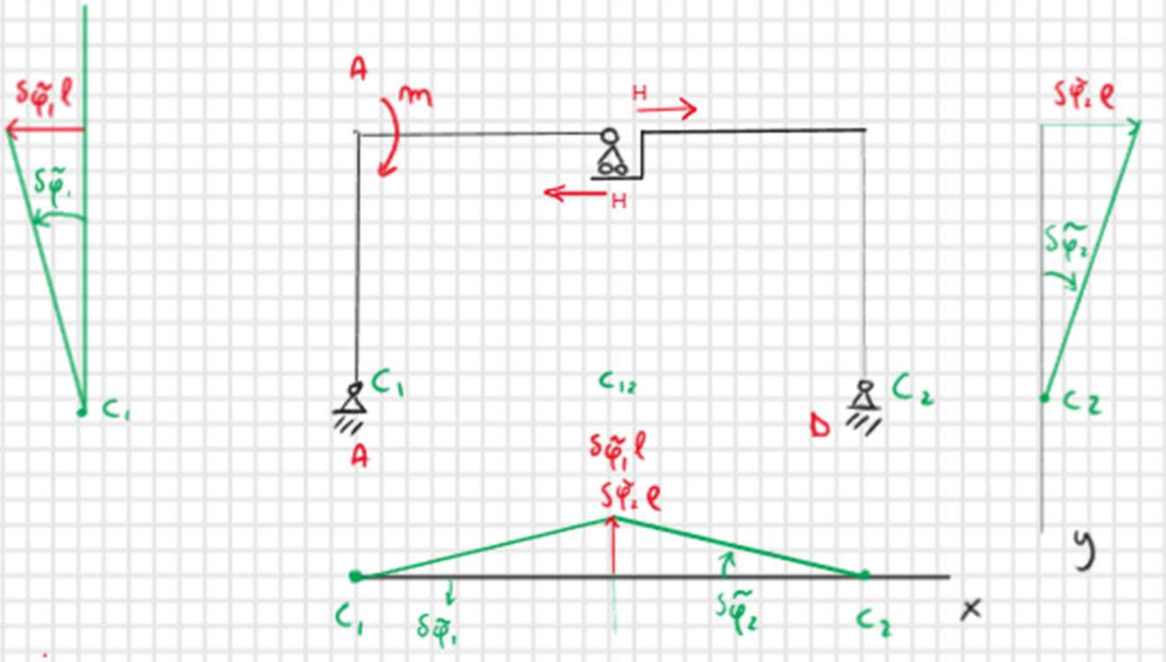






$\delta L = 0 \qquad \delta\tilde{\varphi}_1 = \delta\tilde{\varphi}_2$

$$\boxed{\begin{gathered} \delta L = m\,\delta\tilde{\varphi}_1 + V_B\,\delta\tilde{\varphi}_1 l + V_B\,\delta\tilde{\varphi}_2 l = 0 \\ V_b = -\frac{m}{2l} \end{gathered}}$$



C_1, C_2 e C_{12} sono allineati

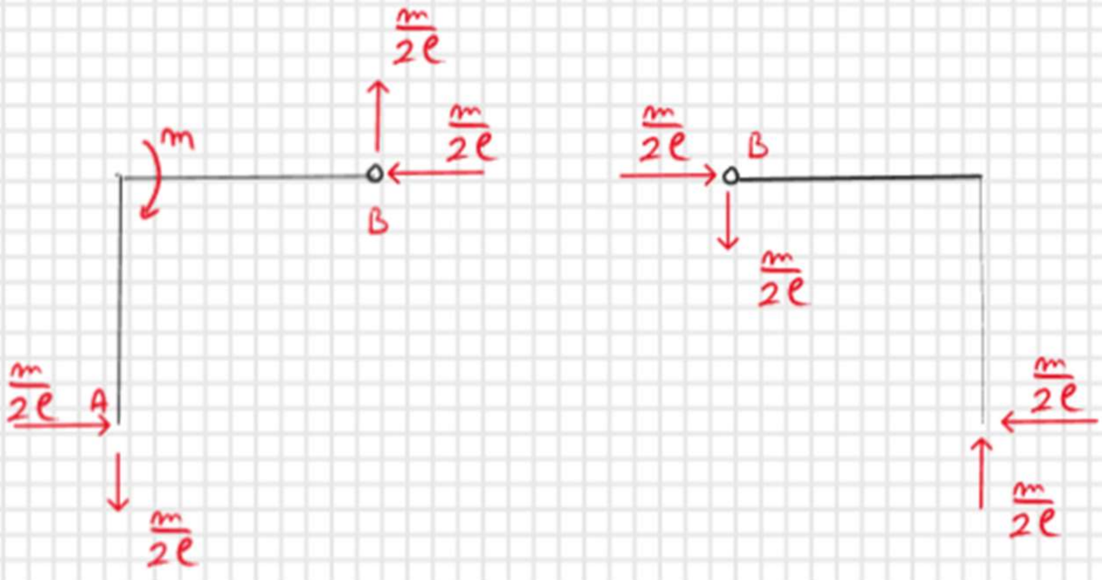
$$\boxed{\begin{gathered}\delta L = -m\,\delta\tilde{\varphi}_1 + H\,\delta\tilde{\varphi}_1 l + H\,\delta\tilde{\varphi}_2 l = 0 \\ H = \frac{m}{2l}\end{gathered}}$$

