Physics Education Laboratory Lecture 05

Pedagogical Content Knowledge - Math/Phys interplay

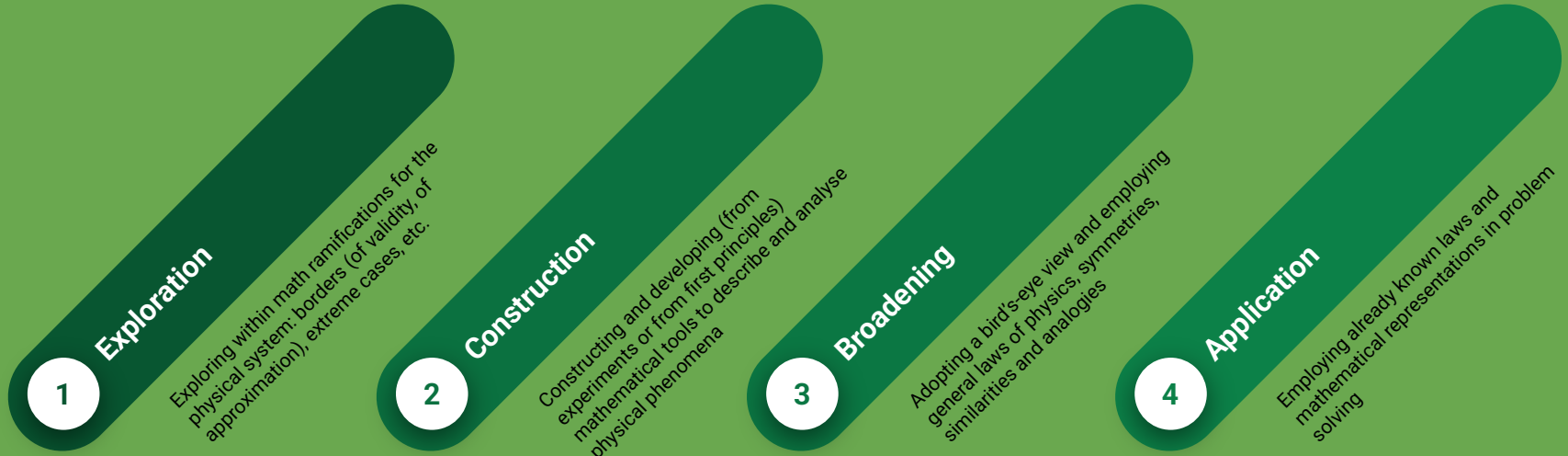
Francesco Longo 06/10/2025

1 PCK FOR SCIENCE TEACHING (Magnusson et al., 1999)

2 PCK FOR PHYSICS TEACHING (Etkina, 2010)
CONTENT KNOWLEDGE FOR PHYSICS TEACHING (Etkina, 2018)

3 DECLINING PCK FOCUSED ON MATH/PHYS INTERPLAY (Lehavi et al., 2014, 2017)

Keep in mind


1
Exploration
Exploring within math ramifications for the physical system: borders (of validity, of approximation), extreme cases, etc.
2
Construction
Constructing and developing (from experiments or from first principles) mathematical tools to describe and analyse physical phenomena
3
Broadening
Adopting a bird's-eye view and employing general laws of physics, symmetries, similarities and analogies
4
Application
Employing already known laws and mathematical representations in problem solving

Developing new patterns for integrating Math/Phys Interplay

REFLECT on the EXERCISE FOR REASONING!

WHICH PATTERN IS PREVALENT?

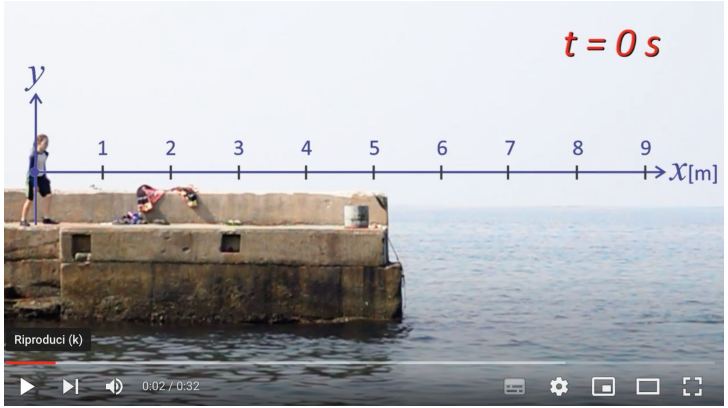
Which are student specific target?

Which are the tasks that the teacher has in mind while teaching in the CKT frameworks?

