GENERALIZED LAGUERRE POLYNOMIAL $L_n^{\alpha}(x) = \sum_m^n (-)^m \binom{n+\alpha}{n-m} \frac{x^m}{m!}$

$$\Rightarrow \langle n'|\dots|n\rangle = e^{-\eta^2/2}\left[\frac{n_<!}{\underbrace{(n_<+\Delta n)!}_{n_>!}}\right]^{1/2}(i\eta)^{\Delta n} L_{n_<}^{\Delta n}(\eta^2)$$

$\Delta n \equiv s$ ← SIDEBAND WE DRIVE

$s=0$ CARRIER

$s=\pm 1$ BLUE OR RED SIDEBAND

This result is particularly importance when we want to know the Rabi frequency. We can absorb the exponential in the definition of the Rabi frequency

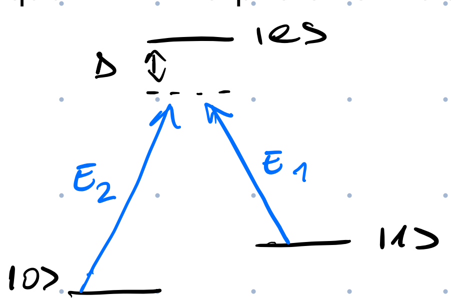
$$H_1^{\pm} = \hbar \underbrace{\frac{\Omega}{2} e^{i\eta(a^\dagger + a)}}_{\Omega_{s,n}} \sigma_+ e^{i\varphi} + h.c$$

$$\Omega_{s,n} = \Omega\,|\langle n'|\dots|n\rangle| = \Omega\, e^{-\eta^2/2}\, \eta^{|s|} \sqrt{\frac{n_<!}{n_>!}}\, L_{n_<}^{|s|}(\eta^2)$$

Important things to remember: LD parameter definition, condition for the LD approximation, interaction Hamiltoninan in the interaction picture, and just remember that the Rabi frequency depends on the motional state.

Raman transitions

If the qubit is encoded in hyperfine transition the qubit frequency is at most at GHz. This means that the qubit can be driven with microwave antennas. However, due to the long wavelength of this drive it becomes quite challenging to have individual qubit addressing. For this purpose it is often useful drive the qubit with two-photon stimulated Raman transitions. We consider the energy level structure



TWO LASERS: E_1 COUPLES $|1\rangle \leftrightarrow |e\rangle$

E_2 COUPLES $|0\rangle \leftrightarrow |e\rangle$

$$H_1 = \frac{\hbar \Omega_1}{2} e^{i\vec{K}_1\cdot\vec{R}} e^{-i\omega_1 t} e^{-i\phi_1} |e\rangle\langle 1| + \frac{\hbar \Omega_2}{2} e^{i\vec{K}_2\cdot\vec{R}} e^{-i\omega_2 t} e^{-i\phi_2} |e\rangle\langle 2| + h.c.$$

INTERACTION HAMILTONIAN

We will not solve the system. However if you want to do it, the best approach is to write the bare Hamiltonian. $H_0 = \hbar\omega_0 |0\rangle\langle 0| + \hbar\omega_1 |1\rangle\langle 1| + \hbar\omega_e |e\rangle\langle e|$ then set the energy 0 at the level of $\omega_0 \Rightarrow \hbar\omega_0 = 0$

Then go in the interaction picture and solve the Schrödinger equation of the population of the three states $|\psi\rangle = C_0(t)|0\rangle + C_1(t)|1\rangle + C_e|e\rangle$

Take then the approximation for large detunings $\Delta \gg \Omega_1, \Omega_2, \Gamma$

THIS IS VERY IMPORTANT

RABI LASER 2

RABI FREQ. OF LASER 1

DECAY RATE OF $|e\rangle$

In this approximation the population of the excited state can be adiabatically eliminated and you can find that

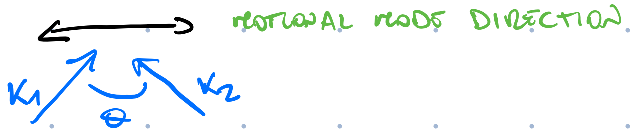
$$\boxed{\Omega_R = \frac{\Omega_1\Omega_2}{2\Delta}}$$

$$H_1 = \frac{\hbar\Omega_R}{2} e^{i\Delta\vec{K}\cdot\vec{R}} e^{-i\Delta\omega t} e^{-i\Delta\varphi t} + h.c.$$

BEHAVES SIMILARLY TO THE HAMILTONIAN OF A SINGLE LASER BUT WITH

$$\Delta\vec{K} = \vec{K}_1 - \vec{K}_2$$
$$\Delta\omega = \omega_1 - \omega_2$$
$$\Delta\varphi = \varphi_1 - \varphi_2$$

