

Image Processing for Physicists

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Image manipulation in the spatial domain



Overview

- coordinate transformations

- translation, rotation, shear, ...

) geometry (pixel positions)

- intensity transformations

- normalization, gamma, thresholding, ...

) pixel values

- image analysis using morphological operations

- dilation, erosion, opening, closing, ...

- image segmentation

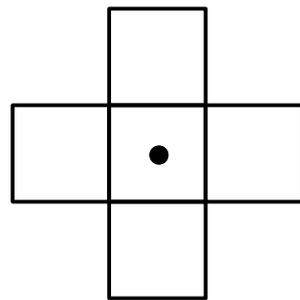
- by morphology, intensity, region, ...

] binary images

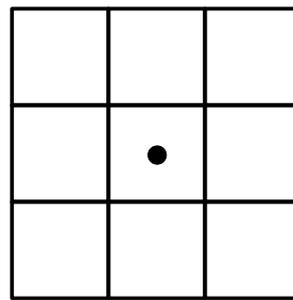
General image transformations

- coordinate transformations *← change pixel positions*
- intensity transformations *↳ interpolation*
- pixel-wise transformations *↘ change pixel value*
- neighborhood transformations *↗ morphology, convolution*

neighborhood / footprint / kernel



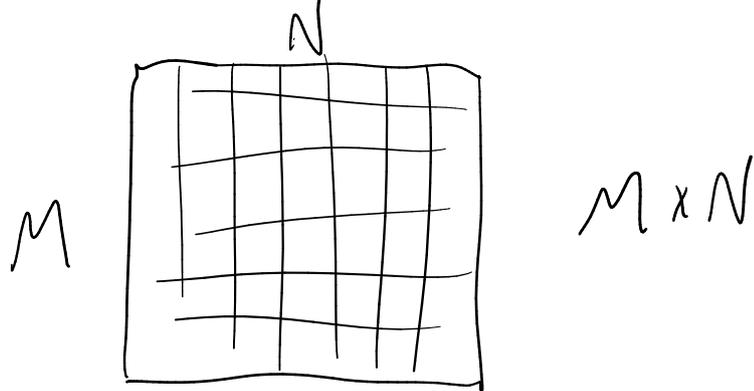
4-neighborhood



8-neighborhood

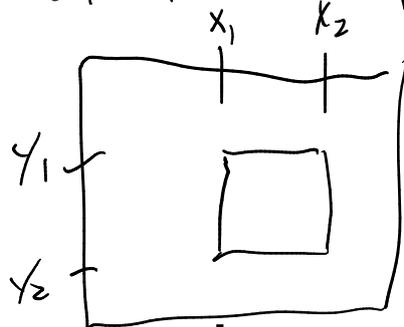
General image transformations

- images as an array



- sub array operations

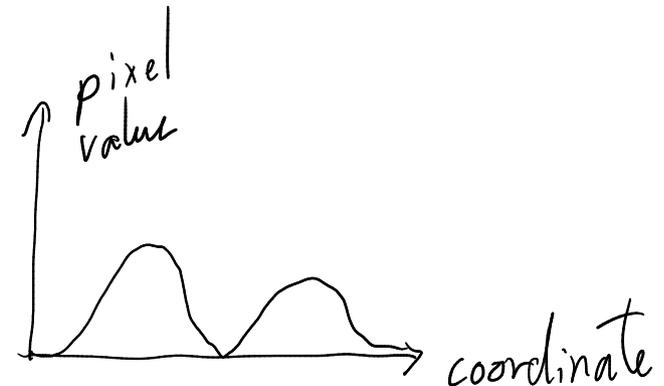
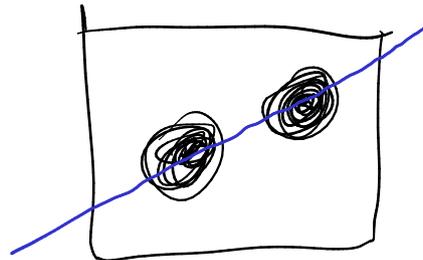
region of interest (ROI)



numpy: $a[y_1:y_2, x_1:x_2]$

- line extractions

"line profile"



General image transformations

- element wise addition

$$\mathcal{I} = \mathcal{I}_1 + \mathcal{I}_2$$

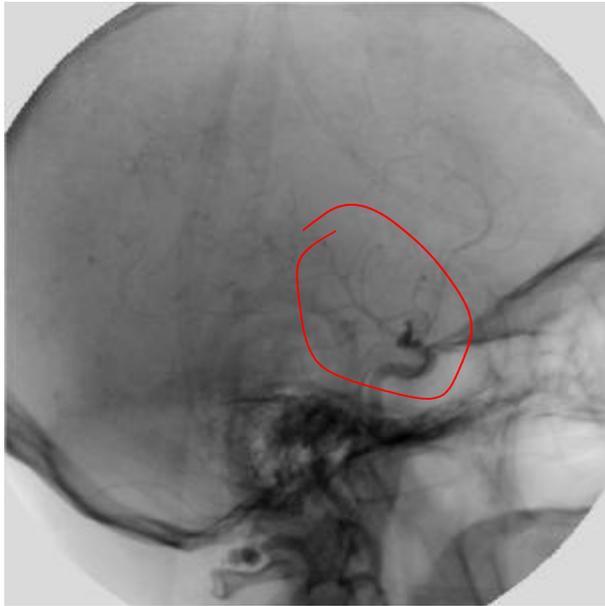
$$\mathcal{I}(i,j) = \mathcal{I}_1(i,j) + \mathcal{I}_2(i,j) \quad \text{for all } (i,j)$$

- element wise multiplication

$$\mathcal{I} = \mathcal{I}_1 \times \mathcal{I}_2$$

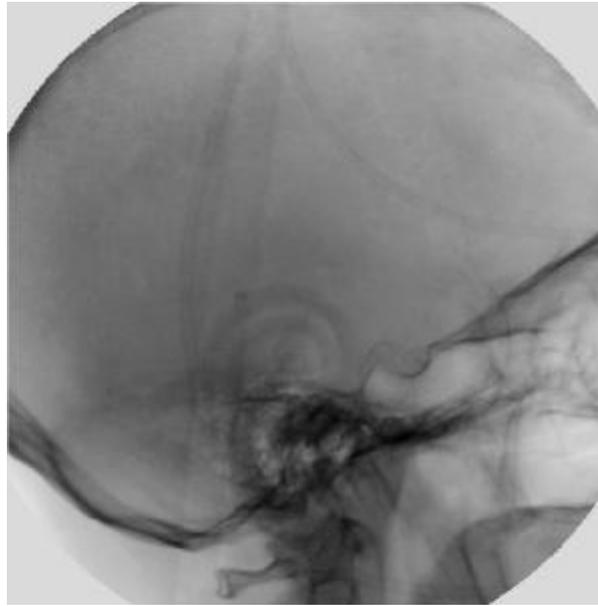
Image Subtraction Example

- Digital Subtraction Angiography
- Xray images before/after contrast agent



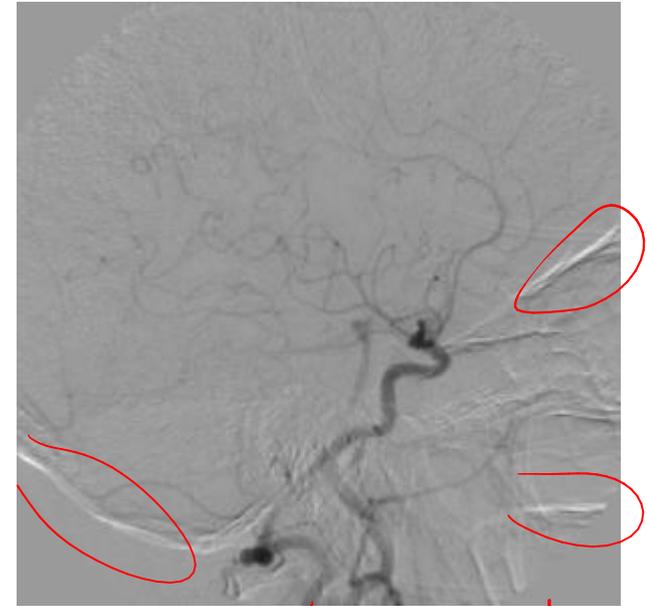
Live or contrast image

-



Mask image

=



DSA image

motion artifacts

Source: Gonzales, Digital Image Processing

Image Addition Example

- Add multiple noisy images of same object
- (More on noise in later lectures)