EXERCISE FOR REASONING

https://www.youtube.com/watch?v=_sCp1igJ3j8

Horizontal Jump



EXERCISE FOR REASONING - Group 01

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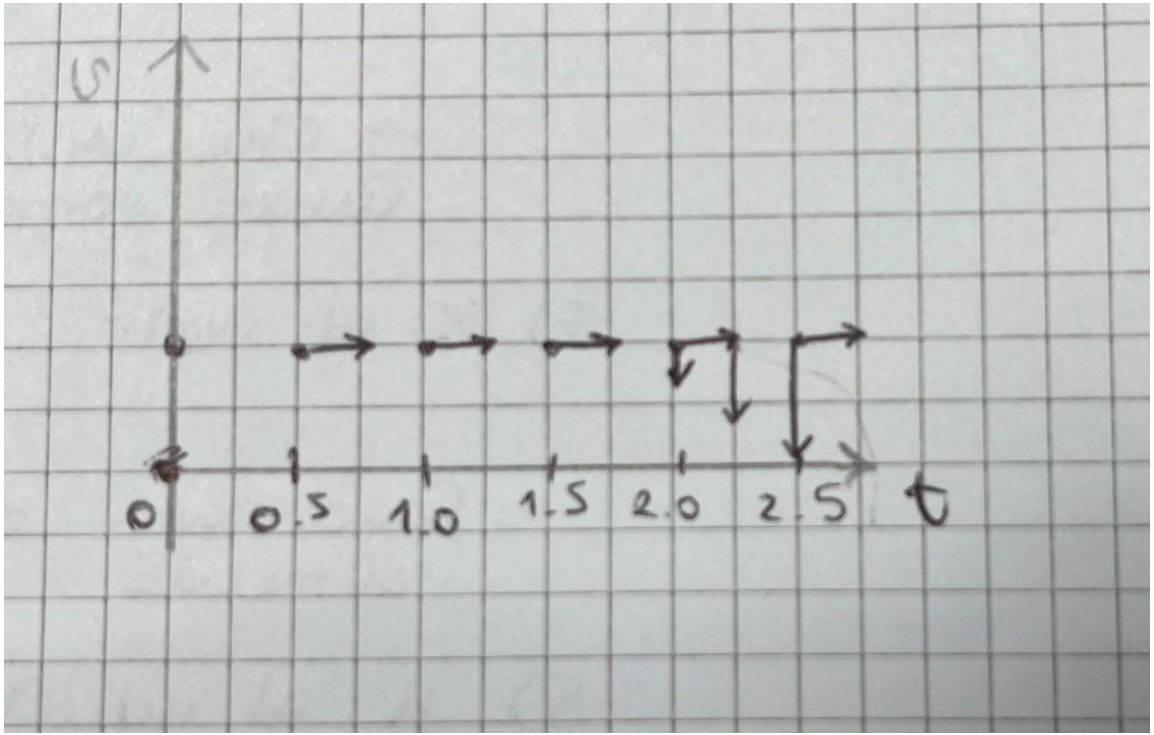
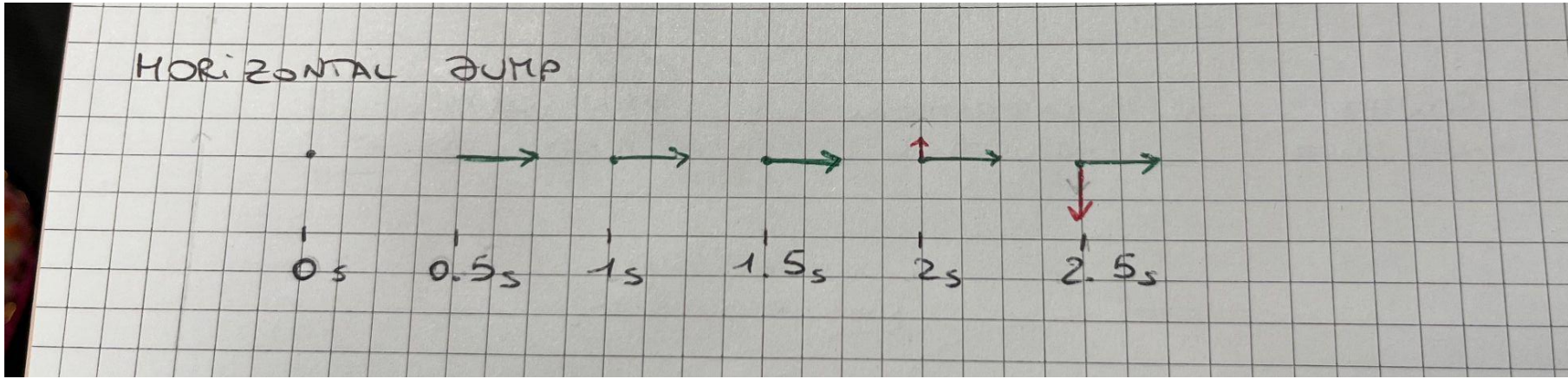
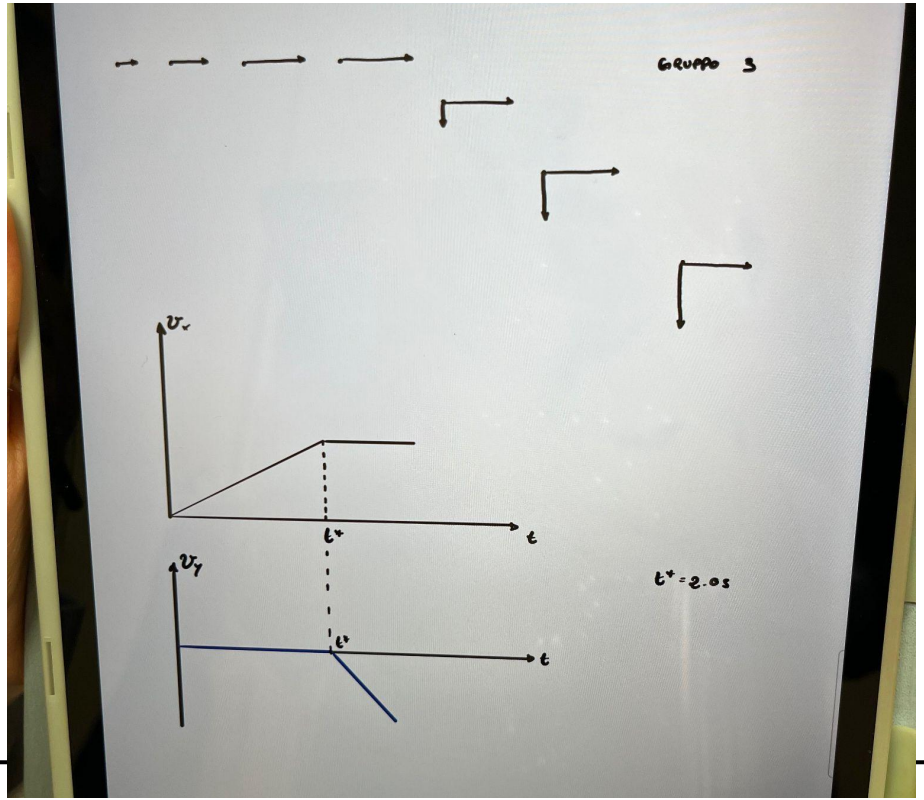