$V_d = \frac{m}{2l}$

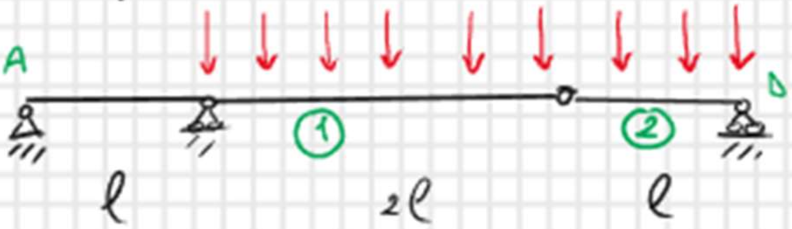
$V_A = -\frac{m}{2l}$

$H_d = -\frac{m}{2l}$

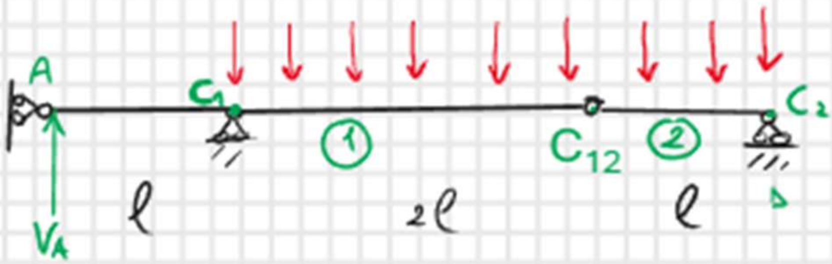
$H_A = \frac{m}{2l}$



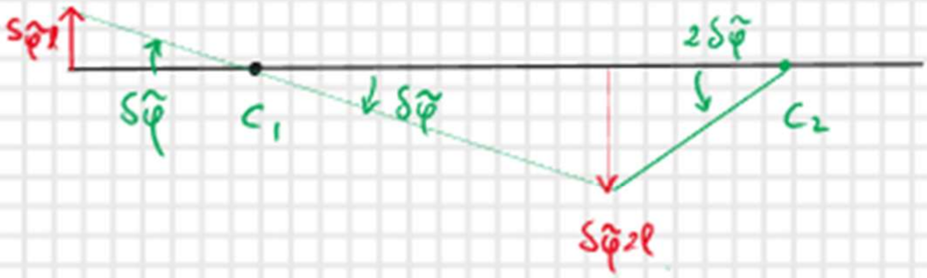
Esempio



Calcolare V_A e V_D

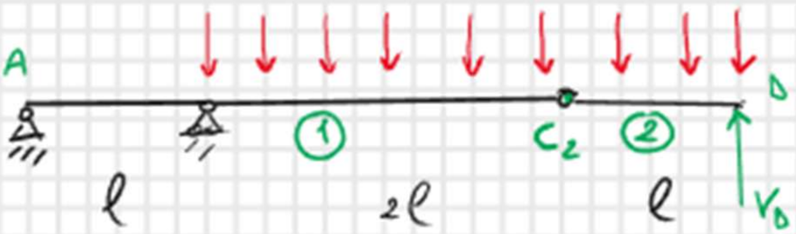


Labile



$$\boxed{\delta L = V_A\,\delta\tilde{\varphi}\,\ell + 2qL\,\delta\tilde{\varphi}\,\ell + qL\,\delta\tilde{\varphi}\,\ell = 0 \qquad V_A = -3qL}$$

