Preference and choice

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Introduction

Choice is the central action of the economic agent.

- A consumer chooses
  - over a set of goods → consumer choice
  - how much to save vs consume / a manager chooses over plans for production → intertemporal choice
- An investor chooses over portfolios → choice under risk
- People choose other people (to work with, to date, to be political representatives, etc)

When is choice ``rational"? what properties should it satisfy?

Introduction

- Starting point: the *set of possible alternatives* from which the individual must choose
- We denote this set by X
- Two different approaches to modelling individual choice behaviour:

  1. ***Preference-based approach.*** *The preference relation* is the primitive characteristic of the individual

     This theory imposes rationality axioms on the decision maker's preferences

     Analysis of the consequences on the individual's choice behaviour

2. ***Choice-based approach.*** Choice behaviour is the primitive characteristic of the individual.

It makes assumptions directly on the behaviour

The weak axiom of revealed preferences imposes an element of consistency

Attractive features of this approach

a. Room of more general forms of individual behaviour
b. Assumptions are on directly observable objects
c. Behavioural foundations of the theory of individual decision making

Preference-based approach

The objective of the decision maker are summarized by a *preference relation* that is denoted by $\succcurlyeq$

$\succcurlyeq$ is a binary relation over the set of alternatives X

It allows the comparison of pairs of alternatives $x, y \in X$.

$x \succcurlyeq y$ means "*x is at least good as y*"

(i) *The strict preference relation* $\succ$ is defined as

$$x \succ y \iff (x \succcurlyeq y \; but \; not \; y \succcurlyeq x)$$

and it means "*x is preferred to y*"

(ii) *The indifference relation* $\sim$ is defined as

$$x \sim y \iff (x \succcurlyeq y \; and \; y \succcurlyeq x)$$

and it means "*x is indifferent to y*"

a *preference relation* $\succcurlyeq$ is rational if the following hypothesis are satisfied:

a. Completeness

For all $x, y \in X$ we have that $x \succcurlyeq y \; or \; y \succcurlyeq x$ (or both)

b. Transitivity

For all $x, y, \mathrm{z} \in X$ if $x \succcurlyeq y$ and $\mathrm{y} \succcurlyeq \mathrm{z}$, then $x \succcurlyeq \mathrm{z}$

About ***Completeness***

Under this assumption an individual has well defined preferences between any two possible alternatives

It is a strong assumption:

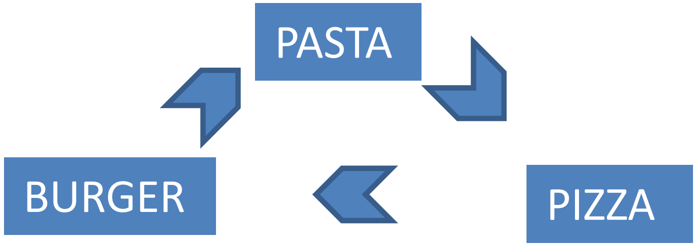
To evaluate alternative that are far from the realm of common experience can be difficult

To do it requires time and work

About ***Transitivity***

Under this assumption is not possible to find a decision maker with a sequence of choices such that her preferences appear to cycle:

For example an individual that prefers a pasta over a pizza, a pizza over a burger and a burger over a pasta.



It is hard to satisfy when alternative that are far from the realm of common experience

This assumption is fundamental for most of the economic theory

The two hypothesis of transitivity and completeness have the following implications.

$\succcurlyeq$ satisfies transitivity and completeness, then:

1. $\succ$ is irreflexive and transitive
2. $\sim$ is reflexive, transitive and symmetric
3. If $x \succ y \succcurlyeq z$ then $x \succ z$

Violations of transitivity

1. Perceptible differences
2. Framing effect
3. Regret theory
4. .....

LICHTENSTEIN, S., AND P. SLOVIC (1971): "**Reversals of Preferences Between Bids and Choices in Gambling Decisions**“, Journal of Experimental Psychology, 89, 46-55.