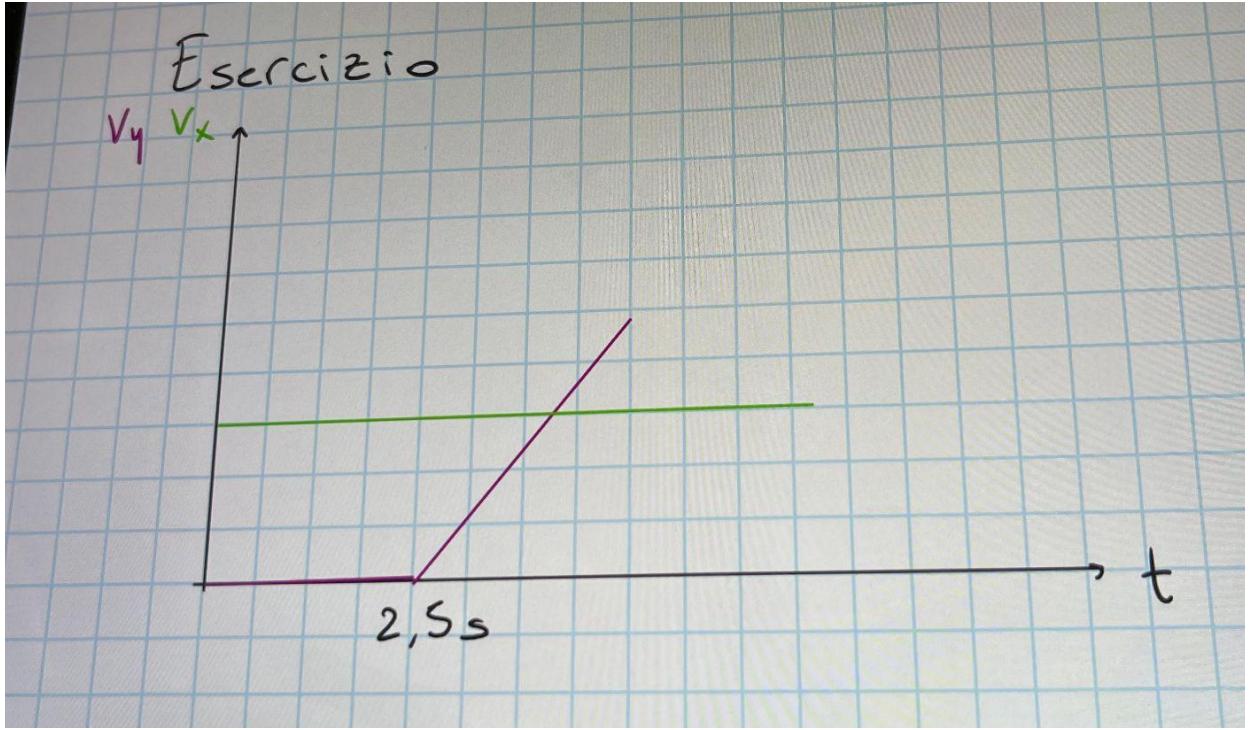
EXERCISE FOR REASONING - Group 02



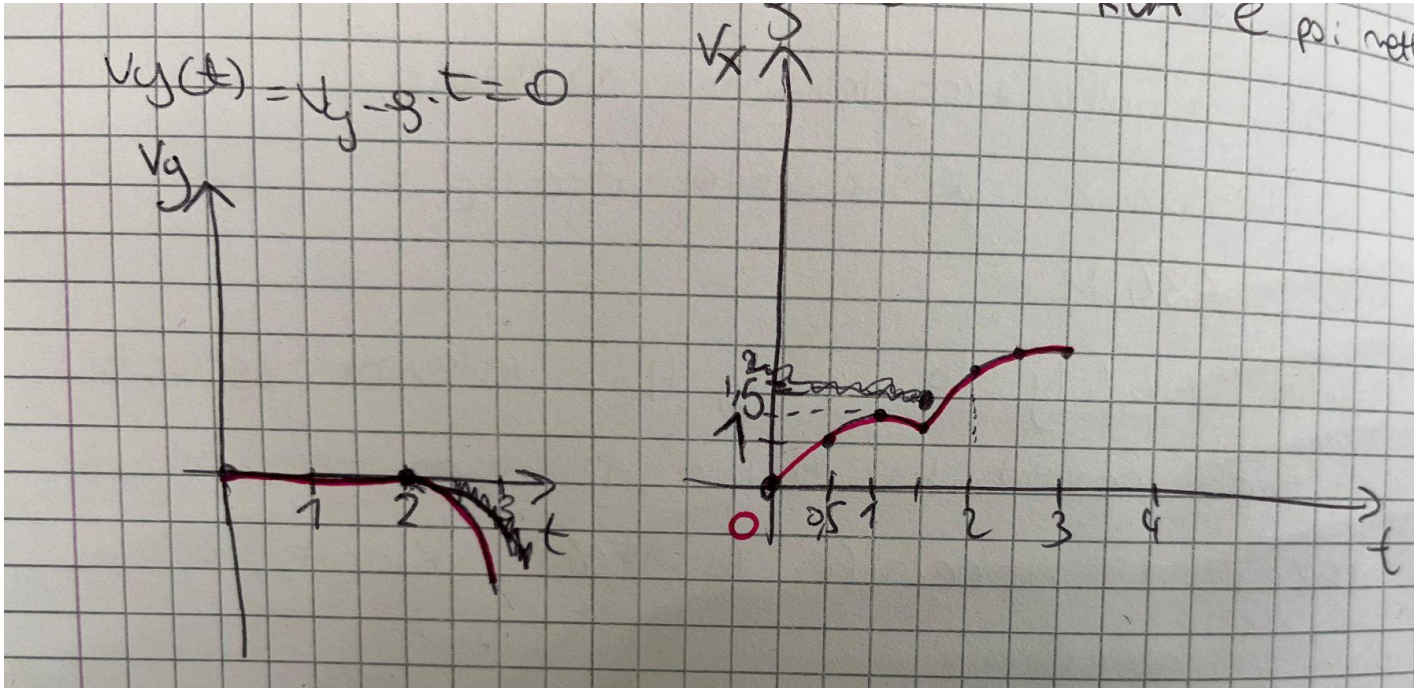
EXERCISE FOR REASONING - Group 03



EXERCISE FOR REASONING - Group 04



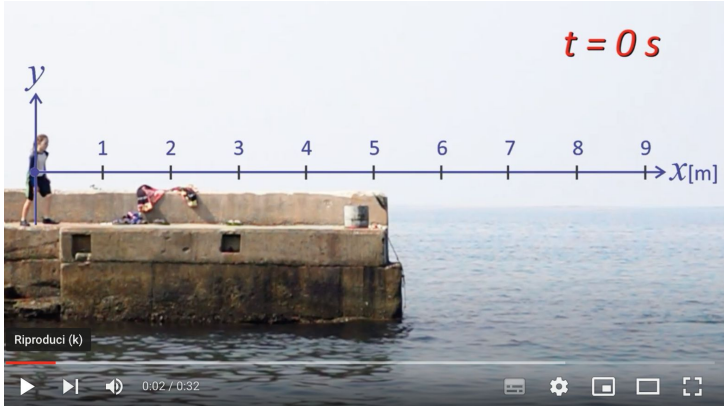
EXERCISE FOR REASONING - Group 05



EXERCISE FOR REASONING

https://www.youtube.com/watch?v=_sCp1igJ3j8

Horizontal Jump



Gorazd Planinsic, University of Ljubljana, Slovenia

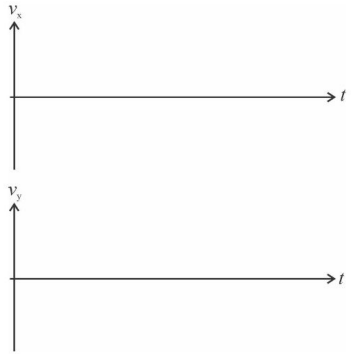
and Eugenia Etkina, Rutgers University, USA

0. FINN'S JUMP

The video https://youtu.be/_sCp1igJ3j8 shows Finn running along the pier and then jumping into the sea.

