Introduction

Conveying a fluid over long distances where it is not naturally available (e.g., oil pipelines).
These systems are characterized by large diameters and a longitudinal extension that can reach tens of thousands of times their diameter. In such cases, they are referred to as *long pipelines*.

For this type of system, hydraulic calculations can be simplified by adopting the following assumptions:

- The horizontal projection of the pipeline can be considered instead of its actual length.
- Kinetic head contributions are negligible compared to piezometric heads.
- Local losses (inlet, outlet, section changes) are negligible compared to continuous (friction) losses.

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Long pipelines are typically characterized by flow velocities on the order of a few meters per second → corresponding to kinetic heads of only a few tens of centimeters. In contrast, the geodetic elevations of the reservoirs being connected usually determine piezometric heads on the order of tens of meters.

Pipelines are usually buried and follow the natural topography; slopes greater than 10 - 20% are rare. For example, assuming a slope of 10% and a horizontal length of L=100m, the actual pipeline length would be 100.5m, introducing an error of only 0.5%.

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When a pipe is a long pipe? Let's introduce the *equivalent length* $L_e = \frac{K_T}{\lambda} D$

Consider K_T the sum of local losses, such as $K_{imbocco}$=0,5, K_{sbocco}=1 and friction index λ=0,025: $L_e = 60D$

We can consider as a threshold the 4% of total lenght $L_e = 60D \leq 0{,}04\ L$

We can find the minimum length, expressing the total length in terms of diameters : $L_{min} = n_{min} D = \frac{40}{0{,}04} K_T D = 1000 K_T D$

Introduction

Long pipelines have long service lives, during which their hydraulic characteristics and roughness may change.

To ensure proper operation during the design phase, pipelines should be calculated under the assumption of *aged pipes*, and their performance may also be checked under *new pipe* conditions.

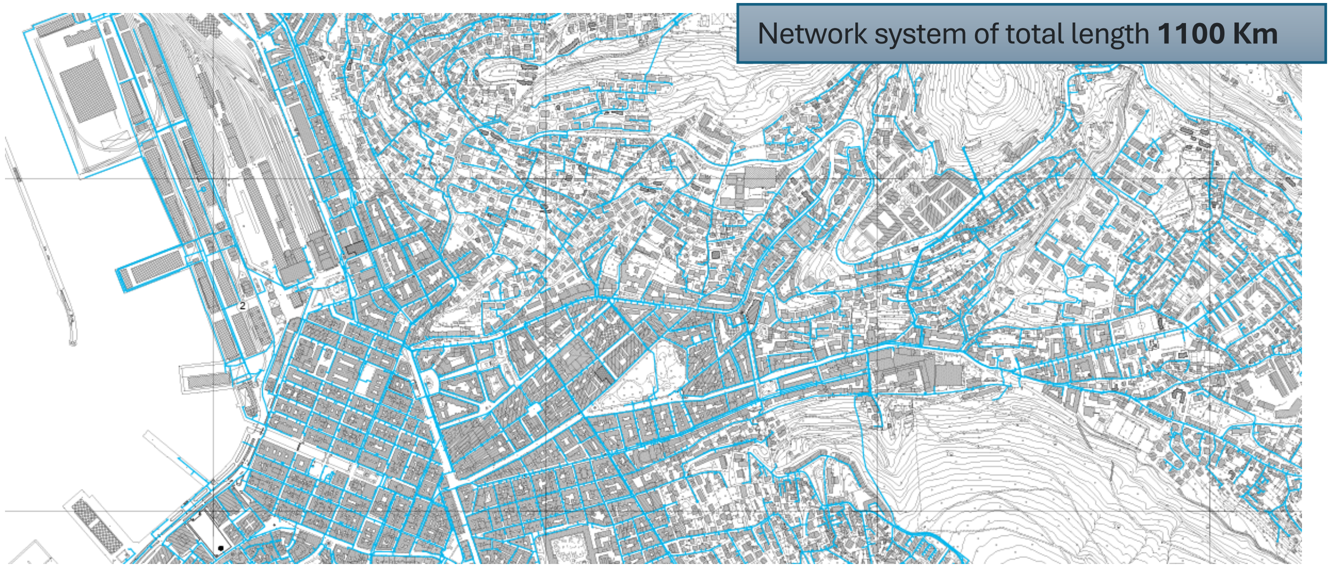
Types of calculations:

- Verification of an existing system (flow rate calculation)
- Design and sizing of the system to meet specific criteria (diameter determination)

In many cases, long pipelines may be combined into a *network system*. In such cases, calculations must also rely on economic evaluation; therefore, among the hydraulically feasible solutions, the economically most advantageous one is adopted.

It is common practice in calculations to rely on empirical formulas of the type:

$$J = k\frac{Q^m}{D^n}$$



Network system of total length **1100 Km**

Examples

Acquedotto Pugliese (Italy)
Description: One of the longest aqueducts in the world, the Apulian Aqueduct, built in the early 20th century, extends for over 500 kilometers. It is used to transport water from the Sele River to the arid regions of Apulia.
Importance: A perfect example of a long water pipeline designed to solve water scarcity in a region with limited surface water resources. → Check the website

Colorado Aqueduct (United States)
Description: This aqueduct stretches for more than 380 kilometers and transports water from the Colorado River to Southern California, crossing deserts and arid areas.
Importance: It represents a major engineering achievement for the management and distribution of water resources in densely populated regions with limited local supplies.

Trans-Mediterranean Pipeline (Italy – Algeria)
Description: This pipeline, more than 2,500 kilometers long, transports natural gas from Algeria to Italy, passing through Tunisia and the Mediterranean Sea.
Importance: An example of a pipeline that spans long distances and complex geographical conditions, both on land and underwater, to ensure energy supply.

Submarine Pipelines (e.g., Nord Stream)
Description: The Nord Stream submarine pipeline, which connects Russia to Germany through the Baltic Sea, extends for about 1,200 kilometers.
Importance: An example of an underwater pipeline designed for the long-distance transport of fluids in challenging environments, with implications for hydraulic design and resistance to extreme conditions.

Examples

Eifel Aqueduct (Germany, Roman era)
Description: An ancient Roman aqueduct about 95 kilometers long that supplied water to the Roman colony of Cologne (Germany) from a source in the Eifel mountains.
Importance: A historical example of a long water conduit demonstrating how Roman engineers were able to transport large volumes of water over long distances using gravity.

Baku–Tbilisi–Ceyhan (BTC) Pipeline
Description: This oil pipeline transports crude oil from the Caspian Sea (Azerbaijan) to the port of Ceyhan in Turkey, along a route of about 1,768 kilometers.
Importance: A relevant example of fluid transport (in this case, oil) through a long pipeline crossing three countries, with significant technical and political challenges.

Suez Canal (Egypt)
Description: Although technically a canal, it can be considered a long navigable conduit. With a length of about 193 kilometers, it is a fundamental hydraulic work for global maritime transport.
Importance: An artificial waterway with a strategic role in global trade, used for ship transit rather than water distribution.

Agricultural Irrigation Pipelines (e.g., California)
Description: In California, there are extensive pipeline systems for agricultural irrigation, such as the Central Valley Project, which supplies water to a vast farming area. These pipelines are designed to transport water from mountains and reservoirs to agricultural plains.
Importance: A practical example of long pipelines for agricultural purposes, demonstrating the importance of efficient water distribution across large territories.

Distribution networks

Pipeline networks connected in series and in parallel, characterized by multiple supply sources and delivery points.

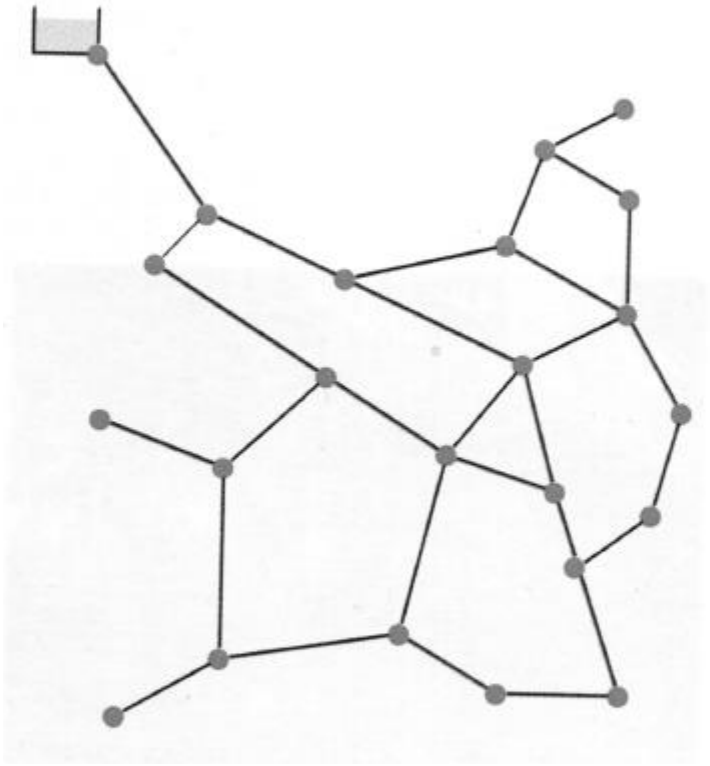
Types of interventions:

- New installation
- Expansion of an existing system
- Maintenance and temporary shutdown of certain sections

Objective: to deliver the required flow rates while maintaining a minimum pressure level. This is a complex calculation that requires appropriate algorithms and the use of computational tools and software → EPANET

System schematization involves defining:

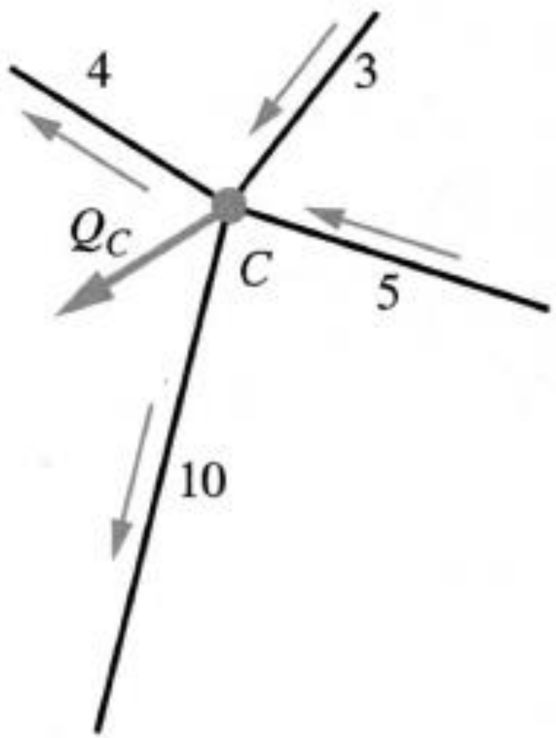
- **Node:** a point where the hydraulic or geometric characteristics of the network change;
- **Link (or branch):** a pipe connecting two nodes, with uniform hydraulic and geometric characteristics;
- **Loop (or mesh):** a sequence of links that, starting from a given node, returns to the same node.



Possible main unknowns: H (head), Q (flow rate), D (diameter).

Distribution networks

First principle: continuity at nodes



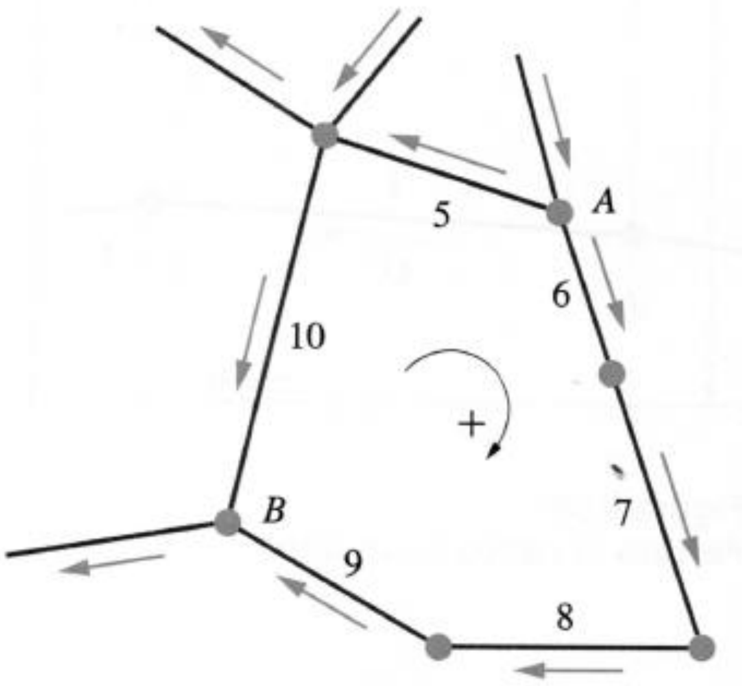
$$Q_5 + Q_3 - Q_4 - Q_{10} - Q_C = 0$$

$$\sum Q_i = 0$$

Continuity equation at the nodes: the sum of the inflows to a node (positive) and the outflows from the node (negative) must be equal to zero.

Distribution networks

Second principle: head loss



$$H_A - H_B = J_5 L_5 + J_{10} L_{10} =$$
$$= J_6 L_6 + J_7 L_7 + J_8 L_8 + J_9 L_9$$

$$J_6 L_6 + J_7 L_7 + J_8 L_8 + J_9 L_9 - J_{10} L_{10} - J_5 L_5 = 0$$

For branch i, that links nodes j and j+1 $H_j - H_{j+1} = J_i L_i$

Consider only continuous head loss

Consider Darcy – Weisbach $H_j - H_{j+1} = \dfrac{8 L_i}{g \pi^2 D_i^5} \lambda_i Q_i^2 = \beta_i \lambda_i Q_i^2$

For a generic mesh:

$$\sum \Delta H_i = \sum \beta_i \lambda_i Q_i^2 = 0$$

Having defined a flow direction, the discharge is taken as positive if it moves in the same direction as the chosen orientation, and negative if it moves in the opposite direction.

[In addition, consider for $\lambda_i(Q_i, Re)$ the Colebrook formula]

Verification of an existing network

The unknown variables are the heads H at the nodes, and the flow rates Q at the branches.
The two principles previously highlighted are used.

N unknown H at nodes
L unknown Q at branches } N+L unknwowns

SYSTEM
SOLVABLE

N eqs. I princ. at nodes
L eqs. II princ. at branches: ΔH=JL } N+L equations

$$\begin{cases} \sum Q_i = 0 \\ \sum \Delta H_i = \sum \beta_i \lambda_i Q_i^2 = 0 \end{cases}$$

Note: To solve the system and simplify the problem, we assumed that the fluid is everywhere fully turbulent, with a flow rate exponent n = 2.

Design a new network

The unknown variables are the heads H at the nodes, and for the branches the flow rates Q and the diameters D.
The two principles previously highlighted are used.

N unknown H at nodes
L unknown Q at branches
L unknown D at branches

} N+2L unknwowns

SYSTEM
INDETERMINATE

N eqs. I princ. at nodes
L eqs. II princ. at branches: $\Delta H = JL$

} N+L equations

Design calculation case instead of **verification calculation** → indeterminate problem.
The number of equations is lower than the number of unknowns.

The complexity is reduced by considering a limited number of available diameters.
To reach a solution, economic principles are applied, choosing among the multiple possibilities the one that results in the lowest cost under otherwise equal conditions.

Verification examples

We apply the theory to two archetypal cases: the case of two reservoirs and the case of three reservoirs.

Verification examples – 2 reservoirs

Consider a long pipeline with a constant elevation difference that connects two reservoirs with free surfaces. The length L diameter D and roughness are known. This particular configuration does not include nodes.

Types of problems:

- Given the elevation difference Y between the two free surfaces, calculate the flowrate Q conveyed.
- Given the flowrate Q conveyed, determine the elevation difference Y between the two surfaces.

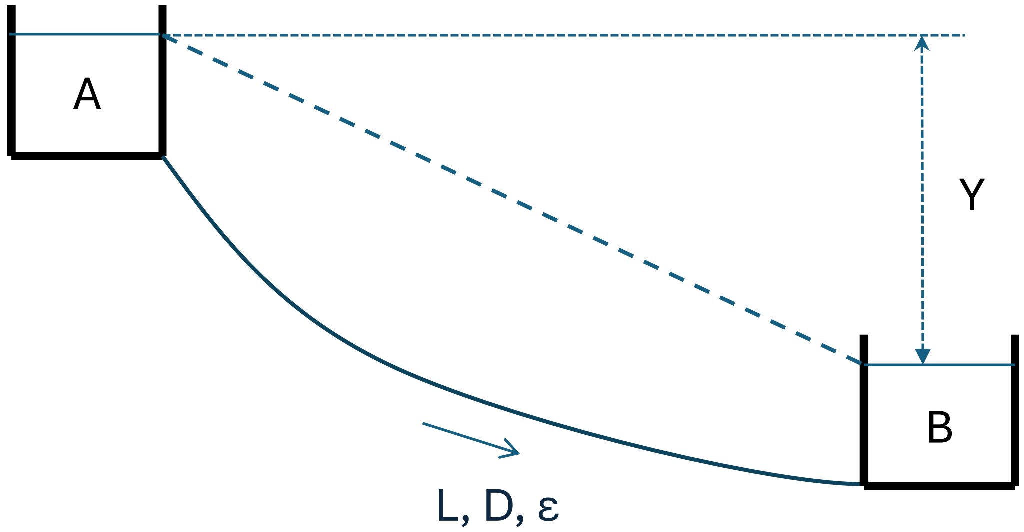
From the assumptions made, the entire elevation difference is converted into continuous (distributed) head losses.

$$Y = JL$$

Using the monomial formulas:

$$Y = JL = k\frac{Q^2}{D^n}L$$

$$J = \frac{Y}{L} = k\frac{Q^2}{D^n} \qquad Q = \sqrt{\frac{JD^n}{k}}$$



Verification examples – 2 reservoirs

Problems related to the progressive fouling of the pipeline and the consequent increase in roughness.

