MADS - 1ª PARTE

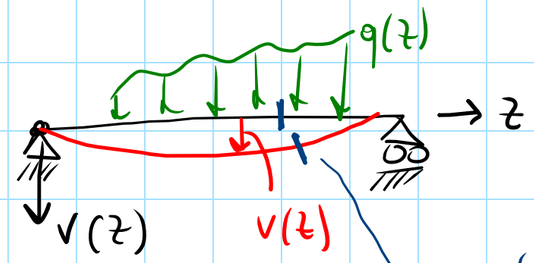
23/9/25

LINEA ELASTICA DEL IV ORDINE (TRAVI SNELLE : E/B)

- LINEA ELASTICA DEL II ORDINE $\begin{cases} v''(z) = -\dfrac{M(z)}{EJ} \\ + 2 \text{ C. LIMITE} \end{cases}$ CURVATURA

VALE PER STRUTT. STAT. DET..

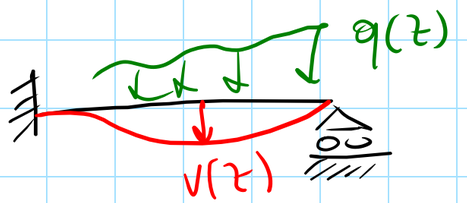
$q(z) \rightarrow M(z)$



$\varphi(z) = -v'(z)$

POSITIVA SE ANTIORARIA

DOMANDA: POSSO RISOLV. LA SEGUENTE STR CON LA L.E. II ORDINE?

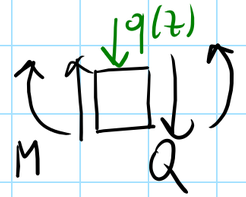


v(z)?

NO PERCHÉ NON SONO IN GRADO DI CALCOLARE M(z) SOLO CON EQ. DI EQUILIBRIO!

PER ORA SO RISOLV. IL PROBL SOLO CON IL METODO DELLE FORZE.

LINEA EL IV ORDINE



- EQUILIBRIO: $\frac{dM}{dz} = Q$ (*) ; $\frac{dQ}{dz} = -q$ (**)

- CONGRUENZA: $K = \frac{d\varphi}{dz}$; $\frac{dV}{dz} = -\varphi$ (C)

  CURVATURA

- LEGAME COSTITUTIVO: $K = \frac{M}{EJ}$

$$\frac{M}{EJ} = K = \frac{d\varphi}{dz} = -\frac{d^2V}{dz^2} \quad ; \quad \boxed{M = -EJV''} \ (B) \Rightarrow M' = -(EJV'')' \xrightarrow{(*)} Q = -(EJV'')' \ (A)$$

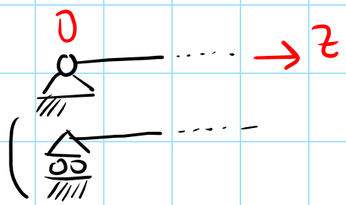
$$\Rightarrow Q' = -(EJV'')'' \xrightarrow{(**)} -q = -(EJV'')'' \Rightarrow \boxed{(EJV'')'' = q}$$

L.E. del IV ORDINE

- PER L'INTEGR SERVONO 4 C. AI LIMITI
- NOTI $EJ(z)$, $q(z)$ $\Rightarrow$ $V(z)$ (INCOGNITA) $\xrightarrow{(A)}$ $Q(z)$ $\xrightarrow{(B)}$ $M(z)$ $\xrightarrow{(C)}$ $\varphi(z)$

SE $EJ = cost$ $\rightsquigarrow$ $EJ\, v^{IV}(z) = q(z)$, $Q(z) = -EJ\, v'''(z)$