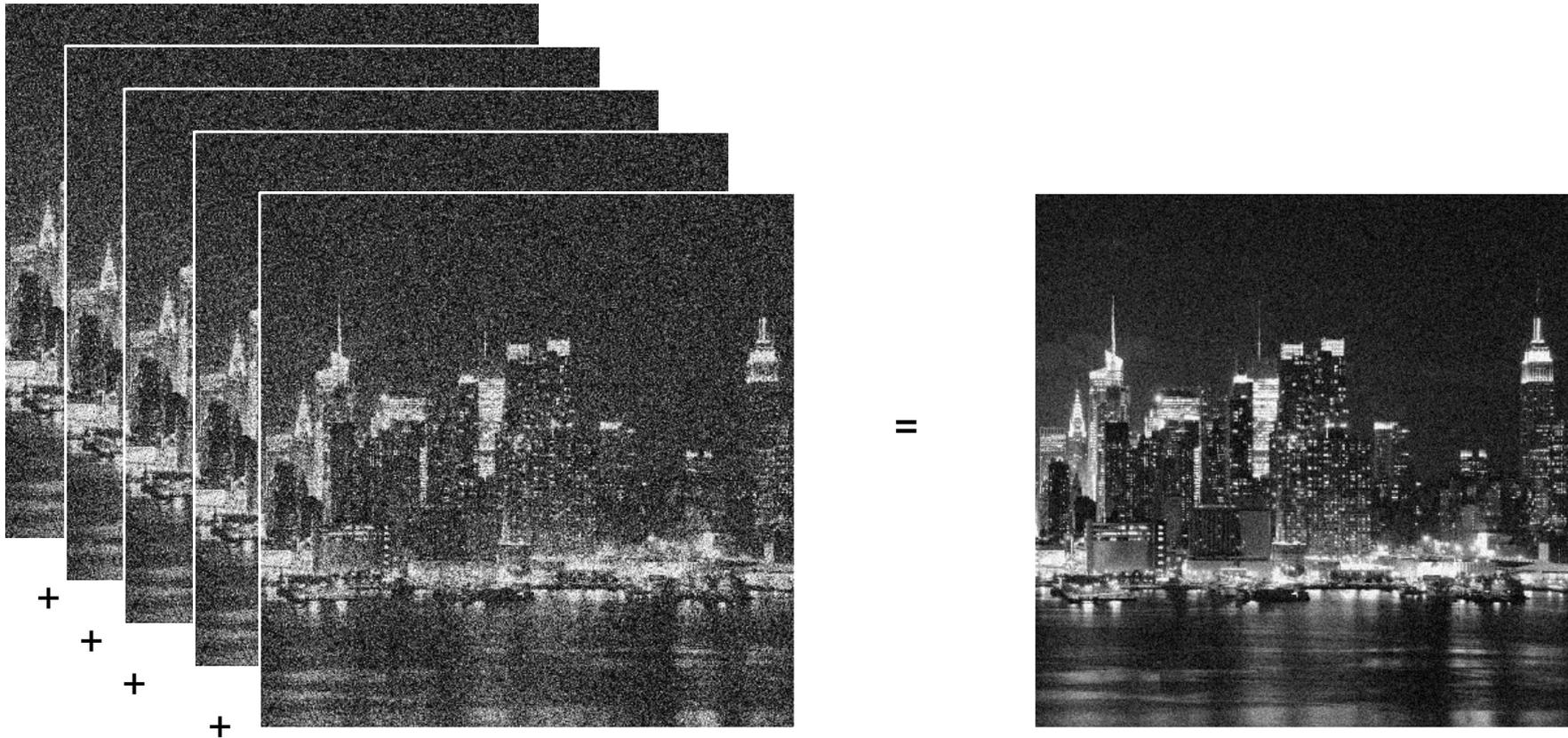


Image Multiplication Example



a b c

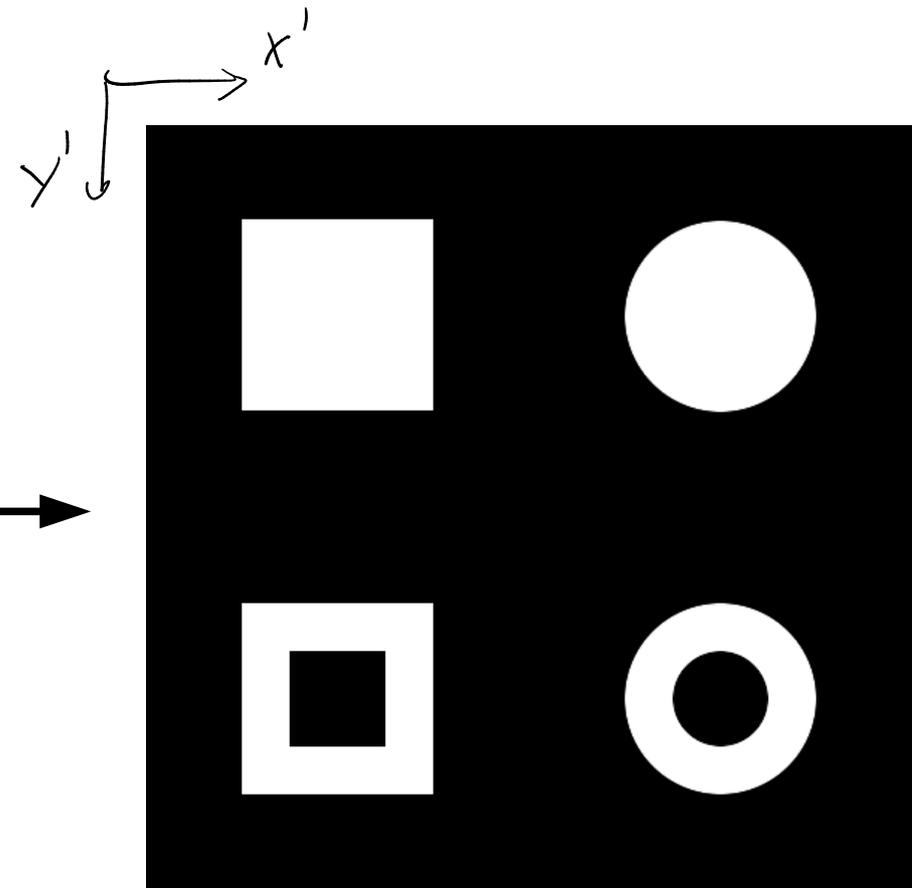
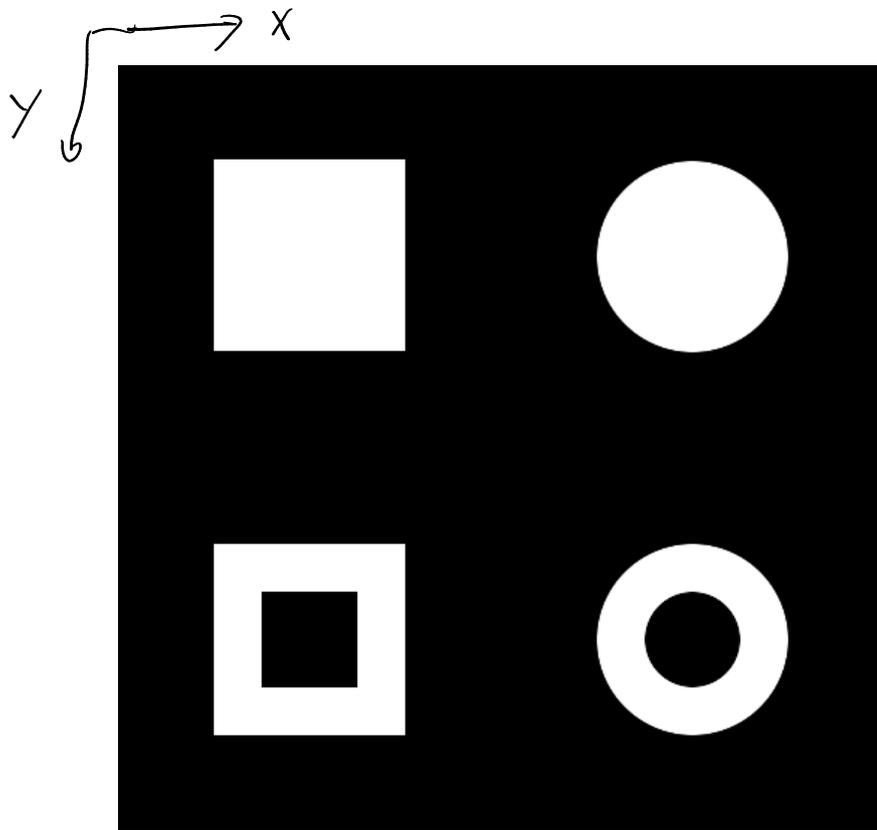
FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Source: Gonzales, Digital Image Processing

Affine transformations

- identity

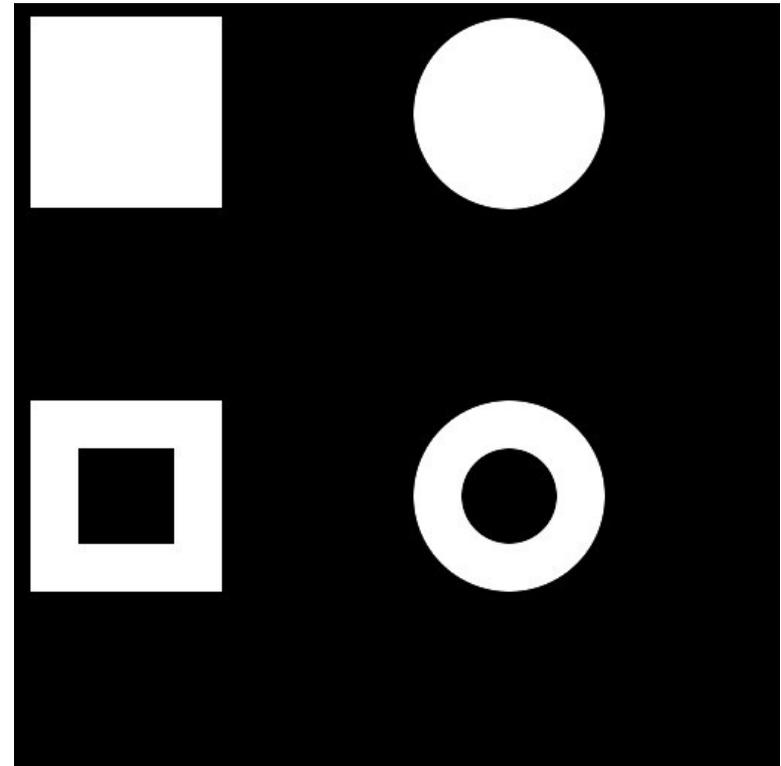
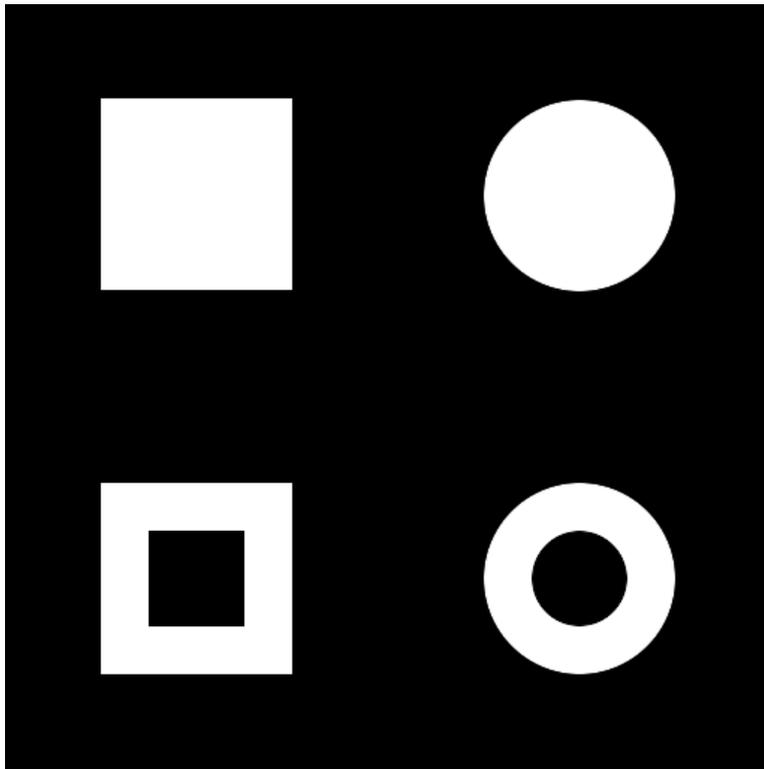
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Affine transformations

- translation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}$$

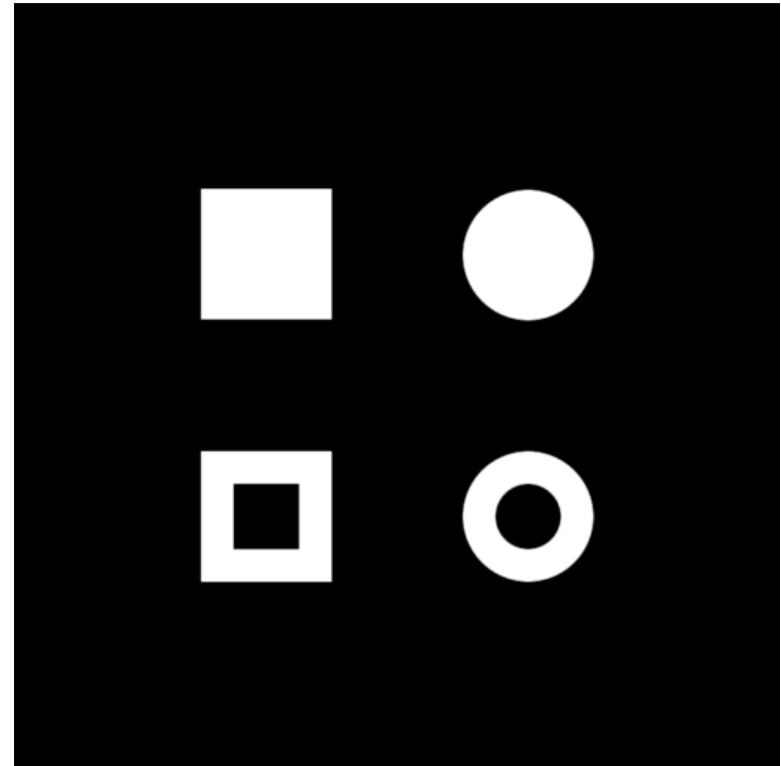
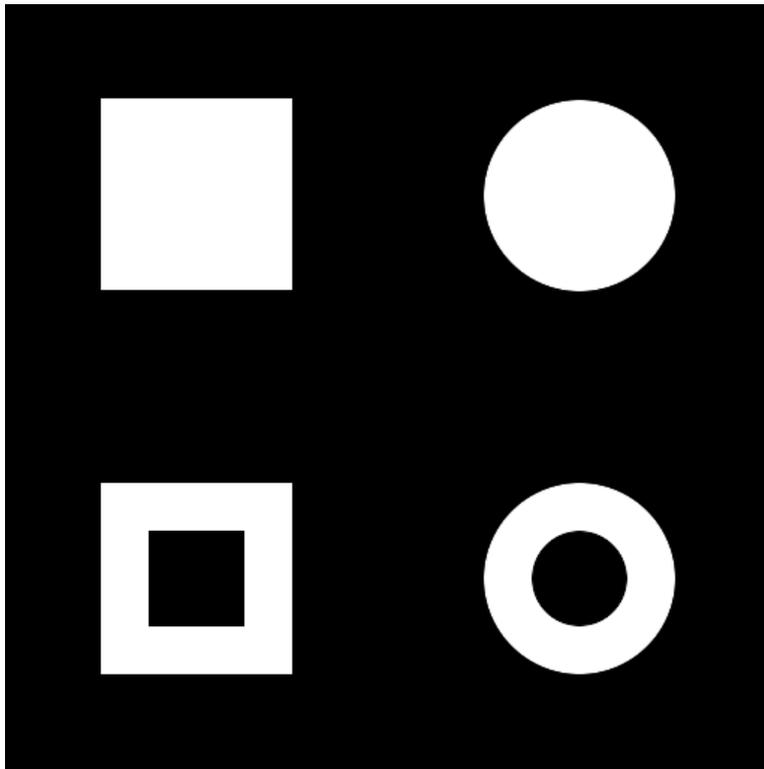