KINEMATICS

a. Draw a qualitative $v_x(t)$ and $v_y(t)$ graphs for Finn's motion, treating him as a point-like object that is positioned at the spot marked on the photo (let's call this point *center of mass*). Indicate any assumptions that you made.



b. Compare your graphs with the actual graphs https://drive.google.com/file/d/15SZnh2IwHAfodD4CGqAQse3P3JIVBhoh/view?usp=sharing that were obtained by tracking the Finn's motion from the video. Do they match? If not, suggest what might be the reasons for the differences (think of the assumptions that you made) and if necessary, revise your graphs.

c. Using data from the actual velocity-versus-time graphs compare the average magnitudes of Finn's acceleration while he is running along the pier and while he is falling. Which one is larger? How do you know? Are the values reasonable? How do you know?

d. Using data from the actual velocity-versus-time graphs and knowing that the distance between Finn's center of mass and the water level is 2.2 m, determine how far from the pier Finn jumped into the water. Indicate any assumptions that you made.

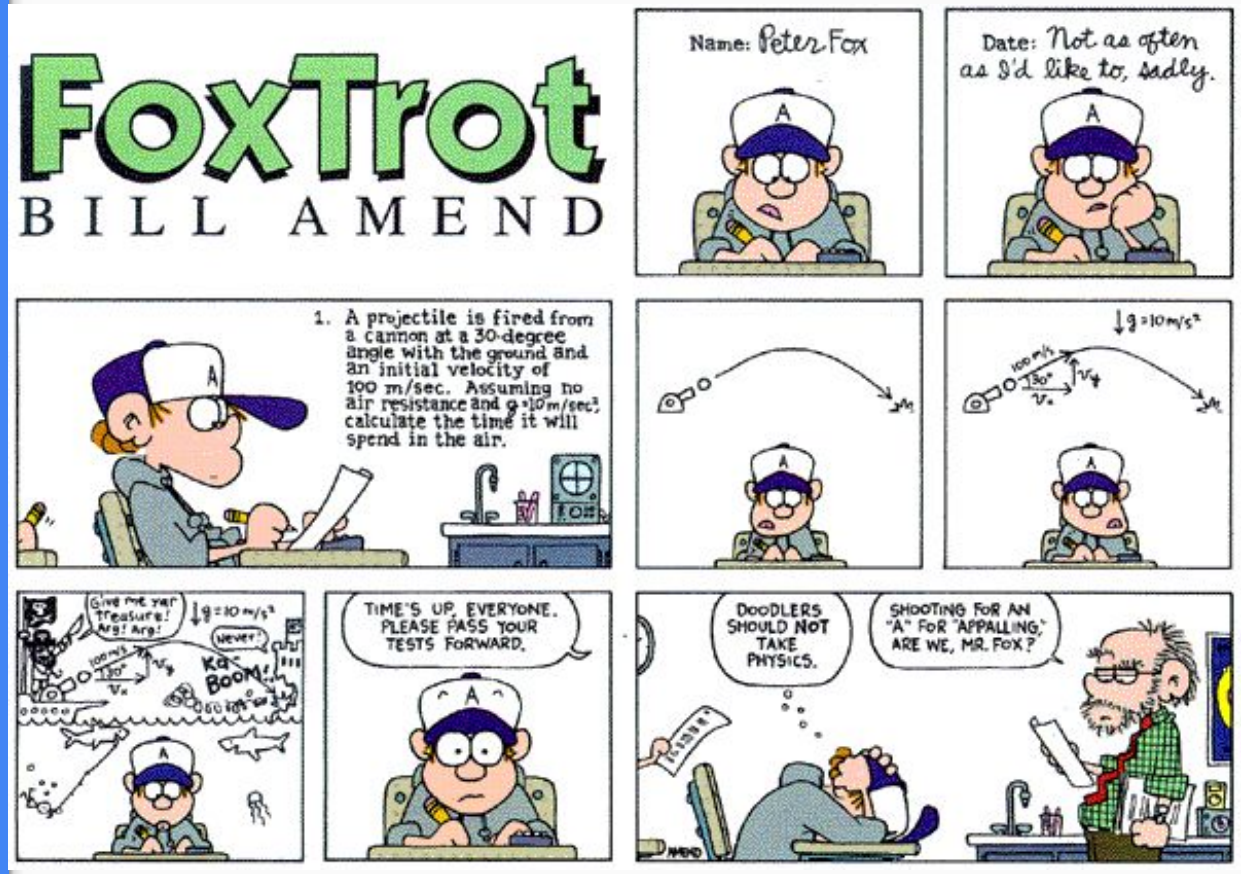
https://drive.google.com/file/d/1wZ71m7GWmmDTsh9dWkPoPxZ4qcLk7Go6/view?usp=sharing

Physics Education Laboratory Lecture 05

Kinematics Concepts and Pedagogical approach with Multiple Representations

Francesco Longo • 06/10/2025

Real world


FoxTrot
BILL AMEND
Name: Peter Fox
Date: Not as often as I'd like to, sadly.
1. A projectile is fired from a cannon at a 30-degree angle with the ground and an initial velocity of 100 m/sec. Assuming no air resistance and g=10m/sec², calculate the time it will spend in the air.
g=10 m/s²
100 m/s
30°
Give me yer treasure! Arg! Arg!
NEVER!
Ka-BOOM!
TIME'S UP, EVERYONE. PLEASE PASS YOUR TESTS FORWARD.
DOODLERS SHOULD NOT TAKE PHYSICS.
SHOOTING FOR AN "A" FOR "APPALLING," ARE WE, MR. FOX?

