Titolo:

Distorsioni termiche costanti

$w_B = \alpha \Delta T l$

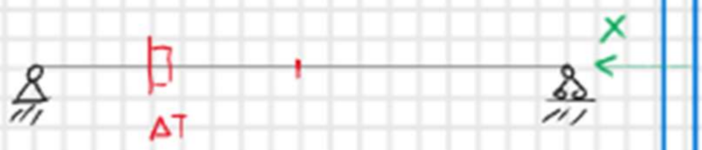
$dw = \alpha \Delta T \, dz = \varepsilon^{\Delta T} dz$

$\varepsilon^{\Delta T}$: dilatazione termica lineare

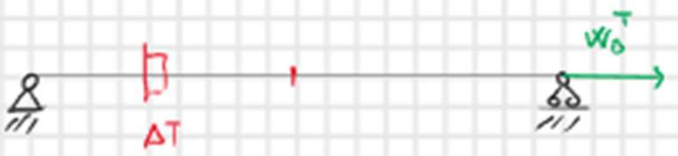
α: coeff. espansione termica

Struttura isostatica ⟶ No CDS

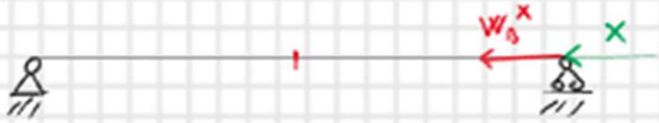
Struttura iperstatica



Eq. congruenza $w_B = 0$



$w_B^T = \alpha \Delta T b$



$w_B^X = \frac{X 2b}{EA}$

$w_B = w_B^T + w_B^X = 0$ · $\alpha \Delta T \cancel{b} - \frac{X 2 \cancel{b}}{EA} = 0$ $X = \frac{\alpha \Delta T EA}{2}$

N compressa

Distorsione termica nelle travature reticolari

$$L_{ve} = L_{vi}$$

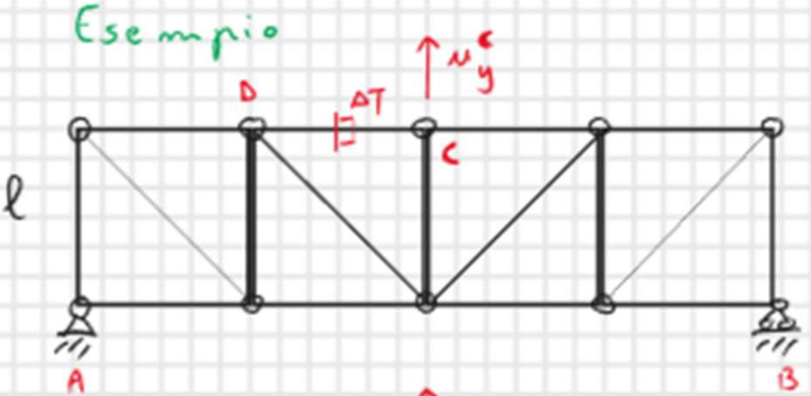
$$L_{vi} = \sum_{i=1}^{m} N_i^* \frac{N_i}{EA} l_i$$

ε_i^{el}

con variazione di temperatura

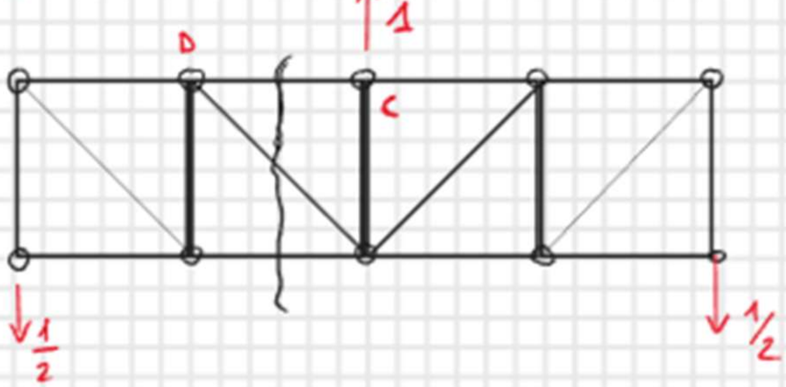
$$L_{vi} = \sum_{i=1}^{m} N_i^* \left(\varepsilon_i^{el} + \varepsilon_i^{\Delta T}\right) l_i = \sum_{i=1}^{m} N_i^* \left(\frac{N_i}{EA} + \underline{\alpha \Delta T_i}\right) l_i$$