Affine transformations

- scaling

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

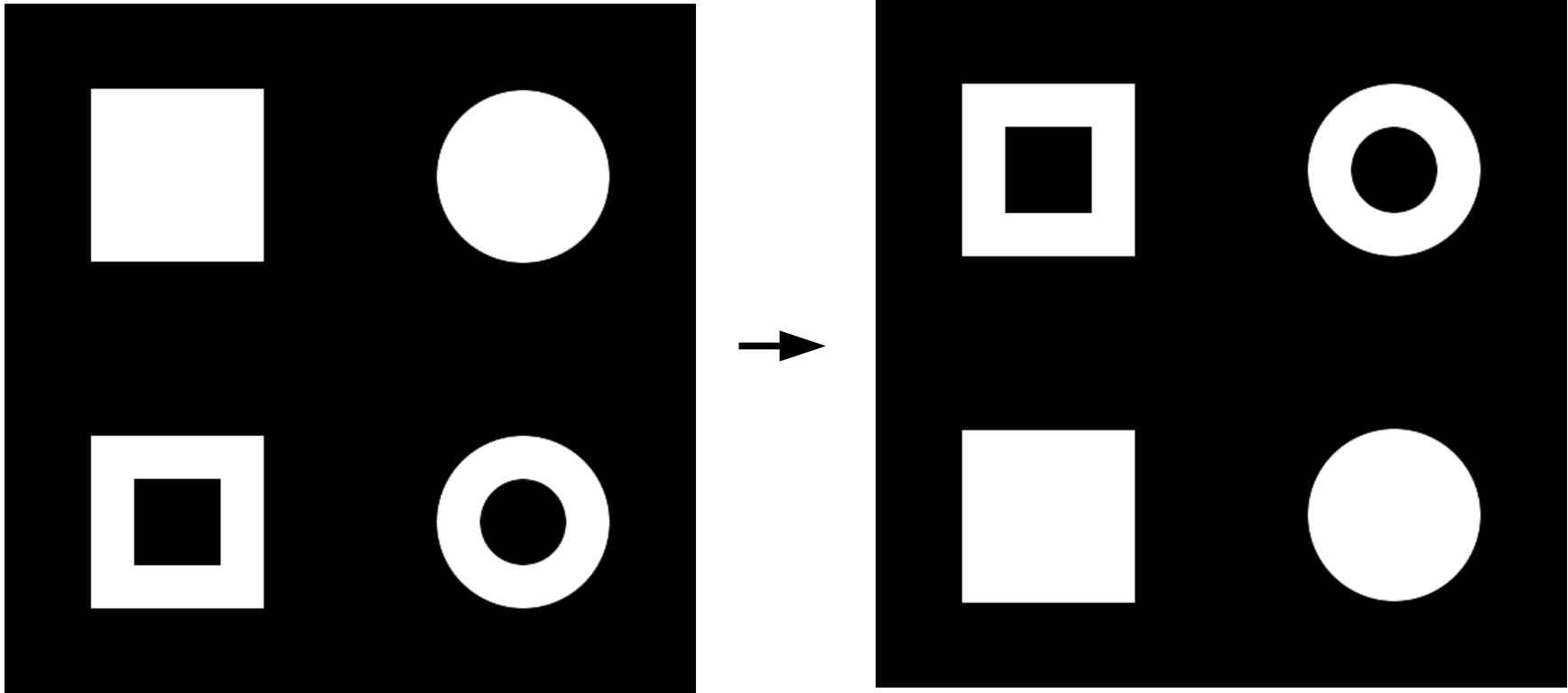
$\rightarrow a = b = \frac{1}{2}$



Affine transformations

- reflections

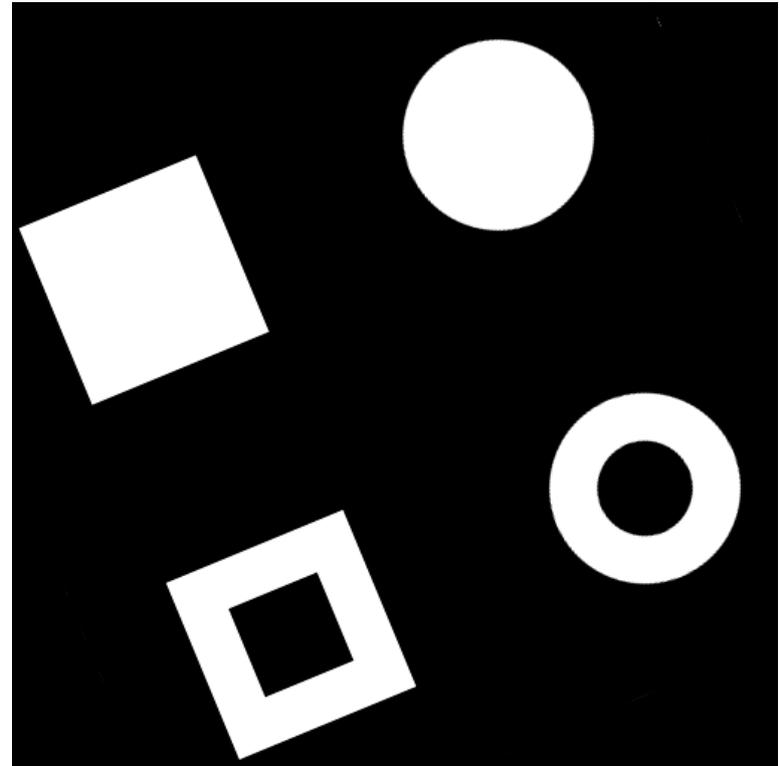
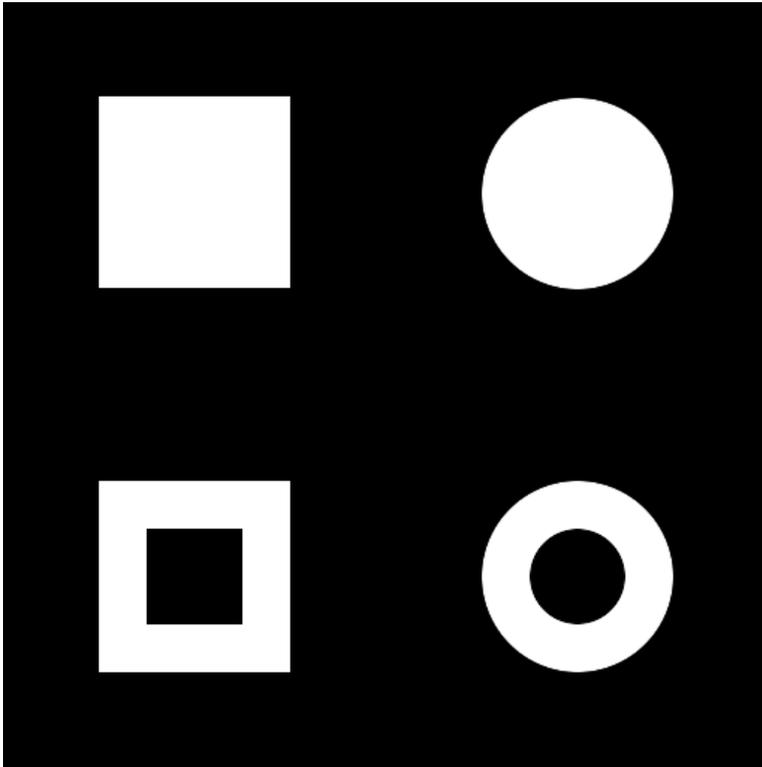
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Affine transformations

- rotation

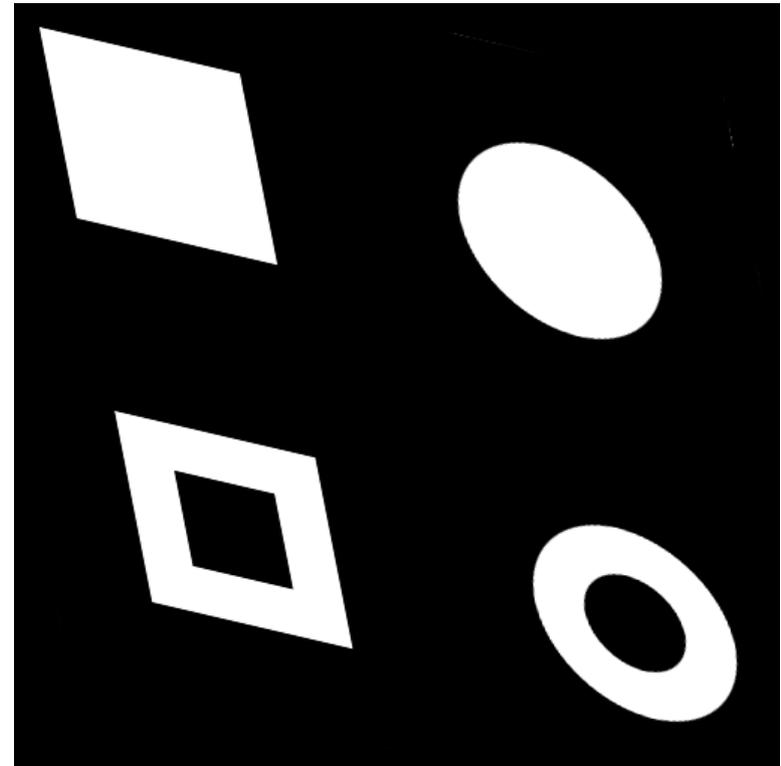
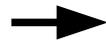
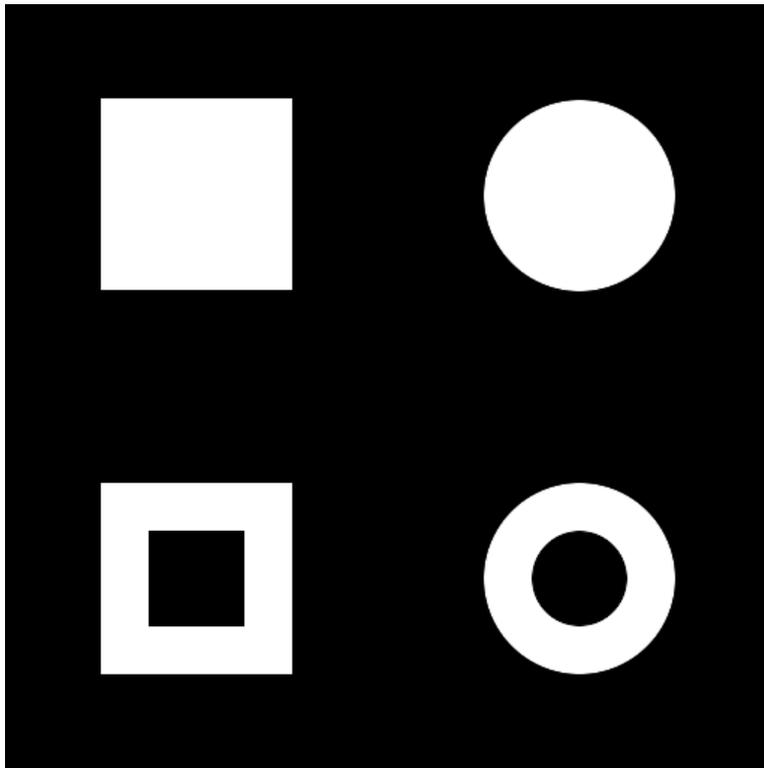
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Affine transformations