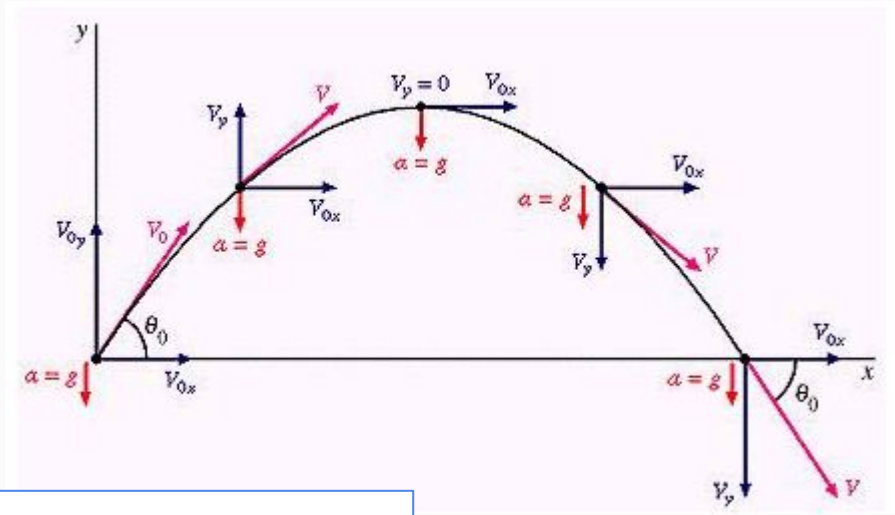
Key-concepts in kinematics

Defining quantities/variables for describing motion (position, displacement, speed, velocity, acceleration)

Frame of reference and observers

From one dimensional to bi-dimensional motion (from scalars to vectors)

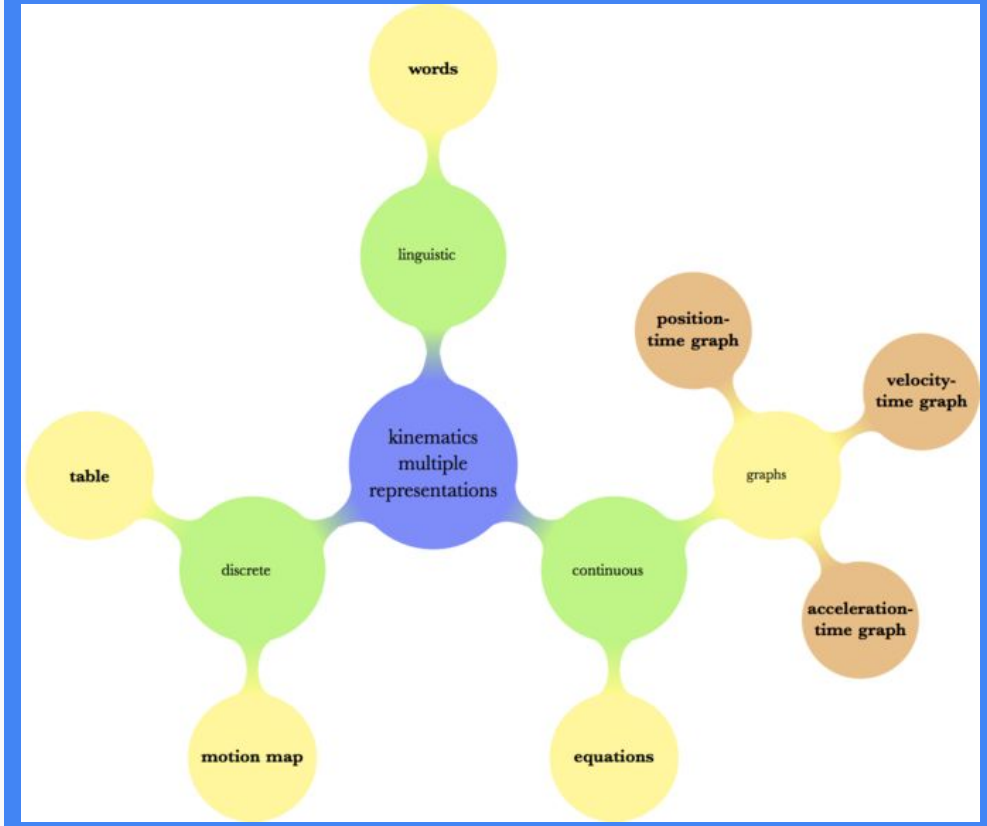
Relative motion



Knowledge of curricula

Key-concepts in Kinematics with Multiple Representations

Orientation toward science teaching



https://fockphysics.wordpress.com/2013/06/15/sbg-and-multiple-representations-in-kinematics/

Misconceptions

Knowledge of students' prior understandings

- Zero velocity implies zero acceleration, even for an instant.
- Velocity decreasing means something slows down. (Failure to account for vector character of velocity.)
- Constant circular speed implies zero acceleration. (#VectorFail)
- Distance and displacement are often confused.
- Position and displacement are often confused when making graphs.
- Position, velocity, and acceleration are "undifferentiated".
- Same velocity means same position.
- The meaning of the slope (in s-t graphs, in v-t graphs, in y-x graphs).

(McDermott et al, 1987)

Math difficulties

- Discriminating the slope and height of a graph and interpreting changes in height and changes in slope
- Identifying slope with the angle between straight line and the x axis
- Evaluate the sign of the slope according to the quadrant in which the line is drawn
- Interval/point confusion

Knowledge of instructional strategies

(Planinic M. et al, 2012)

Phys difficulties

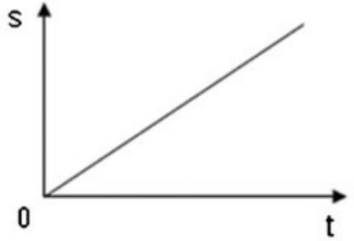
- Interpretation of the meaning of line graph slope in a physics context
- The slope of the line constantly increase (or decrease) in s-t graphs, that means a change in velocity
- Students who have not yet reached the formal operational stage of cognitive development are likely to view graphs as something concrete rather than indicators of abstract trends
- Spatial imagery vs visual imagery

Phys difficulties

Math difficulties

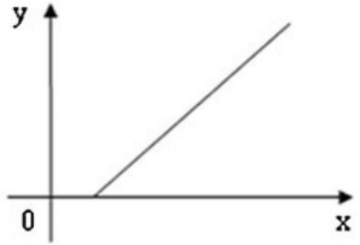
Knowledge of instructional strategies

P1. Distance – time graph of an object's motion is shown below. Which statement best describes this motion?



