Atomic Systems: Qubit and state preparation

It is now time to start looking into our first quantum platform. We will discuss atomic systems in which the qubit is encoded within two particular state of the atomic structure (e.g. two long-lived hyperfine states, two electronic states coupled by and optical transition, a ground and a Rydberg state....). From a conceptual perspective neutral atoms or ions are very similar in the way they interact with light. They only differ in the way we trap them and what is the mechanism that regulates interactions between two qubits. As we will discuss later in the course, neutral atoms are trapped with laser fields using optical dipole forces, while ions are confined through the Coulomb force. The Coulomb force is also the force that mediates the interactions between two ions, while to make neutral atoms interact we will use Van der Waals interactions.

For now we can simply consider atoms or ions as two-level systems that are confined in space.

The two level atom has two states $|e\rangle$ and $|g\rangle$ that are connected by an electric dipole transition with angular frequency ω_e. This system is equivalent to a spin-1/2 evolving in space with a magnetic field aligned along the "vertical" Z axis that splits the energy of the two spin components.

Following this analogy, the atomic Hamiltonian is

$$H_a = \hbar \frac{\omega_e}{2} \sigma_z$$

NOTE THAT ACCORDING TO OUR DEFINITION OF THE QUBIT (PREVIOUS CLASS)

$|e\rangle \rightarrow |0\rangle$

$|g\rangle \rightarrow |1\rangle$

FEELS WEIRD BUT LET'S STICK WITH IT

where we have set the zero energy half-way between the two levels

Let us introduce also the atomic raising and lowering operators $\sigma_\pm$

$$\sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$$

$$\sigma_+ = |0\rangle\langle 1| = |e\rangle\langle g| = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\sigma_- = |1\rangle\langle 0| = |g\rangle\langle e| = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow \sigma_+ |g\rangle = |e\rangle\langle g|g\rangle = |e\rangle \qquad \sigma_- |g\rangle = 0$$

$$\sigma_+ |g\rangle = 0 \qquad \sigma_- |e\rangle = |g\rangle$$

We also said that atoms and ions are confined in space. For now we do not care how we trap them, however we can say that the trap is to a good approximation harmonic. This is generally true if we think of a particle whose motional energy is very small. In this case the particle only explore the bottom of the trapping potential which we can approximate with a parabola (so harmonic). Under this approximation, we can then consider the motion of "cold" particle as that of a quantum harmonic oscillator. We will check the validity of this approximation later on when we will discuss the trapping mechanisms of neutral atoms and trapped ions.

The Hamiltonian governing the atomic motion is thus:

$$H_m = \hbar \omega_m \left(a^\dagger a + \tfrac{1}{2}\right)$$

We can thus describe a trapped atom or ion as a two-level system that is coupled to a quantum harmonic oscillator

The free Hamiltonian for a single trapped particle is thus

$$H_0 = \hbar \frac{\omega_a}{2} \sigma_z + \hbar \omega_m \left(a^\dagger a + \tfrac{1}{2}\right)$$

Qubit definition

In the previous section we investigated how an atom interacts with light. If we want to use atomic systems to build a quantum computer, the first thing we need to do is to define the qubit in these systems (DiVincenzo's first criteria) $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

As we are dealing with atomic transitions we need to find transitions where the decay time is long.

The first transition type that usually comes to mind is a dipole transition whose decay rate is

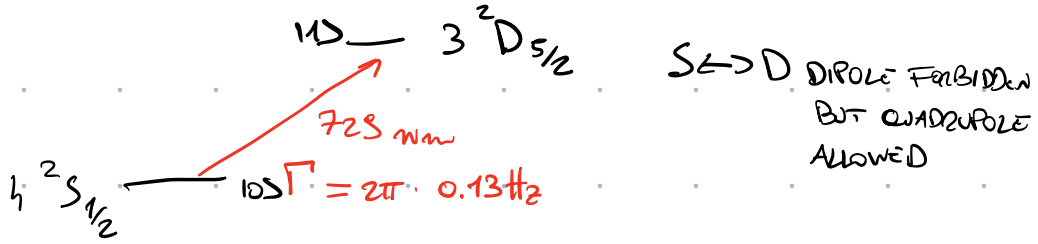
$$\Gamma \propto \omega^3 |\langle 0|\vec{E}\cdot\vec{d}|1\rangle|^2 \propto 10s \text{ of } MHz$$

(ω: 100s THz)

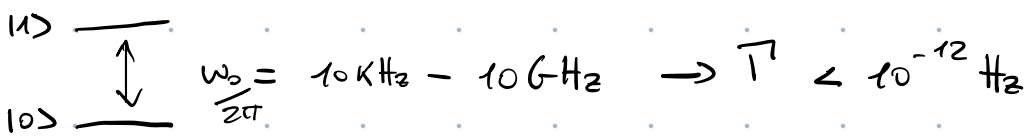
There are two possible ways of reducing the decay rate: the first is to suppress the dipole matrix element, thus using a dipole forbidden transition, and the second is to reduce the qubit transition frequency.

