993SM - Laboratory of Computational Physics week III - Friday 10 October 2025

Maria Peressi
Università degli Studi di Trieste - Dipartimento di Fisica
Sede di Miramare (Strada Costiera 11, Trieste)
e-mail: peressi@units.it
tel.: +39 040 2240242

Random Walks 1D

cosa dobbiamo tenere in memoria
se vogliamo solo i comportamenti medi,
pur istantanei (cioè step per step) $\langle x_i \rangle, \langle \Delta x_i^2 \rangle$?

ogni step di ogni camminatore?

non serve….

Random Walks 1D

The basic algorithm:

ix = position of the walker
x_N, x2_N = cumulative quantities
rnd(N) = sequence of N random numbers

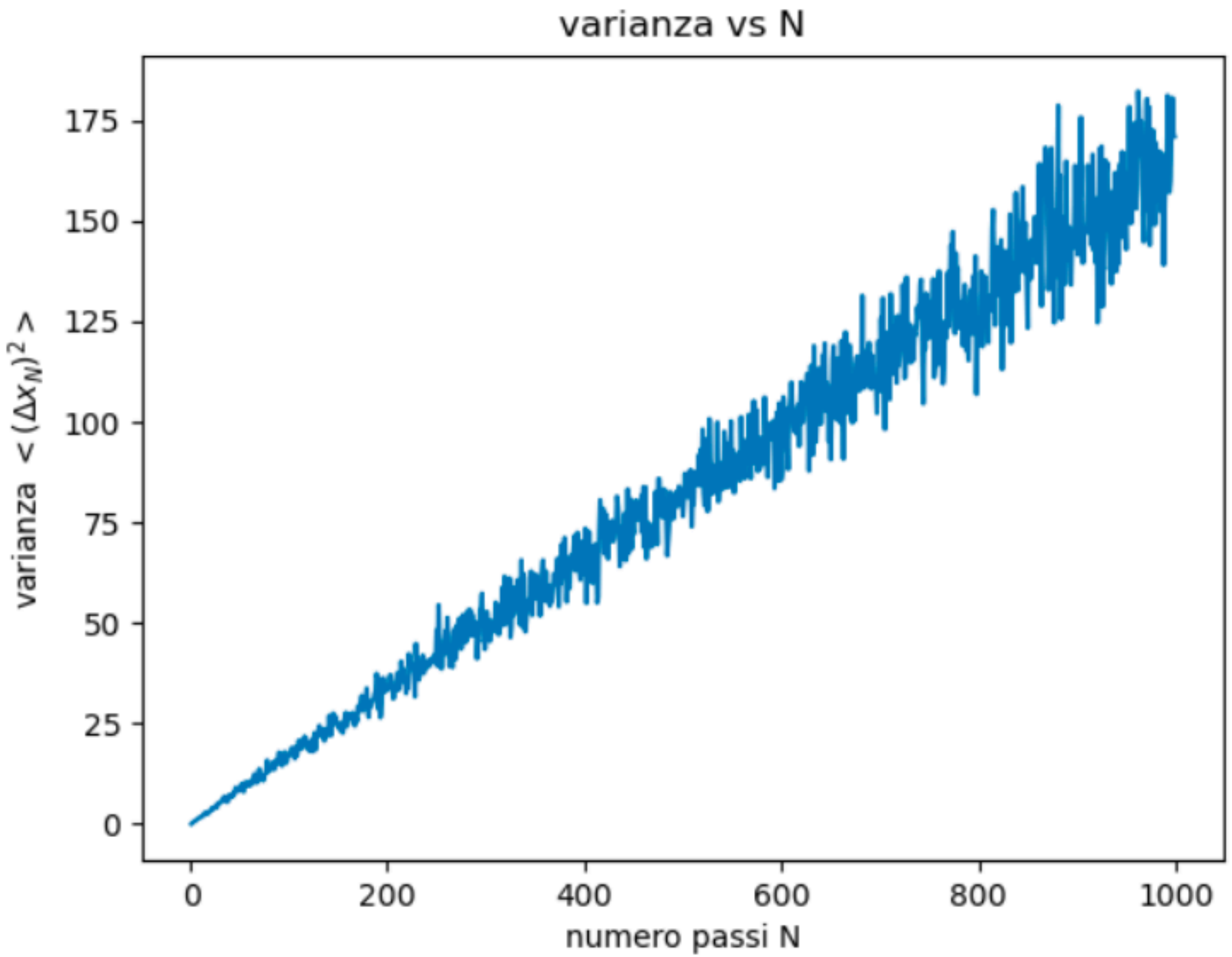
(1 run= 1 particle= 1 walker)

```
do irun = 1, nruns
   ix = 0 ! initial position of each run
   call random_number(rnd) ! get a sequence of random numbers
   do istep = 1, N
      if (rnd(istep) < 0.5) then ! random move
         ix = ix - 1 ! left
      else
         ix = ix + 1 ! right
      end if
      x_N (istep) = x_N (istep) + ix
      x2_N(istep) = x2_N(istep) + ix**2
   end do
   P_N(ix) = P_N(ix) + 1 ! accumulate (only for istep = N)
end do
```

But we can monitor what happens for each intermediate step by using arrays x_N() and x2_N() and including the calculation inside the loop on the steps