Remember that $\Delta\vec{K}\cdot\vec{R} = \sum_s \eta_s (a_s^+ + a_s)$ $\qquad \eta_s = K\cdot K_s \sqrt{\frac{\hbar}{2m\omega_s}}$



$$\vec{\Delta k} = \vec{k}_1 - \vec{k}_2 \Rightarrow |\Delta k| = 2k \sin(\theta/2)$$

CASE 1: COUNTER-PROPAGATING BEAMS $\xrightarrow{k_1} \xleftarrow{k_2}$

$$\theta = \pi \Rightarrow \eta_s = \underbrace{\boxed{\frac{4\pi}{\lambda}}}_{2k} \sqrt{\frac{\hbar}{2m\omega_s}}$$

← MAXIMUM η ⇒ MAXIMUM COUPLING TO THE ATOMIC MOTION

CASE 2: CO-PROPAGATING BEAMS $\xrightarrow{k_1}$ $\xrightarrow[k_2]{}$

$$\theta = 0 \Rightarrow \Delta k \simeq 0 \Rightarrow \eta = 0$$

Co-propagating beams do not couple to the atomic motion and can only drive carrier rotations

Carrier operations

$$\sigma_+ = \sigma_x + i\sigma_y$$
$$\sigma_- = \sigma_x - i\sigma_y$$

$$H^{\pm}_{carr} = \frac{\hbar\Omega}{2}\left(\sigma_+ e^{-i\varphi} + \sigma_- e^{+i\varphi}\right)$$
$$= \frac{\hbar\Omega}{2}\left(\sigma_x e^{-i\varphi} + i e^{-i\varphi}\sigma_y + \sigma_x e^{+i\varphi} - i e^{+i\varphi}\sigma_y\right)$$
$$= \frac{\hbar\Omega}{2}\left(\sigma_x (e^{i\varphi} + e^{-i\varphi}) - i\sigma_y (e^{+i\varphi} - e^{-i\varphi}\right)$$
$$= \hbar\Omega\left[\sigma_x \cos(\varphi) + \sigma_y \sin(\varphi)\right]$$

$$\cos\varphi = \frac{e^{i\varphi} + e^{-i\varphi}}{2}$$

$$\sin\varphi = \frac{e^{i\varphi} - e^{-i\varphi}}{2i}$$

Carrier rotations are rotations around an axis on the equator of the Bloch sphere. This Hamiltonian is true for all transitions (Raman, quadrupole etc) as long as we can neglect decays from the excited state to the ground state and the LD approximation is satisfied.

Operations in the presence of non-negligible motional contribution

In the previous section we assume LD approximation which means that, for carrier operations, we can neglect the role of the motional modes. However, this is rarely the case in real platforms. Before getting into it that let's have a look at some particular motional states.

FOCK STATES: $|n\rangle$ EIGESTATES OF ENERGY AND NUMBER OPERATOR $\hat{N} = a^{\dagger}a$

$$\Rightarrow a^{\dagger}a|n\rangle = n|n\rangle$$

COHERENT STATES: $|\alpha\rangle$ EIGENSTATES OF a

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

GENERATED FROM DISPLACEMENT OPERATOR $\hat{D}(\alpha)|0\rangle$ WITH

$$D(\alpha) = e^{\alpha a^{\dagger} + \alpha^{*} a}$$

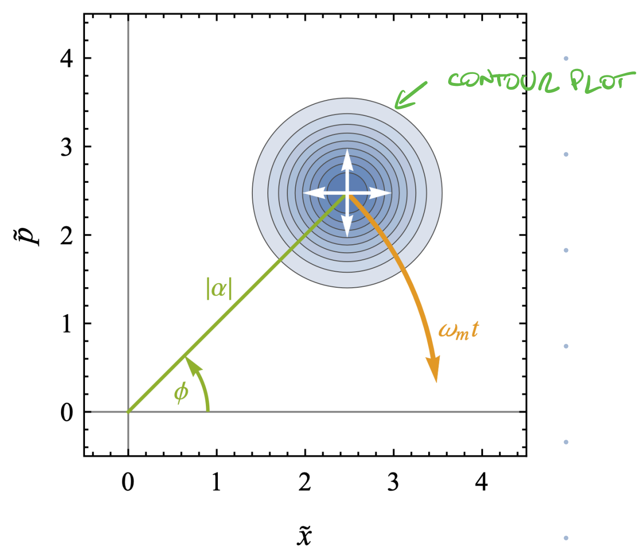
$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

