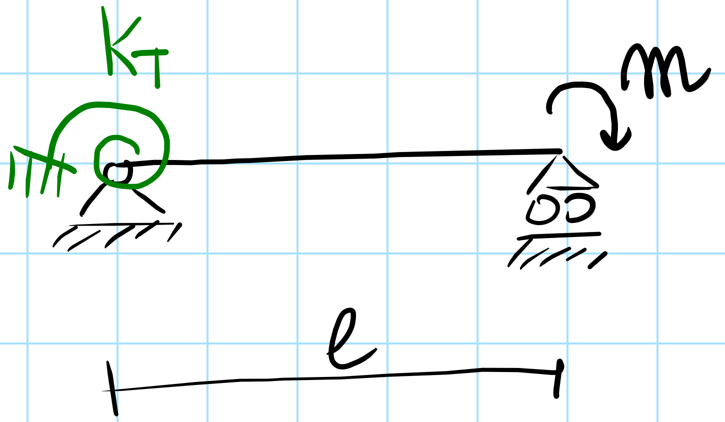
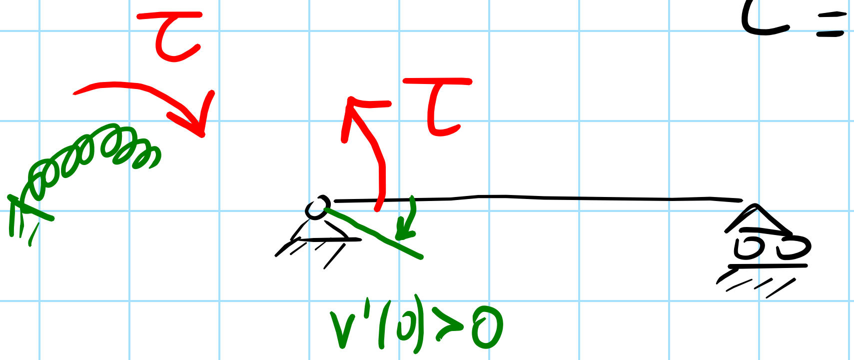
MOLLA ROTAZ. 24/09/25



STR 1 VOLTA IPERST.

INCASTRO CEDEVOLE



$$\tau = K_T\, v'(0)$$

$$[K_T] = [FL]$$

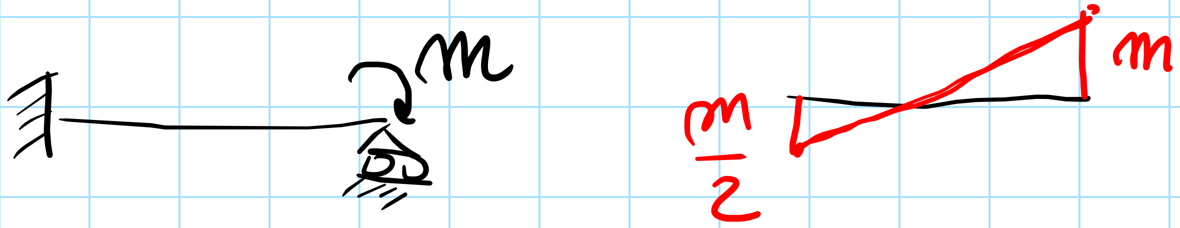
$$M(0) \overset{?}{=} \pm \tau \Rightarrow M(0) = -\tau = -K_T\, v'(0)$$

$$
\begin{cases}
EJ\, v^{IV}(z) = 0 \\
v(\ell) = 0 \\
-EJ\, v''(\ell) = -m \\
v(0) = 0 \\
-EJ\, v''(0) = -K_T\, v'(0)
\end{cases}
$$