Calcolo V_D



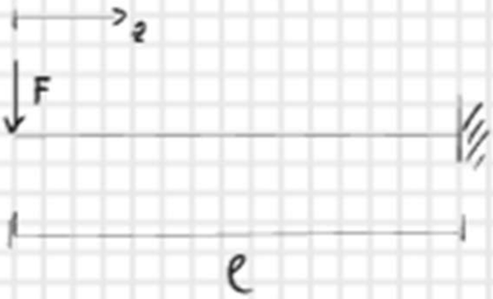
C_1 non esiste

$$\boxed{\delta L = -V_d\,\delta\tilde{\varphi}\,\ell + q\ell\cdot\delta\tilde{\varphi}\,\frac{\ell}{2} = 0 \longrightarrow V_d = q\ell/2}$$

Approfondimento sulla trave di Timoshenko

1) $\varphi'(z) = \dfrac{M(z)}{EI}$

2) $v'(z) = -\varphi(z) + \dfrac{K\,T(z)}{GA}$



$T(z) = -F$

$M(z) = -Fz$

1) $\varphi'(z) = \dfrac{-Fz}{EI} \longrightarrow \varphi(z) = -\dfrac{Fz^2}{2EI} + C$

$\varphi(\ell) = 0 \qquad C = \dfrac{F\ell^2}{2EI} \qquad \varphi(z) = -\dfrac{Fz^2}{2EI} + \dfrac{F\ell^2}{2EI}$

Sostituiamo $\varphi(z)$ nella 2

$v'(z) = \dfrac{Fz^2}{2EI} - \dfrac{F\ell^2}{2EI} - \dfrac{K\,F}{GA}$

$v(z) = \dfrac{Fz^3}{6EI} - \dfrac{F\ell^2 z}{2EI} - \dfrac{KFz}{GA} + D \longrightarrow v(\ell) = 0$

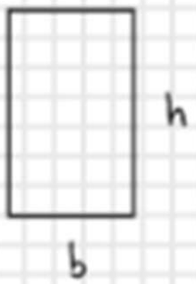
$D = \dfrac{F\ell^3}{3EI} + \dfrac{KF\ell}{GA}$

$v(z) = \dfrac{Fz^3}{6EI} - \dfrac{F\ell^2 z}{2EI} + \dfrac{F\ell^3}{3EI} - \dfrac{KFz}{GA} + \dfrac{KF\ell}{GA}$

Consideriamo la freccia $v(0)$

$f = v(0) = \dfrac{F\ell^3}{3EI} + \dfrac{KF\ell}{GA} \longrightarrow f = \dfrac{F\ell^3}{3EI}\left(1 + \boxed{\dfrac{K\,3EJ}{\ell^2 GA}}\right)$

Contributo Taglio



$$K = \frac{6}{5}$$

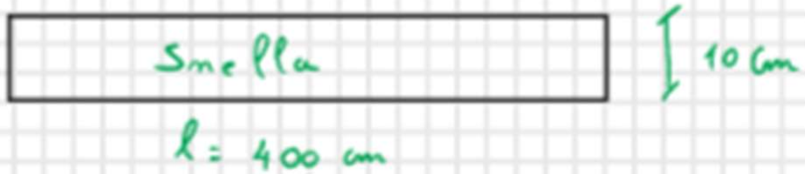
$$I = \frac{1}{12}\, b h^3$$

$$A = b h$$

$$G = \frac{E}{2(1+\nu)}$$

$$\nu = 0{,}3$$

$$\beta = \frac{K\, 3EJ}{\ell^2 GA} = \frac{3}{5}(1+\nu)\frac{h^2}{\ell^2} \qquad \beta \simeq \alpha \frac{h^2}{\ell^2}$$



$\beta \simeq \left(\frac{1}{40}\right)^2 = 0{,}06\%$

β è trascurabile

$\beta \simeq \left(\frac{1}{4}\right)^2 = 6\%$

β può essere significante