(1) $|\langle 0|\vec{E}\vec{d}|1\rangle| = 0$ E.G. QUADRUPOLE TRANSITION

EXAMPLE $^{40}Ca^+$



(2) SMALL ω => HYPERFINE TRANSITION



Quadrupole transitions can be treated in a similar way to dipole transitions with the difference that we need to deal with higher order terms

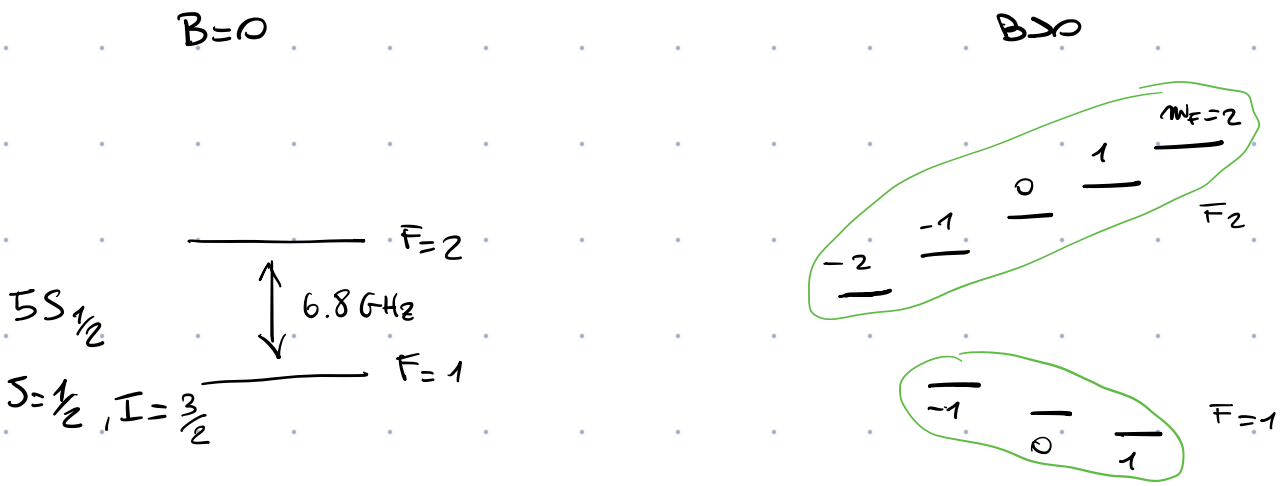
$$\Omega^{DP} = \frac{eE_0}{\hbar} |\langle e| \vec{r}\cdot\vec{\varepsilon} |g\rangle|, \quad \Omega^{QP} = \frac{eE_0 k}{2\hbar} |\langle e| (\vec{r}\cdot\hat{k})(\vec{r}\cdot\vec{\varepsilon}) |g\rangle|$$

DIPOLE RABI FREQ — QUADRUPOLE RABI FREQ

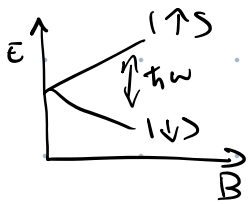
However working with hyperfine transition opens up new interesting way to coherent manipulation of the qubit (i.e. Raman transitions).