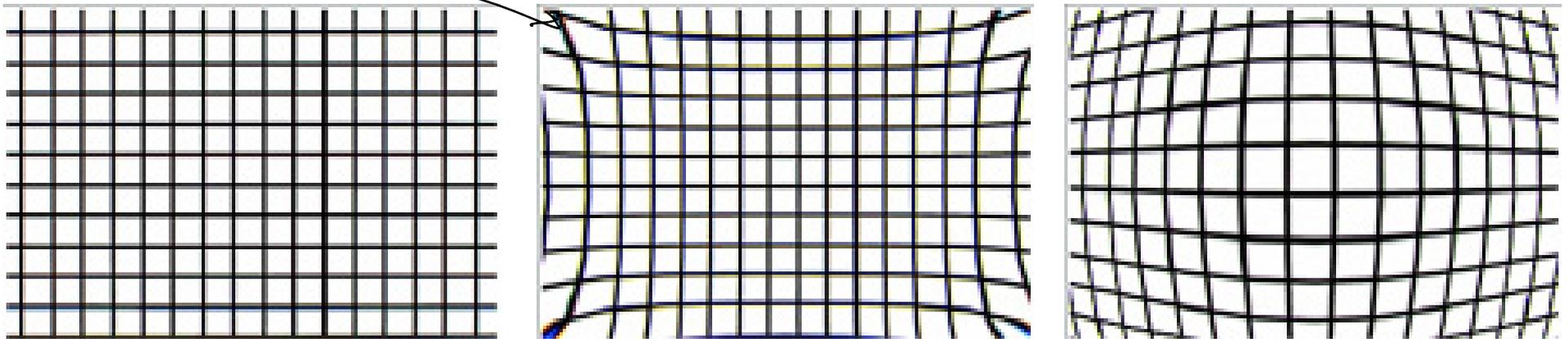
- shear

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



Nonlinear coordinate transformation

- pincushion and barrel distortion



$a > 0$

$a < 0$

$$x' = x + a(x^2 + y^2)$$

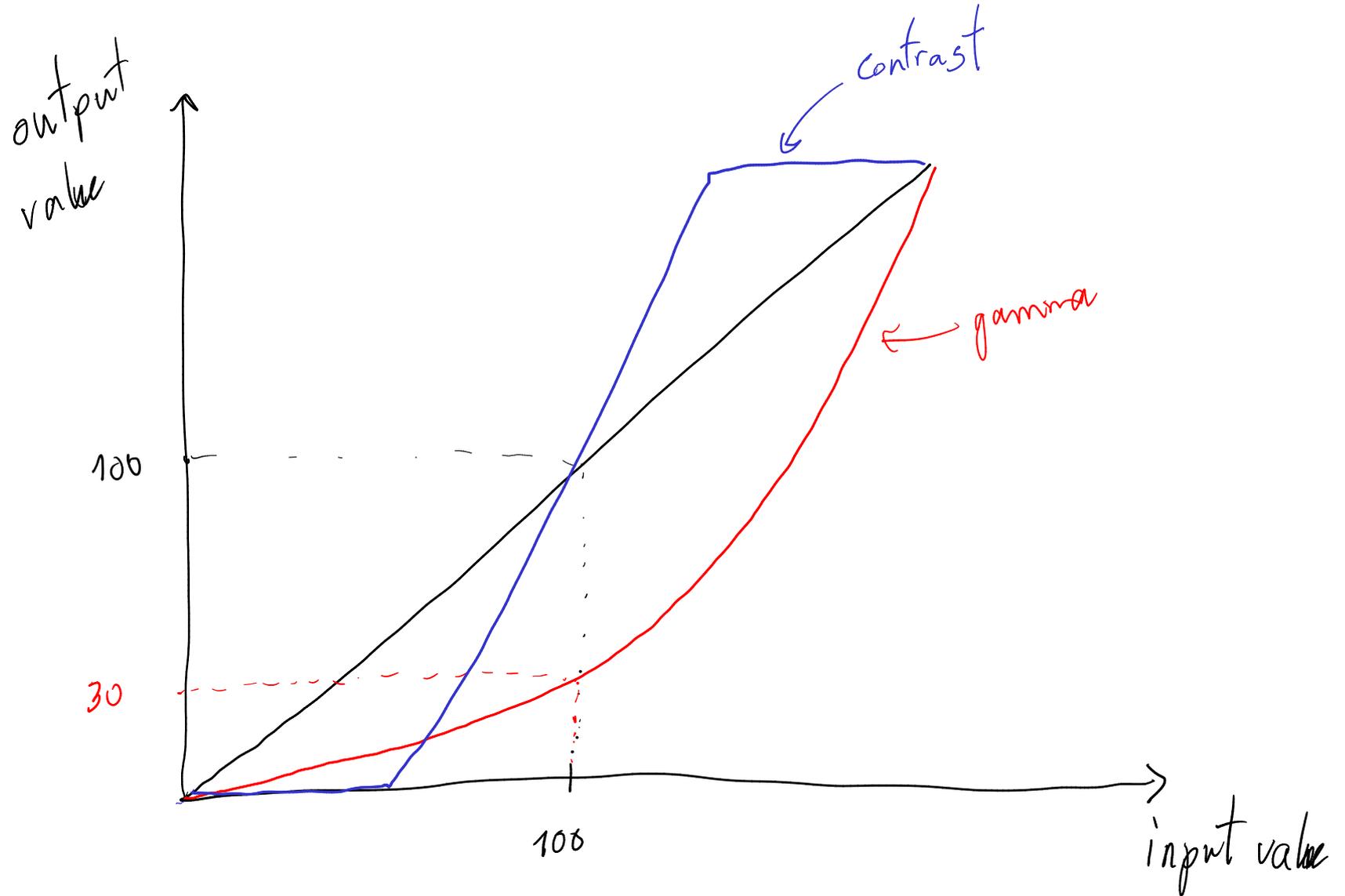
$$y' = y + a(x^2 + y^2)$$

- mapping depends on radial distance from centre

$$x' = x_0 + ax + by + cx^2 + dy^2 + exy + \dots$$

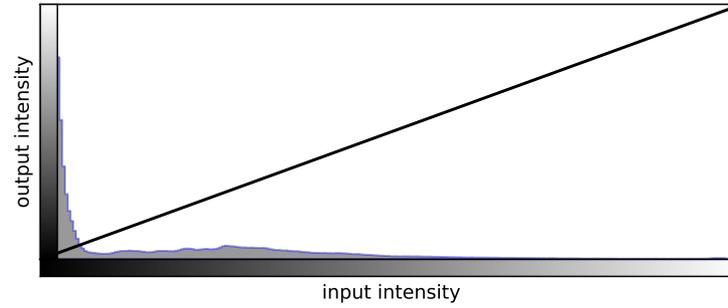
Intensity mapping

(for a given pixel)



Intensity mappings

Identity

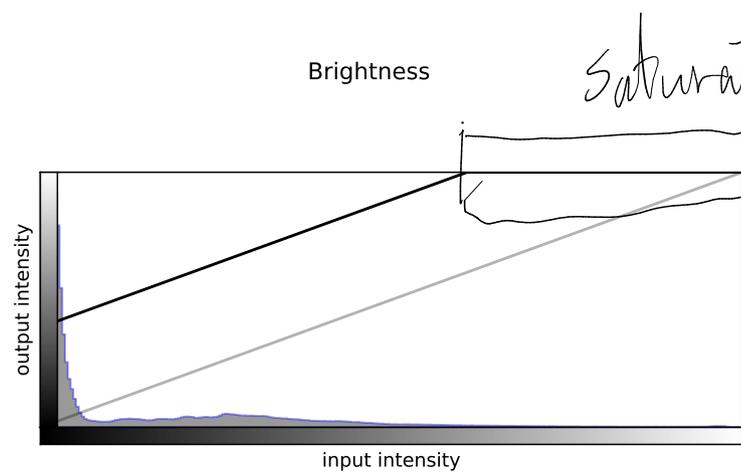


original



remapped

Intensity mappings



$$\text{Output} = \text{Input} + B$$

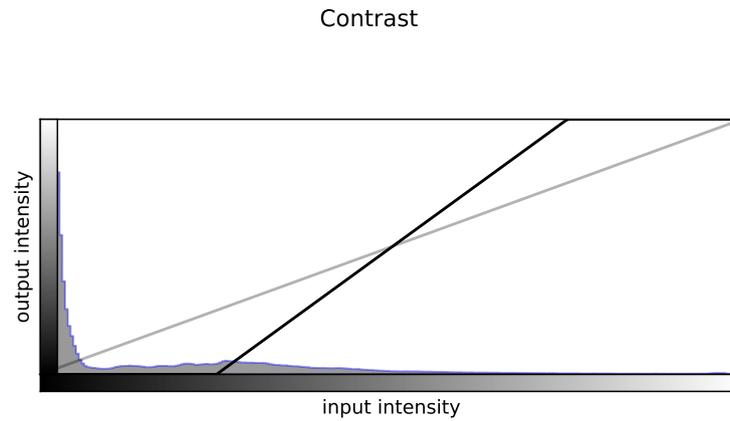


original



remapped

Intensity mappings



" $\text{Output} = \alpha \cdot \text{Input}$ "
8-bit images!

$\text{Output} = \alpha (\text{Input} - 127)$

$+ 127$

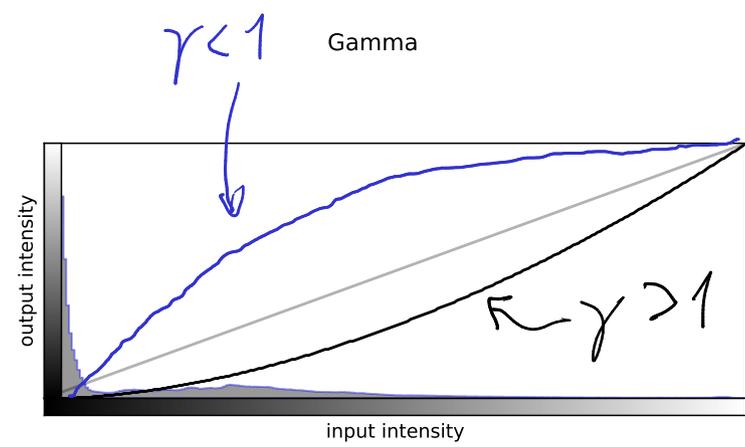


original



remapped

Intensity mappings



"
 $Output = (Input)^\gamma$ "

$$Output = M \cdot \left(\frac{Input}{M}\right)^\gamma$$



original



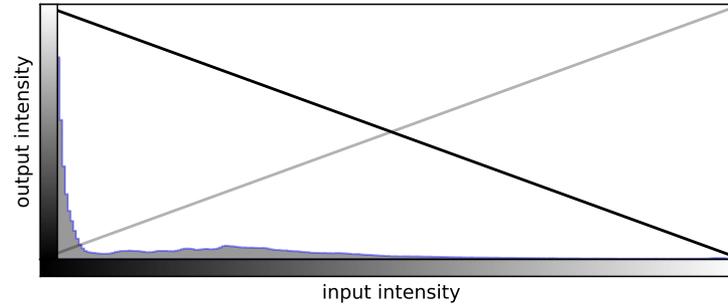
remapped

$M := \max(Input)$
or
max value
of dynamic
range

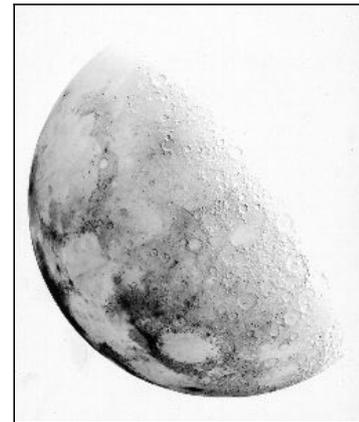
Useful for "non-destructive" contrast adjustment

