Atomic Systems: Qubit control

Coherent control of a trapped two-level system

In the following, we will examine the interaction of the two-level system with a monochromatic laser field of frequency ω_L described by

PHASE

$$E(\vec{r},t) = \frac{E_0}{2}\left[\vec{\varepsilon}\, e^{-i(\omega_L t - \vec{k}\cdot\vec{r} + \varphi)} + \vec{\varepsilon}^{\,*}\, e^{i(\omega_L t - \vec{k}\cdot\vec{r} + \varphi)}\right]$$

POSITIVE FIELD AMPLITUDE

POLARIZATION VECTOR (COMPLEX)

$\vec{k} = k\hat{k}$ WAVE VECTOR

OVERALL PHASE OF THE LASER

ASSUME $\omega_L \simeq \omega_A$

Before we also assumed a dipole transition. It follows that the atom-filed Hamiltonian is:

$$H = H_0 + H_1 \quad \text{WITH} \quad H_0 = \hbar\frac{\omega_0}{2}\sigma_z + \hbar\omega_m\left(a^\dagger a\right)$$

NEGLECT $\frac{1}{2}$

$$H_1 = -\vec{D}\cdot\vec{E}$$

COUPLING OF A LIGHT FIELD TO A DIPOLE TRANSITION

DIPOLE OPERATOR $\vec{D} = e\,\vec{r}$

Remember that a dipole transition is one that occurs between states of opposite parity because the electric dipole moment operator is an odd-parity vector.

SAID DIFFERENTLY IT IS A TRANSITION FROM $|g\rangle \leftrightarrow |e\rangle \Rightarrow$ MUST BE PROPORTIONAL TO $\sigma_\pm$

$$\Rightarrow \vec{D} = d\left(\vec{\varepsilon}_A\,\sigma_- + \vec{\varepsilon}_A^{\,*}\,\sigma_+\right)$$

ATOMIC TRANSITION POLARIZATION

DIPOLE MATRIX ELEMENT

ATOMIC TRANSITION, POLARIZATION

$\sigma^\pm$ (CIRCULARLY POL. TRANSITION) $\quad \vec{\varepsilon}_A = \dfrac{(u_x \pm i u_y)}{\sqrt{2}}$

u_x, u_y, u_z VECTORS IN REAL SPACE

π TRANSITION $\quad \vec{\varepsilon}_A = \vec{u}_z$

Let's now remember that the atom is trapped in an harmonic oscillator. This means that the laser phase seen by the atom is dependent on it's position in the trap. The interaction Hamiltonian can then be written as:

$H_1(t)$ MUST BE OF THE FORM

$$\begin{pmatrix} 0 & \alpha \\ \beta & 0 \end{pmatrix} = \alpha\sigma_+ + \beta\sigma_-$$

BECAUSE THE DIPOLE

MATRIX IS OFF-DIAGONAL

$$H_1(t) = \frac{eE_0}{2} \vec{r}\cdot\vec{\varepsilon}\; e^{-i(\omega_L t - \vec{k}\cdot\vec{R} + \varphi)} + h.c$$

ELECTRON POSITION OPERATOR

EXTERNAL POSITION OPERATOR OF THE ATOM

THIS IS THE α TERM OF THE MATRIX

We said that the matrix contains only off diagonal terms. We can thus evaluate them: $\langle e| \dots |g\rangle$

$$\Omega_{eg} = \frac{eE_0}{2} \langle e| \vec{r}\cdot\vec{\varepsilon} |g\rangle$$

THIS IS TECHNICALLY NOT TRUE BUT THEY WOULD DROP OUT WHEN DOING ROTATING WAVE APPROXIMATION

AND SIMILARLY Ω_{ge}

WE CAN ALSO INTRODUCE THE LAMB-DICKE PARAMETER η

$$\vec{k}\cdot\vec{R} = \sum_s \eta_s (a_s^+ + a_s) \qquad \eta_s = k\,\hat{k}_s \sqrt{\frac{\hbar}{2m\,\omega_{m,s}}}$$

POSITION

$\alpha(a^+ + a)$

ALL MOTION DIRECTION

ROTATIONAL FREQUENCY

MASS

PROJECTION OF WAVE VECTOR ON ROTATIONAL DIRECTION

$$\Rightarrow H_1(t) = \frac{\hbar}{2}(\Omega_{eg}\sigma_+ + \Omega_{ge}\sigma_-)\, e^{-i(\omega_L t + \varphi)} \prod_s e^{i\eta_s(a_s^+ + a_s)} + h.c.$$

$$\Omega_{eg} = \Omega_{ge} = \Omega$$

It is useful to transform this to the interaction picture with respect to H_0 to determine resonances.