Esempio



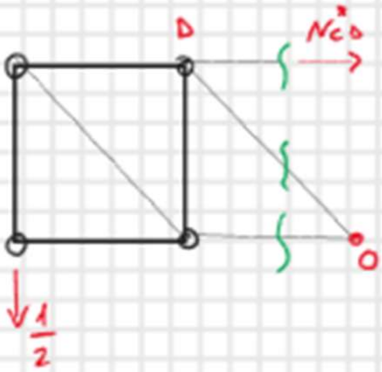
struttura reale

$N_i = 0$



struttura fittizia

Calcoliamo N_i^*



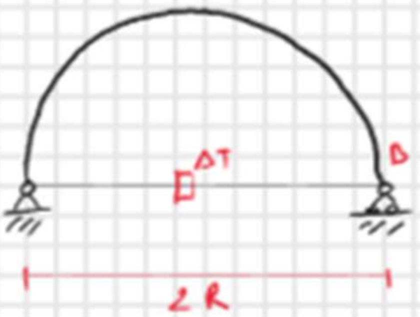
$$N^*_{CD}\, l - \frac{1}{2} 2l = 0 \qquad \text{eq. rispetto ad O}$$

$$N^*_{CD} = 1$$

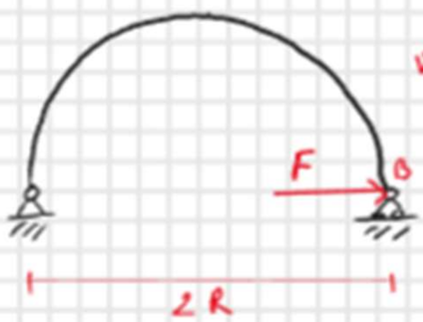
$$L_{ve} = L_{vi}$$

$$1 \cdot u_y^C = 1 \cdot \alpha \Delta T \cdot l_{CD} \longrightarrow u_y^C = \alpha \Delta T l$$

Esempio

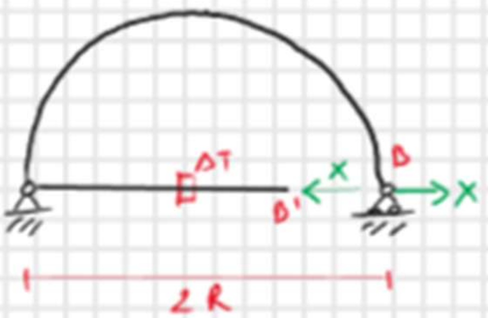


Una volta iperstatica



$$w_B = \frac{\pi}{2}\frac{FR^3}{EI}$$

Lezione scorsa



equazione di congruenza

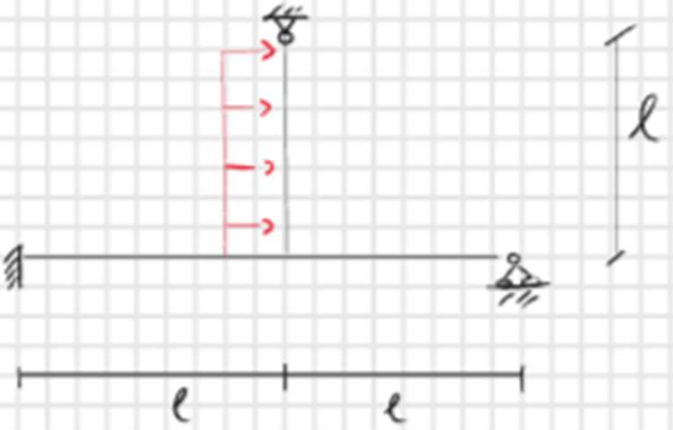
$$w_B = w_{B'}$$

$$\frac{\pi}{2}\frac{XR^3}{EI} = -\frac{X2R}{EA} + \alpha\Delta T 2R \longrightarrow X$$

$$X\left(\frac{\pi}{2}\frac{R^2}{EI} + \frac{2}{EA}\right) = 2\alpha\Delta T \longrightarrow X > 0 \quad N<0$$

