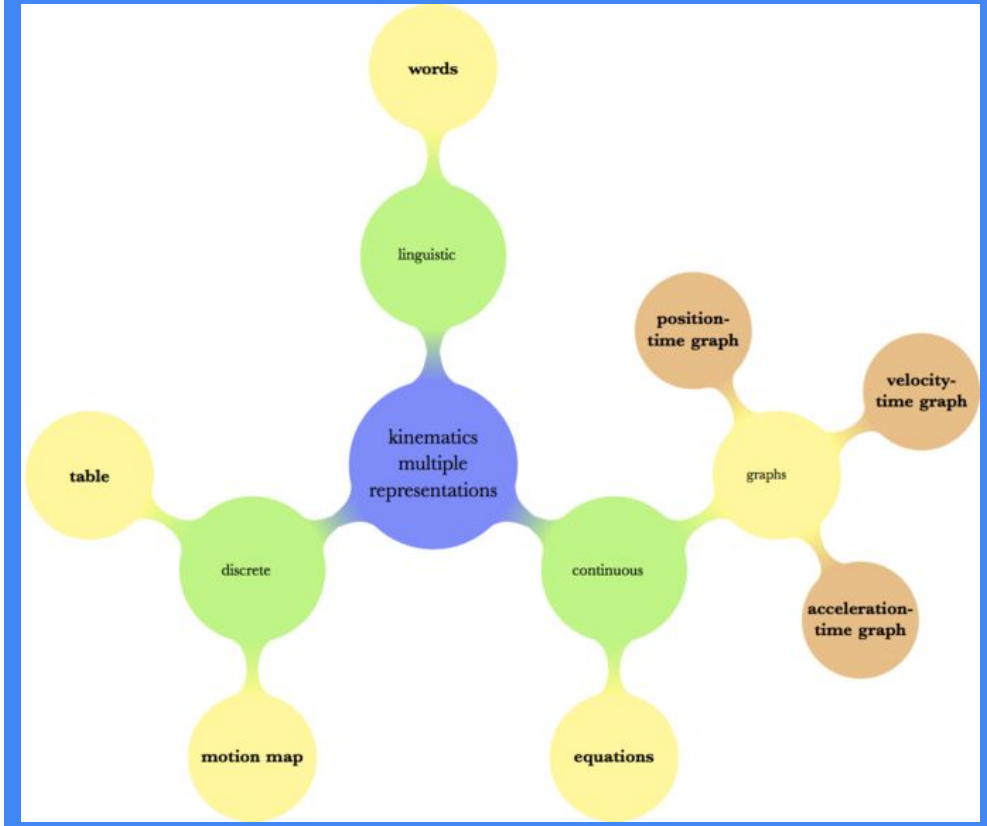
Physics Education Laboratory Lecture 06

The use of multiple representations in kinematics

Francesco Longo • 09/10/2025

Key-concepts in Kinematics with Multiple Representations

Orientation toward science teaching



https://fockphysics.wordpress.com/2013/06/15/sbg-and-multiple-representations-in-kinematics/