Design calculations must be done for a used pipeline, taking the maximum roughness value that that type of pipe can assume → the system must then be checked for operation with new pipes.

Let Q_N be the flow rate with a new pipe and Q_U the flow rate with a used pipe. We have $Q_N > Q_U$ when Y is constant.

The coefficient k increases with roughness, so we will have:

$$\frac{Q_N}{Q_U} = \sqrt{\frac{k_U}{k_N}}; \quad k_U = 2k_N \quad \rightarrow \quad Q_N = \sqrt{2}Q_U$$

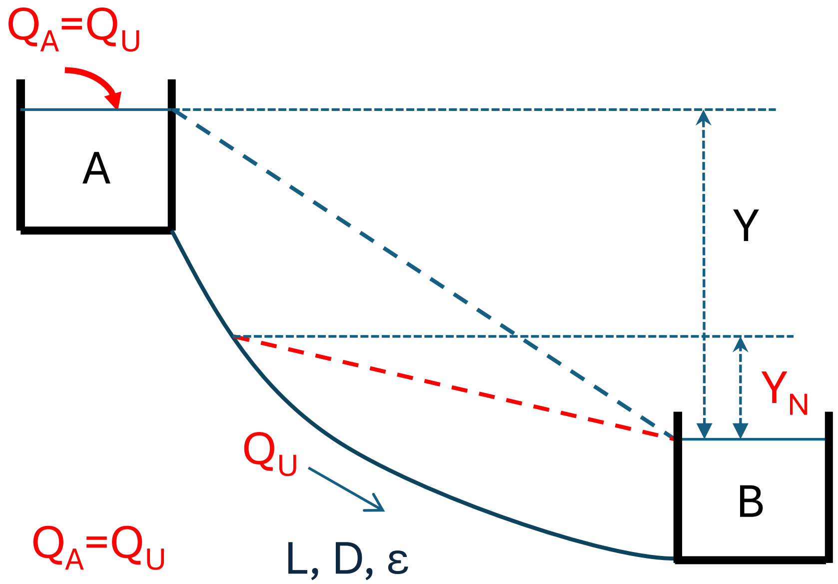
(assumption)

The condition with new pipes would require that the supply reservoir provides a flowrate $Q_A > Q_N$.
In practice, it is usually true that $Q_A >= Q_U$.

Verification examples – 2 reservoirs

The reservoir will begin to empty → Y decreases, and therefore J, and consequently Q_N until a steady-state condition is reached $Q=Q_A=Q_U$

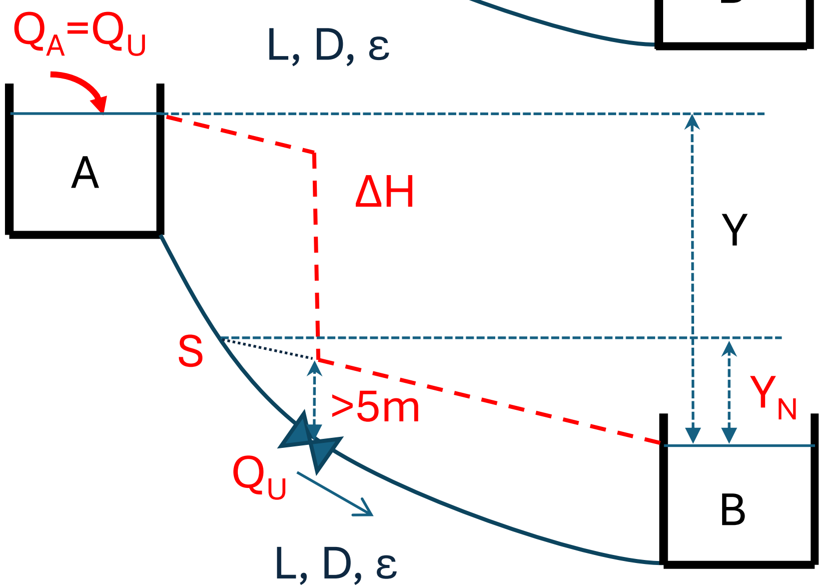
$$J_N = \frac{Y_N}{L} = k_N \frac{Q_U^2}{D^n}$$



It is possible to reach a condition where the piezometric line intersects the pipeline at a section S → upstream of this section, we have free-surface flow.

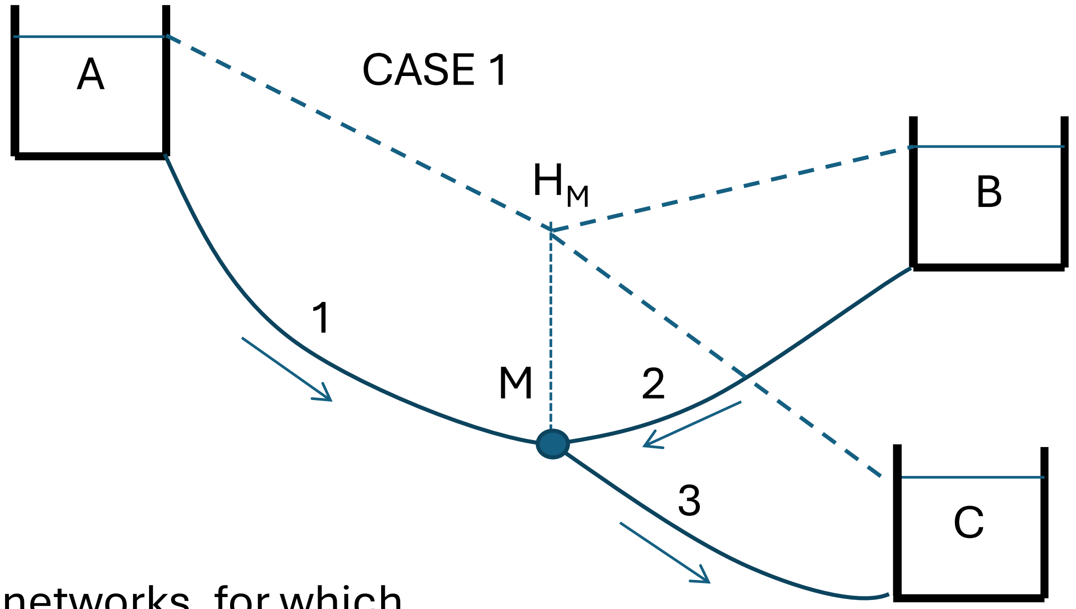
In the case of potable water transport, this is a condition to be avoided due to the potential ingress of contaminants from the outside → a local loss (adjustable valve) is installed downstream of section S.

$$\Delta H = Y - J_N L = Y - \frac{k_N Q_U^2}{D^n} L$$



Verification examples – 3 reservoirs

Let us consider the free-surface elevations of the reservoirs, the diameters, lengths, and roughnesses of the pipelines as known → calculate the flowrate in the three branches and the head at node M.