Random Walks 1D

var



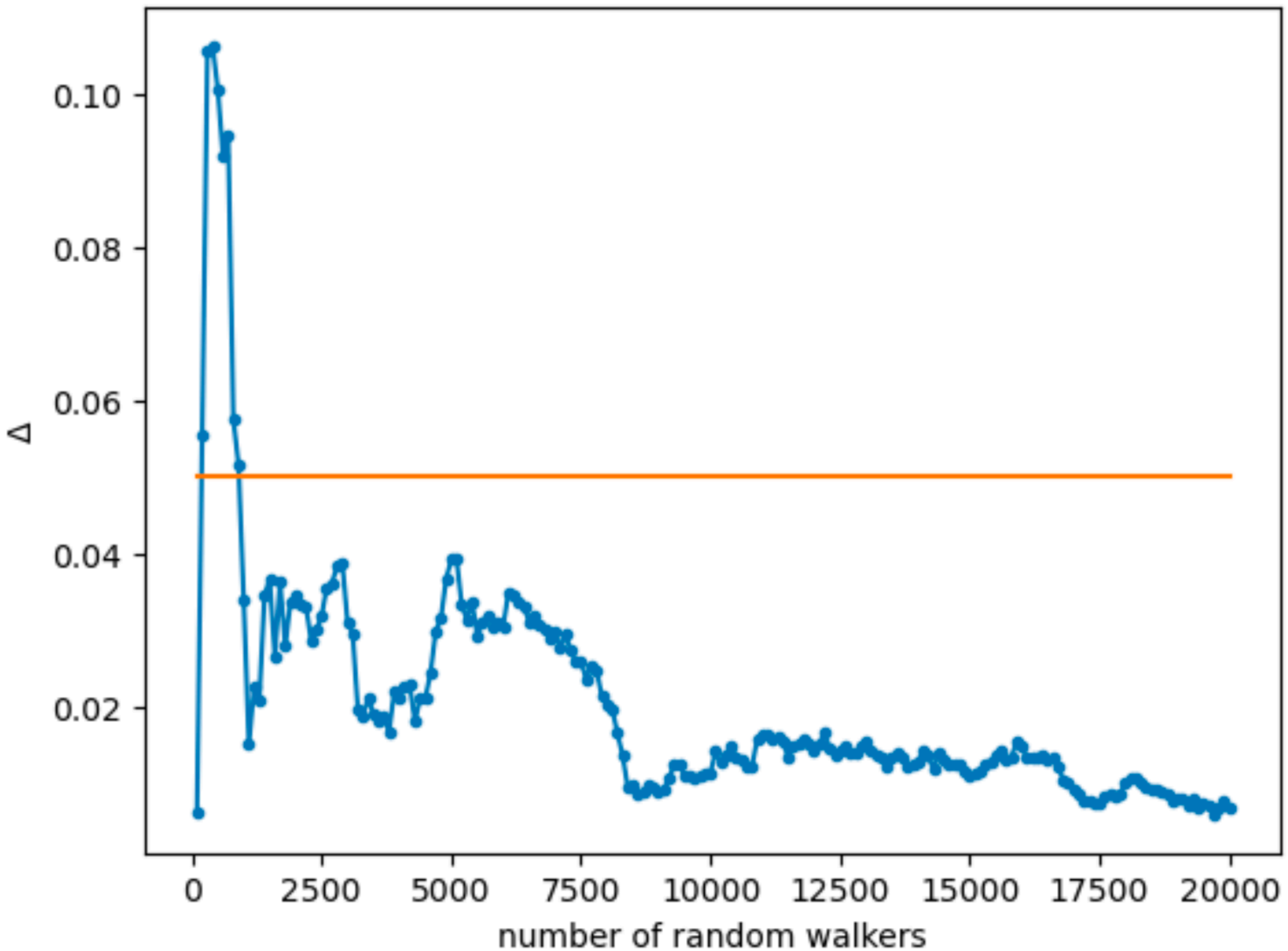
numero di camminatori fissato (qui 150)

Random Walks 1D

accuracy

$$\Delta = \left| \frac{< (\Delta x)^2_N >}{< (\Delta x)^2_N >^{th}} - 1 \right|$$

Δ non è regolare
(non diminuisce
monotonicamente)



Random Walks 1D

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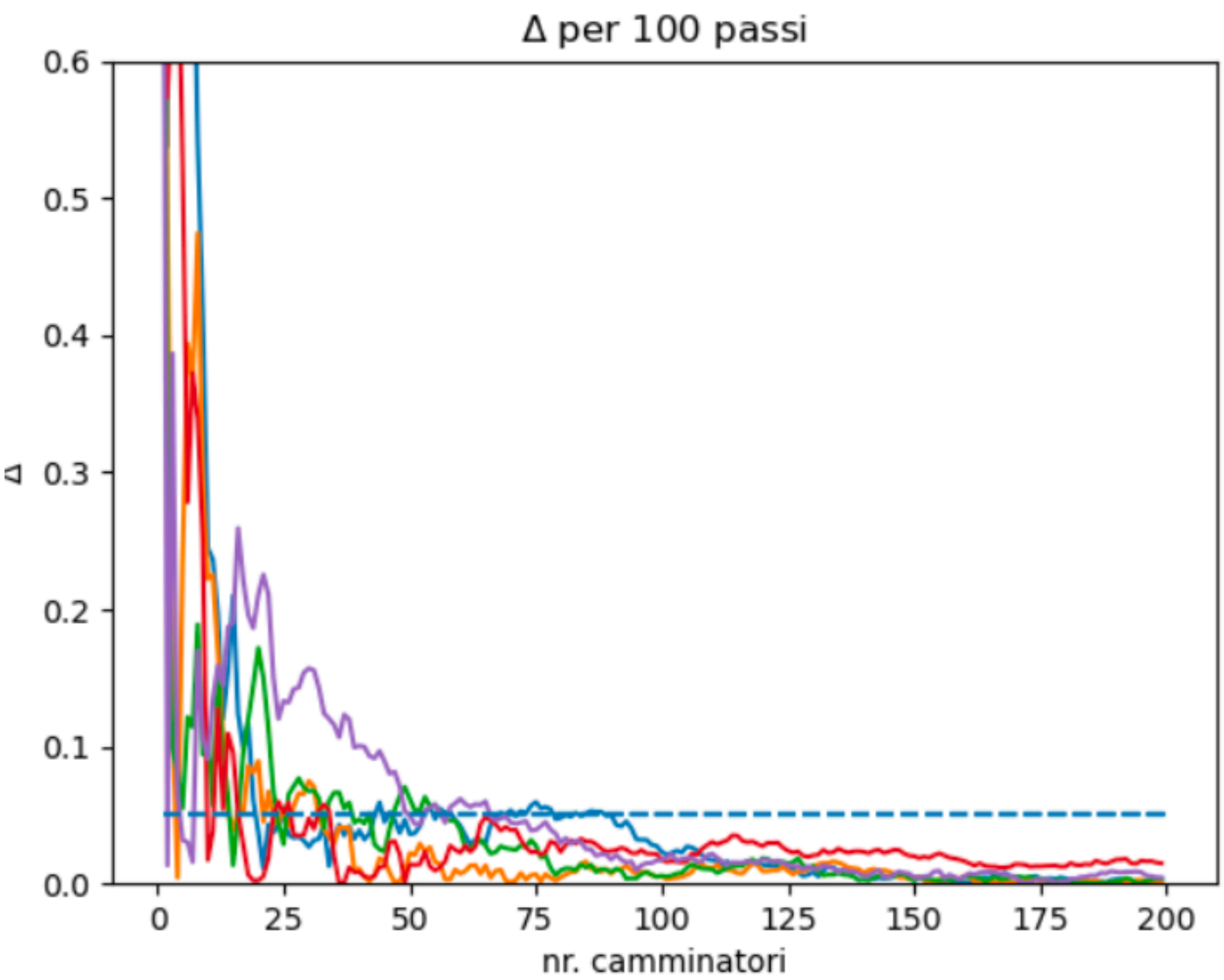