Qubit Coherence

The third DiVincenzo's criteria also requires that the qubit coherence is long compared to the typical experimental timescales. One of the main decoherence channel in atomic systems is noise due to classical fields. Why? In atomic systems we always need to apply a magnetic field to define a quantization axis and to split degeneracies between different levels. Lifting the degeneracy is crucial to isolate qubit states. This is for example the hyperfine ground state of ^{87}Rb

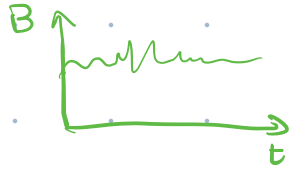


More generally the energy splitting is given by $E = -\mu \cdot B$



A QUBIT $|\psi\rangle = \alpha|0\rangle + \beta e^{i\varphi}|1\rangle$

$e^{i\varphi} = e^{i\int \frac{\mu B}{\hbar} dt}$ — B(t) BECAUSE OF NOISE



WHY:

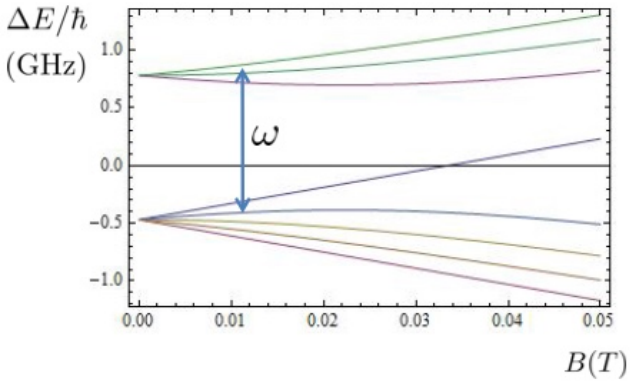
$H_0 = \frac{\hbar\omega}{2}\sigma_z \Rightarrow U(t) = e^{-i\frac{\omega}{2}\sigma_z}$

$\Rightarrow |\psi(t)\rangle = \alpha\, e^{-i\omega/2\, t}|0\rangle + \beta\, e^{i\frac{\omega}{2}t}$

$= e^{-i\omega/2\, t}\left(\alpha|0\rangle + \beta\, e^{i\omega t}|1\rangle\right)$

BUT $\hbar\omega$ CHANGES IN TIME DUE TO B FLUCTUATIONS.

There are two ways in which one can improve the coherence other than improving the stability of the magnetic field. In hyperfine interactions, depending on the atomic structure, there can be certain magnetic field values for which first-order magnetic field insensitive transition appears. The Hamiltonian of the system is driven by: $H_{HF} = A\,\vec{I}\cdot\vec{S} + \vec{\mu}\cdot\vec{B}$

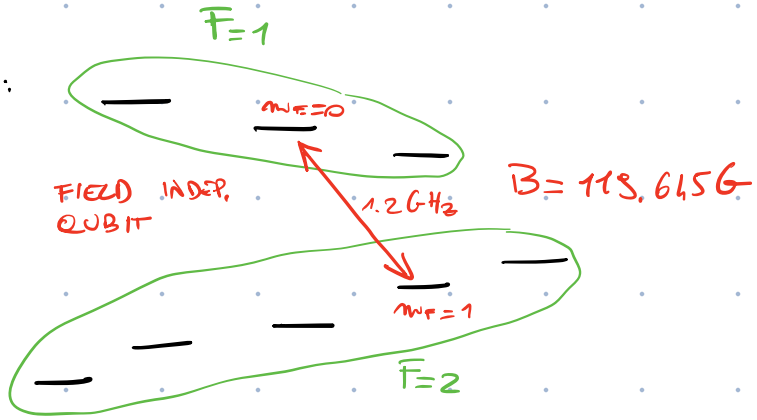


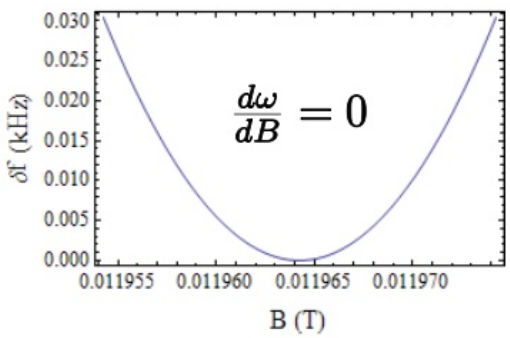
FOR CERTAIN VALUES OF B THE GRADIENT OF TWO CURVES IS THE SAME ⇒ IF B CHANGES THE CHANGE IS THE SAME FOR BOTH STATES ⇒ GOOD COHERENCE