$$\sigma_+ \rightarrow e^{i\omega_e t}\sigma_+ \qquad a^+ \rightarrow e^{+i\omega_m t} a^+$$

$$\sigma_- \rightarrow e^{-i\omega_e t}\sigma_- \qquad a \rightarrow e^{-i\omega_m t} a$$

REMEMBER THAT $\omega_L \approx \omega_e \Rightarrow$ ROTATING WAVE APPROXIMATION NEGLECTS TERMS OSCILLATING AT $\omega_L + \omega_e$

$$\Rightarrow H_1^I(t) = \frac{\hbar\Omega}{2}\sigma_+ e^{-i[(\omega_L - \omega_A)t + \phi]} \prod_s \mathrm{EXP}\left[i\eta(a^+ e^{i\omega_m t} + a e^{-i\omega_m t})\right] + h.c.$$

Two more approximations can be made to simplify things: First, I assume that the laser is tuned close to a resonance involving only one motional mode, and that other possible resonances are far away compared to .   such that no significant excitations on off-resonant transitions occur. Second, motional excitations in the motional modes are assumed to be low such that $\eta\sqrt{\langle (a^\dagger + a)^2 \rangle} \ll 1$ holds at all times, which is called Lamb-Dicke regime

$\omega_L = \omega_a$ CARRIER TRANSITION

$$H^I_{1,C} = \frac{\hbar\Omega}{2}\left(1 - \eta^2 (a^\dagger a + a a^\dagger) + \dots\right)\sigma_+ e^{-i\varphi} + h.c$$

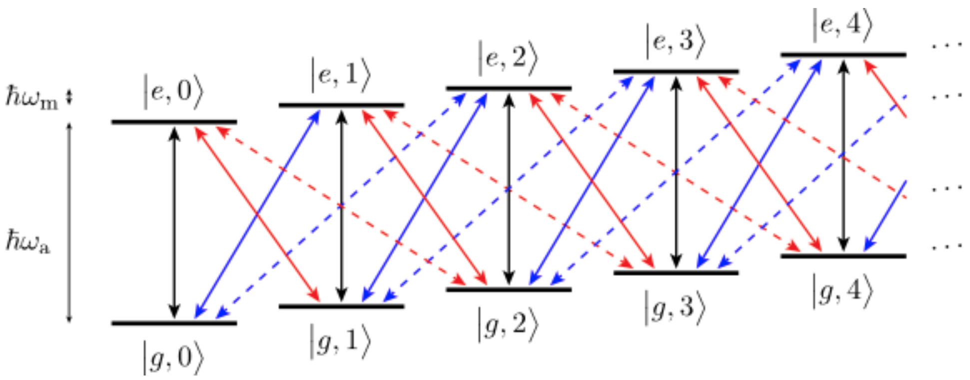
ROTATING WAVE APPROXIMATION

$\omega_L = \omega_a + \omega_1$ BLUE SIDEBAND

$$H^I_{1BSB} = \frac{\hbar\Omega}{2}\left(i a^\dagger + \dots\right)\sigma_+ e^{-i\varphi} + h.c$$

$\omega_L = \omega_a - \omega_1$ RED SIDEBAND

$$H^I_{1RSB} = \frac{\hbar\Omega}{2}\left(i a + \dots\right)\sigma_+ e^{-i\varphi} + h.c$$



— CARRIER
— RED SIDEBAND
— BLUE SIDEBAND

In the LD regime we only consider the first term of the exponential. We can thus see that

CARRIER : $|g, n\rangle \leftrightarrow |e, n\rangle$

RSB : $|g, n\rangle \leftrightarrow |e, n-1\rangle$

BSB $|g, n\rangle \leftrightarrow |e, n+1\rangle$

The coupling strength can be calculated as

CARRIER: $\langle e,n| H_1^I |g,n\rangle = \frac{\hbar\Omega}{2}$

RSB $\langle e,n-1| H_1^I |g,n\rangle = \frac{\hbar\Omega}{2}\eta \langle n-1| \underbrace{a|n\rangle}_{\sqrt{n}\,|n-1\rangle} = \frac{\hbar}{2} \Omega\sqrt{n}\,\eta$ $\quad \Omega_{RSB}$

BSB $\langle e,n+1| H_1^I |g,n\rangle = \frac{\hbar\Omega}{2}\eta \langle n+1| \overbrace{a^+|n\rangle}^{\sqrt{n+1}\,|n+1\rangle} = \frac{\hbar}{2} \Omega\eta\sqrt{n+1}$ $\quad \Omega_{BSB}$

Notice that the LD approximation is valid only for small LD parameters or for very low occupation of the motional state. Experimentally, assuming that one cannot change the mass of the atom or its motional frequency, a small LD parameter can be achieved if the laser k-vector does not couple well with the motional mode direction.