Multiple Representations definition

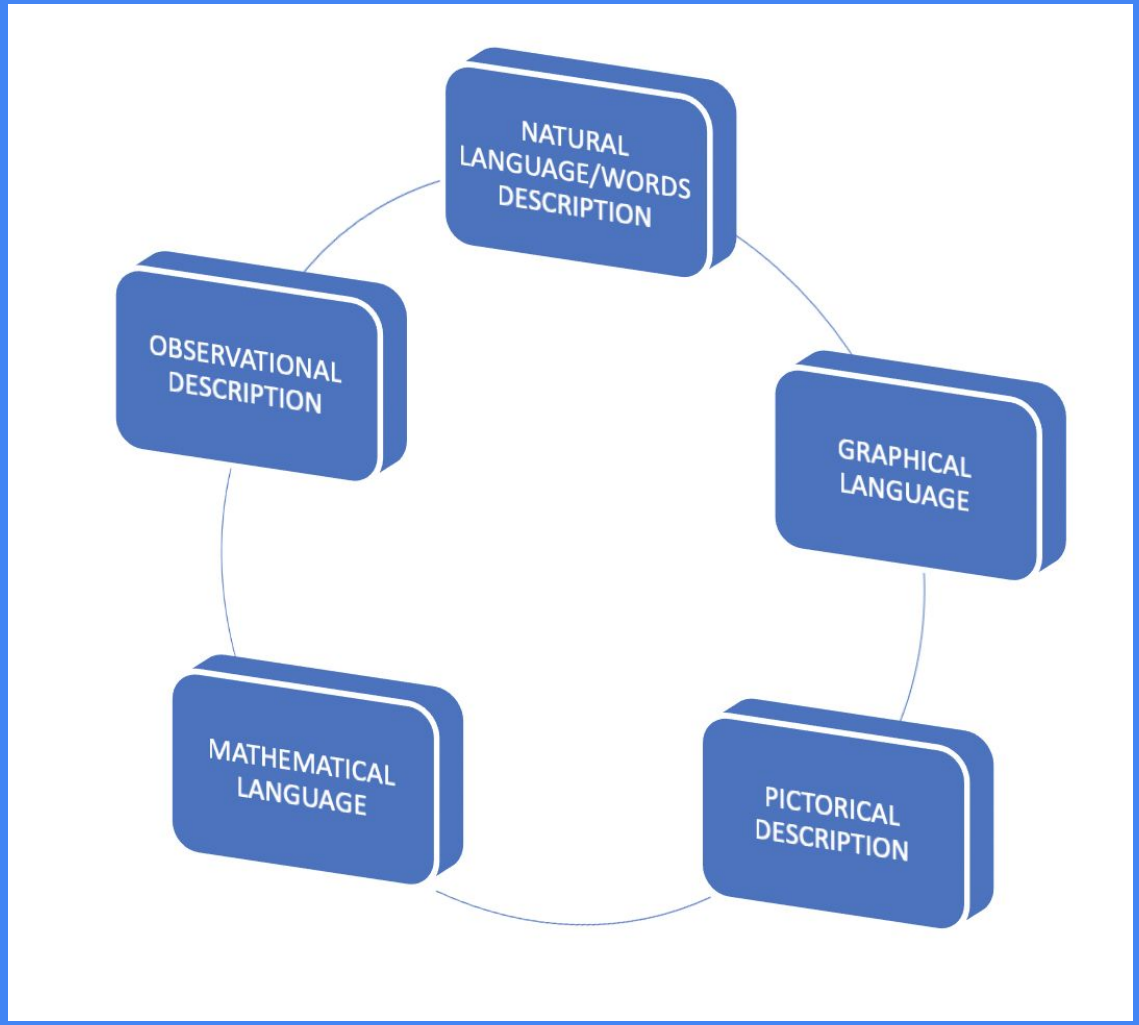
MR refer to the capacity of scientific discourse to represent the same concepts or processes in different modes (e.g., verbal, visual, mathematical, graphical) while multi-modal representation refers to the integration of different modes to represent scientific processes, findings, and scientific explanations