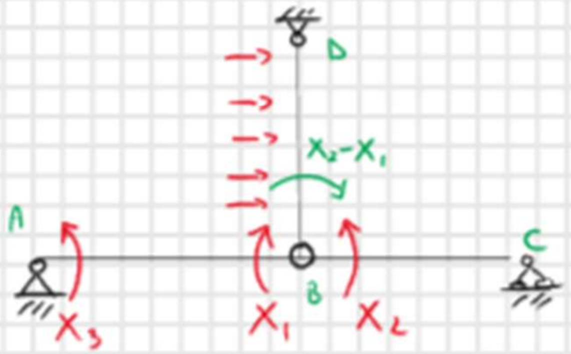
catena compressa

Esempio tema esame

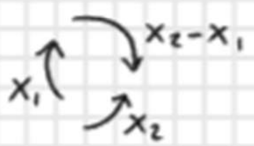


3 volte iperstatica

Servono 3 equazioni di congruenza



Devo avere



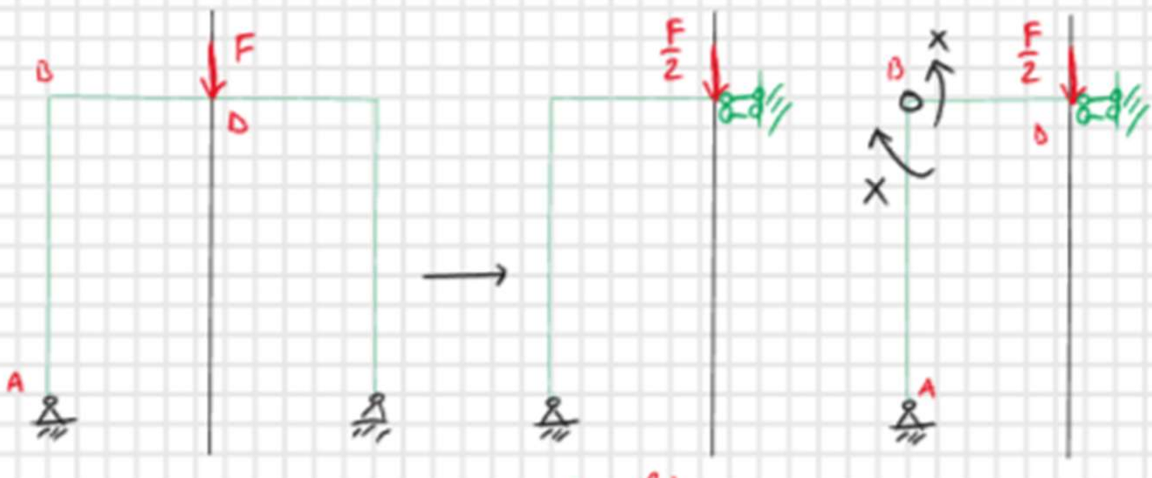
$= 0$

1) $\varphi_{AB} = 0$ $\quad \frac{X_3 l}{3EJ} + \frac{X_1 l}{6EJ} = 0$

2) $\varphi_{BA} = \varphi_{BC}$ $\quad -\frac{X_1 l}{3EJ} - \frac{X_3 l}{6EJ} = \frac{X_2 l}{3EJ}$