G. Loomes, C. Starmer and R. Sugden: “**Observing Violations of Transitivity by Experimental Methods**”, Econometrica, Vol. 59, No. 2 (Mar., 1991), pp. 425-439

Utility functions

In economics we often describe preference relation by means of a *utility function*

An utility function $u(x)$ assigns a numerical value to each element in X

Definition:

A function $u: X \to R$ is a utility function representing preference relation $\succcurlyeq$ if for all $x, y \in X$,

$$x \succcurlyeq y \leftrightarrow u(x) \geq u(y)$$

Utility functions

A utility function representing $\succcurlyeq$ is not unique

For any strictly increasing function $f: R \rightarrow R$, $v(x) = f(u(x))$ is a new utility function representing the same preferences as $u(.)$.

Properties of $u(.)$ that do not change for any increasing transformation are called *ordinal.*

Properties of $u(.)$ that change for some increasing transformation are called *cardinal.*

Then a preference relation associated to an utility function is an ordinal property.

The magnitude of any differences in the utility measure between alternatives are cardinal properties

Proposition

A preference relation $\succcurlyeq$ can be represented by a utility function only if it is rational

Proof:

Completeness

$u(.)$ is a real valued function defined on X. Then for any $x, y \in X$ either $u(x) \geq u(y)$ or $u(y) \geq u(x)$. Then it implies that either $x \succcurlyeq y$ or $y \succcurlyeq x$. Hence $\succcurlyeq$ must be complete.

Transitivity

Suppose $x \succcurlyeq y$ or $y \succcurlyeq z$. Then must be $u(x) \geq u(y)$ and $u(y) \geq u(z)$. Therefore $u(x) \geq u(z)$. This implies $x \succcurlyeq z$

Previous proposition provides only necessary conditions, not sufficient ones

Then not all rational preferences can be represented by an utility function

Example: lexicographic preferences

Proposition (Representation theorem , Debreu)

A preference relation $\succcurlyeq$ *can be represented by a utility function if it is satisfying rationality, continuity and strict monotonicity*

continuity

$\forall y \in X, \{x \in X | x \succcurlyeq y\}$ *and* $\{x \in X | y \succcurlyeq x\}$ *are closed sets*

strict monotonicity

$\forall \boldsymbol{x} \neq \boldsymbol{y}, x_l \geq y_l \, \forall l \rightarrow \boldsymbol{x} \succ \boldsymbol{y}$

lexicographic preferences do not satisfy continuity

$x_n = \left\{1 + \frac{1}{n}, 1\right\} x_0 = \{1, 3\}$

For $n > 0$ $x_n \succ x_0$ but for $n \rightarrow \infty$ $x_0 \succ x_1$

Choice-based approach

Choice behaviour is represented by means of a *choice structure* $(\mathcal{R}, C(\cdot))$ where:

$\mathcal{R}$ is a family of nonempty subsets of X, i.e. every element of $\mathcal{R}$ is a set $B \subseteq X$.

Sometime B is called budget set.

$C(\cdot)$ is a choice rule that assigns a nonempty set of chosen elements $C(B) \subset B$ for every budget set $B \subseteq X$

Note, $C(B)$ could contain more than one elements

Example

Suppose $X = \{x, y, z\}$, $\mathcal{R} = \{\{x, y\}, \{x, y, z\}\}$ and the choice rule $C_1(\cdot)$ is $C_1(\{x, y\}) = \{x\}$ and $C_1(\{x, y, z\}) = \{x\}$

In this case x is chosen in all budget sets consumer faces

Suppose a choice rule $C_2(\cdot)$ is $C_2(\{x, y\}) = \{x\}$ and $C_2(\{x, y, z\}) = \{x, y\}$.

In this case x is chosen when the decision maker faces $\{x, y\}$. But we could observe either x or y chosen when the decision maker faces $\{x, y, z\}$

Weak axiom of revealed preferences

This axiom imposes a certain amount of consistency to the individual's observed choices:

If an individual chooses (only) x when she faces a budget set $\{x, y\}$, she will not choose y when she faces $\{x, y, z\}$.