Random Walks 2D

Generating 2-D random unit steps

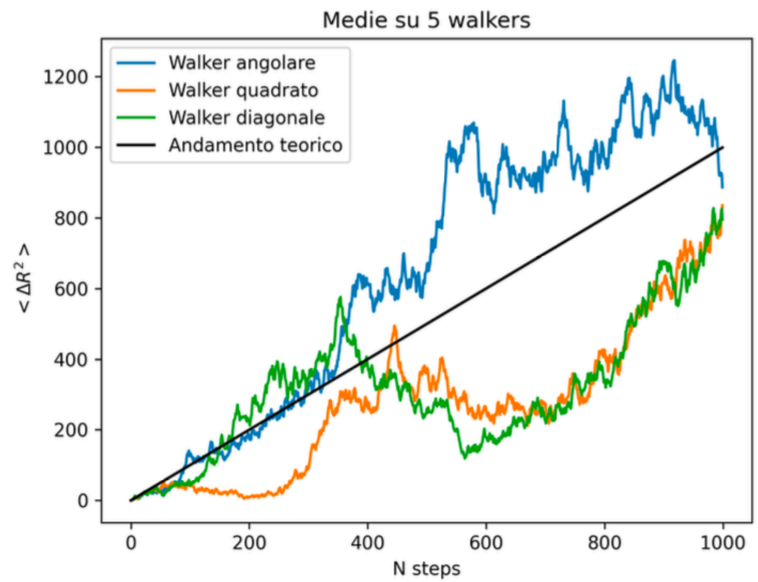
1. Choose θ a random number in the range $[0, 2\pi]$ and then set $x = \cos\theta, y = \sin\theta$.
2. Choose a random value for Δx in the range $[-1, 1]$ and $\Delta y = \pm\sqrt{1 - \Delta x^2}$ (choose the sign randomly too).
3. Choose separate random values for $\Delta x, \Delta y$ in the range $[-1, 1]$ (but not $\Delta x = 0, \Delta y = 0$). Normalize $\Delta x, \Delta y$ so that the step size is 1.
4. Choose a direction (N, E, S, W) randomly as the step direction (no trigonometric functions are then needed). Note, choosing one of four directions is equivalent to choosing a random *integer* on [0,3].
5. Choose separate random values $\Delta x, \Delta y$ in the range $\left[-\sqrt{3/2}, \sqrt{3/2}\right]$

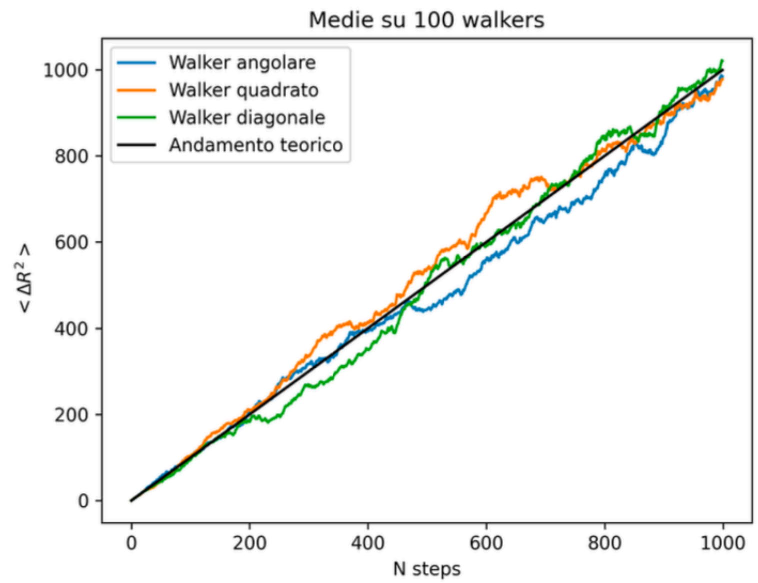
TEST DIFFERENT ALGORITHMS!