4 unknown H_M,Q_1,Q_2,Q_3

Once again, we refer to distribution networks, for which the following apply at the node and along the branches:

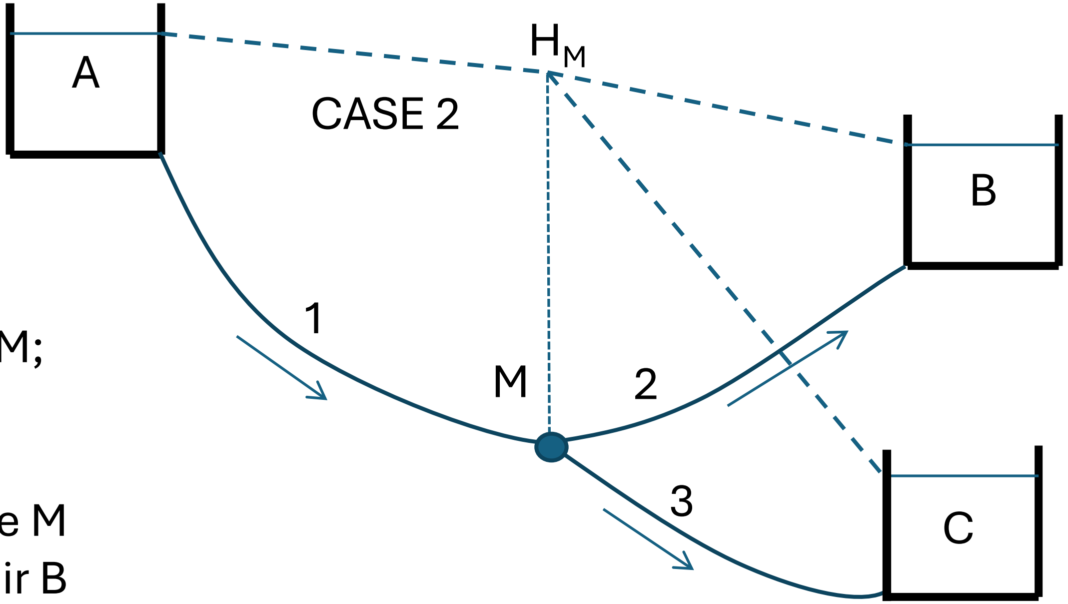
$$\sum Q_i + Q_j = 0$$

$$H_j - H_{j+1} = J_i L_i$$

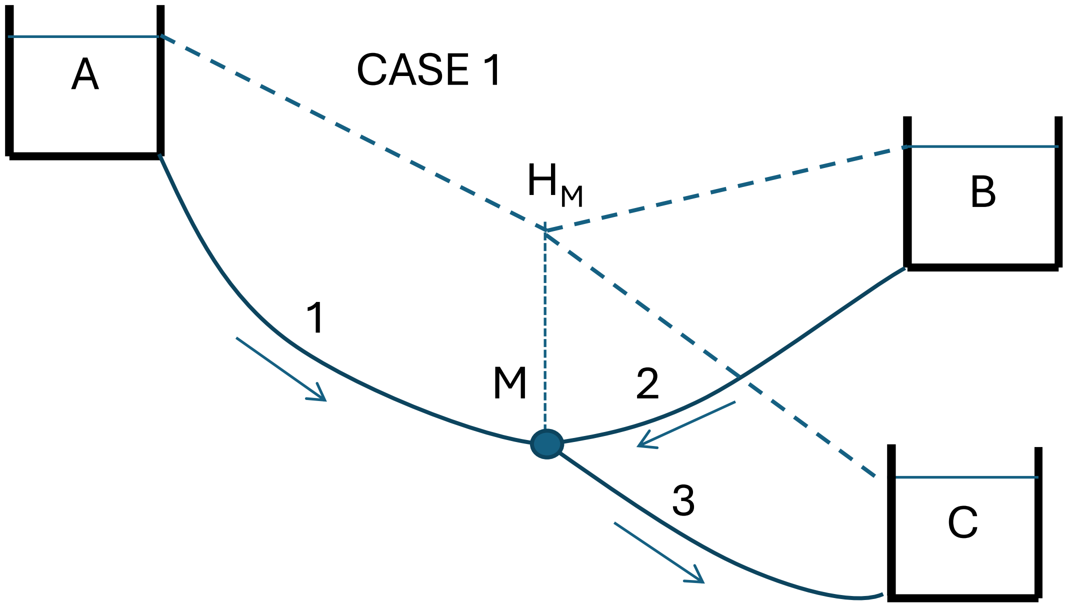
4 equations → solvable system

Define the direction. For branch A, the flow must go from A to node M; for branch 3, it must go from node M to reservoir C; the direction for branch 2 is not known a priori.

Case 1: $H_M < z_B$ → from reservoir B to node M
Case 2: $H_M > z_B$ → from node M to reservoir B



Verification examples – 3 reservoirs



Case 1, the system is:

$$\begin{cases} z_A - H_M = J_1 L_1 = \beta_1 Q_1^2 \\ z_B - H_m = J_2 L_2 = \beta_2 Q_2^2 \\ \mathrm{H}_M - z_C = J_3 L_3 = \beta_3 Q_3^2 \\ Q_1 = Q_2 + Q_3 \end{cases}$$

With $\beta = kD^{-n}L$ from monomial formulas

Case 2, the system is:

$$\begin{cases} z_A - H_M = J_1 L_1 = \beta_1 Q_1^2 \\ \mathrm{H}_M - z_B = J_2 L_2 = \beta_2 Q_2^2 \\ \mathrm{H}_M - z_C = J_3 L_3 = \beta_3 Q_3^2 \\ Q_1 = Q_2 + Q_3 \end{cases}$$

If the assumption is wrong, a real value for Q_2 is not obtained.

Project problems

Principle of minimum liability, with liability being the annual expenditure required to maintain the system.

Liability is given by the sum of the capital depreciation used to build the plant and the interest accrued on the capital, maintenance costs, and operating costs.

Liability or annual cost per unit length C_A:

$$C_A = rC_T + C_E$$

The installation cost is generally constant, while the actual cost of the pipeline depends on the pipe class (in terms of operating pressures).

For pipes belonging to the same class, the cost depends on the diameter: $C_T = c_0 + c_1 D^c$

The coefficients vary depending on the type of pipe and the pressure class.

Project problems

Operating cost depends on the presence or absence of machines and thus on electricity consumption.

$$C_E = \eta c_k E_d = \frac{1}{3600} \eta c_k \int_{t_A} P_d dt$$

η efficiency of the machine
c_k cost per kilowatt-hour
E_d energy [kWh] dissipated per year per unit length
P_d power dissipated [kW]
t_A annual time interval [s]

The dissipated power is given by the weight flow $\rho g Q$ multiplied by the unit head loss J.
We consider a practical monomial-type formula.

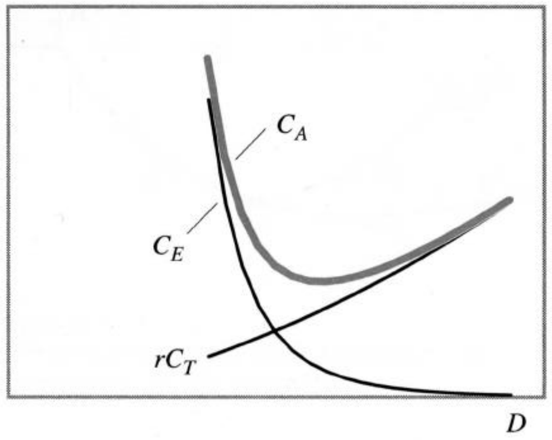
$$J = k \frac{Q^m}{D^n}$$

Consider m=2 for a pure turbolence flow

Project problems

Sobstituting:

$$P_d = \frac{1}{1000}\rho g Q J = \frac{1}{1000}\rho g Q k \frac{Q^2}{D^n} = \frac{1}{1000}\rho g k \frac{Q^3}{D^n}$$



The operating cost:

$$C_E = \frac{1}{3,6} 10^{-6}\, \eta c_k \frac{\rho g k}{D^n} \int_{t_A} Q^3 dt$$

We have reformulated the maintenance and operating costs in terms of diameter D and flowrates Q.
We integrate my solution system with as many liability conditions as there are degrees of indeterminacy.

Project system:

$$\begin{cases} \sum Q_i + Q_j = 0 \\ \sum \Delta H_i = \sum \beta_i \lambda_i Q_i^2 = 0 \\ C_A = rC_T + C_E \quad \begin{cases} C_T = c_0 + c_1 D^c \\ C_E = \frac{1}{3,6} 10^{-6}\, \eta c_k \frac{\rho g k}{D^n} \int_{t_A} Q^3 dt \end{cases} \end{cases}$$

Complex system to be solved. We need to evaluate minimum of C_A.

Project examples

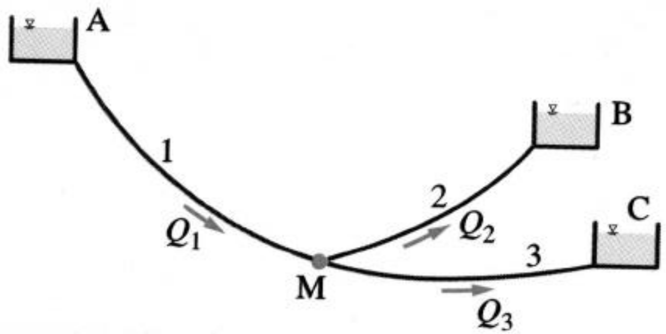
Liability or annual cost given by the sum of liabilities related to all branches linked to a node

$$C_A = rC_T + C_E \qquad C_T = c_0 + c_1 D^c$$

One equation, containing $D_1, D_2, \ldots, D_L$ unknown variables.