A. The object is not moving.
B. The object is moving at a constant velocity.
C. The object is moving with a uniformly decreasing velocity.
D. The object is moving with a uniformly increasing velocity.

M1. Consider the following line (ray) in the coordinate system. Which statement is correct?



A. The slope of the line is constant and different from zero.
B. The slope of the line is constant and equal to zero.
C. The slope of the line is constantly increasing.
D. The slope of the line is constantly decreasing.

(Planinic M. et al, 2012)

Students' difficulties in kinematics graphs

Knowledge of students' prior understanding

(Beichner, 1994)

Graph as Picture Errors
The graph is considered to be like a photograph of the situation. It is not seen to be an abstract mathematical representation, but rather a concrete duplication of the motion event.

Slope/Height Confusion
Students often read values off the axes and directly assign them to the slope.

Variable Confusion
Students do not distinguish between distance, velocity, and acceleration. They often believe that graphs of these variables should be identical and appear to readily switch axis labels from one variable to another without recognizing that the graphed line should also change.

Nonorigin Slope Errors
Students successfully find the slope of lines which pass through the origin. However, they have difficulty determining the slope of a line (or the appropriate tangent line) if it does not go through zero.

Area Ignorance
Students do not recognize the meaning of areas under kinematics graph curves.

Area/Slope/Height Confusion
Students often perform slope calculations or inappropriately use axis values when area calculations are required.

Quantitative/Qualitative problem solving in symbol representation

formats, are crucial. It is well known that the application of quantitative strategy (manipulation of equations for attempting problems) does not imply comprehension of concepts presented in the tasks as well as the underlying physics principles of the equations used. According to the Johnson-Laird cognitive framework of sense making [45], it is argued that comprehension occurs with the construction of a mental model which is a key element in the learning process [46,47]. However, although the application of external representations promotes the construction of a mental model, the strategies used by the students when attempting tasks with different representational formats play a crucial role. The study by Greca and Moreira [48] characterized students with a mental model as focusing on comprehension and identifying physics ideas, using a qualitative appr
ing visual repre
with the equati
problem solving
taught and pro
various ways i
representations
on the use of q
central role of v
ing as well as f
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These are some
et al. [17] whic

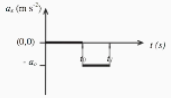


FIG. 1. Kinematic task with graphical format requesting qualitative solution.

immediately apply the brakes and your car starts slowing down at 0.8 m s^{-2}. Determine whether a collision will take place.

ACKNOWLEDGMENTS

This work is supported in part by the U.S. National Science Foundation under Grant No. 0816207.

APPENDIX A

Question 1: Kinematics—Qualitative—Symbolic
The equation of motion for an object moving along a straight horizontal path is given by

$$x(t) = 30 + 5t + 2t^2$$

Write down, in words, everything you can say about the motion of the object.

Question 2: Kinematics—Quantitative—Linguistic
You are driving at a speed of 60 m s^{-1} when suddenly you see a van 60 m directly ahead of you also travelling in the same direction at a constant speed of 40 m s^{-1}. You

Question 6: Work—Qualitative—Symbolic
Write down everything you can say from the force equation $\vec{F}(x) = (-4 + x^2)\,\hat{i}$ N applied to move the box from an initial position of 0 m to final position of 4 m.

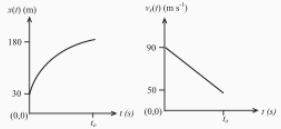


FIG. 2. Kinematic task with graphical format requesting quantitative solutions.

Specific strategies to assess students' understandings

The equation of motion for an object moving along a straight horizontal path is found to be

$$v_x(t) = 3 + 2t$$

If the object was in motion for 5 s, what is the distance travelled and acceleration during this time?

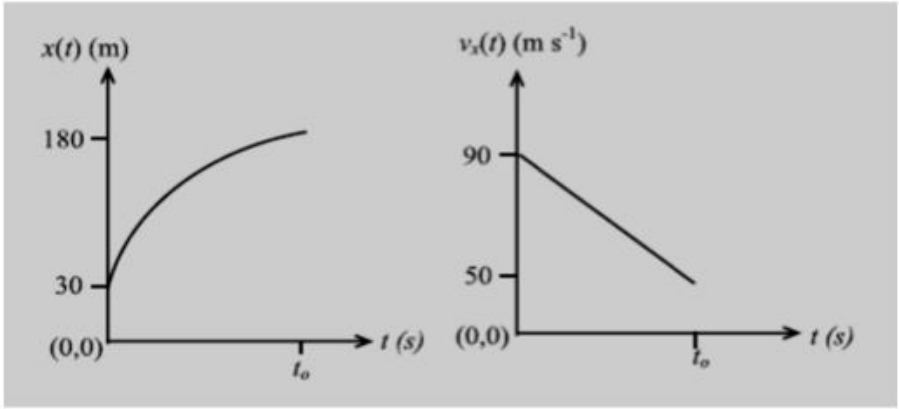
The equation of motion for an object moving along a straight horizontal path is given by

$$x(t) = 30 + 5t + 2t^2$$

Write down, in words, everything you can say about the motion of the object.

(Ibrahim & Rebello, 2012) (Saputra et al. 2019)

Quantitative/Qualitative problem solving in Graphic representation

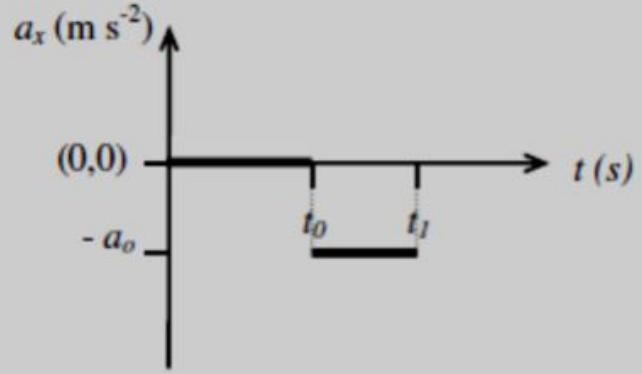


The motion of a truck along a straight horizontal path is shown by the graphs below. Determine the time taken and acceleration of the truck to complete the whole journey

(Ibrahim & Rebello, 2012) (Saputra et al. 2019)

The acceleration-time graph for an object moving along a straight horizontal path is shown in Fig. 4.