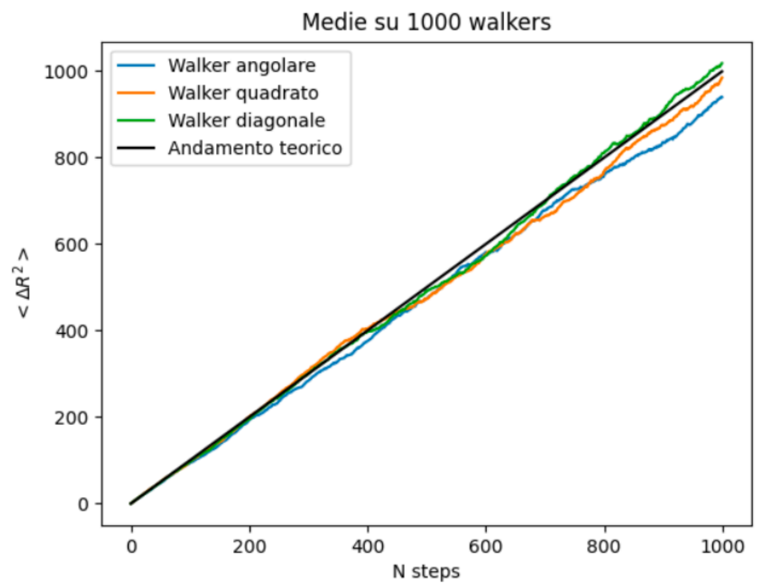
WHAT IS THE BEST? THE ONE WHICH GIVES THE BEST BEHAVIOR?

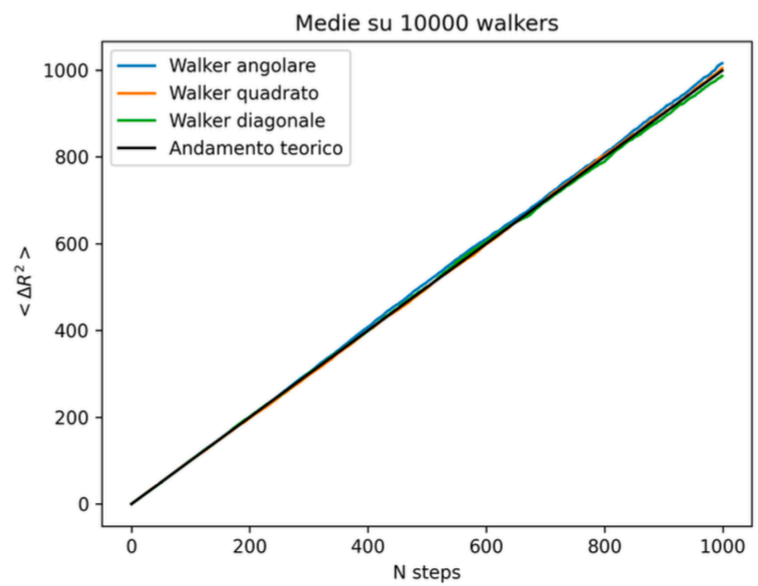
WHAT IS THE MOST EFFICIENT?