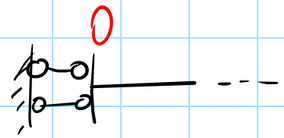
CONDIZIONI LIMITI



$v(0) = 0$

$M(0) = -EJ\, v''(0) = 0$

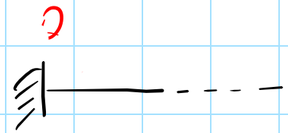
CERNIERA
CARRELLO
HINGE, PIN



$\varphi(0) = -v'(0) = 0$

$Q(0) = -EJ\, v'''(0) = 0$

DOPPIO PENDOLO
PATTINO
SLIDER



$v(0) = 0$

$-v'(0) = 0$

INCASTRO

FIXED-END

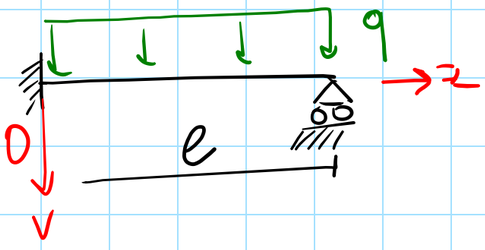
$M(0) = -EJ\, v''(0) = 0$

$Q(0) = -EJ\, v'''(0) = 0$

ESTREMO LIBERO

FREE-END

ES



EJ: cost

$$\begin{cases} EJ\, v''''(z) = q \\ v(0)=0 \\ v'(0)=0 \\ v(\ell)=0 \\ v''(\ell)=0 \end{cases} \Rightarrow \begin{cases} C_3=0 \\ C_4=0 \\ C_1=-\frac{5}{8}\frac{q\ell}{EJ} \\ C_2=\frac{1}{8}\frac{q\ell^2}{EJ} \end{cases}$$

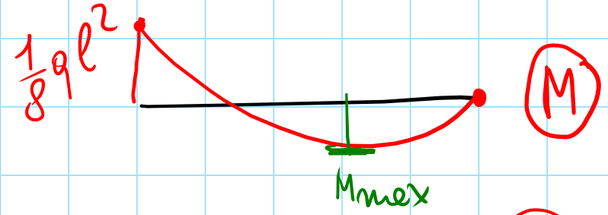
$$v'''(z)=\frac{q}{EJ}z+C_1$$

$$v''(z)=\frac{q}{EJ}\frac{z^2}{2}+C_1 z+C_2$$

$$v'(z)=\frac{q}{EJ}\frac{z^3}{6}+C_1\frac{z^2}{2}+C_2 z+C_3$$

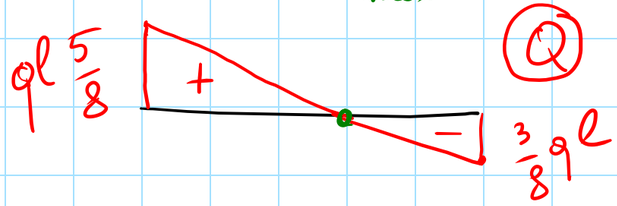
$$v(z)=\frac{q}{EJ}\frac{z^4}{24}+C_1\frac{z^3}{6}+C_2\frac{z^2}{2}+C_3 z+C_4$$

$$M(z)\,?\,=-EJ\,v''(z)=-EJ\left(\frac{q}{EJ}\frac{z^2}{2}-\frac{5}{8}\frac{q\ell}{EJ}z+\frac{1}{8}\frac{q\ell^2}{EJ}\right)$$



$$M(0)=-\frac{1}{8}q\ell^2$$

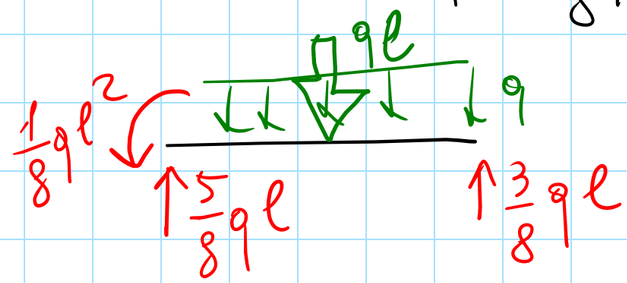
$$M(\ell)=0$$



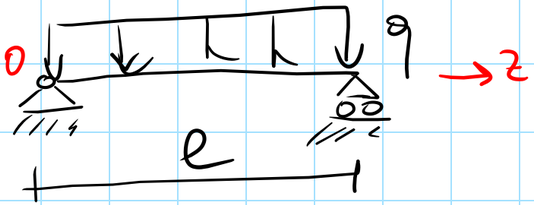
↑↑⊟↓↑

$$Q(z)=-EJ\,v'''(z)=-EJ\left(\frac{q}{EJ}z-\frac{5}{8}q\frac{\ell}{EJ}\right)$$

$$Q(0)=+\frac{5}{8}q\ell \quad ; \quad Q(\ell)=-\left(q\ell-\frac{5}{8}q\ell\right)=-\frac{3}{8}q\ell$$