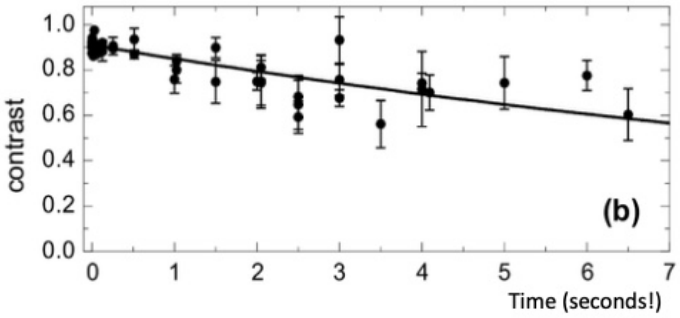
THIS IS FOR $^9Be^+$

THE GROUND STATE IS:

$H_{HF} = A\,\vec{I}\cdot\vec{S}$ ($I = 3/2$, $S = 1/2$, $A \approx 4.6\,GHz$)







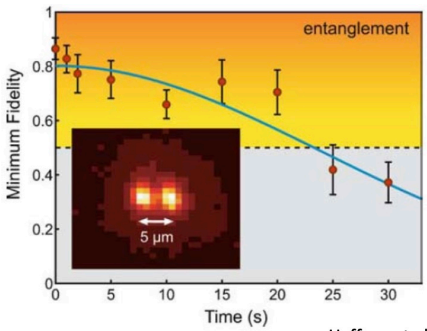
The second way of improving the coherence is to use the entanglement.

QUBIT 1: $|0\rangle + e^{i\omega'(t)t}|1\rangle$

QUBIT 2: $|0\rangle + e^{i\omega(t)t}|1\rangle$

$\Rightarrow$ ENTANGLED STATE $e^{i\omega(t)t}|01\rangle + e^{i\omega(t)}|10\rangle = e^{i\omega(t)t}\left(|01\rangle + e^{i(\omega'(t)-\omega(t)t)}|10\rangle\right)$

If the noise is common mode then entangled states can have very long coherence times



Haffner et al., Appl. Phys. B 81, 151-153 (2005)

THE TRIPLET STATE $(|01\rangle+|10\rangle)$ IS A DECOHERENCE FREE SUBSPACE

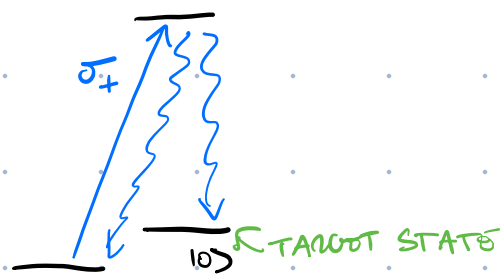
Qubit initialization

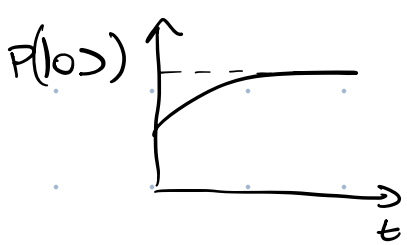
Before any coherent control of the qubit it is necessary to initialize the qubit to a fiducial state. Most atomic systems have complex atomic structures where the ground state is composed of multiple substates (fine structure and hyperfine structure). When atoms are initially trapped all states are commonly occupied.

To initialize the qubit to a particular state we typically perform **optical pumping**

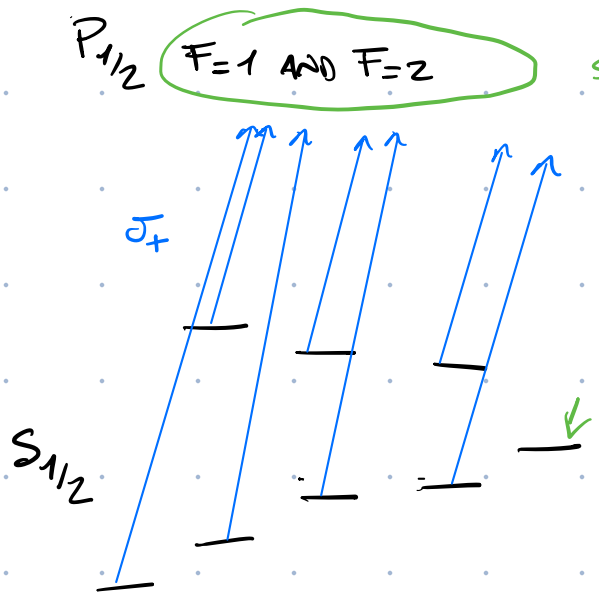
Idea: excite all occupied states except for the target one to an excited state that has a decay component to the target state. The typical approach is to use σ polarized light.