Write down, in words, everything you can say about the motion of the object



Specific strategies to assess students' understandings

Questionnaire on Kinematics

https://nhrhs.instructure.com/files/6518/download?download_frd=1

Questionnaire on Kinematics - Let’s try it

https://forms.gle/wWg4n2y8dmyGEZDBA

Multiple Representations definition

MR refer to the capacity of scientific discourse to represent the same concepts or processes in different modes (e.g., verbal, visual, mathematical, graphical) while multi-modal representation refers to the integration of different modes to represent scientific processes, findings, and scientific explanations

Tytler et al. (2007) p.314

DEFINING REPRESENTATIONS:

External representations: which are in a form that can be seen by others, such as pictures, text narrations, graphs, symbols, etc.

External representations refer to “the knowledge and structure in the environment, as physical symbols, objects, or dimensions”

Internal representations or **mental models**, which act out as structural analogies of situations or processes.

Ainsworth (1999,2008)

Internal representations refer to “the knowledge and structure in memory, as propositions, productions, schemas, neural networks, or other forms” Zhang (1997)

Building deep learning

Information in external representations can be processed by perceptual operation and refers to information that is directly perceived.

Information in internal representations refers to information retrieved directly from working memory that involves cognitive operation.

Through memorization, an internal representation can be transformed to an external representation.

Multiple Representations

In what ways does the use of MR in instruction support student learning?

MR might serve as a powerful learning strategy in physics education and has a positive impact on undergraduate students' conceptual understanding of physics concepts.

Different representations might be used for different purposes and also depending on students' prior experience with the use of a specific kind of representation.

Multiple Representations

What kinds of representations do students use?

Students use different kinds of representations and the combination or a series of kinds of representations has the potential to support student problem solving and consequently their development of conceptual understandings.

Multiple Representations

What difficulties do students face in using MR?

Even though MR provide an empowering learning tool or learning strategy, students face different difficulties in using MR during the learning process, especially in switching between kinds of representations.

Multiple Representations

What is the relation between students' use of MR and students' problem-solving skills?

The kind of representation used impacts students' problem-solving success.

Multiple Representations

What is the added value of technology integration in teaching with MR?

The potential value of computer-based MR in supporting students' physics learning such as supporting students' conceptual understandings as well as their representational competence.

SUMMARIZING:
how multiple representations (MR) can be used in undergraduate physics education and in physics education research

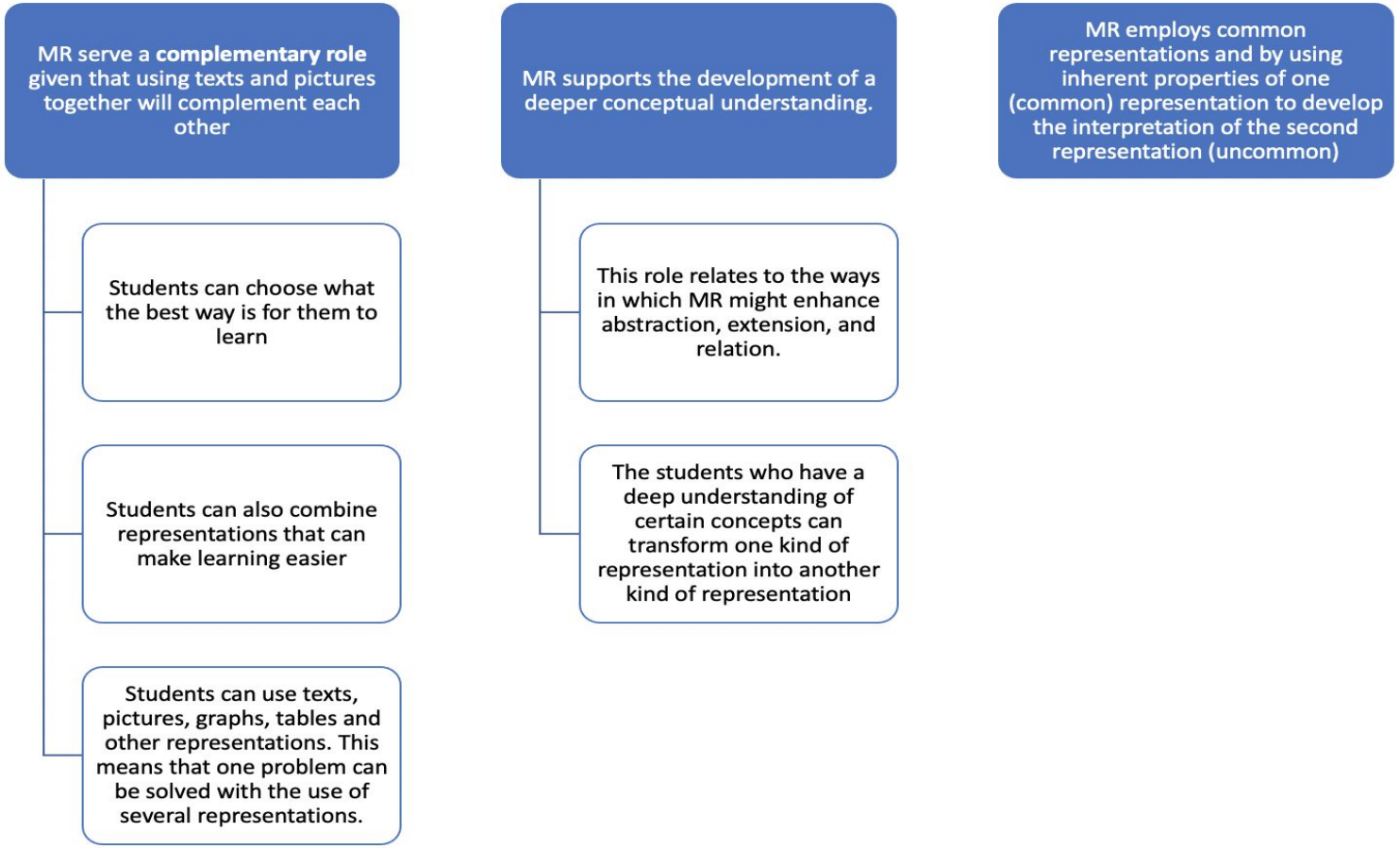
- MR might serve as a powerful learning strategy in physics education and has a positive impact on undergraduate students' understanding of physics concepts.
- There is no evidence in the literature that one kind representation is better than another in supporting students develop their understandings of physics concepts.
- Students use different kinds of representations and the combination of kinds of representations has the potential to support student problem solving and consequently their development of conceptual understanding.
- Students face different difficulties in using MR during the learning process, especially in switching between kinds of representations.
- Computer-based representations can support students' physics learning, as for example, conceptual understanding and representational competence.