Consider the case of 3 reservoir – no machines $C_E = 0$

Assume all pipes belong to the same class, thus c_0 c_1 and c constant



$$r(c_0 + c_1 D_1^c)L_1 + r(c_0 + c_1 D_2^c)L_2 + r(c_0 + c_1 D_3^c)L_3 = min$$

$$D_1 = \left(\frac{\alpha_1}{z_A - H_M}\right)^{1/n} \qquad z_A - H_M = J_1 L_1 = \beta_1 Q_1^2 = \alpha D_1^{-n}$$

$$rL_1\left[c_0 + c_1\left(\frac{\alpha_1}{z_A - H_M}\right)^{c/n}\right]$$

Derive with respect to H_M

Project examples

$$-rL_1c_1\alpha_1^{\frac{c}{n}}\frac{\frac{c}{n}(z_A-H_M)^{\frac{c}{n}-1}}{(z_A-H_M)^{\frac{2c}{n}}}=-rL_1c_1\alpha_1^{\frac{c}{n}}\frac{c}{n}\frac{1}{(z_A-H_M)^{\frac{c}{n}+1}}=$$

$$=-rL_1c_1\frac{c}{n}\,\alpha_1^{\frac{c}{n}-1}\,\alpha_1\,\frac{1}{(z_A-H_M)^{\frac{c}{n}-1}}\frac{1}{(z_A-H_M)^2}=$$

$$=-rL_1c_1\frac{c}{n}\frac{1}{\alpha_1}\frac{\alpha_1^{\frac{c}{n}-1}}{(z_A-H_M)^{\frac{c}{n}-1}}\frac{\alpha_1^2}{(z_A-H_M)^2}=-rL_1c_1\frac{c}{\alpha_1 n}\,D_1^{c+n}$$

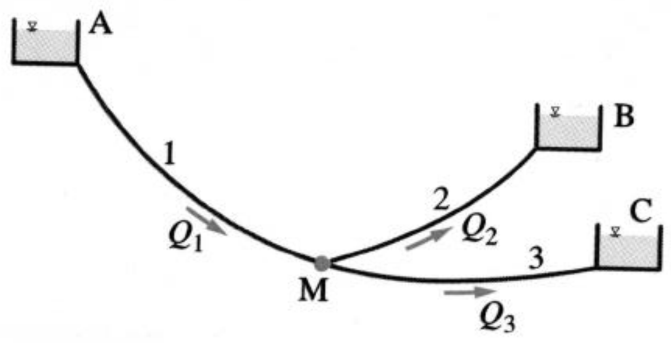
Applying the same method to other two terms

$$D_1=\left(\frac{\alpha_1}{z_A-H_M}\right)^{1/n}$$

$$\frac{1}{\alpha_1}L_1D_1^{c+n}=\frac{1}{\alpha_2}L_2D_2^{c+n}+\frac{1}{\alpha_3}L_3D_3^{c+n}$$

Project examples

Final solvable system



$$\begin{cases} z_A - H_M = J_1 L_1 = \beta_1 Q_1^2 = \alpha_1 D_1^{-n} \\ \mathrm{H}_M - z_B = J_2 L_2 = \beta_2 Q_2^2 = \alpha_2 D_2^{-n} \\ \mathrm{H}_M - z_C = J_3 L_3 = \beta_3 Q_3^2 = \alpha_3 D_3^{-n} \\ \frac{1}{\alpha_1} L_1 D_1^{c+n} = \frac{1}{\alpha_2} L_2 D_2^{c+n} + \frac{1}{\alpha_3} L_3 D_3^{c+n} \end{cases}$$

4 equations 4 unknown → D_1 D_2 D_3 H_M

General condition of minimul annual cost: $$\sum_{n_e} \frac{1}{\alpha_i} L_i D_i^{c+n} = \sum_{n_u} \frac{1}{\alpha_i} L_i D_i^{c+n}$$

Incoming flow rate

Outgoing flow rate

Consider pipes of the same type k=cost: $\frac{\alpha}{L} = kQ^2$

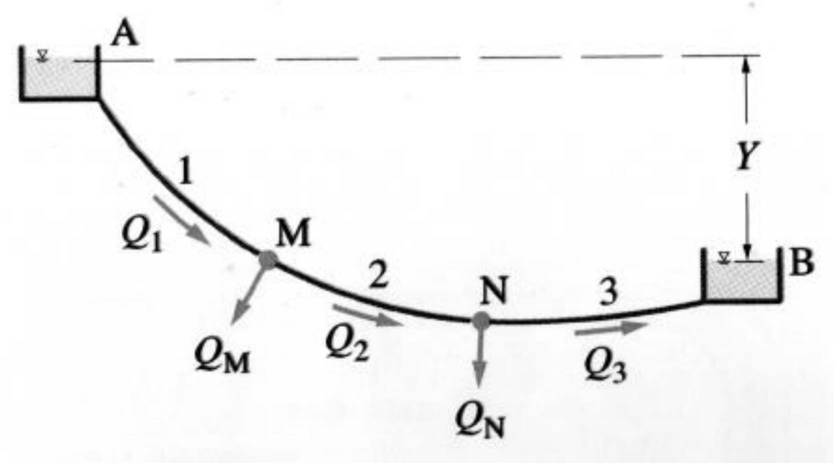
We obtain $$\sum_{n_e} \frac{1}{Q_i^2} D_i^{c+n} = \sum_{n_u} \frac{1}{Q_i^2} D_i^{c+n}$$

Simple case of a node with one incoming and one outgoing flow rate: $$\frac{D_e^{c+n}}{Q_e^2} = \frac{D_u^{c+n}}{Q_u^2}$$

Project examples

Another example: 2 reservoir with demands at two nodes:

$$\begin{cases} Y = \alpha_1 D_1^{-n} + \alpha_2 D_2^{-n} + \alpha_3 D_3^{-n} & \text{Carichi lati} \\ \dfrac{D_1^{c+n}}{Q_1^2} = \dfrac{D_2^{c+n}}{Q_2^2} & \text{Nodo M} \\ \dfrac{D_2^{c+n}}{Q_2^2} = \dfrac{D_3^{c+n}}{Q_3^2} & \text{Nodo N} \end{cases}$$



For the general case of pipelines of different types with varying roughness values, and with cost parameters c_0, etc., the minimum dissipation condition can be rewritten as:

$$\sum_{n_e} \frac{c_{1i} c_i}{\alpha_i n_i} L_i D_i^{c+n} = \sum_{n_u} \frac{c_{1i} c_i}{\alpha_i n_i} L_i D_i^{c+n}$$

Project examples – 2 reservoirs

Normally, the system under a design condition is indeterminate and requires additional conditions (minimum annual cost). An exception is the case of two reservoirs connected by a pipeline.

Note the elevation difference between the reservoirs and the pipeline length L → J=Y/L .

The calculation of the diameter to assign to a pipe of known roughness remains, so that it conveys the flowrate Q with slope J.

Using practical formulas, under the assumption of fully turbulent flow.

$$D = \left(k\frac{Q^2}{J}\right)^{1/n}$$

Once the theoretical diameter D is found, the next higher commercial diameter D_1 is chosen.

As a result, $J_1 < J$ and it is necessary to install a valve to dissipate the residual head.

Using a smaller diameter D_2 would result in a lower flowrate.

Project examples – 2 reservoirs

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Using practical formulas, under the assumption of fully turbulent flow.

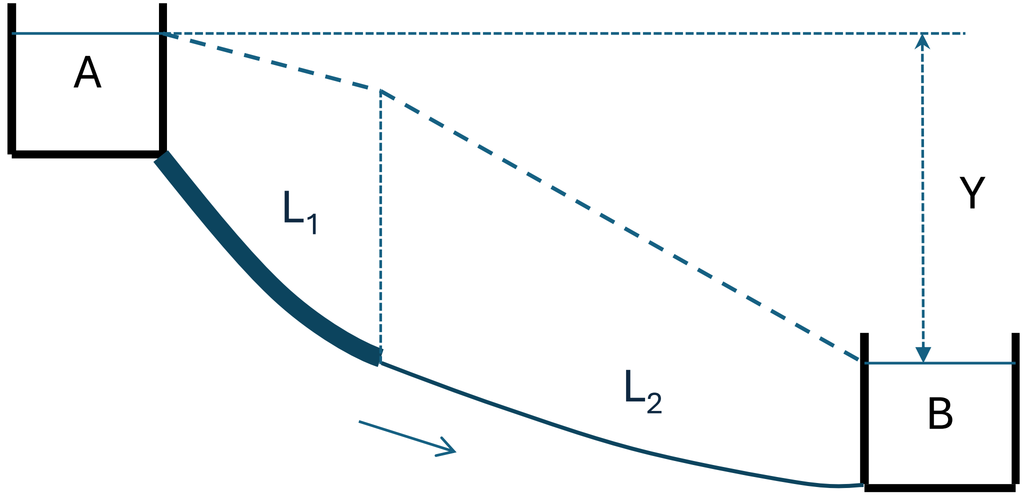
Practical solution

Use both diameters D1 and D2 with appropriate pipeline sections to produce a total head loss equal to Y