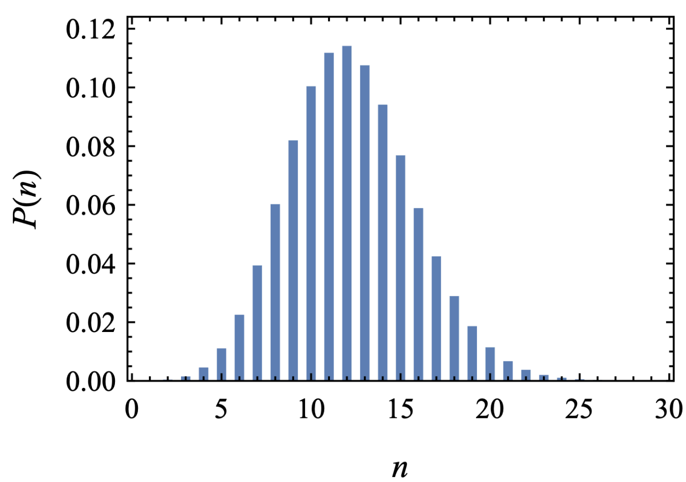
The time-evolution of a coherent state in the Schrödinger picture reads

THE NUMBER STATE DISTRIBUTION IS POISSONIAN $P_\alpha(n) = |\langle n|\alpha\rangle|^2 = e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}$

$$|\alpha(t)\rangle = e^{-iHt/\hbar}|\alpha(0)\rangle = |\alpha(0)e^{-i\omega_m t}\rangle$$

$H = \hbar\omega_m(a^{\dagger}a + 1/2)$





THERMAL STATES

A harmonic oscillator in thermal equilibrium with a heat bath is in a mixed state with the number-state populations weighted by the Boltzmann factor $\exp[-n\hbar\omega/(k_B T)]$

FOCK STATE NUMBER

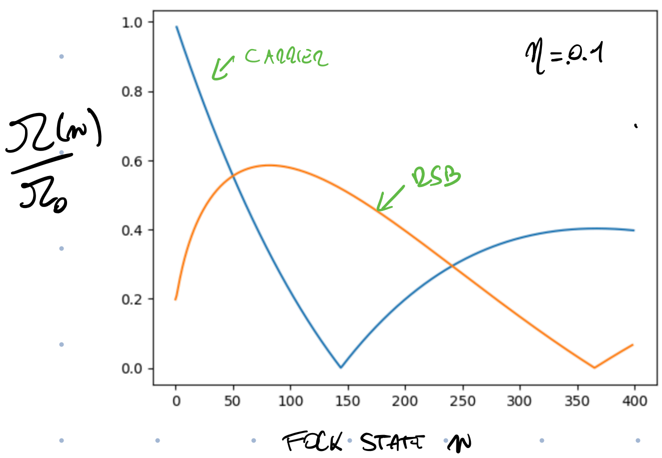
$$\Rightarrow \langle n \rangle = \bar{n} = \frac{1}{e^{\hbar\omega_m/k_B T}-1} \qquad \Rightarrow P(n) = \frac{1}{1+\bar{n}}\left(\frac{\bar{n}}{1+\bar{n}}\right)^n$$

The state is a mixed state, so we have to write it with the density matrix formalism

$$\rho = \sum P(n)\,|n\rangle\langle n| = \frac{1}{1+\bar{n}}\sum\left(\frac{\bar{n}}{1+\bar{n}}\right)^n |n\rangle\langle n|$$

All motional states seen above are important for the rest of the course

Rabi frequency dependence as a function of Fock states

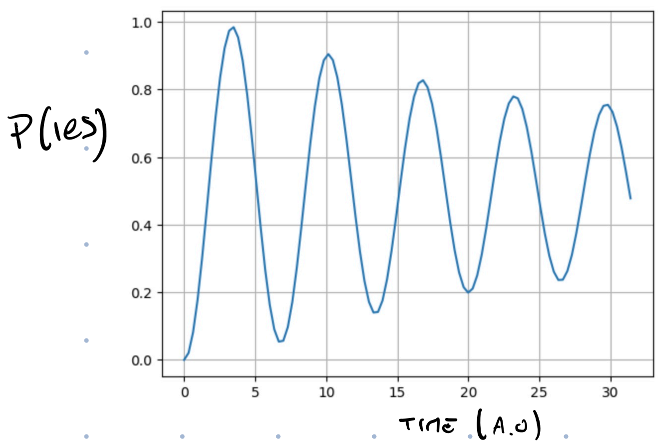


IN LD APPROXIMATION

$$\frac{\Omega_{carr}(n)}{\Omega_0} = 1$$

$$\frac{\Omega_{RSB}(n)}{\Omega_0} = \eta\sqrt{n}$$

Working outside the LD approximation has a direct effect on qubit operations. This is because each Fock state evolves at a different frequency. This causes beating