Intensity mappings

Inversion



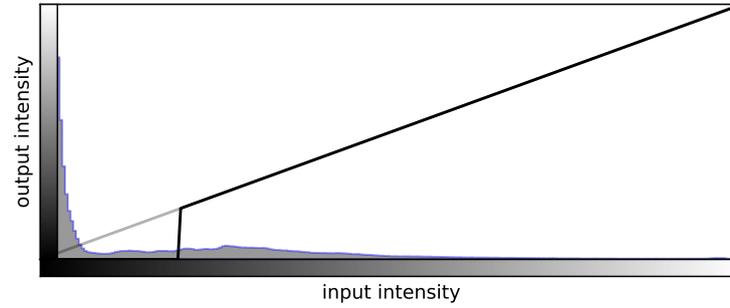
original



remapped

Intensity mappings

Threshold



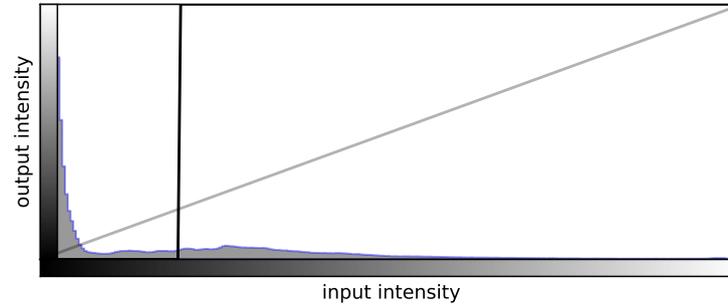
original



remapped

Intensity mappings

Binary threshold



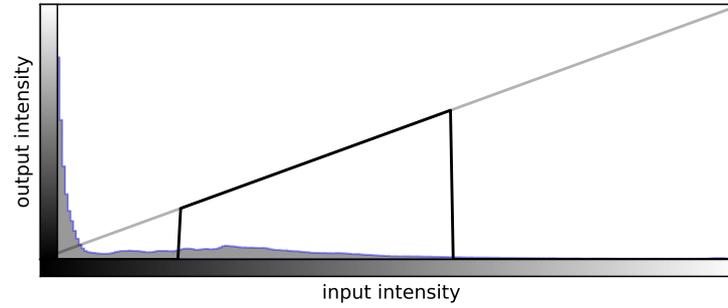
original



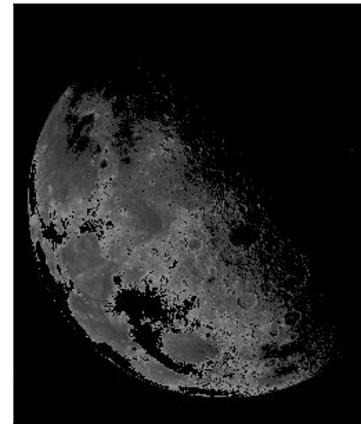
remapped

Intensity mappings

Window



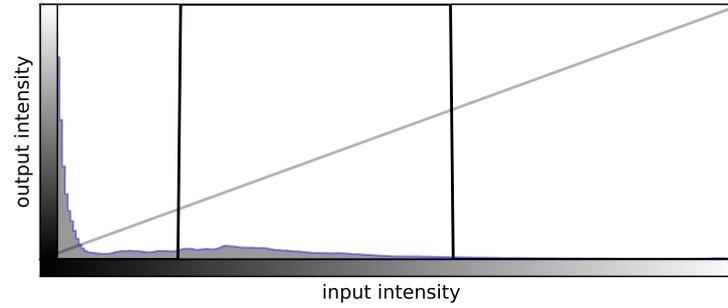
original



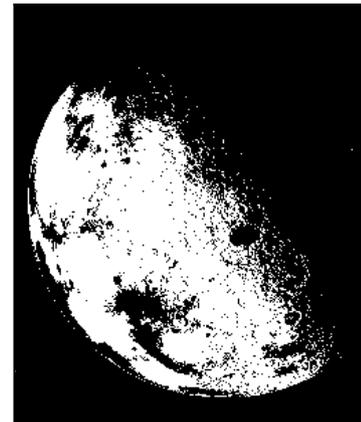
remapped

Intensity mappings

Binary window



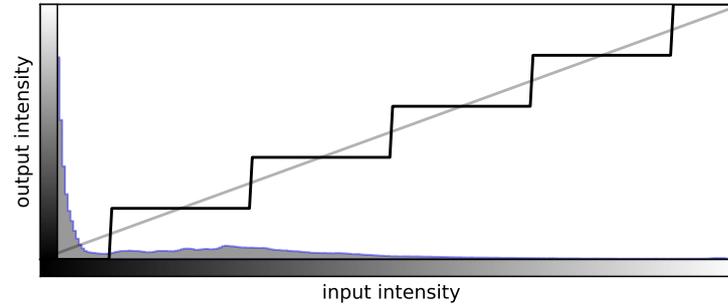
original



remapped

Intensity mappings

Posterization



original



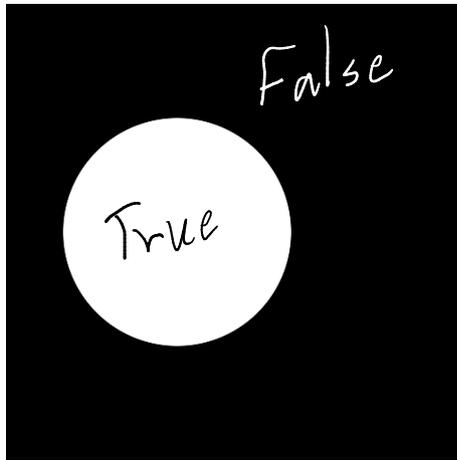
remapped

Morphological operations

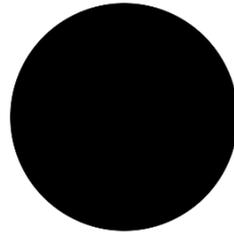
- analyze morphology of image structures
 - based on set theory and topology
- extract image information
 - shape
 - size
 - connectivity
 - number
 - boundary
- mostly on binary images

Set operations

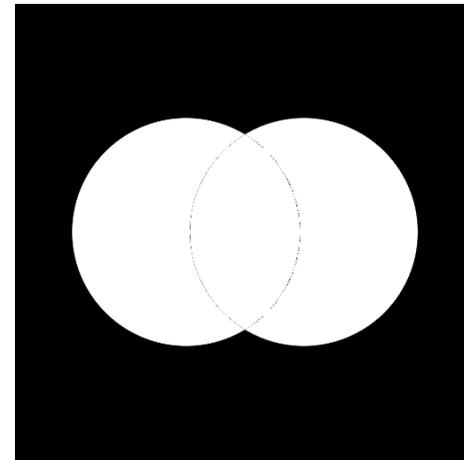
binary mask
 A



complement (background)
 A^c

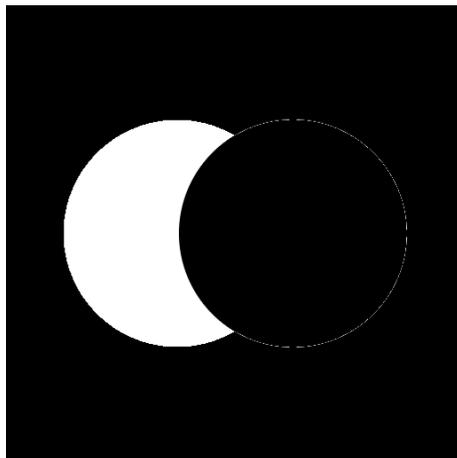


union
 $A \cup B$



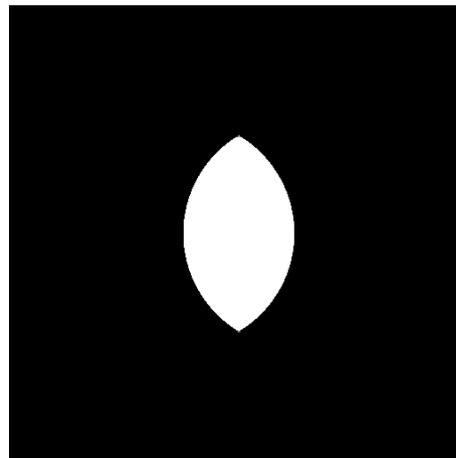
and not
↓

difference
 $A \setminus B$



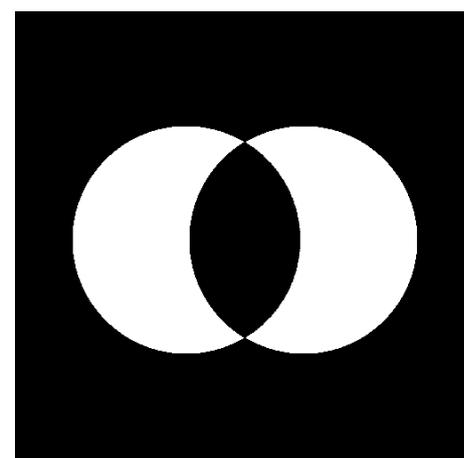
and
↓

intersection
 $A \cap B$



symmetric difference
 $A \Delta B$

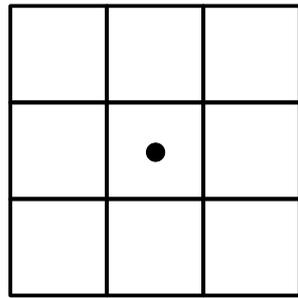
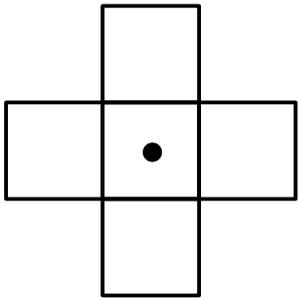
xor
↓



Structuring elements

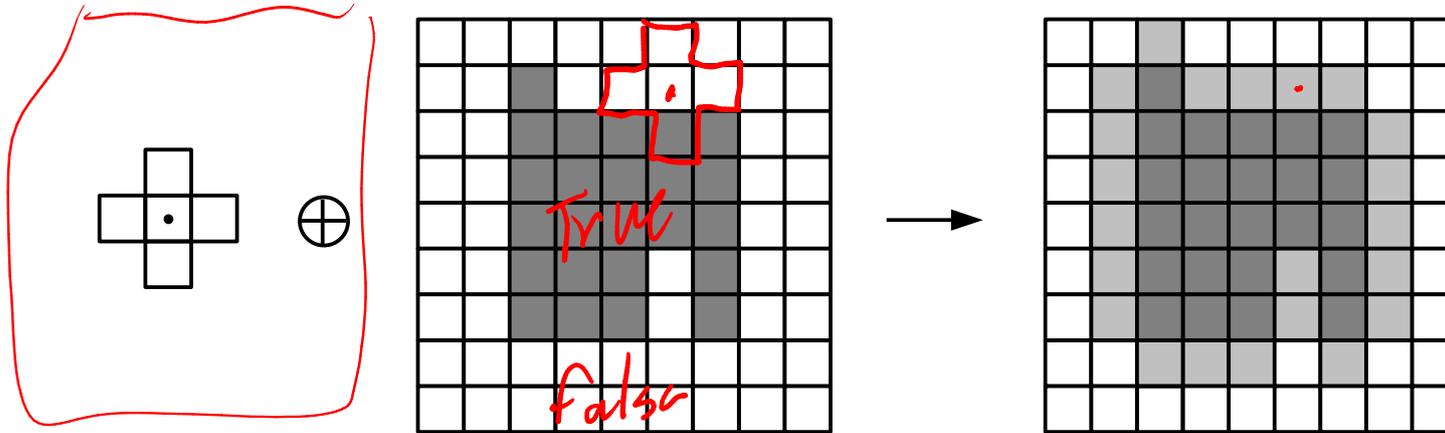
"footprint"

- small bit mask to probe the image
- scan origin of SE over image
- check overlap between SE and image
- set pixel(s) to zero (or one)