Most experiments try to work in the LD regime. However if this is not possible (too large LD parameter or too large motional occupation) we can still find an exact solution to the exponential. In this case we notice that we are not simply restricted to rotations between neighboring motional states but we can couple any $|n\rangle$ to any $|n'\rangle$

$$\langle n'| e^{i\eta(a^+ + a)} |n\rangle \qquad \text{WE USE} \quad e^{A+B} = e^A e^B e^{-[A,B]/2}$$

$$[a^+, a] = -1$$

$$\Rightarrow \langle n'| e^{i\eta a^+ + i\eta a} |n\rangle = e^{-\eta^2/2} \langle n'| e^{i\eta a^+} e^{i\eta a} |n\rangle$$

$$a^K |n\rangle = \begin{cases} \left[\frac{n!}{(n-K)!}\right]^{1/2} |n-K\rangle \\ 0 \qquad \text{IF } K > n \end{cases} \qquad \Leftarrow \quad a|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$\Rightarrow e^{i\eta a} |n\rangle = \sum_K \frac{(i\eta)^K}{K!} a^K |n\rangle = \sum_K \frac{(i\eta)^K}{K!} \frac{n!}{(n-K)!} |n-K\rangle$$

$$\Delta n = |n' - n|$$

$$\Rightarrow \langle n'| \dots |n\rangle = e^{-\eta^2/2} (n!\, n'!)^{1/2} (i\eta)^{\Delta n} \sum_{K=0}^{n_<} \frac{(-1)^K (\eta)^{2K}}{K!\,(K+\Delta n)!\,(n_< - K)!}$$

$n_<$ LESSER BETWEEN n AND n'

GENERALIZED LAGUERRE POLYNOMIAL $L_n^{\alpha}(x) = \sum_m^n (-)^m \binom{n+\alpha}{n-m} \frac{x^m}{m!}$

$$\Rightarrow \langle n'|\dots|n\rangle = e^{-\eta^2/2}\left[\frac{n_<!}{\underbrace{(n_<+\Delta n)!}_{n_>!}}\right]^{1/2} (i\eta)^{\Delta n} L_{n_<}^{\Delta n}(\eta^2)$$

$\Delta n \equiv s \leftarrow$ SIDEBAND WE DRIVE

$s=0$ CARRIER

$s=\pm 1$ BLUE OR RED SIDEBAND

This result is particularly importance when we want to know the Rabi frequency. We can absorb the exponential in the definition of the Rabi frequency

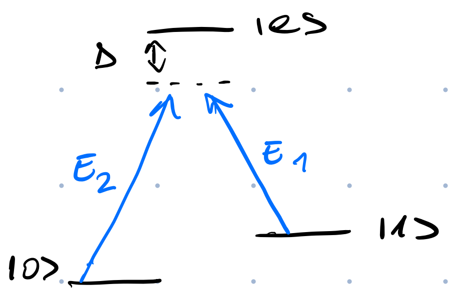
$$H_1^{\pm} = \hbar \underbrace{\frac{\Omega}{2} e^{i\eta(a^\dagger+a)}}_{\Omega_{s,n}} \sigma_+ e^{i\varphi} + h.c$$

$$\boxed{\Omega_{s,n} = \Omega\,|\langle n'|\dots|n\rangle| = \Omega\, e^{-\eta^2/2}\, \eta^{|s|} \sqrt{\frac{n_<!}{n_>!}}\, L_{n_<}^{|s|}(\eta^2)}$$

Important things to remember: LD parameter definition, condition for the LD approximation, interaction Hamiltoninan in the interaction picture, and just remember that the Rabi frequency depends on the motional state.

Raman transitions

If the qubit is encoded in hyperfine transition the qubit frequency is at most at GHz. This means that the qubit can be driven with microwave antennas. However, due to the long wavelength of this drive it becomes quite challenging to have individual qubit addressing. For this purpose it is often useful drive the qubit with two-photon stimulated Raman transitions. We consider the energy level structure



TWO LASERS: E_1 COUPLES $|1\rangle \leftrightarrow |e\rangle$

E_2 COUPLES $|0\rangle \leftrightarrow |e\rangle$