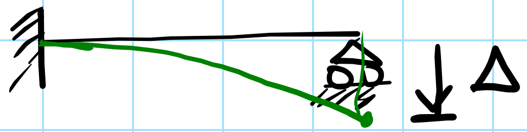
CASI LIMITE

$K_T = 0$

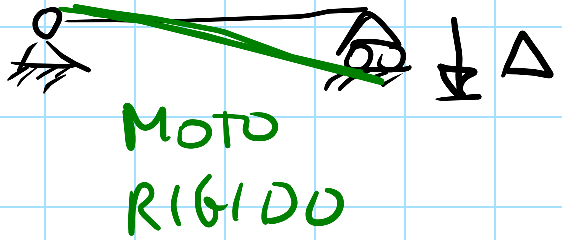
$K_T \to \infty$



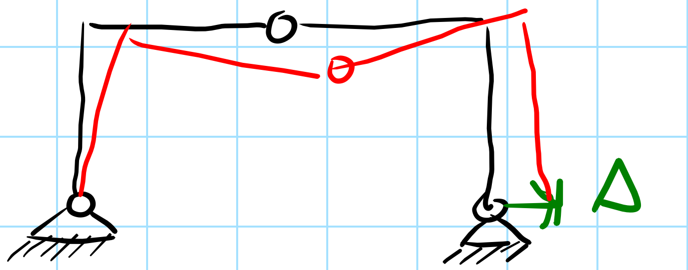
CEDIMENTI ANELASTICI (CEDIMENTI DI FONDAZ.)



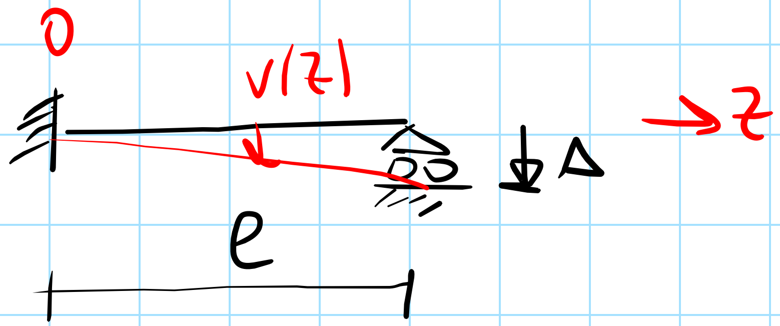
STR IPERST: SOLLECITATA DAL CEDIMENTO



STR ISOST: SOLO MOTI RIGIDI NO SOLLECITAZ.



(CATENE CINEMATICHE)



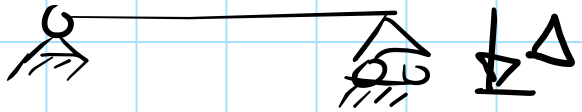
$$EJ\, v^{IV}(z) = 0$$

$$v(0) = 0$$

$$v'(0) = 0$$

$$v(\ell) = +\Delta$$

$$v''(\ell) = 0$$



$$EJ\, v^{IV}(z) = 0$$

$$v(0) = 0$$

$$v''(0) = 0$$

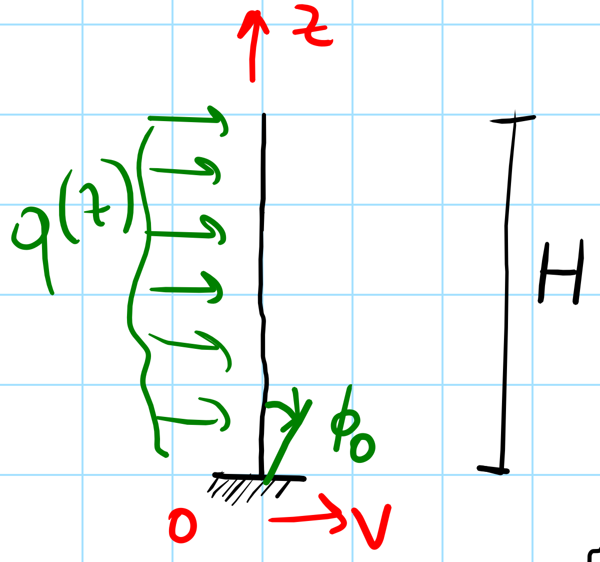
$$v(\ell) = +\Delta$$

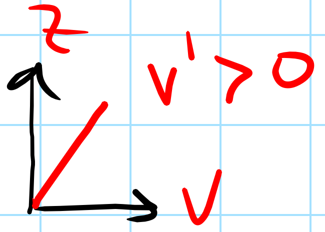
$$v''(\ell) = 0$$

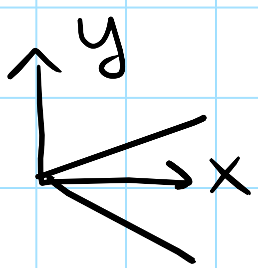
$$v(z) = \alpha z \quad , \quad \alpha = \frac{\Delta}{\ell}$$