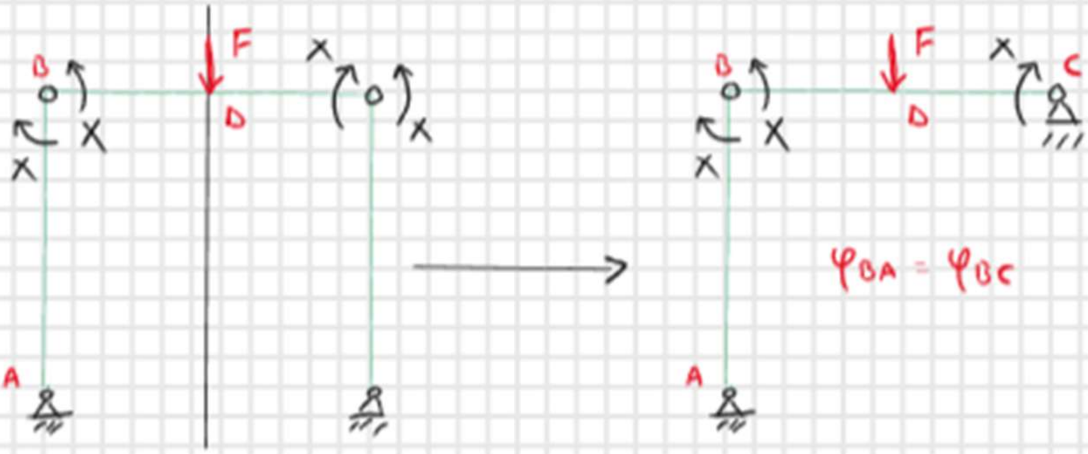
3) $\varphi_{BA} = \varphi_{BD}$ $\quad -\frac{X_1 l}{3EJ} - \frac{X_3 l}{6EJ} = \frac{-q l^3}{24EJ} - \frac{(X_2 - X_1) l}{3EJ}$

Nota: strutture simmetriche



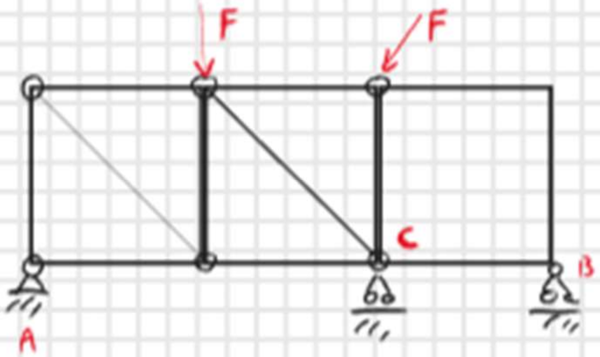
1 volta iperstatica

$\varphi_{BA} = \varphi_{BD}$

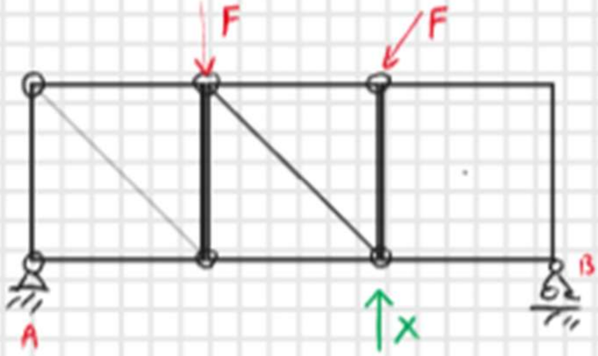


$\varphi_{BA} = \varphi_{BC}$

Strutture reticolari iperstatiche

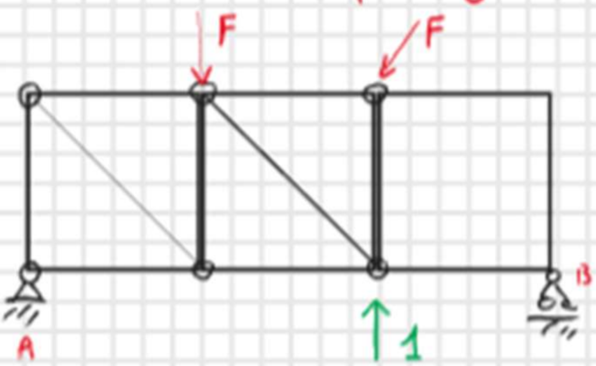


1 volta iperstatica



Struttura principale (staticamente determinata)

- eq. congruenza $u_y^c = 0$



struttura fittizia*

$$L_{vi} = \sum_{i=1}^{m} N_i^* \frac{N_i}{EA} l_i$$

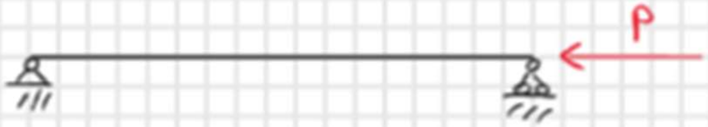
$$L_{ve} = L_{vi}$$

$$1 \cdot u_c^y = \sum_{i=1}^{m} N_i^* \frac{N_i}{EA} l_i$$

$$\Big\downarrow \; u_c^y (F, X)$$

Imponiamo $u_c^y = 0 \longrightarrow X$

Instabilità dell'equilibrio elastico



Aumentare il carico P, le condizioni di equilibrio con le cds non siano più stabili ma instabili.