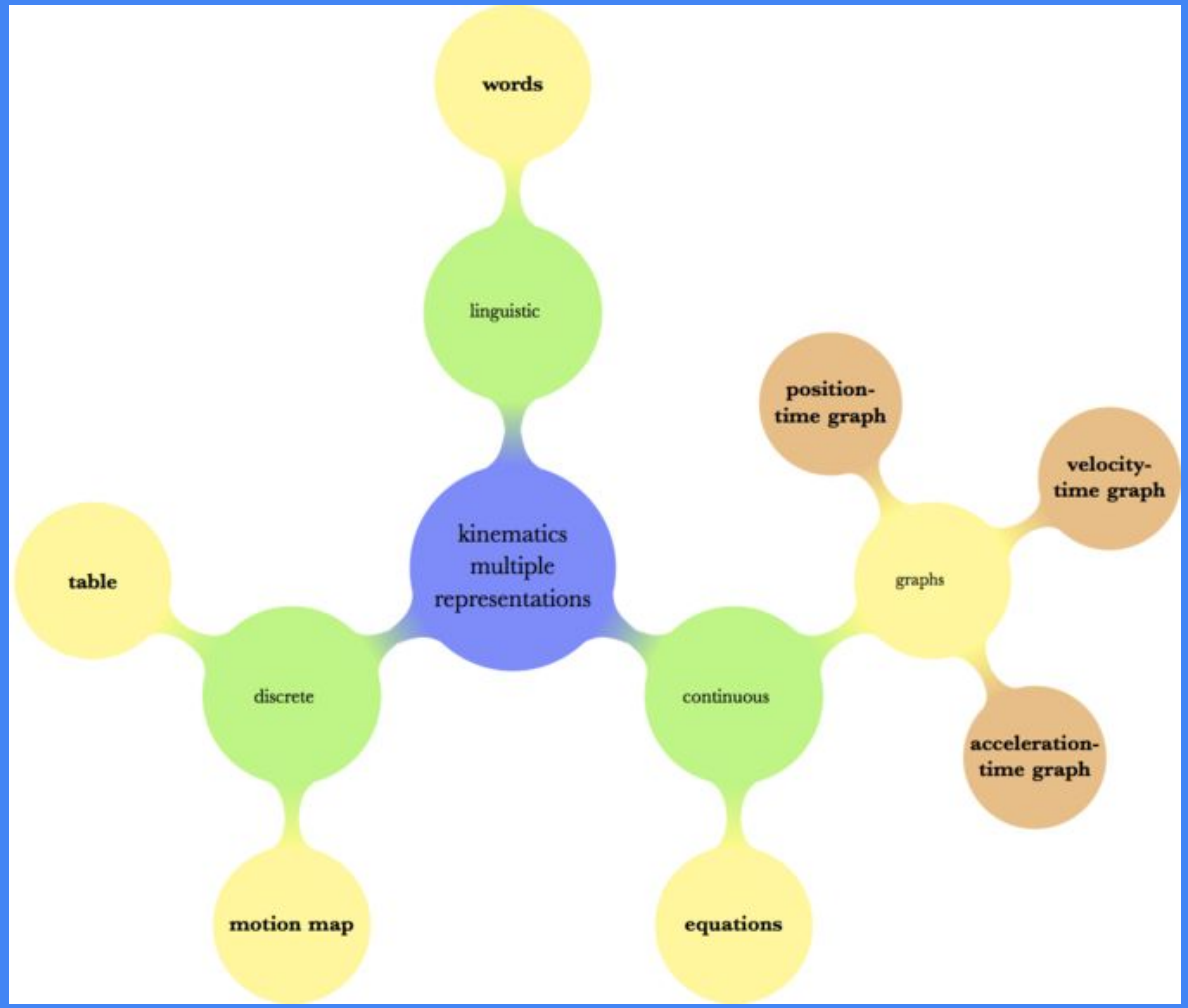
Tytler et al. (2007) p.314

SUMMARIZING:
how multiple representations (MR) can be used in undergraduate physics education and in physics education research

- MR might serve as a powerful learning strategy in physics education and has a positive impact on undergraduate students' understanding of physics concepts.
- There is no evidence in the literature that one kind representation is better than another in supporting students develop their understandings of physics concepts.
- Students use different kinds of representations and the combination of kinds of representations has the potential to support student problem solving and consequently their development of conceptual understanding.
- Students face different difficulties in using MR during the learning process, especially in switching between kinds of representations.
- Computer-based representations can support students' physics learning, as for example, conceptual understanding and representational competence.

Kinematics with Multiple Representations DESCRIPTIONS OR DISCIPLINE'S LANGUAGES?