Formally:

<u>Definition</u>

The choice structure $\left(\mathcal{R}, C(\cdot)\right)$ *satisfies the weak axiom of revealed preferences if for some* $B \in \mathcal{R}$ *with* $\{x, y\} \in B$ *we have* $x \in C(B)$*, then for any* $B' \in \mathcal{R}$ *with* $\{x, y\} \in B'$ *and* $y \in C(B')$*, we must also have* $x \in C(B')$*.*

Weak axiom of revealed preferences

In words:

If x is chosen when y is available then there is no budget set containing x and y for which y is chosen and x is not.

If $C(\{x, y\}) = \{x\}$ we cannot have $C(\{x, y, z\}) = \{y\}$.

Revealed preference relation $\succcurlyeq^*$

Definition:

Given a choice structure $(\mathcal{R}, C(\cdot))$ the *revealed preference relation* $\succcurlyeq^*$ is defined as:

$$x \succcurlyeq^* y \leftrightarrow \ \exists\, B \in \mathcal{R}\ s.t.\ x, y \in B\ and\ x \in C(B)$$

We read $x \succcurlyeq^* y$ as:

x is revealed at least as good as y

To say that

x is revealed preferred to y we need that $\exists\, B \in \mathcal{R}\ s.t.\ x, y \in B\ and\ y \notin C(B)$

Revealed preference relation $\succcurlyeq^*$

We can restate the weak axiom of revealed preferences as:

If x is revealed at least as good as y, then y cannot be revealed preferred to x.

Example:

$X = \{x, y, z\}, \mathcal{R} = \{\{x, y\}, \{x, y, z\}\}$

i) choice rule $C_1(\cdot)$ is $C_1(\{x, y\}) = \{x\}$ and $C_1(\{x, y, z\}) = \{x\}$

The axiom is satisfied

ii) Choice rule $C_2(\cdot)$ is$C_2(\{x, y\}) = \{x\}$ and$C_2(\{x, y, z\}) = \{x, y\}$.

The axiom is violated

The relation between the two approaches

Two fundamental questions:

1) Suppose a decision maker with rational preference relation $\succcurlyeq$. Do her decisions satisfy the weak axiom?
2) Suppose a decision maker with a choice structure $\left(\mathcal{R}, C(\cdot)\right)$ satisfying the weak axiom. Is there a rational preference relation that is consistent with this choice structure?

Answer to question 1

Suppose an individual has rational preference relation $\succcurlyeq$ on X.

If this individual faces a subset of alternatives $B \subset X$ his behaviour is represented by:

$$C^*(B, \succcurlyeq) = \{x \in B : x \succcurlyeq y \; for \; every \; y \in B\}$$

Then in $C^*(B, \succcurlyeq)$ there are the decision maker's most preferred alternatives in B.

Suppose that preference relation $\succcurlyeq$ and families of budget sets $\mathcal{R}$ such that $C^*(B, \succcurlyeq) \neq \emptyset \; \forall B \in \mathcal{R}$.

Then we say that rational preference relation $\succcurlyeq$ generates the choice structure $\left(\mathcal{R}, C^*(B, \succcurlyeq)\right)$

Proposition:

Suppose that $\succcurlyeq$ is a rational preference relation. Then the choice structure generated by $\succcurlyeq$, $C^*(B, \succcurlyeq)$ satisfies the weak axiom

Proof.

Suppose that for some $B \in \mathcal{R}$ we have $x, y \in B$ and $x \in C^*(B, \succcurlyeq)$. It implies that $x \succcurlyeq y$. Suppose now that for some $B' \in \mathcal{R}$ we have $x, y \in B'$ and $y \in C^*(B', \succcurlyeq)$. It implies that $y \succcurlyeq z \, \forall z \in B'$. So, by transitivity (we already know $x \succcurlyeq y$) must be that $x \succcurlyeq z \, \forall z \in B'$. Therefore $x \in C^*(B', \succcurlyeq)$

Answer to question 2

Definition:

Given a choice structure $\left(\mathcal{R}, C(\cdot)\right)$ the rational preference relation $\succcurlyeq$ rationalizes $C(\cdot)$ relative to $\mathcal{R}$ if:

$$C(B) = C^*(B, \succcurlyeq)$$

for all $B \in \mathcal{R}$. That is $\succcurlyeq$ generates the choice structure $\left(\mathcal{R}, C(\cdot)\right)$.