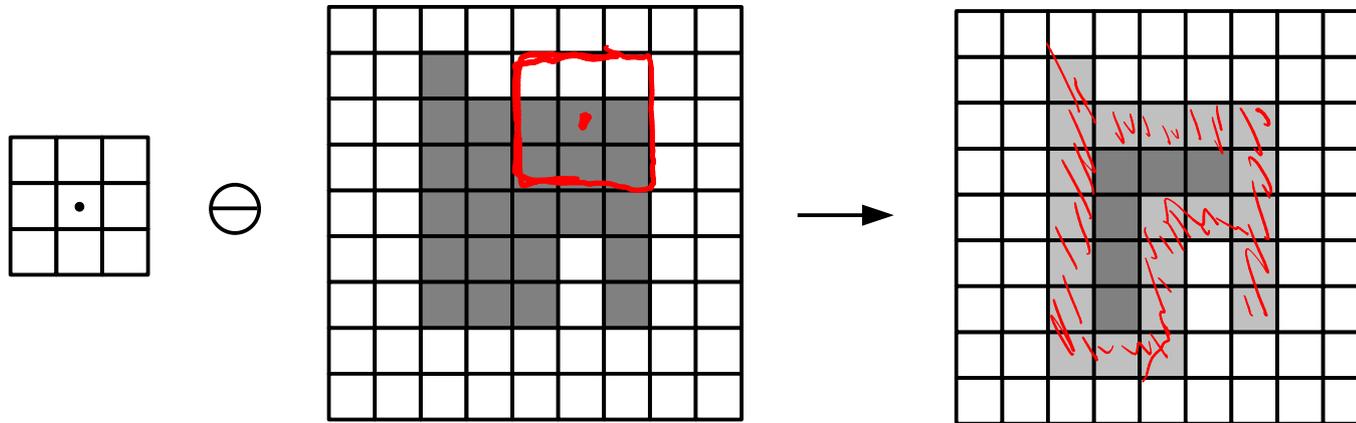


Basic operations

- Dilation: expand region

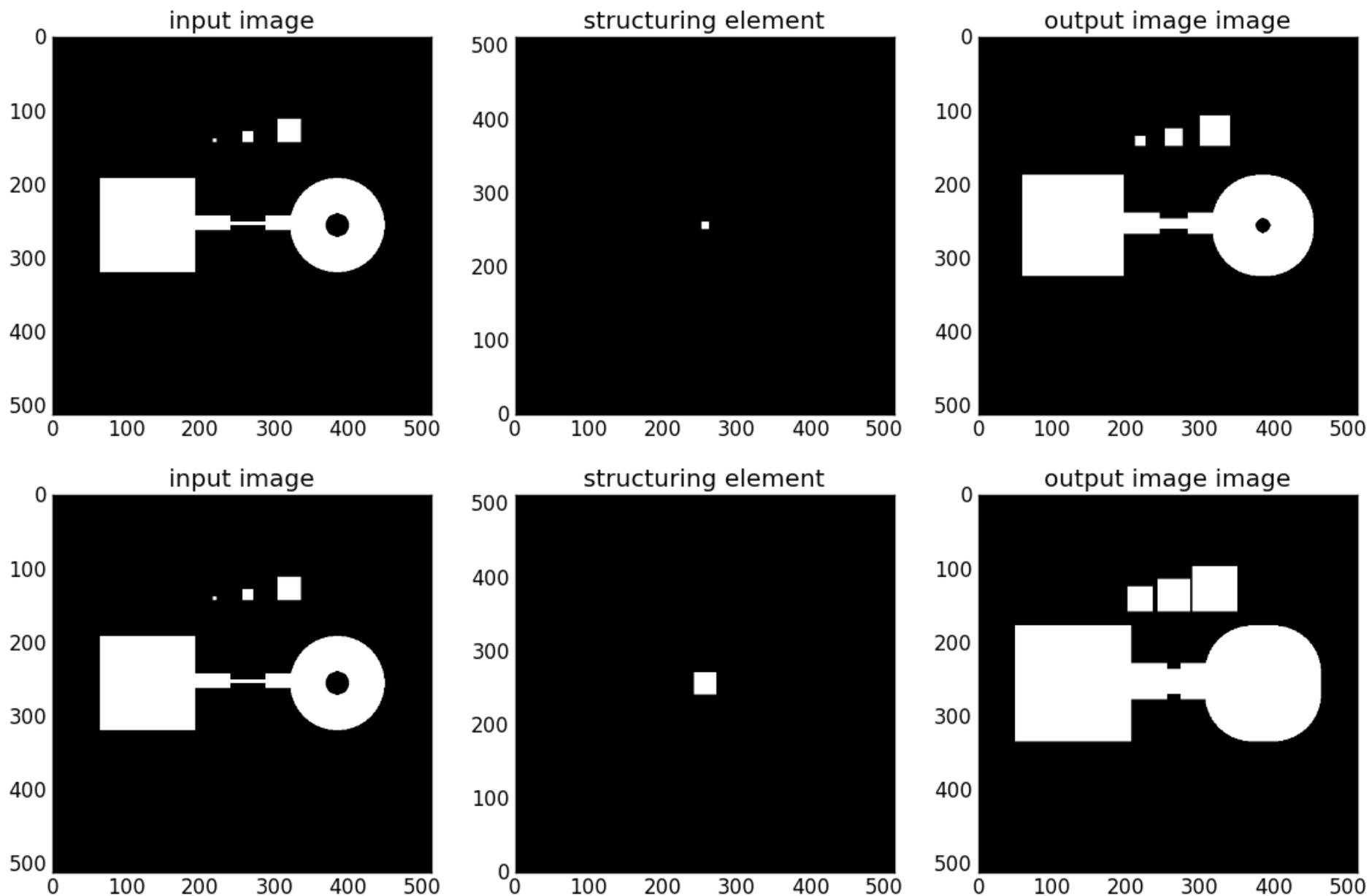


- Erosion: shrink region



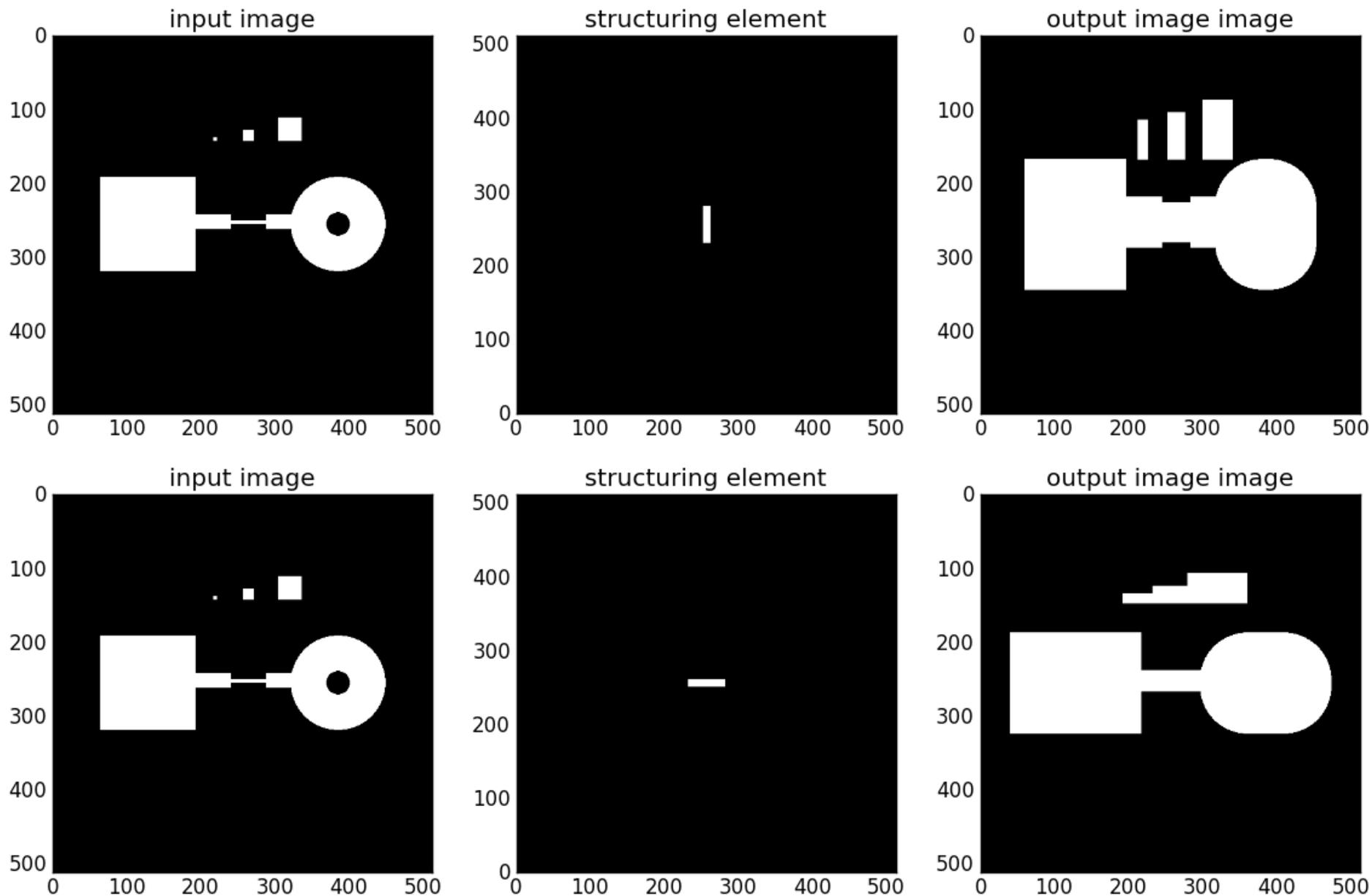
Morphological operations

- dilation



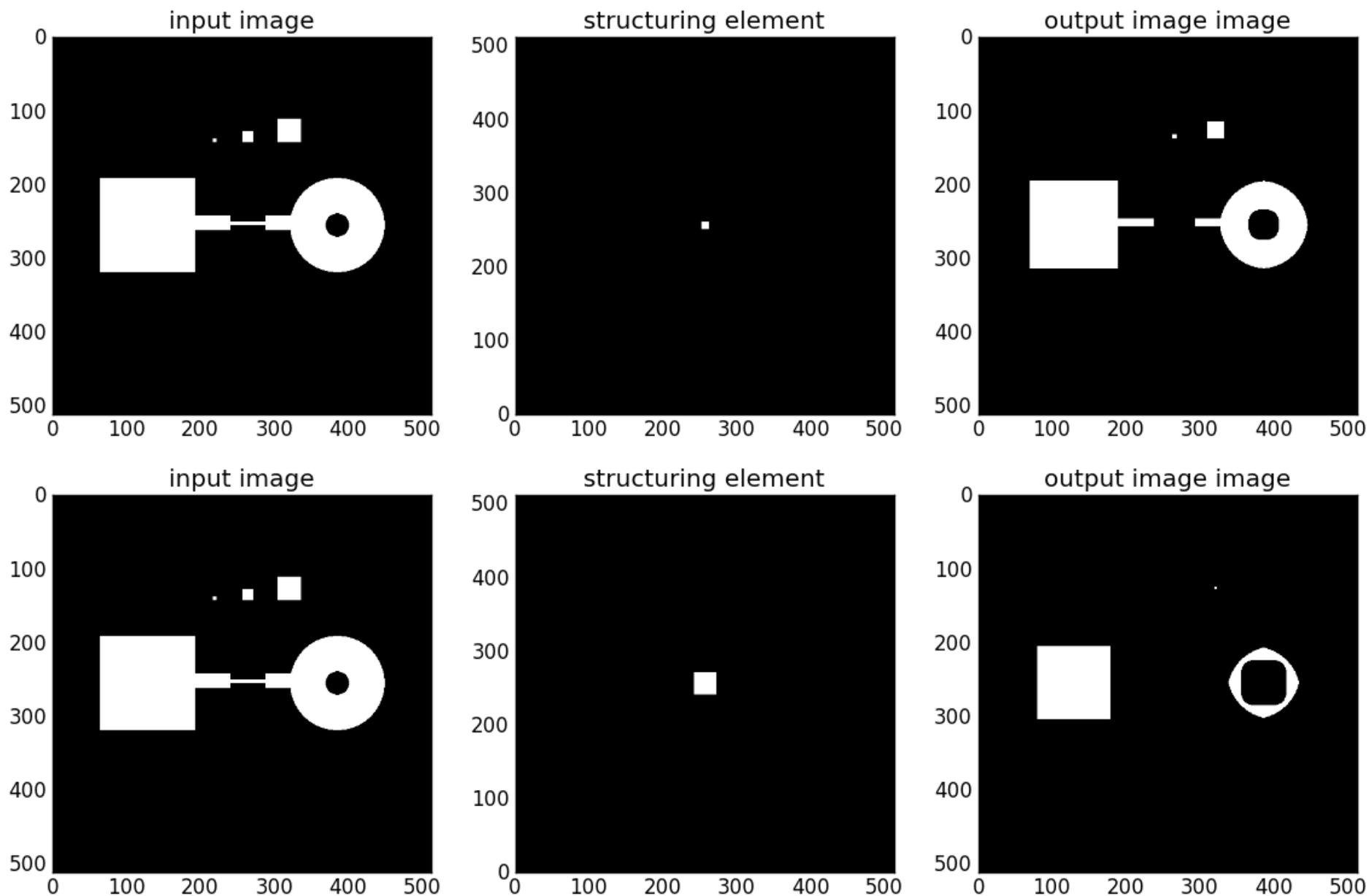
Morphological operations

- dilation



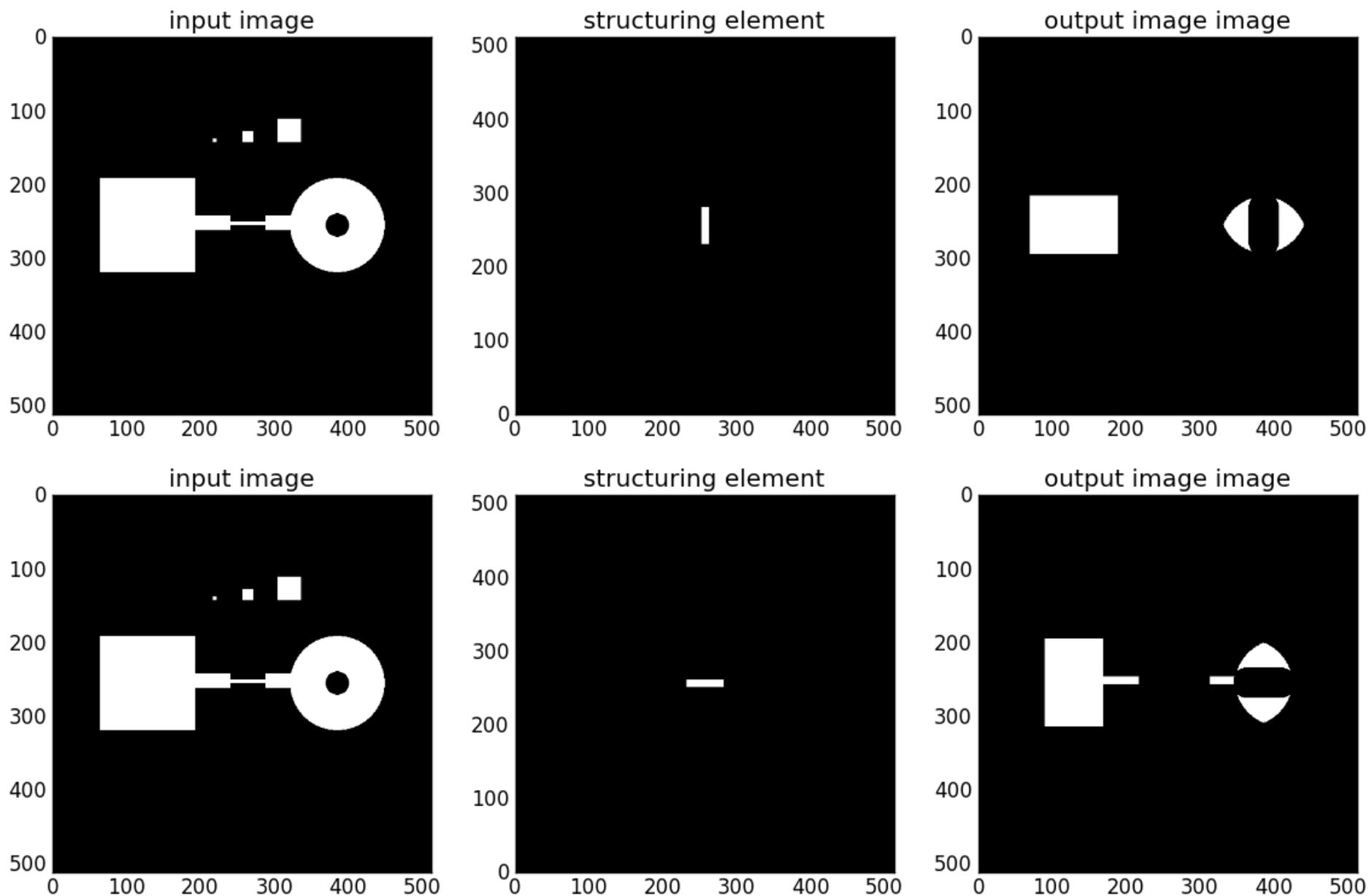
Morphological operations

- erosion



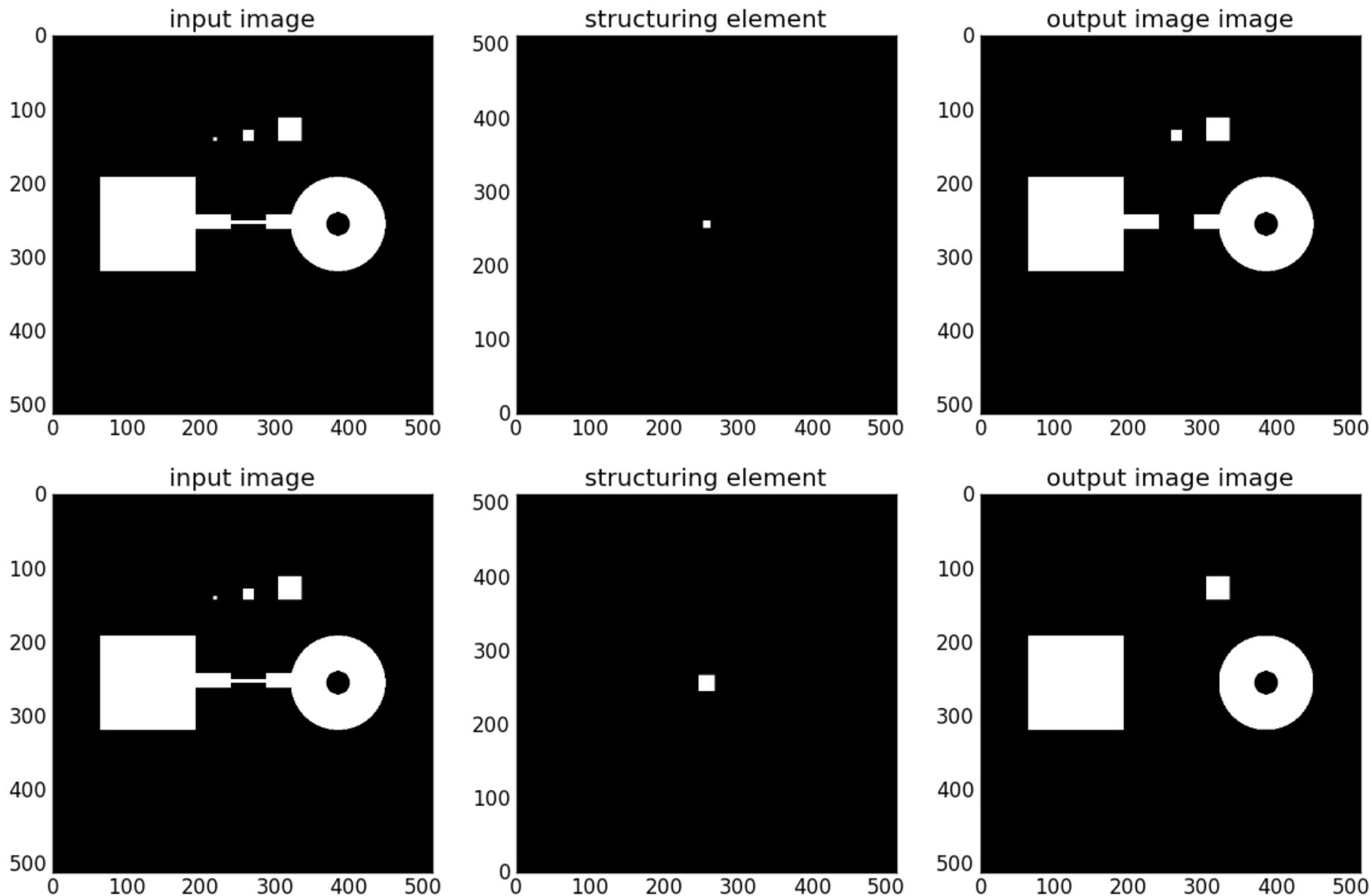
Morphological operations

- erosion



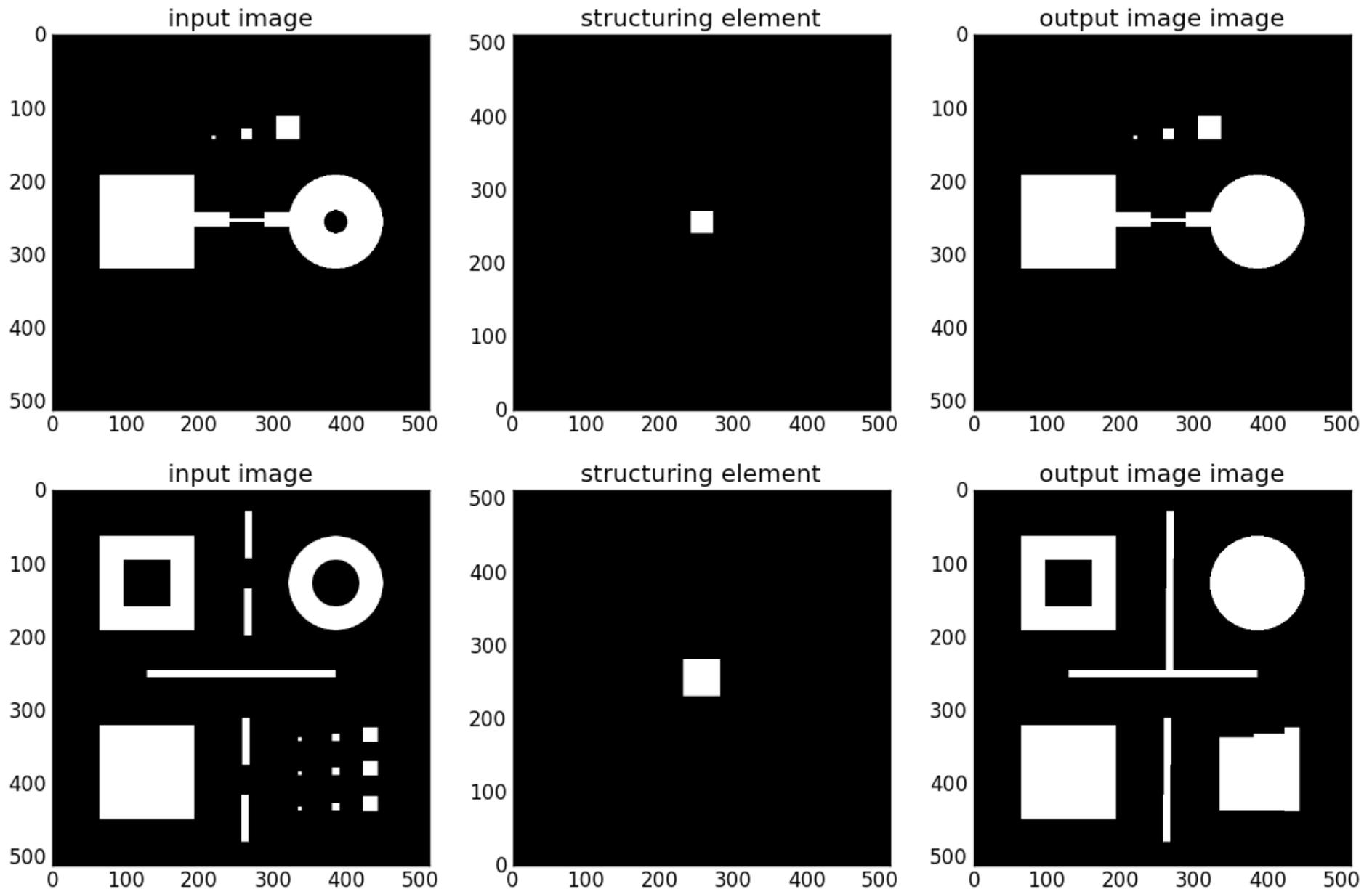
Morphological operations

- opening: first erosion, then dilation