$$M(z) = 0$$

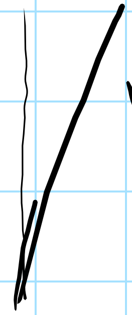
$$Q(z) = 0$$







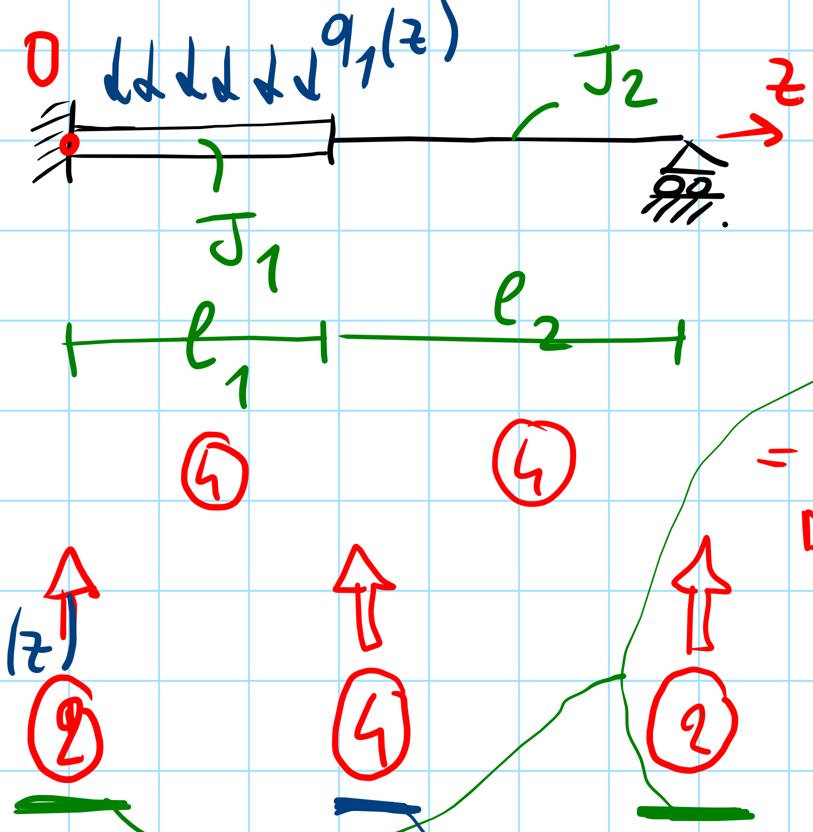
solo ϕ_0 :



$v(z) = \phi_0 z$

$$\begin{cases} EJ v^{IV}(z) = q(z) \\ v(0) = 0 \\ v'(0) = +\phi_0 \\ v''(H) = 0 \\ v'''(H) = 0 \end{cases}$$

L.E. DEL IV ORDINE QUANDO SONO PRESENTI PIÙ CAMPI DI INTEGRAZ. (NECESSARIE CONDIZ. DI RACCORDO / SALTO / CONTINUITÀ)



E : cost

= 8 COST. DI INTEGR.

COND. AI LIMITI

$$\begin{cases} v_1(0) = 0 \ ; \ v_1'(0) = 0 \\ v_2(\ell_1+\ell_2) = 0 \ ; \ v_2''(\ell_1+\ell_2) = 0 \end{cases}$$