The meaning is that preferences can explain behaviour and we can think of the decision maker as a preference maximizer.

Note that only a choice rule that satisfy the weak axiom can be rationalized.

But the weak axiom is not enough to ensure the existence of a rationalizing preference relation.

Example:

$X = \{x, y, z\},\ \mathcal{R} = \big\{\{x, y\}, \{x, z\}, \{z, y\}\big\}$ and

$C(\{x, y\}) = \{x\}, C(\{z, y\}) = \{y\}, C(\{x, z\}) = \{z\}$

This choice structure satisfies the weak axiom

To rationalize the first two choices we need that $x \succ y$ and $y \succ z$. Then a rational preference relation implies that $x \succ z$ but this contradicts the third choice.

Proposition:

If $(\mathcal{R}, C(\cdot))$ is a choice structure such that:

i. The weak axiom is satisfied

ii. $\mathcal{R}$ includes all subsets of X up three element

Then there is a rational preference relation $\succ$ that rationalize $C(\cdot)$ relative to $\mathcal{R}$, i.e. $C(B) = C^*(B, \succcurlyeq)$ $\forall B \in \mathcal{R}$

Furthermore this is the unique preference relation that does so.

Proof.

Consider the revealed preferences relation $\succcurlyeq^*$.

We have to prove that:

1. $\succcurlyeq^*$ is a rational preference relation
2. $\succcurlyeq^*$ rationalizes $C(\cdot)$ on $\mathcal{R}$
3. $\succcurlyeq^*$ is unique

1. We check completeness and transitivity of $\succcurlyeq^*$

a. Completeness. By assumption ii) all pairs $\{x, y\} \in \mathcal{R}$. Since either x or y must be an alement of $C(\{x, y\})$ we have $x \succcurlyeq^* y$ or $y \succcurlyeq^* x$ or both. Then $\succcurlyeq^*$ is complete

b. Transitivity. Let $x \succcurlyeq^* y$ and $y \succcurlyeq^* z$. We have to prove that $x \succcurlyeq^* z$. Suppose that $y \in C(x, y, z)$. Given that $x \succcurlyeq^* y$ and the weak axiom holds we have that $x \in C(x, y, z)$. Suppose that $z \in C(x, y, z)$. Given that $y \succcurlyeq^* z$ and the weak axiom holds we have that $y \in C(x, y, z)$ as in the previous case

2. We check that $\succcurlyeq^*$ rationalizes $C(\cdot)$ on $\mathcal{R}$

i.e. $C(B) = C^*(B, \succcurlyeq^*)$ $\forall B \in \mathcal{R}$

Suppose that $x \in C(B)$, then $x \succcurlyeq^*$y $\forall y \in B$, so we have that $x \in C^*(B, \succcurlyeq^*)$. It means that $C(B) \subset C^*(B, \succcurlyeq^*)$

Next suppose that $x \in C^*(B, \succcurlyeq^*)$. It implies that $x \succcurlyeq^*$y $\forall y \in B$; for each $y \in B$ it exists some set B_y such that $x, y \in B_y$ and $x \in C(B_y)$. It implies that $\mathrm{x} \in C(B)$. Hence $C^*(B, \succcurlyeq^*) \subset C(B)$. Therefore $C^*(B, \succcurlyeq^*) = C(B)$.

3. Uniqueness. It derives from the observation that $\mathcal{R}$ includes all subsets of two element. Then the choice behavior of $C(.)$ completely determine the pairwise preference relation over X.