NATURAL LANGUAGE/WORDS DESCRIPTION
GRAPHICAL LANGUAGE
PICTORICAL DESCRIPTION
MATHEMATICAL LANGUAGE
OBSERVATIONAL DESCRIPTION

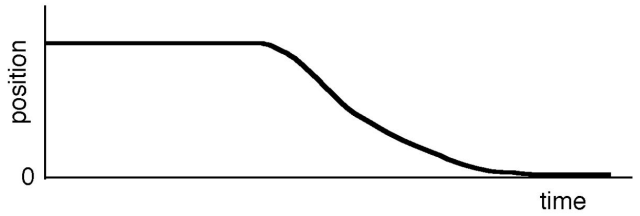

words
linguistic
kinematics
multiple
representations
table
discrete
motion map
continuous
equations
graphs
position-
time graph
velocity-
time graph
acceleration-
time graph

Interpreting representation and translating

FROM GRAPHICAL TO WORDS

Internal representation → External representation

6. Here is a graph of an object's motion. Which sentence is a correct interpretation?



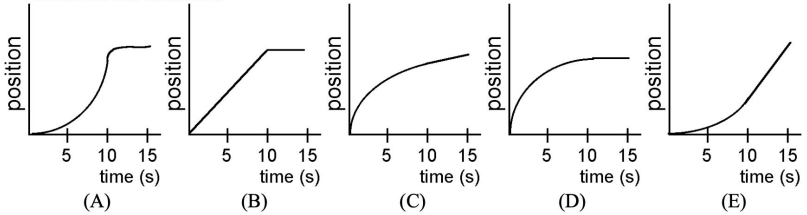
A) The object rolls along a flat surface. Then it rolls forward down a hill, and then finally stops.
B) The object doesn't move at first. Then it rolls forward down a hill and finally stops.
C) The object is moving at a constant velocity. Then it slows down and stops.
D) The object doesn't move at first. Then it moves backwards and then finally stops.
E) The object moves along a flat area, moves backwards down a hill, and then it keeps moving.

Interpreting representation and translating

FROM WORDS TO GRAPHICAL

Internal representation → External representation

7. An object starts from rest and undergoes a positive, constant acceleration for ten seconds. It then continues on with constant positive velocity. Which of the following graphs correctly describes this situation?



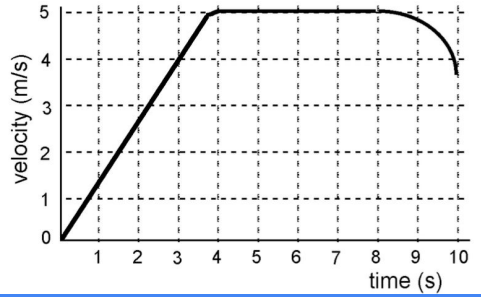
Interpreting representation and translating

FROM WORDS THROUGH GRAPHICAL TO MATHEMATICAL

Internal representation → External representation

3. An elevator moves from the basement to the tenth floor of a building. The mass of the elevator is 1000 kg and it moves as shown in the velocity-time graph below. How far does it move during the first three seconds of motion?

A) 0.75 m
B) 1.33 m
C) 4.0 m
D) 6.0 m
E) 12.0 m



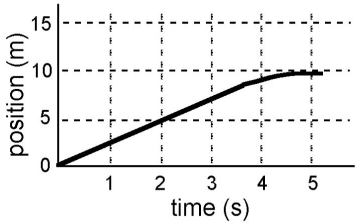
Collecting data and elaborating

FROM GRAPHICAL TO MATHEMATICAL

Internal representation → External representation

4. The velocity at the 2 second point is:

A) 0.4 m/s
B) 2.0 m/s
C) 2.5 m/s
D) 5.0 m/s
E) 10.0 m/s

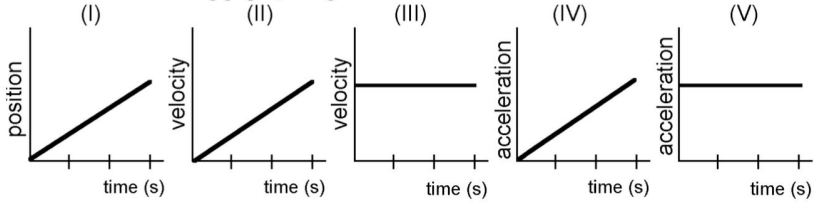


Interpreting representation and modelling concept

FROM GRAPHICAL TO CONCEPT MODEL

External representation → Internal representation

9. Consider the following graphs, noting the different axes:

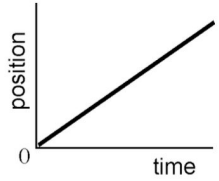


Which of these represent(s) motion at constant velocity?