Random Walks 2D









Random Walks 2D

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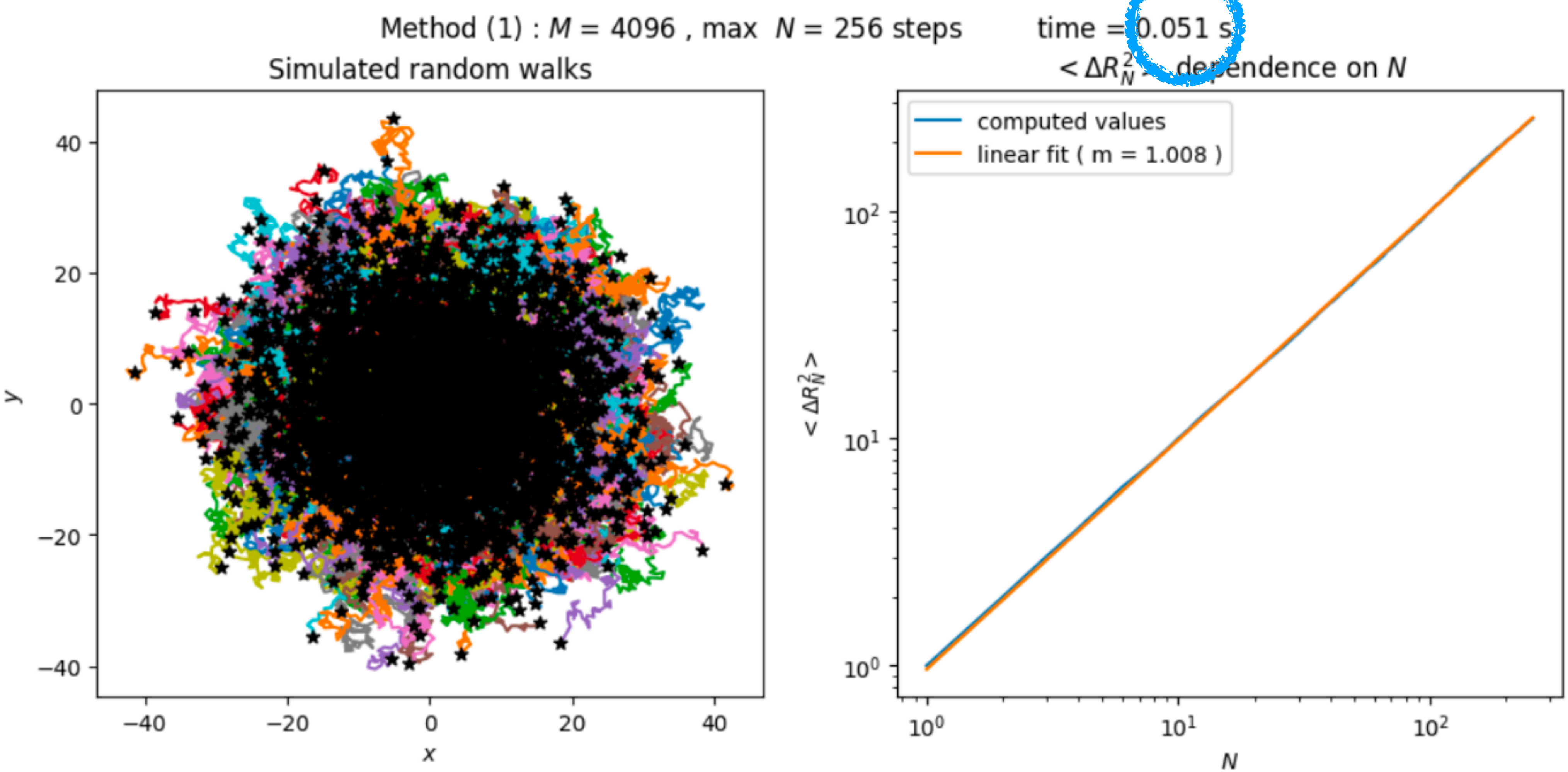
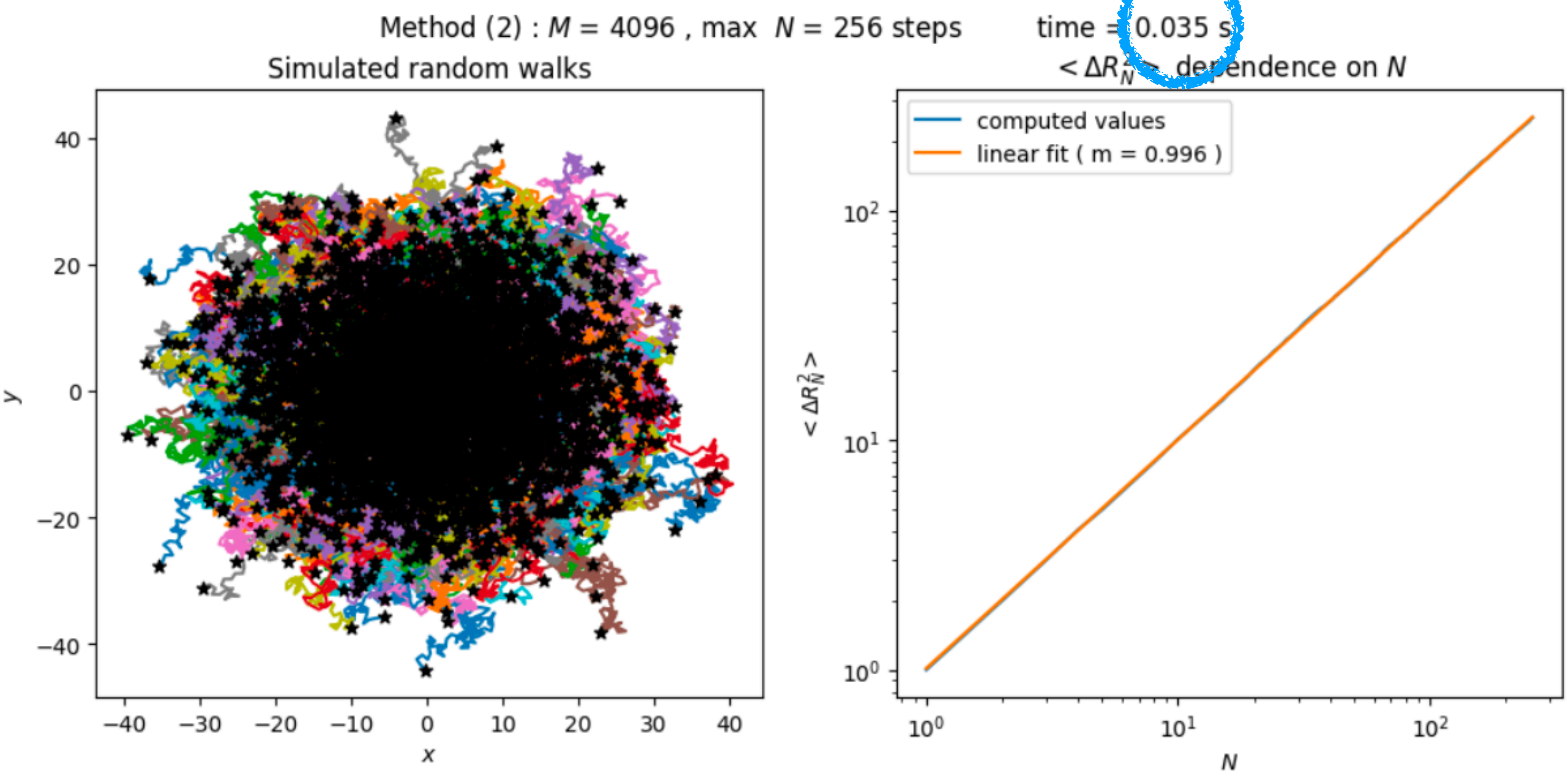