L_1 with $D_1>D$ L_2 with $D_2<D$

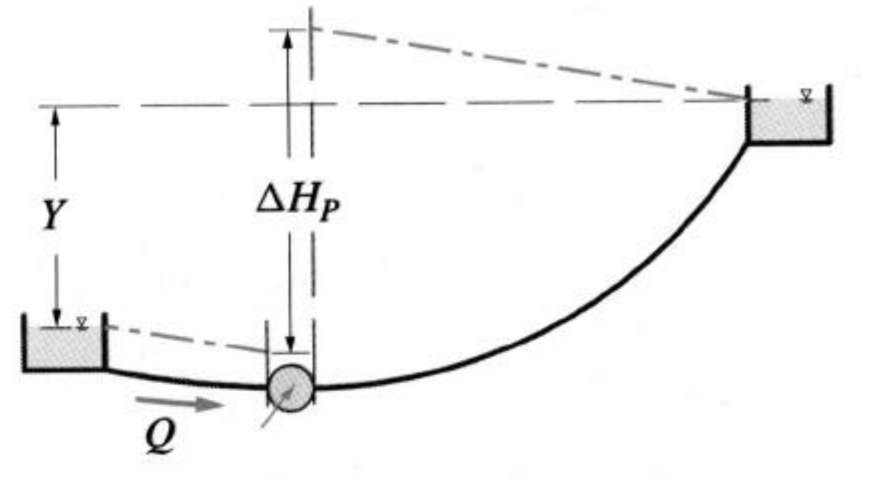
We have additional 2 equations, to determine the lengths L_1, L_2

$J_1L_1 + J_2L_2 = Y$ $L_1 + L_2 = L$



Project examples

Pumping system design



Known: L, Q and geodetic head Y → ΔH_P

The pump head must be equal to the geodetic head plus the losses (continuous losses for long pipelines).

$$\Delta H_p = Y + JL = Y + k\frac{Q^2}{D^n}L$$

Once we know ΔH_P we can evaluate the diameter D

The presence of an external source allows the use of any pipe diameter.

The energy required from the pump increases with higher continuous losses and smaller pipe diameters.

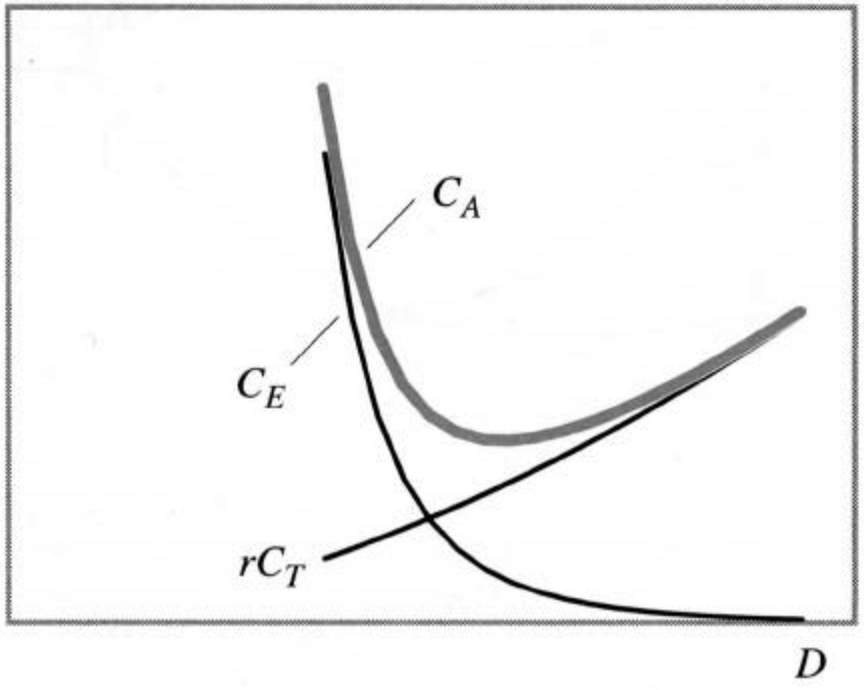
Smaller D → lower pipe cost but higher energy cost.

Project examples

$$C_T = c_0 + c_1 D^c$$

$$C_E = \frac{1}{3,6} 10^{-6}\, \eta c_k \frac{\rho g k}{D^n} \int_{t_A} Q^3 dt$$

$$C_E = \varphi\, \overline{Q^3} D^{-n} \quad con \quad \overline{Q^3} = \frac{1}{T}\int_0^T Q^3 dt$$



As operating costs increase, the minimum shifts toward larger diameters.

The cost depends on the operating time of the system:

- Irrigation system, used 4 months/year → small diameters
- Water supply system, continuous use → larger diameters
- Firefighting system → small diameters

Project examples

To select the best diameter, according to the principle of minimum annual cost:

$$L\left[r(c_0 + c_1 D^c) + \varphi \overline{Q^3} D^{-n}\right] = min$$

Derive with respect to D and set equal to =0

$$rLc_1 c D^{c-1} = -L\varphi \overline{Q^3}(-n) D^{-n-1}$$

$$rc_1 c D^{c-1} = n\varphi \overline{Q^3} D^{-n-1}$$

$$\frac{D^{c-1}}{D^{-n-1}} = D^{c+n} = \frac{n\varphi \overline{Q^3}}{rc_1 c}$$

$$D = \left[\frac{n\varphi \overline{Q^3}}{rc_1 c}\right]^{\frac{1}{c+n}}$$

Diameter of maximum benefit:

- It does not depend on the length LLL of the section and can be considered in terms of cost per unit length.
- Larger diameter is required when operating costs are higher than the pipe costs.

Other types of operations in networks

In the next slides we focus on some additional/particular case, that we may encounter in dealing with hydraulic networsk:

1) Continuous supply
2) Series and parallel connections
3) Closed mesh – Cross method

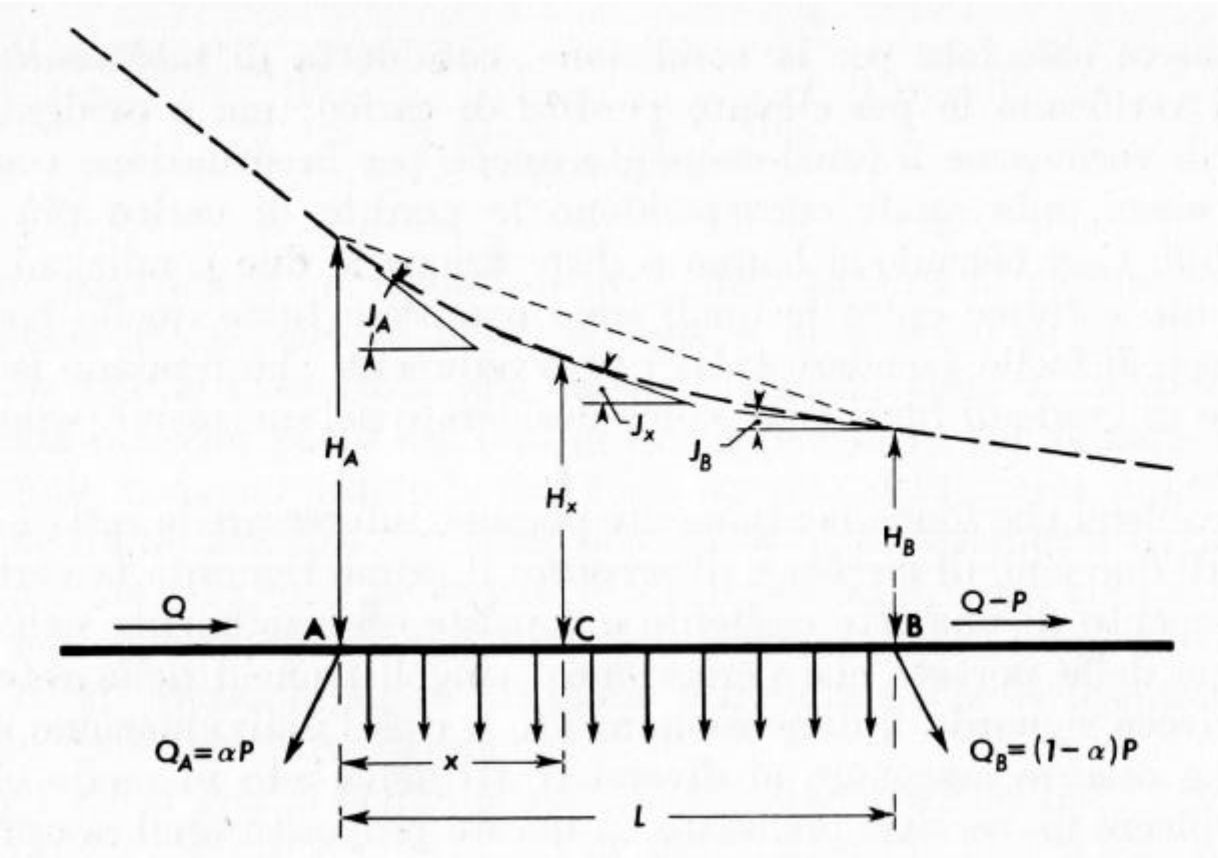
1 - Continuous supply

Consider a pipeline with constant diameter D, with a section AB, with length L, where demands are present.