$\bar{n} = 0.5$, $\eta = 0.3$

$$P(|e\rangle) = \sum_n \frac{P(n)}{2}\left(1-\cos(\Omega(n)\,t)\right)$$

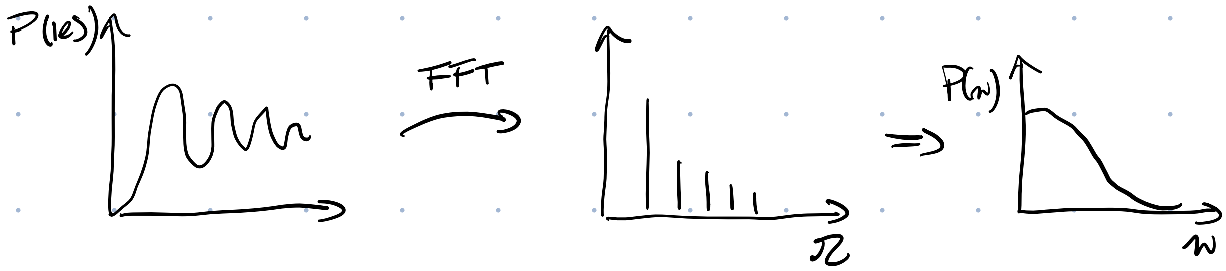
Diagnosing the motional state

We can make use of the Rabi frequency dependence of the carrier or the sidebands to diagnose the thermals state population. For this use the sideband is more convenient as it is proportional to the first order with η , while the carrier transition is second order.

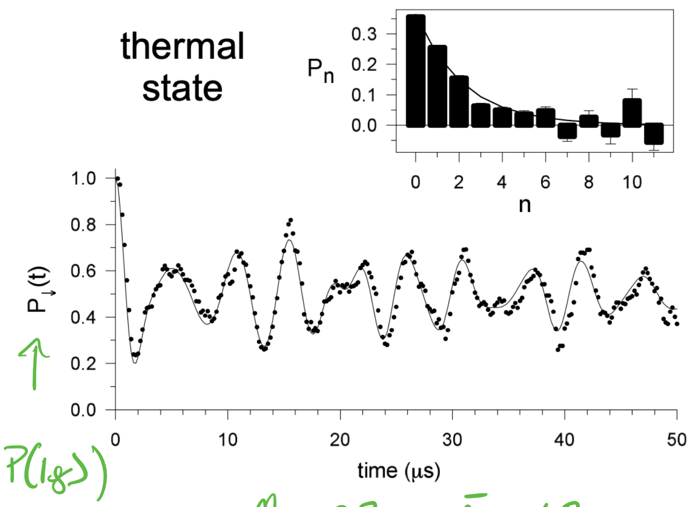
A commonly used tool is the to perform Rabi oscillations using the blue sideband transition where the Rabi frequency, in the LD approximation scales as $\sqrt{n+1}$.

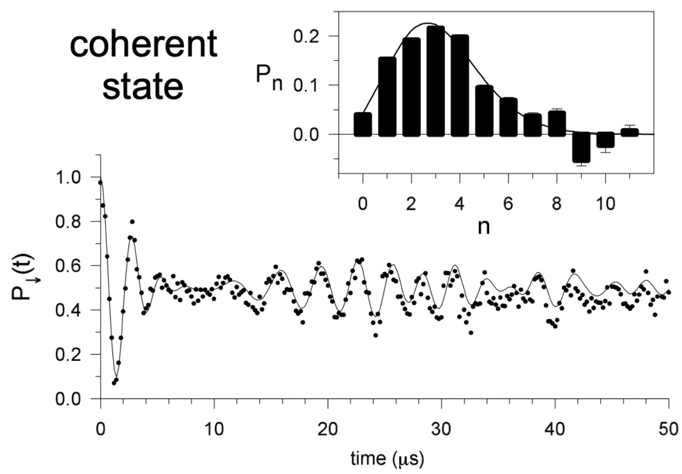
$$P(|e\rangle) = \frac{1}{2} \sum P(n) \left(1 - \cos(\Omega_{BSB}(n)\, t)\right)$$

By doing a Fourier transformation we can extract the different Rabi frequency and so reconstruct the motional state occupancy



Example: Itano et al http://tf.nist.gov/general/pdf/1173.pdf





$\eta = 0.2$ $\bar{n} = 1.3$ $\bar{n} = 3.1$