A) I, II, and IV
B) I and III
C) II and V
D) IV only
E) V only

2. To the right is a graph of an object's motion.
Which is the best interpretation?

A) The object is moving with a constant, non-zero acceleration.
B) The object does not move.
C) The object is moving with a uniformly increasing velocity.
D) The object is moving at a constant velocity.
E) The object is moving with a uniformly increasing acceleration.

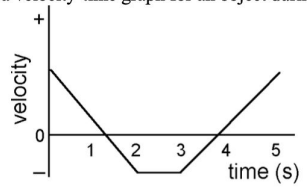


Interpreting representation and modelling concept

FROM GRAPHICAL TO CONCEPT MODEL

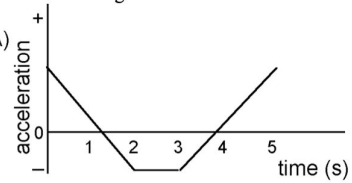
External representation → Internal representation

11. The following represents a velocity-time graph for an object during a 5 s interval.

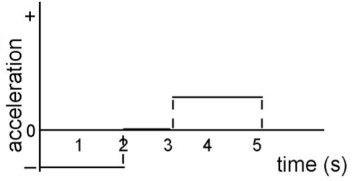


Which one of the following graphs of acceleration vs time would best represent the object's motion during the same time interval

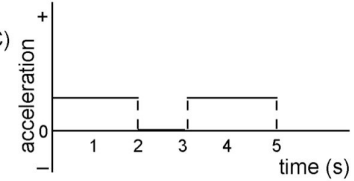
(A)



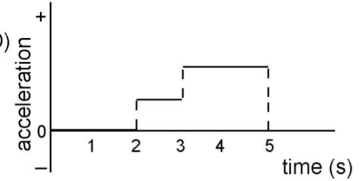
(B)



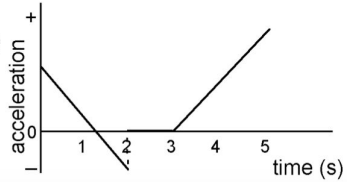
(C)



(D)

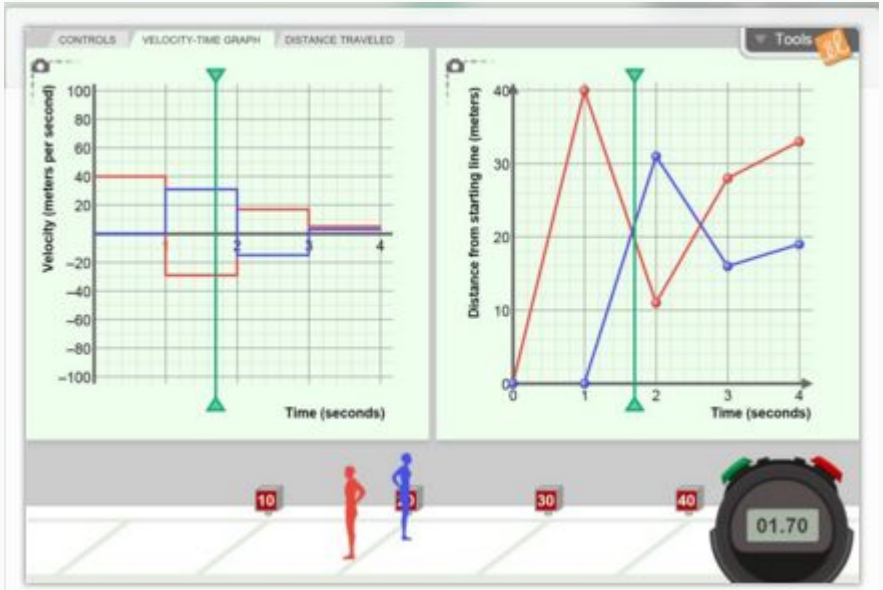


(E)