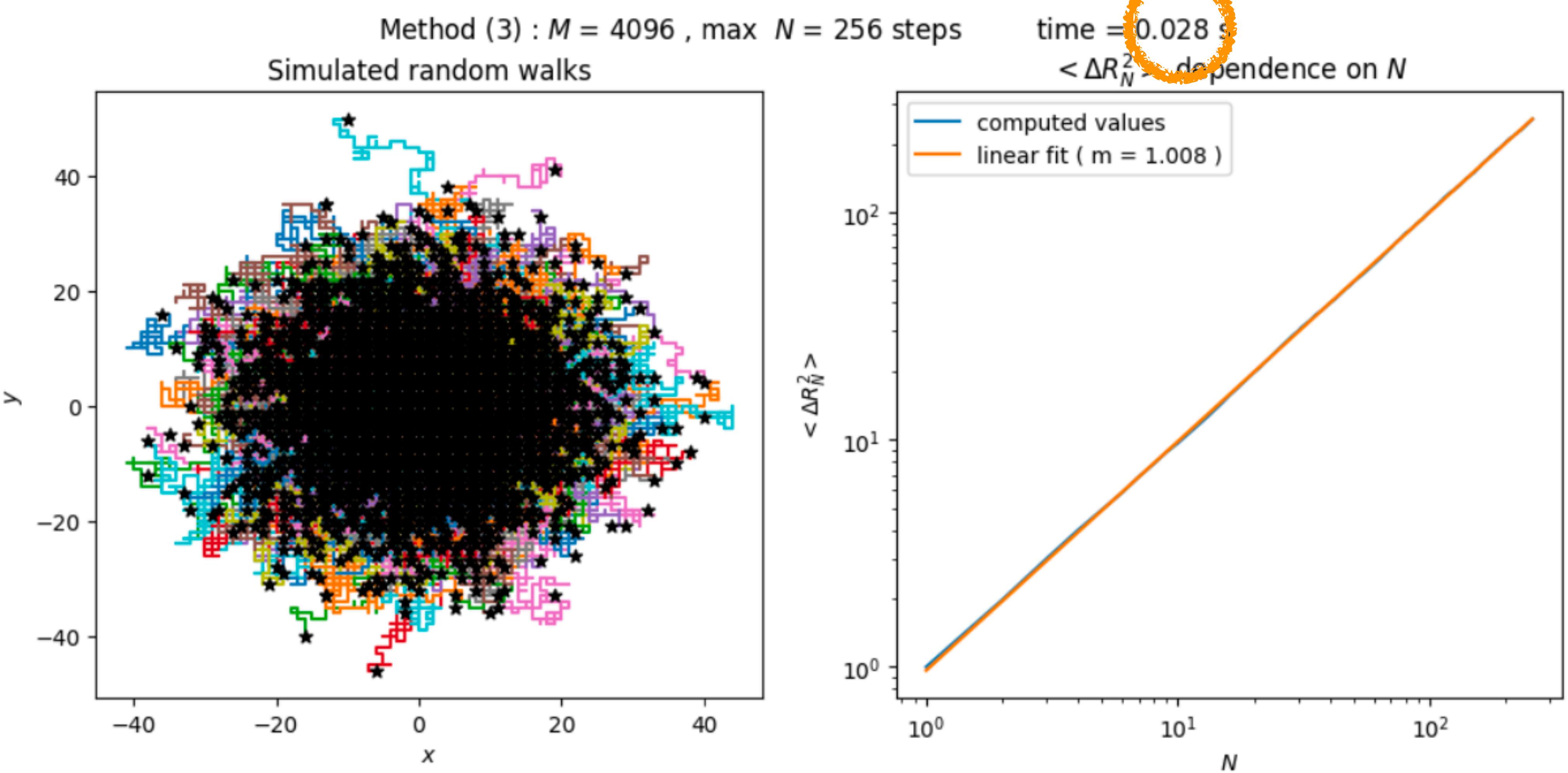
Random Walks 2D

3. Choose separate random values for $\Delta x, \Delta y$ in the range $[-1, 1]$ (but not $\Delta x = 0, \Delta y = 0$). Normalize $\Delta x, \Delta y$ so that the step size is 1.