The withdrawals are considered to be uniformly distributed with a flowrate per unit length q.

Q_E=qL,
Q>Q_E in A having total head H_A.

What is the piezometric prifile along AB and head in H_B?



At section C the flow rate is Q-qx, in a successive section (distant dx from the one considered) we have

$$dH = J_x dx = k(Q - qx)^2 D^{-n} dx$$

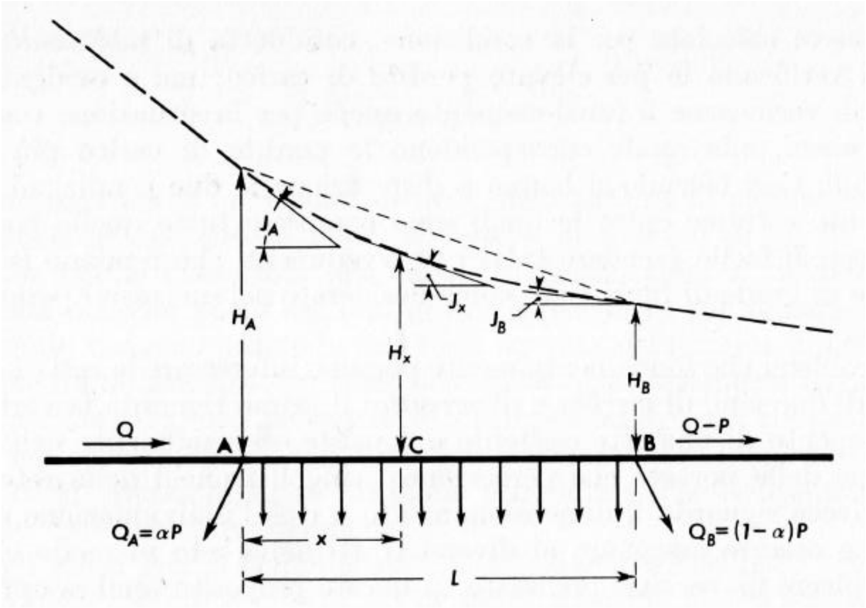
1 - Continuous supply

The total head H_x in C

$$H_x = H_A - \int_0^x dH$$

$$H_x = H_A - \int_0^x kD^{-n}(Q^2 - 2Qqx + q^2x^2)dx$$

$$H_x = H_A - kD^{-n}\left[Q^2x - Qqx^2 + \frac{1}{3}q^2x^3\right]$$



The piezometric line has the shape of a cubic parabola; at the endpoints A and B the tangent is equal to the hydraulic gradient (slope):

$$J_A = kQ^2D^{-n}$$

$$J_B = k(Q - Q_E)^2D^{-n}$$

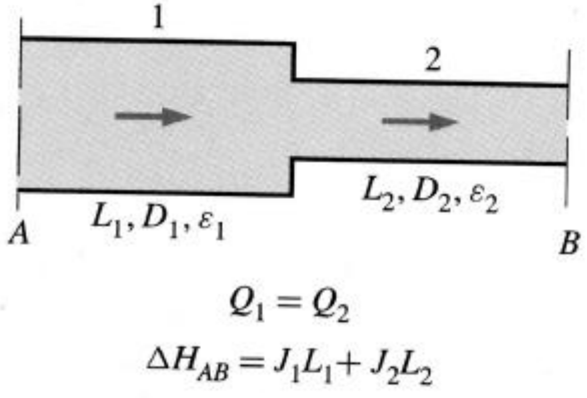
The total head in B

$$H_B = H_A - kD^{-n}L\left[Q^2 - QQ_E + \frac{1}{3}Q_E^2\right]$$

2 - Series and parallel connections

Series connections

Series of pipes connected in series: the flow rate remains constant. The total head loss is given by the sum of the head losses in each pipe.



$$Q_1 = Q_2 = \ \dots = Q_n$$

$$\Delta H = J_1L_1 + J_2L_2 + \cdots + J_nL_n$$

Parallel connections

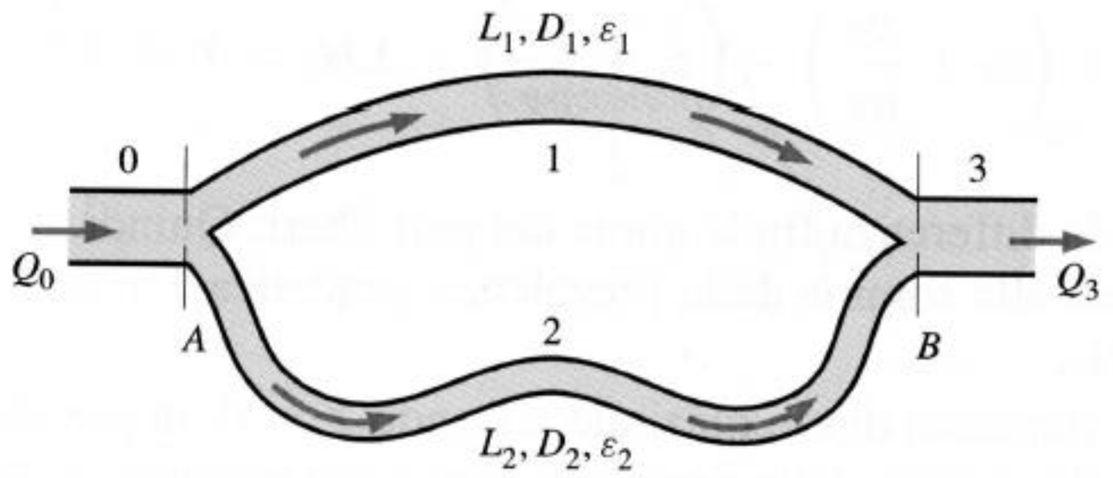
For a pipeline that splits into two or more parallel branches and then rejoins downstream at a node, the total flow rate is given by the sum of the flow rates of the individual pipes.
The head loss is the same in each parallel pipe, since they connect two nodes that are common to all branches.

2 - Series and parallel connections

Regardless of the path from A to B, the head at B cannot be different.

$$\Delta H_1 = \Delta H_2$$

$$\lambda_1 L_1 \frac{V_1^2}{2gD_1} = \lambda_2 L_2 \frac{V_2^2}{2gD_2}$$



$$Q_0 = Q_1 + Q_2 = Q_3$$

$$\Delta H_{AB} = J_1 L_1 = J_2 L_2$$

The ratio between the flow rates

$$\frac{Q_1}{Q_2} = \frac{A_1 V_1}{A_2 V_2} = \frac{D_1^2}{D_2^2}\sqrt{\frac{\lambda_2 L_2}{\lambda_1 L_1}\frac{D_1}{D_2}} = \sqrt{\frac{\lambda_2 L_2}{\lambda_1 L_1}\frac{D_1^5}{D_2^5}}$$

Given n branches in parallel connection the flow rate at the i-branch is:

$$Q_i = Q_1 \sqrt{\frac{\lambda_1 L_1}{\lambda_i L_i}\frac{D_i^5}{D_1^5}} \quad \rightarrow \quad Q = \sum_{i=1}^{n} Q_i \quad \rightarrow \quad Q_1 = \frac{Q}{\sqrt{\frac{\lambda_1 L_1}{D_1^5}} \sum_{i=1}^{n} \sqrt{\frac{D_i^5}{\lambda_i L_i}}}$$