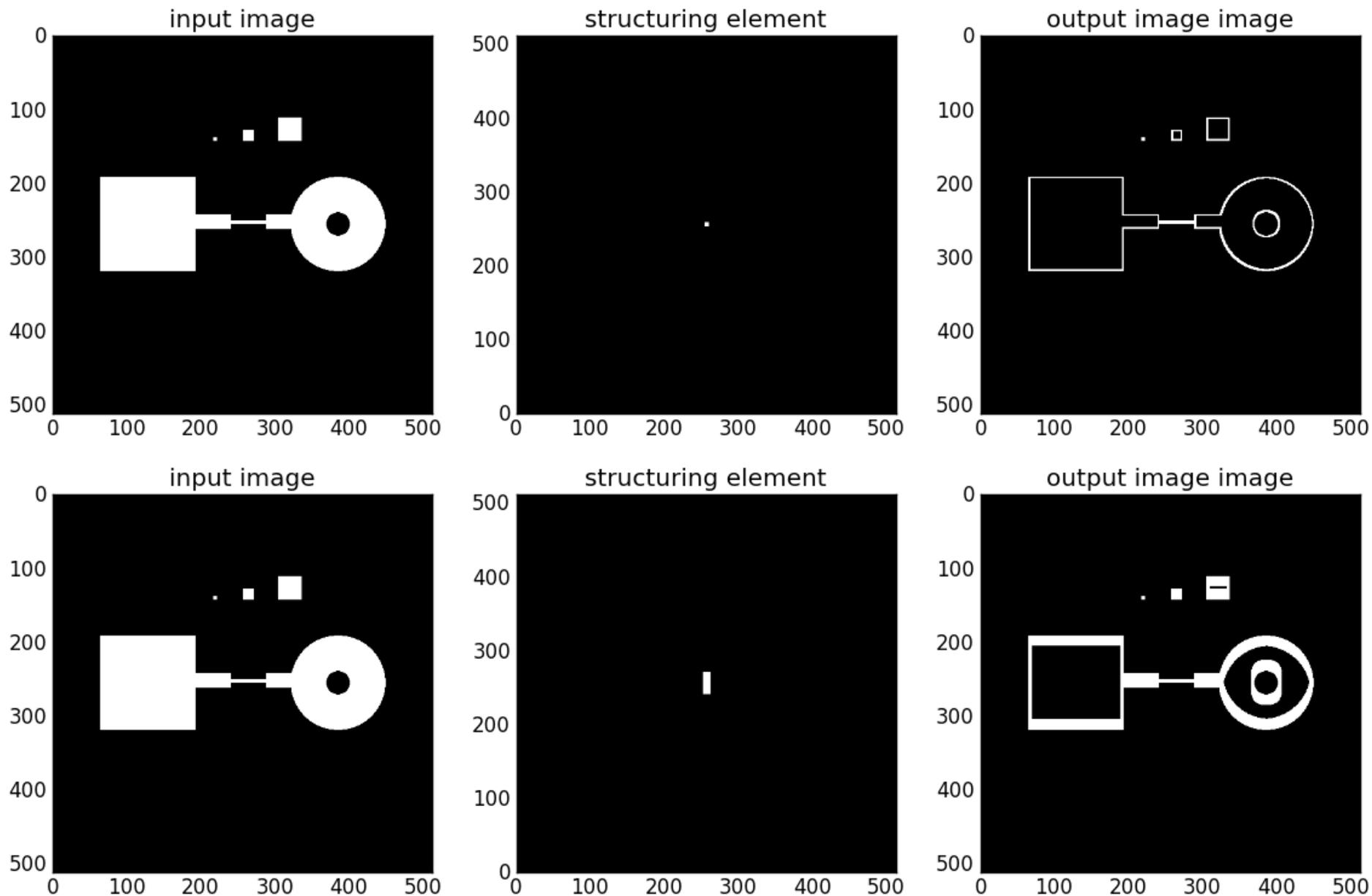
Morphological operations

- closing: first dilation, then erosion



Morphological operations

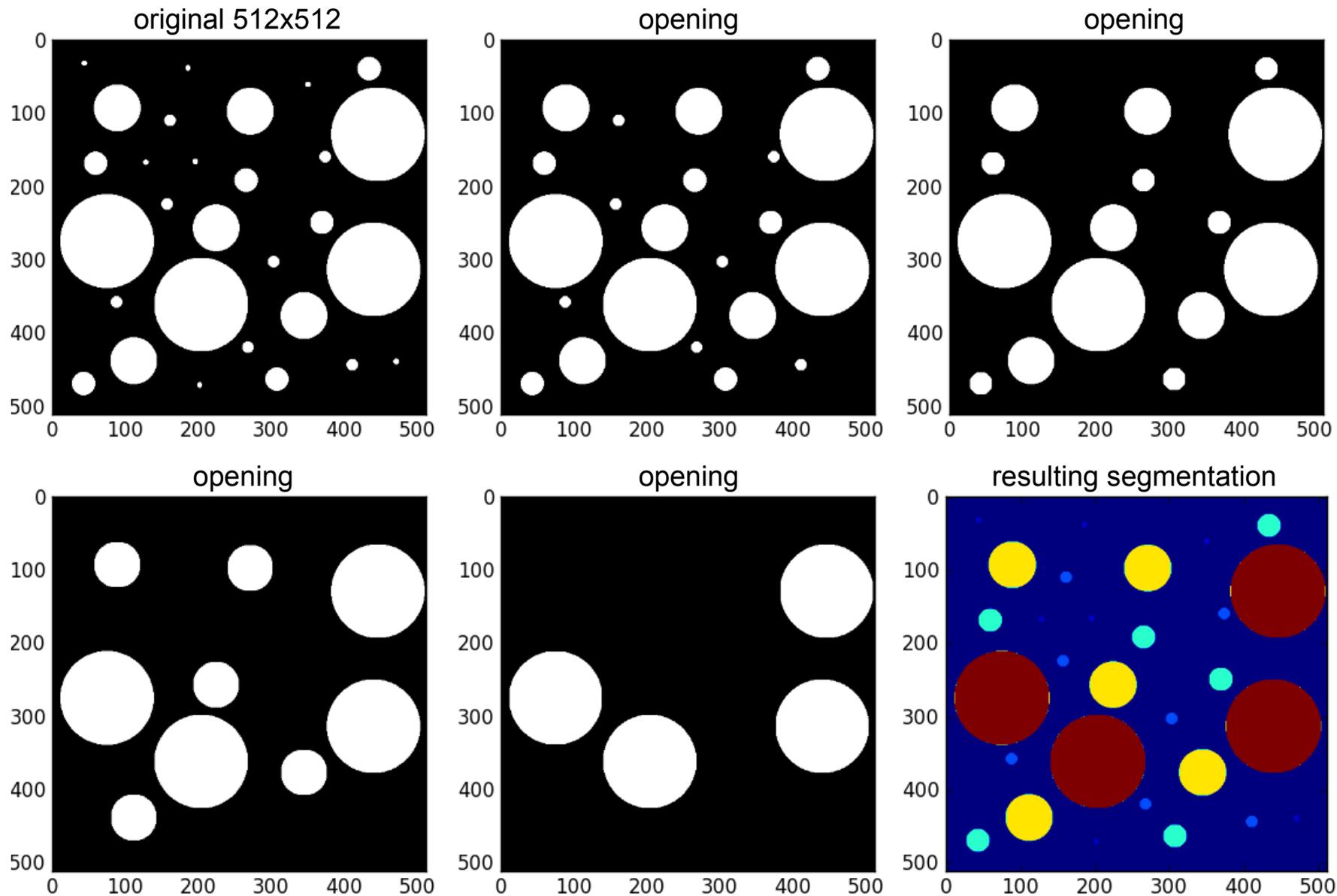
- boundary: original - erosion



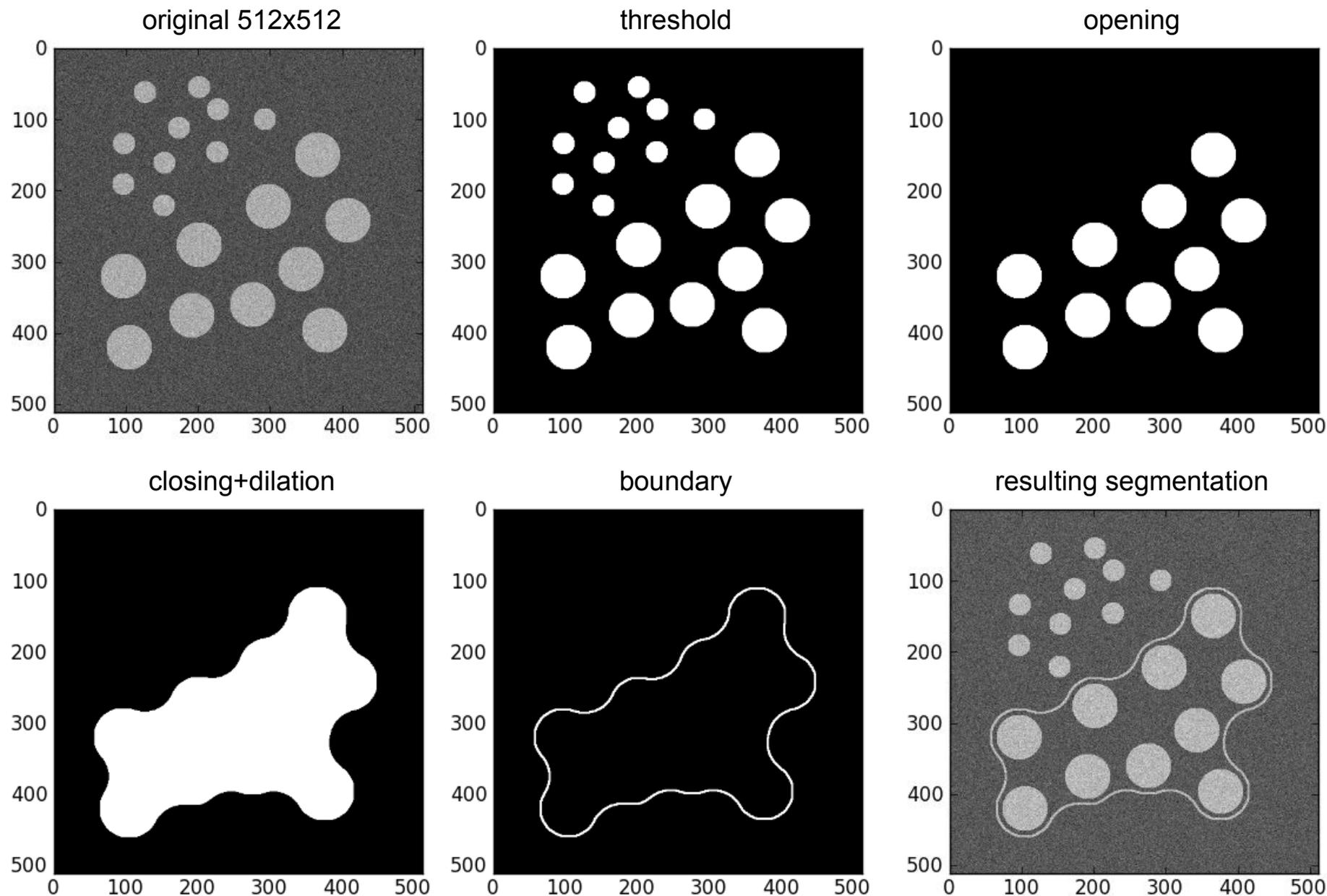
Segmentation: Motivation

- Partitioning of image by regions-of-interest
- various methods available
 - by morphology
 - by intensity
 - by region
 - by boundary
 - ...

Segmentation by morphology



Segmentation by morphology



Segmentation by intensity

- easy
- widely used

original



high window