$v_1 \in [0, \ell_1]$ $\quad EJ_1 v_1^{IV}(z) = q_1(z)$

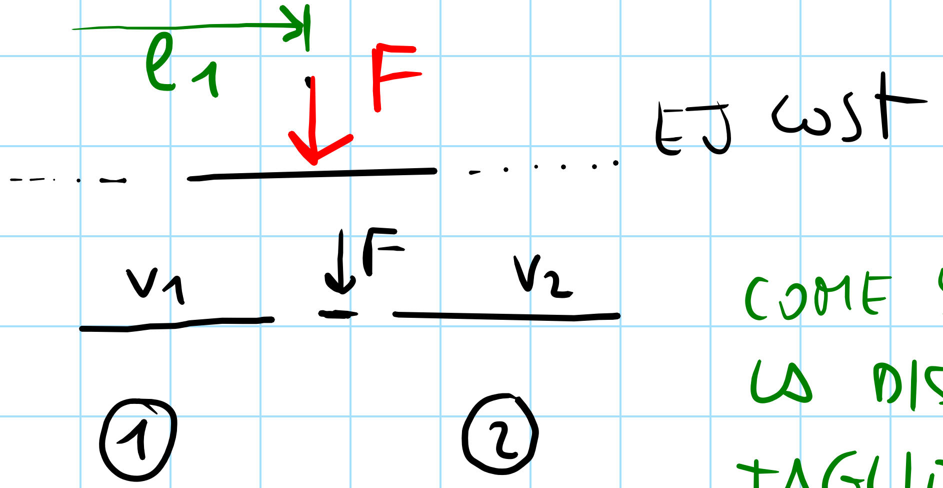
$v_2 \in [\ell_1, \ell_1+\ell_2]$

$EJ_2 v_2^{IV}(z) = 0$

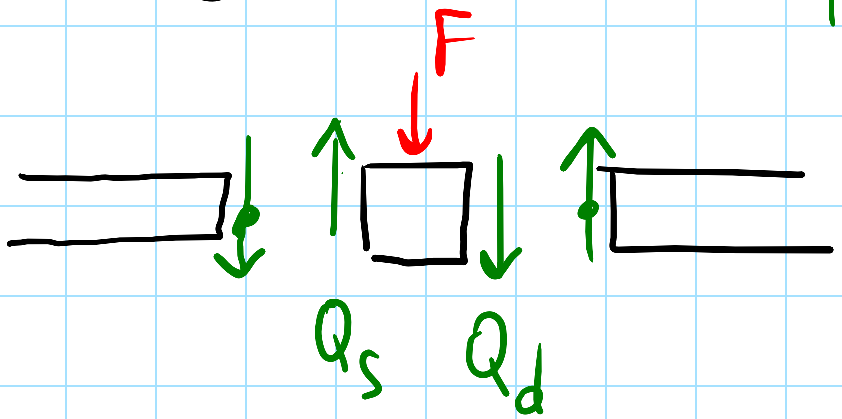
$$\begin{cases} v_1(\ell_1) = v_2(\ell_1) \\ v_1'(\ell_1) = v_2'(\ell_1) \\ -EJ_1 v_1''(\ell_1) = -EJ_2 v_2''(\ell_1) \\ -EJ_1 v_1'''(\ell_1) = -EJ_2 v_2'''(\ell_1) \end{cases}$$

M CONTINUO

Q CONTINUO



COME SCRIVERE
LA DISC. DEL
TAGLIO Q

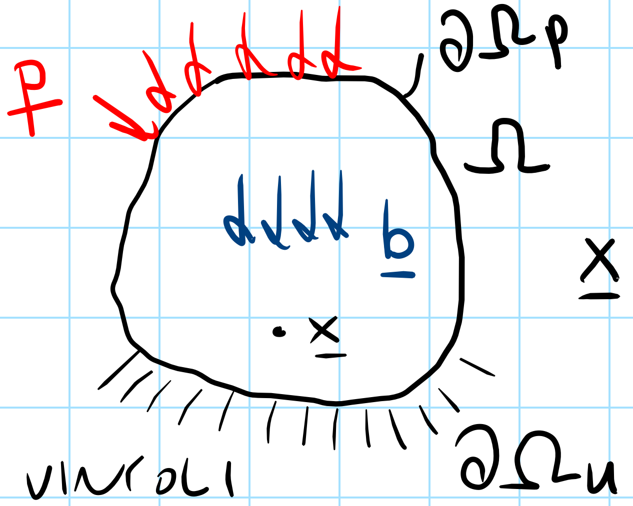


EQUIL. CONCIO: $+\uparrow$: $Q_s - F - Q_d = 0$ (EQ RACCORDO TAGLIO)

$$-EJ\, v_1'''(\ell_1) - F + EJ\, v_2'''(\ell_1) = 0$$

PRINCIPIO DI STAZION. DELL'EN. POTENZ. TOTALE

Nelle MECC. DEI SIST. DISCRETI QUESTO PRINCIPIO PERMETTE DI STUDIARE LE CONFIG. DI EQUIL. DEL SISTEMA



$\underline{u}(\underline{x})$: CAMPO DI SPOSTAMENTO

$$\underset{\sim}{\varepsilon} = \frac{1}{2}\left(\nabla\underline{u} + \nabla\underline{u}^T\right) \quad ; \quad \underset{\sim}{\sigma} = \mathbb{C}\,\underset{\sim}{\varepsilon}$$

$\underline{x} \in \Omega$

$$\Pi(\underline{u}) = \underbrace{\frac{1}{2}\int_\Omega \underset{\sim}{\sigma}\cdot\underset{\sim}{\varepsilon}\, dV}_{\text{EN. ELASTICA NEL CORPO}} - \underbrace{\left\{\int_\Omega \underline{b}\cdot\underline{u}\, dV + \int_{\partial\Omega_p} \underline{p}\cdot\underline{u}\, dA\right\}}_{\text{POTENZ. DEI CARICHI}}$$

IMMAGINO CHE $\underline{u}(\underline{x})$ SIA IL CAMPO DI SPOST. EFFETTIVO ALL'EQUILIBRIO.