Distance -Time, Velocity-Time Graphs Metric

Create a graph of a runner's position versus time and watch the runner run a 40-meter dash based on the graph you made. Notice the connection between the slope of the line and the velocity of the runner. Add a second runner (a second graph) and connect real-world meaning to the intersection of two graphs. Also experiment with a graph of velocity versus time for the runners, and also distance traveled versus time.



https://gizmos.explorelearning.com/find-gizmos/lesson-info?resourceId=260

Students' Exploration Sheet

Analyze the sheet

Observe the use of Multiple Representations

Student Exploration: Distance-Time and Velocity-Time Graphs

[NOTE TO TEACHERS AND STUDENTS: This lesson was designed as a follow-up to the Distance-Time Graphs Gizmo. We recommend you complete that activity before this one.]

Vocabulary: displacement, distance traveled, slope, speed, velocity

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)
Dora runs one lap around the track, finishing where she started. Clark runs a 100-meter dash along the straight side of the track.

1. Which runner traveled a greater distance? _____

2. Which runner had a greater change in position, start to finish?

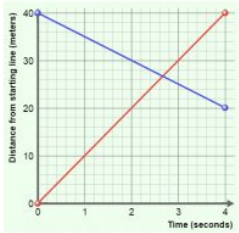
_____

Gizmo Warm-up
The *Distance-Time Graphs* Gizmo shows a dynamic graph of the position of a runner over time. The *Distance-Time and Velocity-Time Graphs* Gizmo includes that same graph and adds two new ones: a velocity vs. time graph and a distance traveled vs. time graph.

The graph shown below (and in the Gizmo) shows a runner's position (or distance from the starting line) over time. This is most commonly called a *position-time graph*.

Check that the **Number of Points** is 2. Turn on **Show graph** and **Show animation** for both **Runner 1** and **Runner 2**.

1. Drag the points to create the graph shown to the right.
   - **Runner 1**'s line (the red one) should have endpoints at (0, 0) and (4, 40).
   - **Runner 2**'s line (the blue one) should have endpoints at (0, 40) and (4, 20).