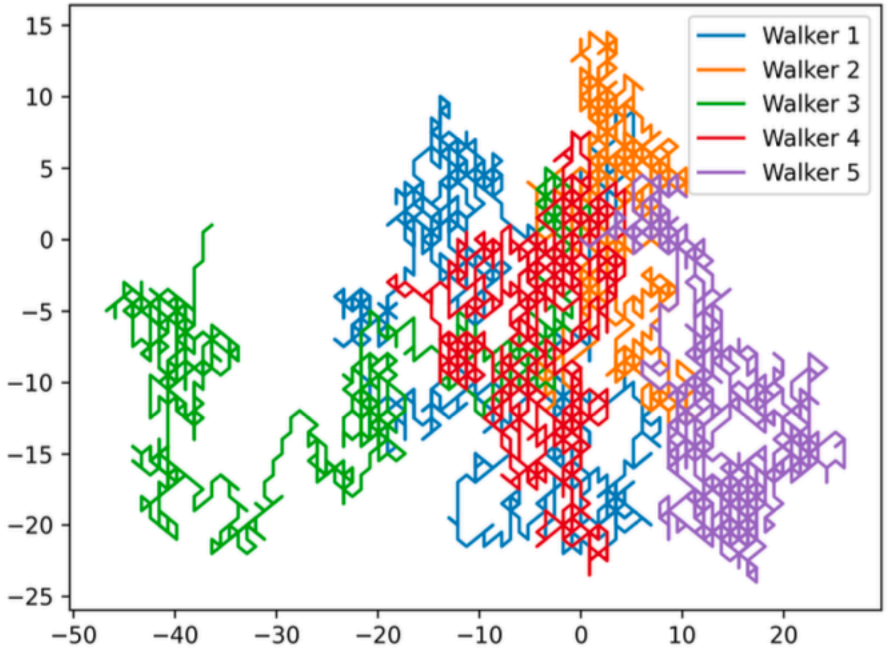
Random Walks 2D

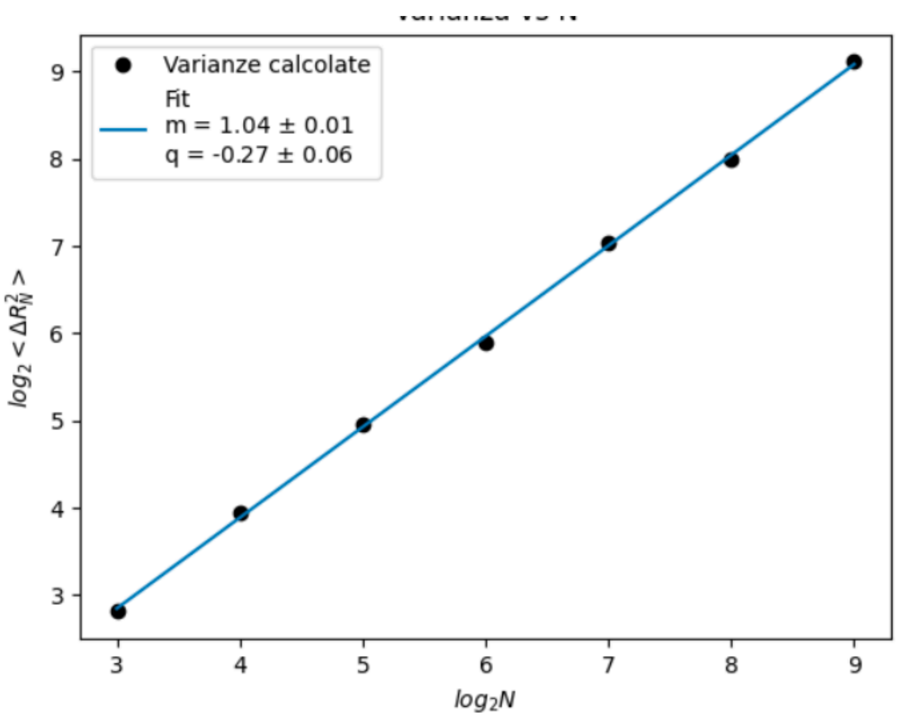
4. Choose a direction (N, E, S, W) randomly as the step direction (no trigonometric functions are then needed). Note, choosing one of four directions is equivalent to choosing a random *integer* on [0,3].