APPLICO A $\underline{u}(\underline{x})$ UNA VARIAZIONE $\delta\underline{u}$ E STUDIO LE MODIFICHE CHE QUESTA VARIAZ. INDUCE NELL'EN. POT. Π

$$\underline{u} \rightarrow \underline{u} + \delta\underline{u}$$

$$\Pi \rightarrow \Pi + \Delta\Pi$$

$$\Pi(\underline{u}+\delta\underline{u}) \stackrel{!}{=} \frac{1}{2}\int_{\Omega} \mathbb{C}(\underset{\sim}{\varepsilon}+\delta\underset{\sim}{\varepsilon})\cdot(\underset{\sim}{\varepsilon}+\delta\underset{\sim}{\varepsilon})\,dV - \int_{\Omega}\underline{b}\cdot(\underline{u}+\delta\underline{u})\,dV - \int_{\partial\Omega_p}\underline{p}\cdot(\underline{u}+\delta\underline{u})\,dA$$

$$\left(\delta\underset{\sim}{\varepsilon}=\frac{1}{2}\left(\nabla\delta\underline{u}+\nabla\delta\underline{u}^T\right)\right)$$

$$\stackrel{!}{=} \frac{1}{2}\int_{\Omega}\mathbb{C}\underset{\sim}{\varepsilon}\cdot\underset{\sim}{\varepsilon} + 2\,\mathbb{C}\underset{\sim}{\varepsilon}\cdot\delta\underset{\sim}{\varepsilon} + \mathbb{C}\delta\underset{\sim}{\varepsilon}\cdot\delta\underset{\sim}{\varepsilon}\,dV - \int_{\Omega}\underline{b}\cdot\underline{u}\,dV - \int_{\Omega}\underline{b}\cdot\delta\underline{u}\,dV - \int_{\partial\Omega_p}\underline{p}\cdot\underline{u}\,dA - \int_{\partial\Omega_p}\underline{p}\cdot\delta\underline{u}\,dA$$

$$\Pi(\underline{u}+\delta\underline{u}) = \Pi(\underline{u}) + \underbrace{\delta\Pi}_{\text{VARIAZ. PRIMA}} + \underbrace{\delta^2\Pi}_{\text{VARIAZ. SECONDA}}$$

$\delta\Pi \stackrel{?}{=} 0$ (VEDI PAGINA SUCCESSIVA): E.P.T. È STAZIONARIA IN CORRISPONDENZA DI UNA CONF. DI EQUILIBRIO

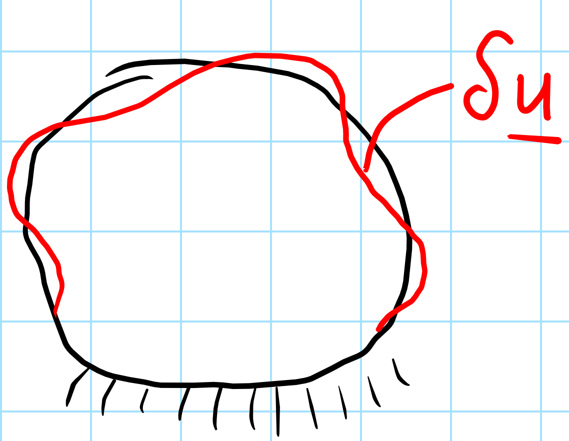
STUDIO $\delta^2\Pi$: $\frac{1}{2}\int_{\Omega}\mathbb{C}\delta\underset{\sim}{\varepsilon}\cdot\delta\underset{\sim}{\varepsilon}\,dV$

VISTO CHE $\mathbb{C}$ è DEFINITO POSITIVO ALLORA $\delta^2\Pi>0$, $\forall\delta\underline{u}\neq\underline{0}$

ALLORA $\Pi(\underline{u})$ HA UN MINIMO ALL'EQUILIBRIO.