Superato un P_{crit} perdiamo l'unicità della soluzione del problema elastico

Sono possibili configurazioni della trave diverse da quella iniziale che verificano:

- le eq di equilibrio
- le eq di congruenza
- il legame costitutivo del materiale

Ipotesi: materiale elastico lineare
spostamenti infinitesimi
eq. equilibrio scritte nella confi indeformata

$$M(z) = P\,v(z)$$

Eq. linea elastica $\quad v''(z) = -\dfrac{M(z)}{EJ} \longrightarrow v''(z) = -\dfrac{P\,v(z)}{EJ}$

$$v''(z) + \frac{P}{EJ}\,v(z) = 0 \xrightarrow{K^2 = P/EJ} v''(z) + K^2 v(z) = 0$$

Soluzione : $v(z) = C_1 \operatorname{sen}(kz) + C_2 \cos(kz)$

Impongo le condizioni al contorno

$$v(0) = 0 \qquad v(\ell) = 0$$

$$v(0) = C_2 = 0 \longrightarrow C_2 = 0$$

$$v(\ell) = C_1 \operatorname{sen}(k\ell) + C_2 \cos(k\ell) = 0 \longrightarrow C_1 \operatorname{sen} k\ell = 0$$

$$\begin{array}{l} v(0)=0 \\ v(\ell)=0 \end{array} \begin{bmatrix} 0 & 1 \\ \operatorname{sen} k\ell & \cos k\ell \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Soluzione banale $C_1 = 0 \quad C_2 = 0 \longrightarrow v(z) = 0$

Soluzioni non banali

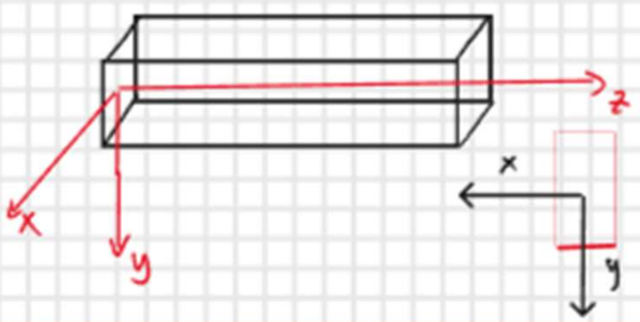
$$\det \begin{bmatrix} 0 & 1 \\ \operatorname{sen} k\ell & \cos k\ell \end{bmatrix} = 0 \longrightarrow -\operatorname{sen} k\ell = 0$$

$$\operatorname{sen} k\ell = 0 \longrightarrow k\ell = n\pi \qquad n = 1, 2, \ldots$$

$$k = \frac{n\pi}{\ell} \qquad k^2 = \frac{n^2\pi^2}{\ell^2} = \frac{P}{EJ} \longrightarrow P = \frac{n^2\pi^2}{\ell^2} EJ$$