State of PER results:

Research shows that the use of MR might enhance student learning

Research also shows that students have difficulties in relating and integrating the representations and translating information between them

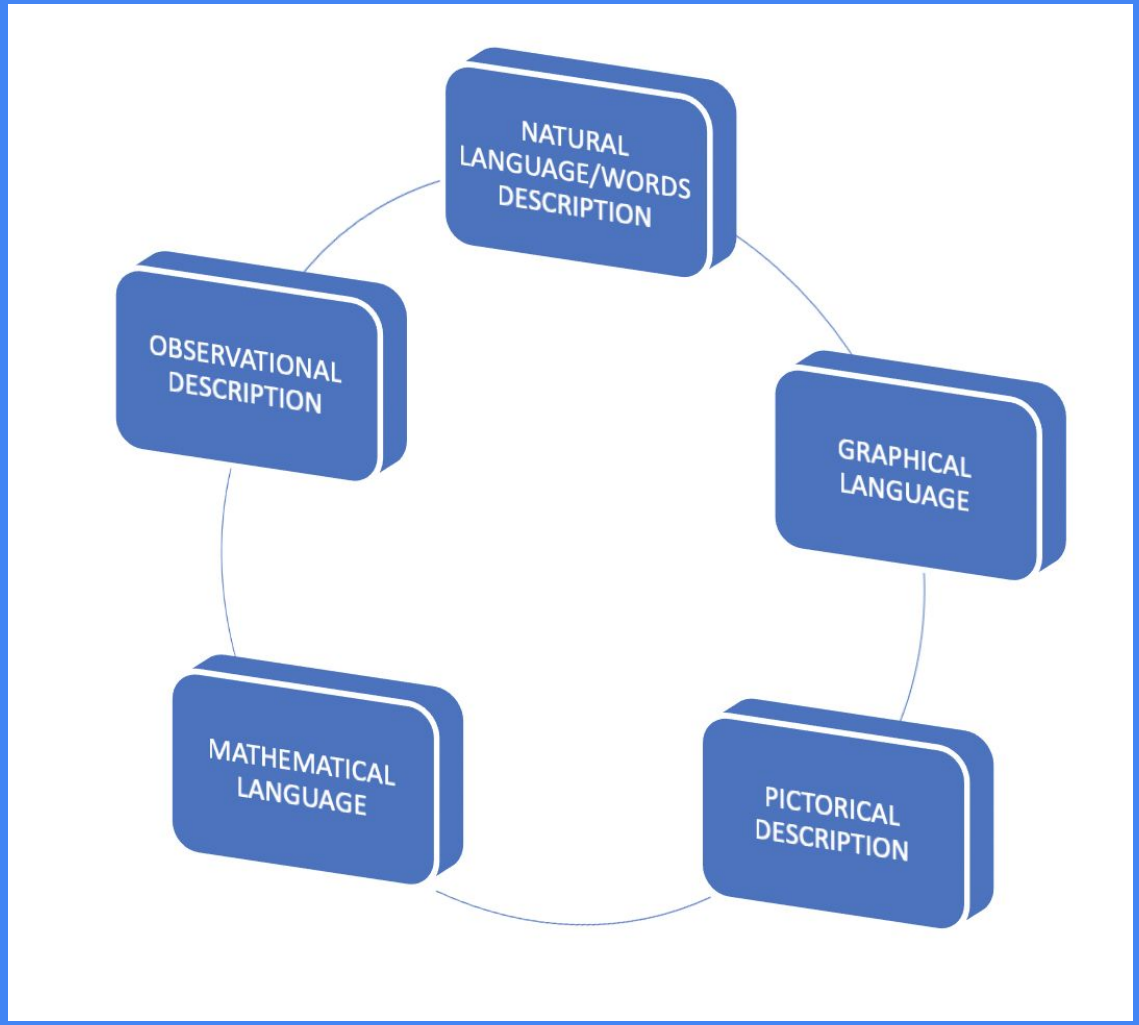
Learning outcomes: MR might have **three** distinct roles in learning.



“MR have a great potential in supporting student learning of physics concepts because students learn easier when problems include MR, hence, the use of MR can maximize the results of students’ learning processes. “

Opfermann M., Schmeck A., Fischer H.E. (2017) Multiple Representations in Physics and Science Education – Why Should We Use Them?. In: Treagust D., Duit R., Fischer H. (eds) **Multiple Representations in Physics Education. Models and Modeling in Science Education**, vol 10. Springer, Cham. https://doi.org/10.1007/978-3-319-58914-5_1

Kinematics with Multiple Representations DESCRIPTIONS OR DISCIPLINE'S LANGUAGES?

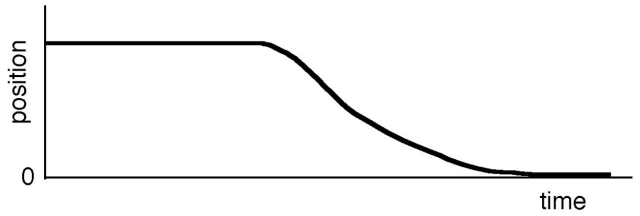

NATURAL LANGUAGE/WORDS DESCRIPTION
GRAPHICAL LANGUAGE
PICTORICAL DESCRIPTION
MATHEMATICAL LANGUAGE
OBSERVATIONAL DESCRIPTION

Interpreting representation and translating

FROM GRAPHICAL TO WORDS

Internal representation → External representation

6. Here is a graph of an object's motion. Which sentence is a correct interpretation?



A) The object rolls along a flat surface. Then it rolls forward down a hill, and then finally stops.
B) The object doesn't move at first. Then it rolls forward down a hill and finally stops.
C) The object is moving at a constant velocity. Then it slows down and stops.
D) The object doesn't move at first. Then it moves backwards and then finally stops.
E) The object moves along a flat area, moves backwards down a hill, and then it keeps moving.