mid window



low window



segmented



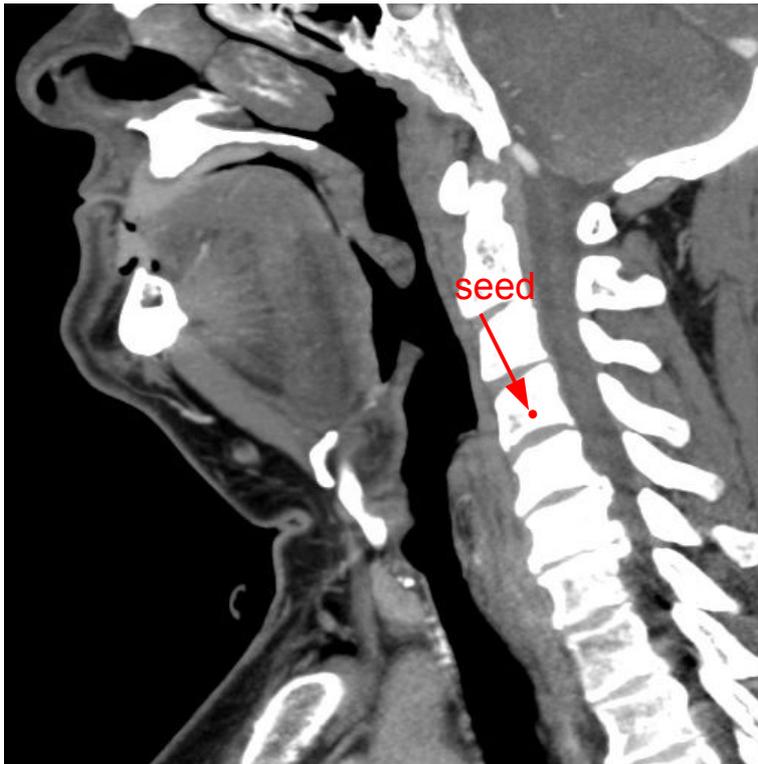
- noise prone
- no connectivity

Segmentation by region growth

- start with seed
- check intensity in neighborhood
- if intensity within window, set to 1
- iterate until no change

*often used in
combination with
filtering*

original



Segmentation by boundaries

- look for sharp changes in intensity
- more next week...

original

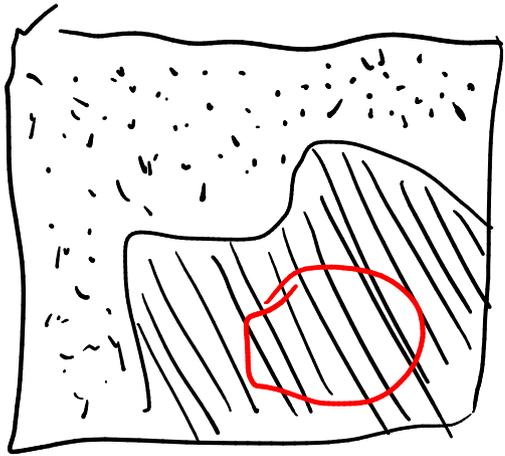


laplace



More advanced segmentation methods

→ texture-based segmentation



↓
Machine learning

↑
paint by hand → learn