Random Walks 2D

RW su reticolo esagonale



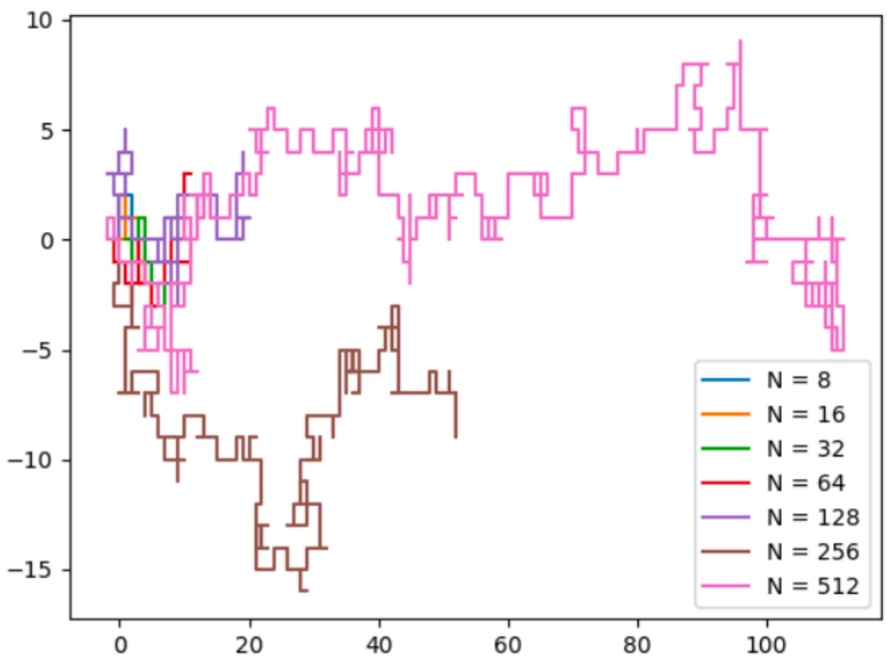


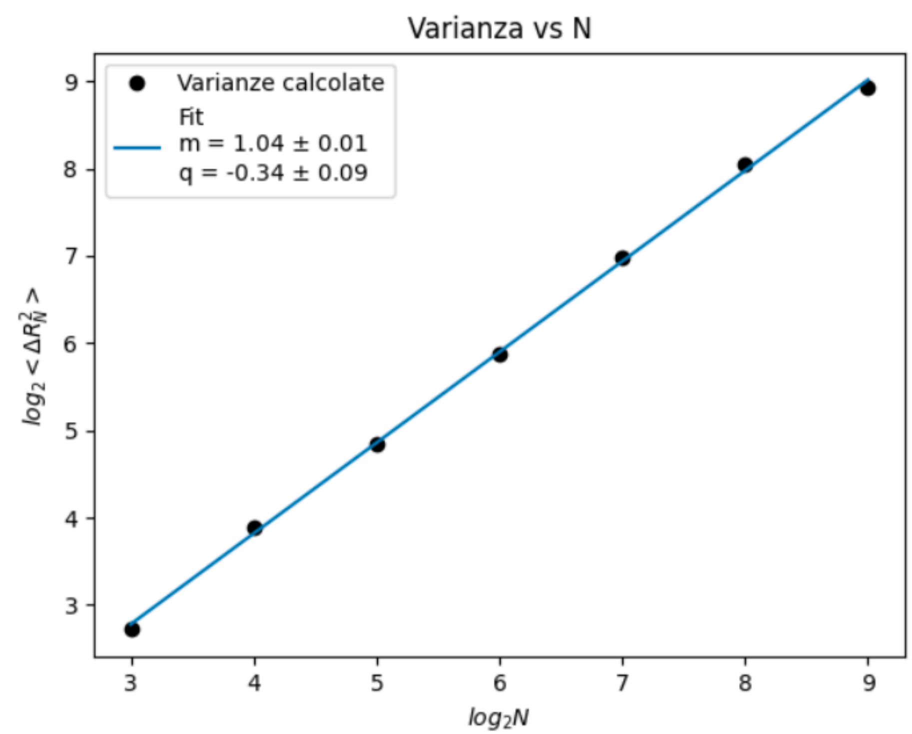
stessa legge

Random Walks 2D

RW su reticolo quadrato con drift vs dx

$$p_{\rightarrow} = 0.4$$



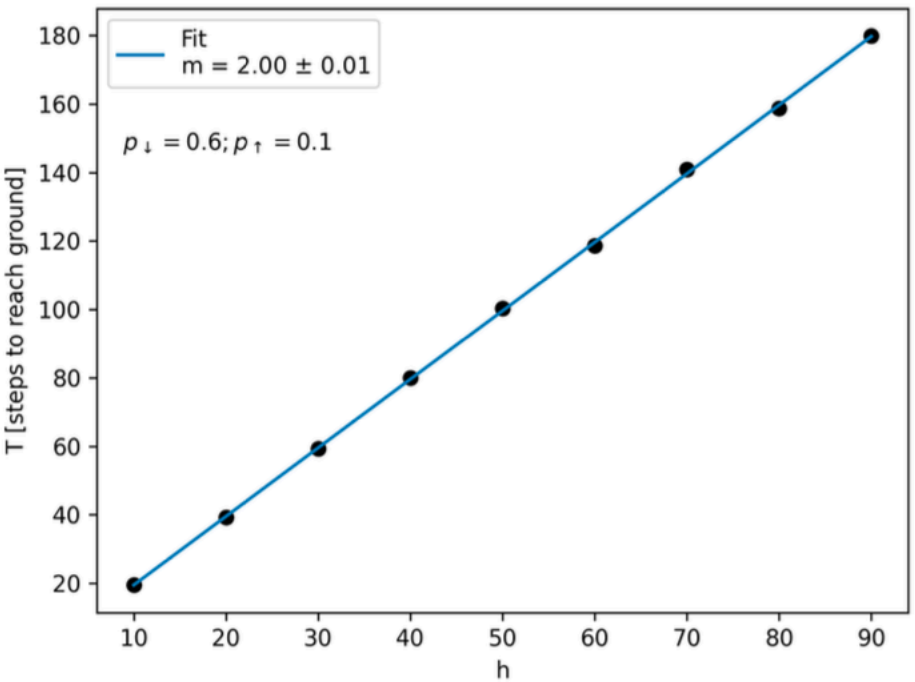


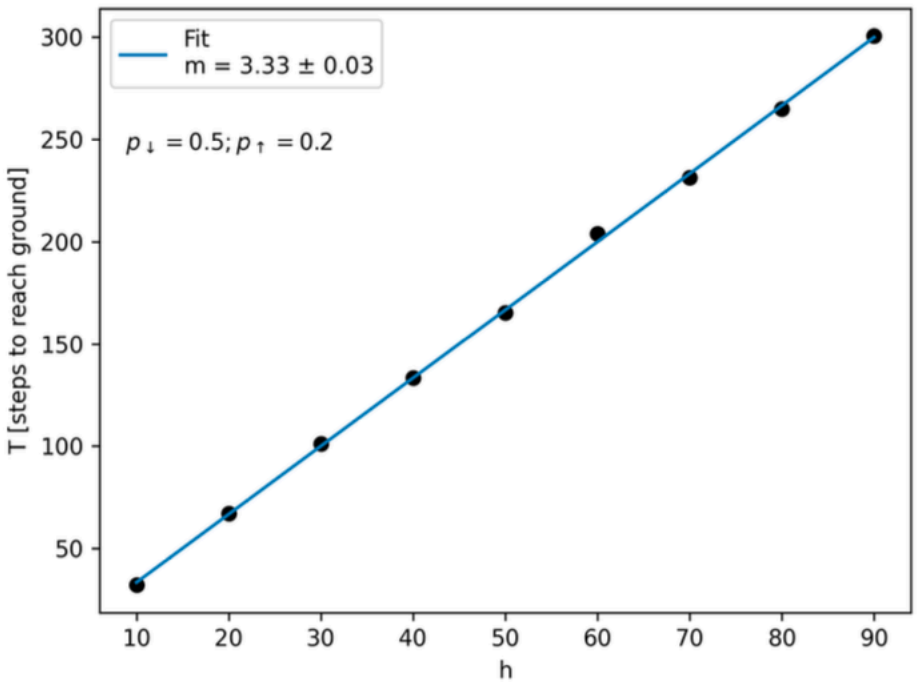
stessa legge

Random Walks 2D - pioggia

T il tempo medio necessario a raggiungere il suolo (ovvero il numero di steps del RW per raggiungere y=0)

$$T(h) = \frac{h}{p_{\downarrow} - p_{\uparrow}}$$





stessa legge

Random Walks 2D - pioggia

dispersione al suolo cioè $\langle \Delta x_0^2 \rangle$ ("0" ad indicare y=0)

$\langle \Delta x_N^2 \rangle = 4 p_{\leftarrow} p_{\rightarrow} N l^2$

considerare N_x, il numero di steps eseguiti in direzione x $\langle N_x \rangle = (p_{\leftarrow} + p_{\rightarrow}) N$

$p_{\leftarrow}, p_{\rightarrow}$ devono essere rinormalizzate per considerare solo i passi lungo $\hat{x}$.

$$p_{\leftarrow} = p_{\rightarrow} = 0.15 \Rightarrow p'_{\leftarrow} = p'_{\rightarrow} = 0.5; \; \langle N_x \rangle = 0.3N$$

$$\left\langle (\Delta x_0)^2 \right\rangle = 4 p'_{\leftarrow} p'_{\rightarrow} \left[(p_{\leftarrow} + p_{\rightarrow}) \langle N_x \rangle \right] l^2$$