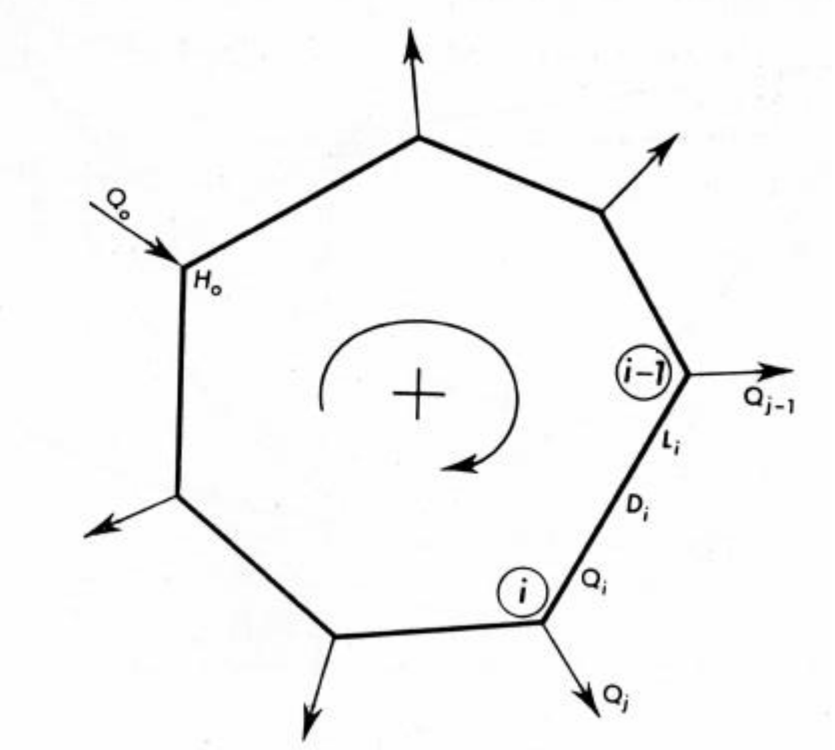
Continuity eq. at nodes

3 - Cross method

Case of a closed loop with supply at the nodes

Known data:
- The inflow rate Q_0 and the head H_0
- The outflow rates Q_j at the nodes
- The characteristics of the branches: length, diameters, roughness

Unknowns:
- The heads H_J at the nodes
- The flow rates Q_J in the branches



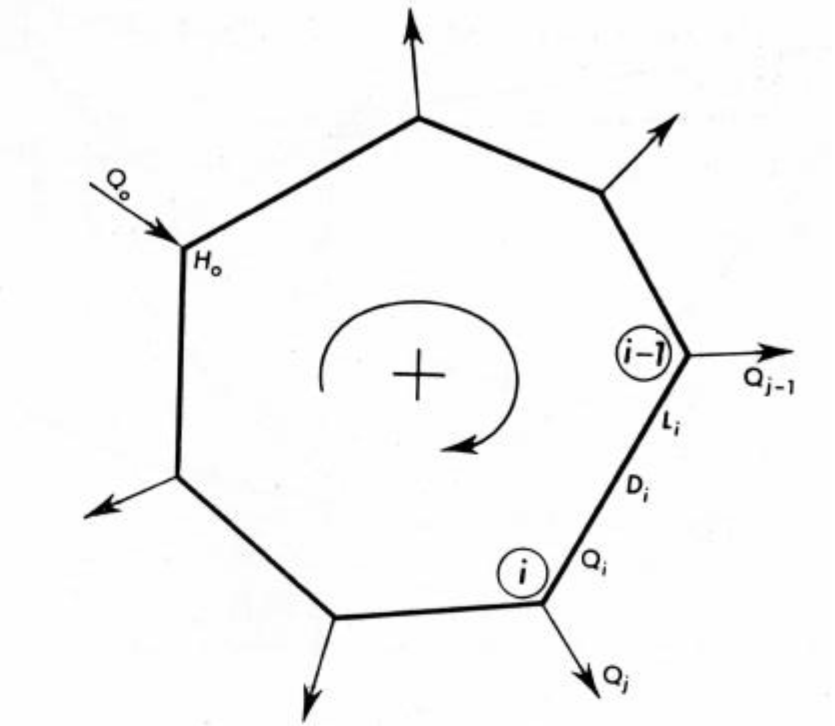
Available equations:

At the nodes $$\sum Q_i + Q_j = 0$$ with $$Q_0 = \sum_{1}^{n-1} Q_j$$

Through the mes $$\sum \beta_i Q_i^2 = 0$$

3 - Cross method

Consider an «attempt» solution Q_i' that satisfies continuity at nodes



$$\sum Q_i{}' + Q_j = 0$$

When we verify this solution Q_i' considering mesh equation, we get:

$$\sum \beta_i Q_i'^2 \neq 0$$

Assume that the attempt Q_i' differs from Q_i of a small quantity Q_e with $Q_e < Q_i$'

$$Q_i = Q_i' + Q_e$$

Since Q_i' satisfies the continuity equation, the quantity Q_e must necessarily remain constant throughout the loop to ensure the continuity equation is satisfied. This results in a flow circulating around the loop in the direction we have chosen.

3 - Cross method

Thus, for the first principle results

$$\sum (Q_i' + Q_e) + Q_j = 0$$

While, for the second principle we have

$$\sum \beta_i (Q_i' + Q_e)^2 = 0$$

$$\sum \beta_i (Q_i'^2 + 2Q_i' Q_e + Q_e^2) = 0$$

Assuming $Q_e < Q_i'$ then $Q_e^2 << Q_i'^2$

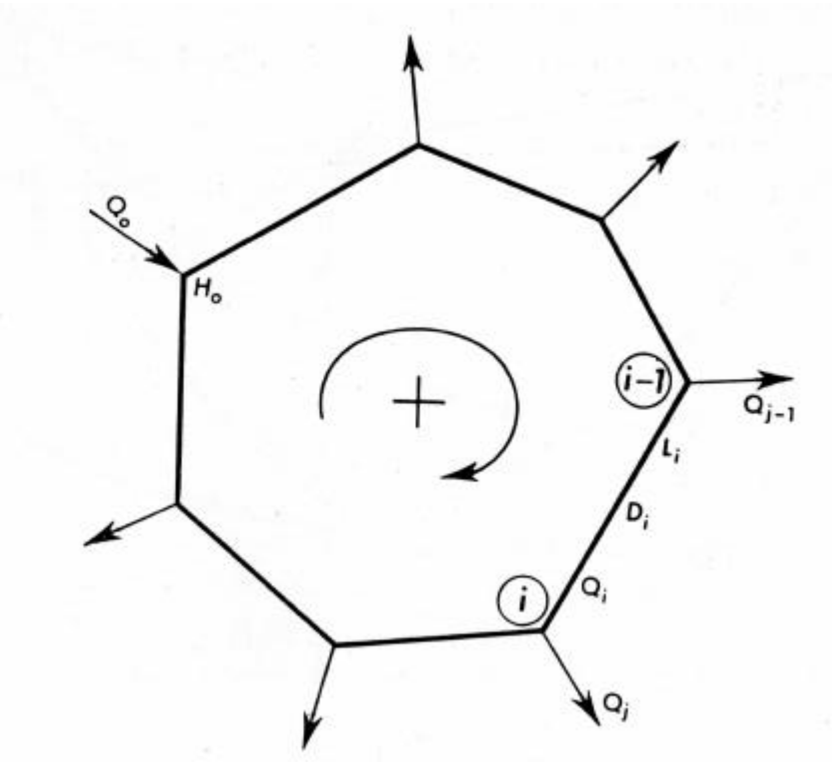
$$\sum \beta_i (Q_i'^2 + 2Q_i' Q_e + Q_e^2) = 0$$

Thus I can compute Q_e as

$$Q_e = -\frac{\sum \beta_i Q_i'^2}{2 \sum \beta_i Q_i'}$$

We obtain a new solution

$$Q_i'' = Q_i' + Q_e$$



Iterate the process until a solution is obtained that satisfies both principles.

Table of terms

Italiano	Inglese tecnico	Note / Uso tipico
Impianto idraulico di produzione di energia	Hydropower plant	Centrale idroelettrica; termine standard nei testi tecnici
Centrale idroelettrica	Hydroelectric power station / Hydroelectric facility	Sinonimi di “hydropower plant”
Condotta	Pipeline / Conduit / Penstock	“Pipeline” per lunghe condotte generiche, “penstock” per condotte che alimentano turbine
Tubazione di grande diametro	Large-diameter pipeline	Usato in contesti di trasporto di fluidi su lunghe distanze
Turbina idraulica	Hydraulic turbine	Es. Francis turbine, Kaplan turbine
Bacino / Serbatoio	Reservoir	Spesso collegato a dighe o centrali idroelettriche
Quota geodetica	Geodetic elevation	Altezza rispetto a un riferimento; usata in calcolo delle teste piezometriche
Altezza piezometrica	Piezometric head	Parte dell’equazione di Bernoulli, misura pressione + quota
Altezza cinetica	Velocity head	$v^2 / 2g$ in Bernoulli
Indice di scabrezza	Roughness coefficient	Es. Manning’s **n** o Darcy–Weisbach **λ**
Cadente J	Hydraulic gradient / Head loss gradient	Pendenza della linea piezometrica / perdite di carico
Problemi di verifica e di progetto	Verification and design projects	Terminologia del corso ita/eng
Impianto di sollevamento	Pumping system	O «lift station» spesso usato per reti fognarie o acquedotti, dove l’acqua viene sollevata da un punto più basso a uno più alto.