$\delta\Pi$ APPLICO IL T.L.V. NELLA SITUAZ. DI EQUILIBRIO

SIST. STAT. AMMISS. | SIST. CINEM. AMM.



$$\delta\underset{\sim}{\varepsilon} = \frac{1}{2}\left(\nabla\delta\underline{u} + \nabla\delta\underline{u}^T\right)$$

$\underset{\sim}{\sigma}(\underline{x})$ VERO

EQUIL. $\begin{cases} \mathrm{div}\,\underset{\sim}{\sigma} + \underline{b} = \underline{0} \\ \underset{\sim}{\sigma}\,\underline{n} = \underline{p} \end{cases}$

$$L_{ve} = L_{vi} \quad ; \quad \int_\Omega \underline{b}\cdot\delta\underline{u}\,dV + \int_{\partial\Omega_p} \underline{p}\cdot\delta\underline{u}\,dA = \int_\Omega \underset{\sim}{\sigma}\cdot\delta\underset{\sim}{\varepsilon}\,dV$$

$$L_{vi} - L_{ve} = 0$$

$L_{vi} - L_{ve}$ è $\delta\Pi$, QUINDI ALL'EQUILIBRIO $\boxed{\delta\Pi = 0}$

OSS.: POSSIAMO FORMUL. UN PRINCIPIO DI STAZIONARIETA' "EQUIVALENTE" PARTENDO DAL POTENZIALE COMPL. $\varphi^c(\underset{\sim}{\sigma})$:

E. P. COMPL. T. $\Pi^c(\underset{\sim}{\sigma})$

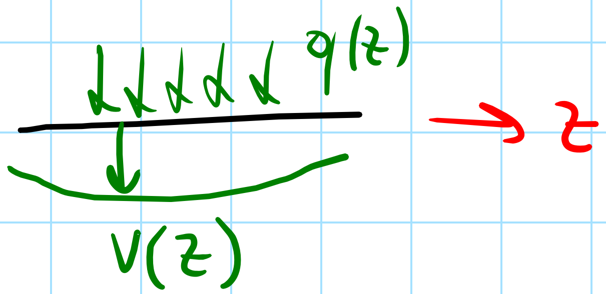
$$\Pi^c(\underset{\sim}{\sigma}) = \frac{1}{2}\int_{\Omega} \underset{\sim}{\sigma} \cdot \mathbb{C}^{-1}\underset{\sim}{\sigma} \, dV - \int_{\Omega} \overbrace{(-\operatorname{div}\underset{\sim}{\sigma})}^{\underline{b}} \cdot \underline{u} \, dV - \int_{\partial\Omega p} \overbrace{\underset{\sim}{\sigma}\underline{n}}^{\underline{p}} \cdot \underline{u} \, dA$$

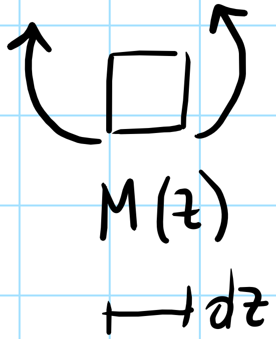
$\underline{\delta \Pi^c = 0}$ ALL'EQUIL.

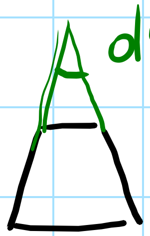
E.P.T. $(\Pi(u))$ PER LE STR. INFLESSE

DA VINCI

(EULERO – BERNOULLI)







$d\varphi = K\,dz$ ↖ CURVATURA

$$K = -v''(z)$$

$$M = -EJ\,v''(z)$$

$$\Pi(v) = \underbrace{\text{EN. EL. TRAVE}}_{\oplus} - \underbrace{\text{POTENZ. CARICHI}}$$

$$d\Phi = \frac{1}{2} M\,d\varphi = \frac{1}{2} M\left(-v''\,dz\right) = -\frac{1}{2} M\,v''\,dz = \frac{1}{2} EJ\,v''^2\,dz$$

$d\Phi$: EN. EL. NEL CONCIO ELEM

$$\Pi(v) = \underbrace{\frac{1}{2}\int_0^{\ell} EJ\,v''^2\,dz}_{\oplus} - \underbrace{\int_0^{\ell} q\,v\,dz}$$

E.P.T. TRAVI INFLESSE

$$\delta\Pi = 0 \Longrightarrow (EJv'')'' = q$$

(L.E. DEL IV ORD.)