EXAMPLE:





The exact optical pumping scheme depends on the atomic species. Here is an example for $^9Be^+$ since we talked about it before.



THE LARGEST m_F STATE IN $P_{1/2}$ IS $F=2, m_F=2$
BUT THE $|S_{1/2}, F=2, m_F=2\rangle$ WOULD REQUIRE AN $m_F=3$ TO BE EXCITED
$\Rightarrow$ DARK STATE!

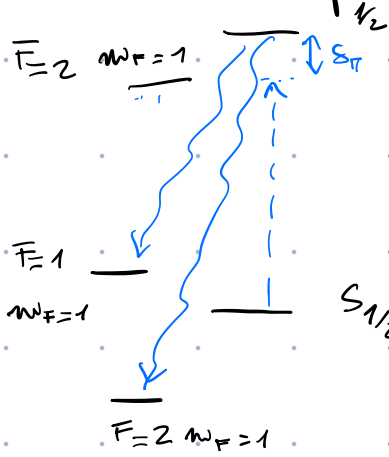
In real systems, the state preparation infidelity arises form polarization impurities. In the previous example, if the polarization is not perfectly σ_+ but also has some σ_- component, then the state $|S_{1/2}, F=2, m_F=2\rangle$ can be excited to $|P_{1/2}, F=2, m_F=2\rangle$

The optical pumping rate from an initial state i to f is

$$\Gamma_{i\to f} = \frac{\Gamma}{2}\frac{S_0}{1+S_0+4\delta^2/\Gamma^2}\, c_{i\to f}$$

Γ: LINEWIDTH; S_0: SATURATION I/I_{SAT}; δ: DETUNING; $c_{i\to f}$: COUPLING COEFFICIENT BETWEEN i AND f

THE FORMULA COMES FROM SOLVING THE RATE EQUATIONS AND ASSUMING LORENTZIAN LINESHAPES



DECAYS TO UNWANTED STATES EMITTING σ_- PHOTONS

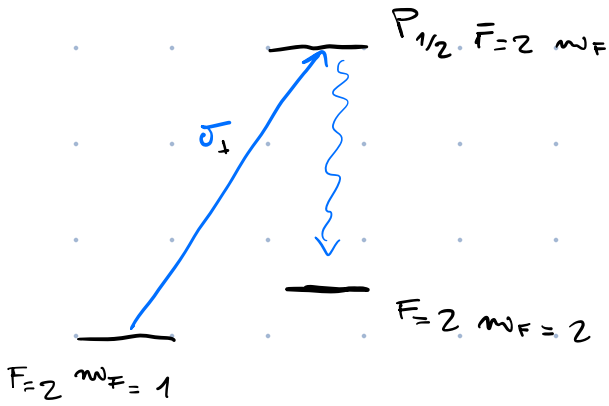
UNWANTED
$|S_{1/2}, F=2, m_F=2\rangle \to |P_{1/2}, F=2, m_F=2\rangle$
THAT DECAYS TO $|S_{1/2}, F=1, m_F=1\rangle$
$|S_{1/2}, F=2, m_F=1\rangle$

FOR THIS CASE $\sum c_{i\to f} = 2/9$

$$\Rightarrow \Gamma_\pi = \frac{\Gamma}{2} \frac{\varepsilon_\pi s_0}{1 + s_0 + 4\delta_\pi^2/\Gamma^2} \frac{2}{9}$$

ε_π: AMOUNT OF IMPURE π POLARIZATION

In the ideal case scenario



$c_{i\to f} = 1/18$

$$\Rightarrow \Gamma_{\sigma_+} = \frac{\Gamma}{2} \frac{s_0}{1+s_0} \frac{1}{18}$$

IDEAL CASE $\delta = 0$

$$\Rightarrow \text{ERROR} \quad e_{WIT} = \frac{\Gamma_\pi}{\Gamma_{\sigma_+}} = \frac{4(1+s_0)\varepsilon_\pi}{1 + s_0 + 4\delta_\pi^2/\Gamma^2}$$

FOR Be^+ AT $B = 119,45\,G$ $\quad \delta_\pi = 2\pi \times 102.8\,MHz$

$\Rightarrow$ IF $s_0 = 1$ AND $\varepsilon_\pi = 0.1\%$ $\Rightarrow$ $e_{WIT} \approx 10^{-5}$