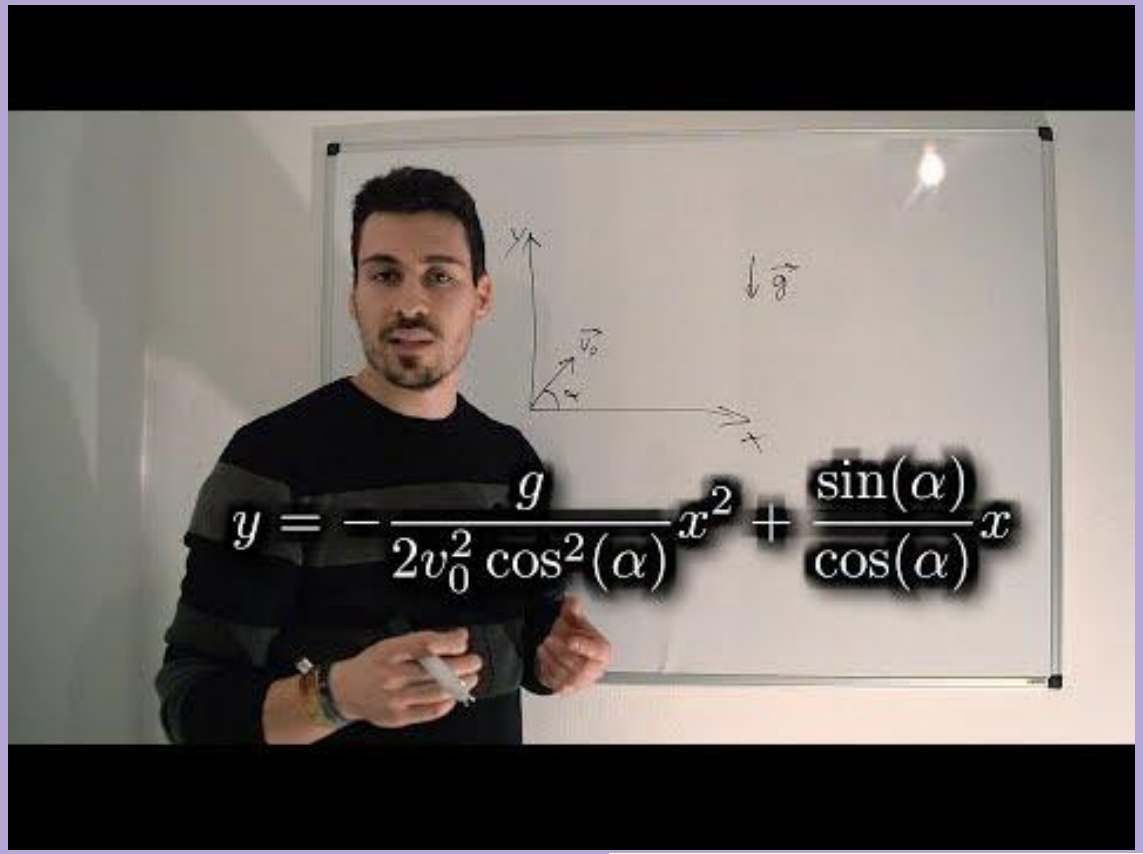


https://docs.google.com/document/d/1cHU2Ao0lNca0lbBKnoj_oWEwggXJpYQhWOFH2b7zThw/edit?usp=sharing

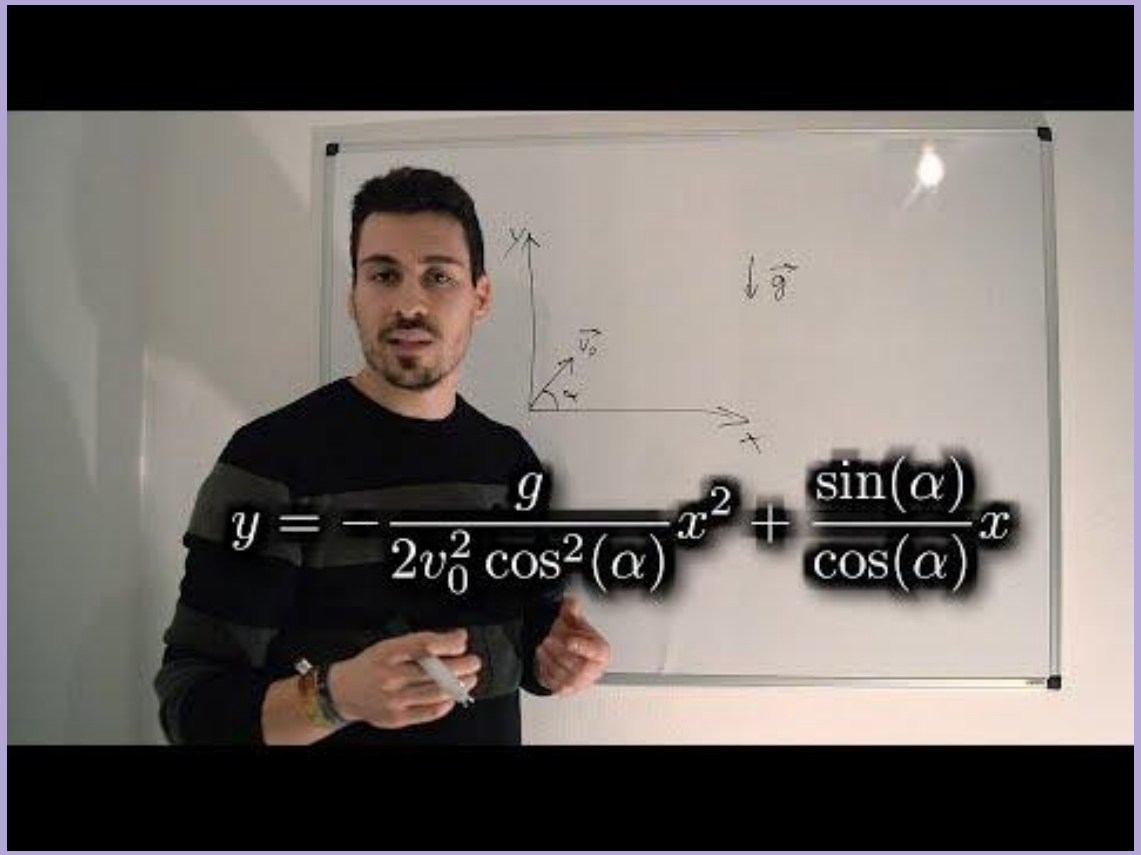
Physics Education Laboratory
Lecture 06
Content Knowledge for teaching Dynamics