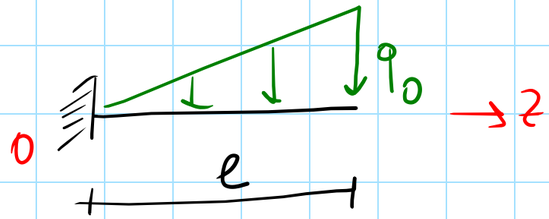
LES



$$\begin{cases} EJ\, v^{IV}(z) = q \\ v(0) = 0 \qquad v(\ell) = 0 \\ v''(0) = 0 \qquad v''(\ell) = 0 \end{cases}$$

VERIF CHE $v(z)$ RISULTI LA STESSA CHE SI OTT. CON LA L.E. II ORDINE

LES

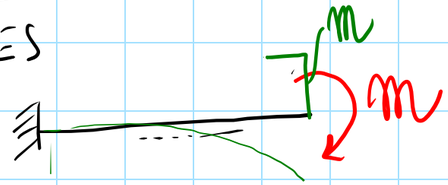


$$q(z) = \frac{q_0}{\ell} z \quad (+)$$

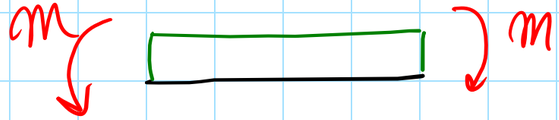
EJ cost

$$\begin{cases} EJ\, v^{IV}(z) = \dfrac{q_0}{\ell} z \\ v(0) = 0 \qquad v''(\ell) = 0 \\ v'(0) = 0 \qquad v'''(\ell) = 0 \end{cases}$$

ES



$$M(\ell) \sim m \Rightarrow M(\ell) = -m$$



$$M(z) = -m$$

$$Q(z) = 0$$

$$\begin{cases} v^{IV}(z) = 0 \\ v(0) = 0 \\ v'(0) = 0 \end{cases}$$

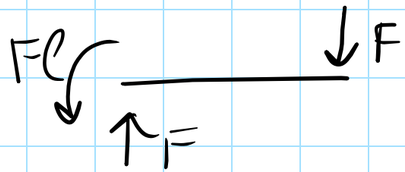
$$-EJ v''(\ell) = M(\ell) = -m$$

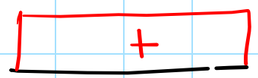
$$\begin{cases} EJ\, v''(\ell) = m \\ v'''(\ell) = 0 \end{cases}$$

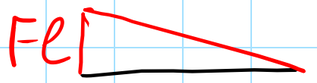
$$\begin{cases} v^{IV}(z) = 0 \\ v(0) = 0 \\ v'(0) = 0 \end{cases} \qquad v''(\ell) = 0 \qquad -EJ\, v'''(\ell) = F$$

$Q(\ell) \sim F$  $Q(\ell) = +F$

$$-EJ\, v'''(\ell) = +F$$

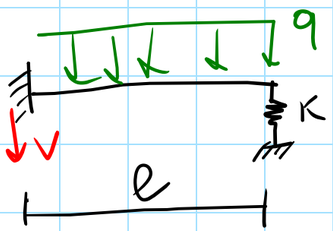


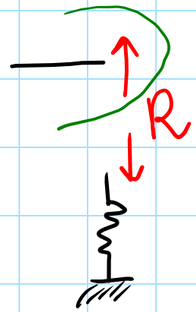




CEDIMENTI ELASTICI

$F = K\Delta\ell$





$v(\ell) > 0$

$R = K\, v(\ell)$

$Q(\ell) = -R = -K\, v(\ell)$

$$\begin{cases} EJ\, v^{IV}(z) = q \\ v(0) = 0 \\ v'(0) = 0 \\ v''(\ell) = 0 \\ -EJ\, v'''(\ell) = -K\, v(\ell) \end{cases}$$