Il carico critico sarà per $n = 1$ $\quad P_{cr} = \frac{\pi^2}{\ell^2} EI$

Considerazione



$$I \longrightarrow I_{min}$$

$$P_{cr} = \frac{\pi^2 E I_y}{\ell^2}$$

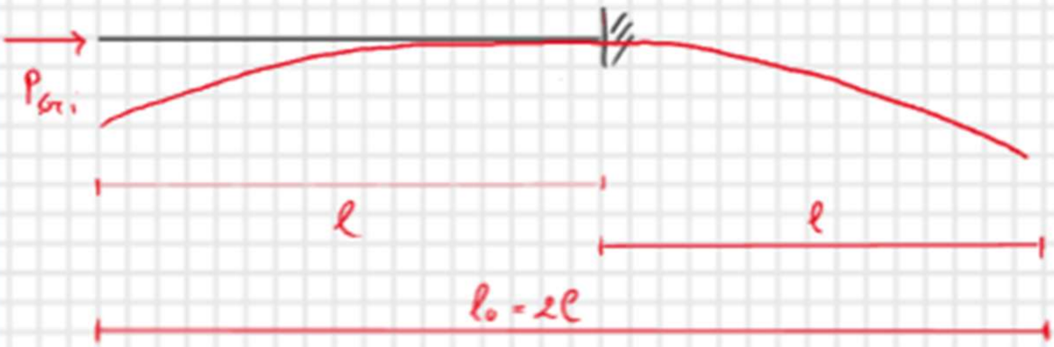
$$P_{cr} = \frac{\pi^2}{l_0^2} E I$$

$$P_{cr} = \frac{\pi^2 E I}{l^2}$$

$l = l_0$

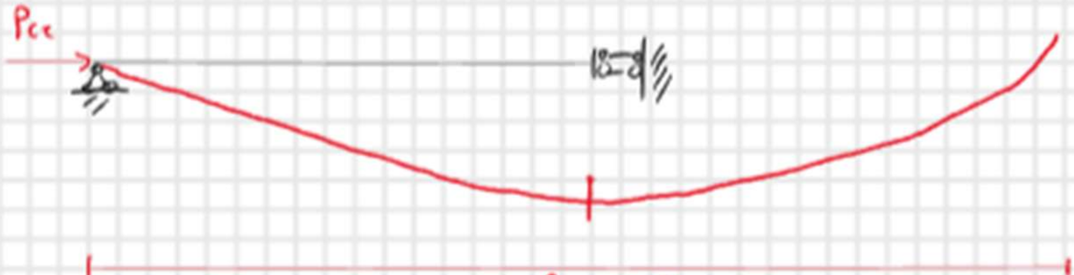
l_0 = lunghezza di libera inflessione

Condizioni di vincolo differenti

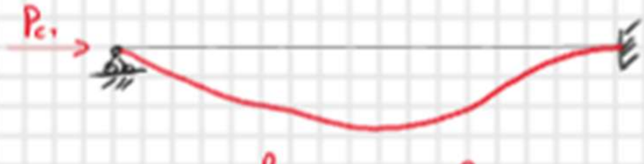


$$P_{cr} = \frac{\pi^2}{l_0^2} E I$$

$$P_{cr} = \frac{\pi^2 E I}{4 l^2}$$



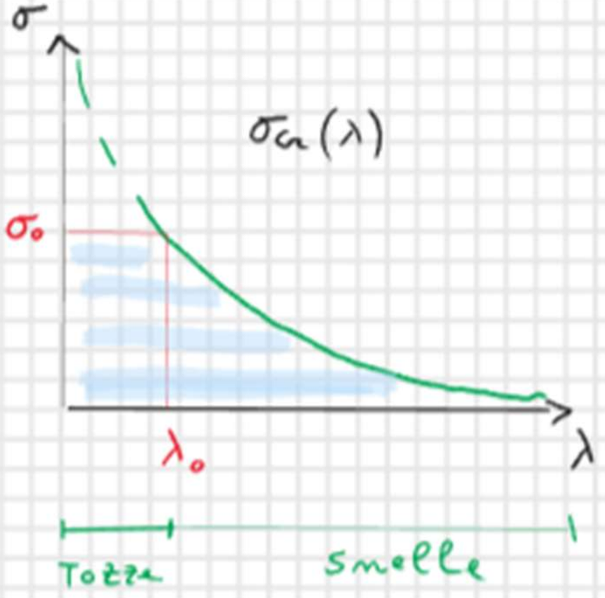
$l_0 = 2l$



$l_0 = 0,69\, l$

$$P_{cr} = \frac{\pi^2 E I}{l^2} \longrightarrow \sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E I}{l^2 A} \xrightarrow{\rho^2 = \frac{I}{A}} \sigma_{cr} = \pi^2 E \frac{\rho^2}{l^2}$$

$\lambda = \frac{l}{\rho}$ smellezza dell'asta $\qquad \sigma_{cr} = \frac{\pi^2 E}{\lambda^2}$ Iperbole di Eulero



σ_0 = snervamento del materiale

$\lambda_0 \longrightarrow$ snellezza limite

$\lambda > \lambda_0$ $\sigma_{cr} < \sigma_0$ snelle

Collasso avviene per instabilità

$\lambda < \lambda_0$ $\sigma_{cr} > \sigma_0$ Tozza

Collasso per cedimento del materiale