Francesco Longo - 09/10/25

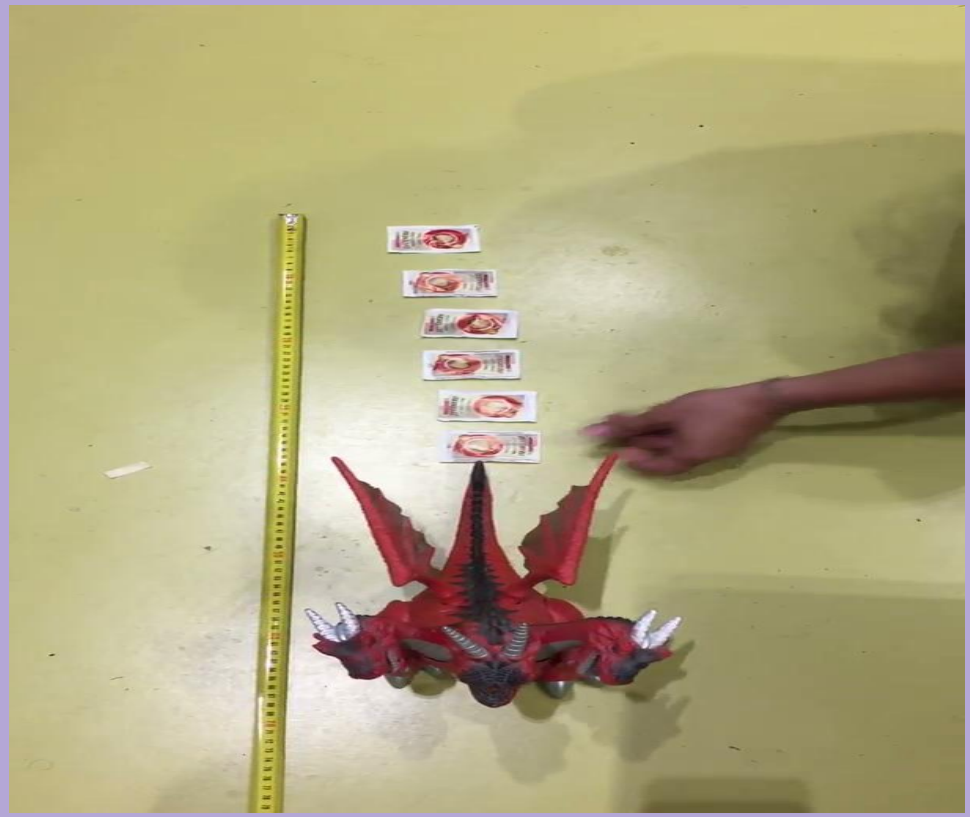
https://www.youtube.com/watch?v=A2cYcQkcJ08



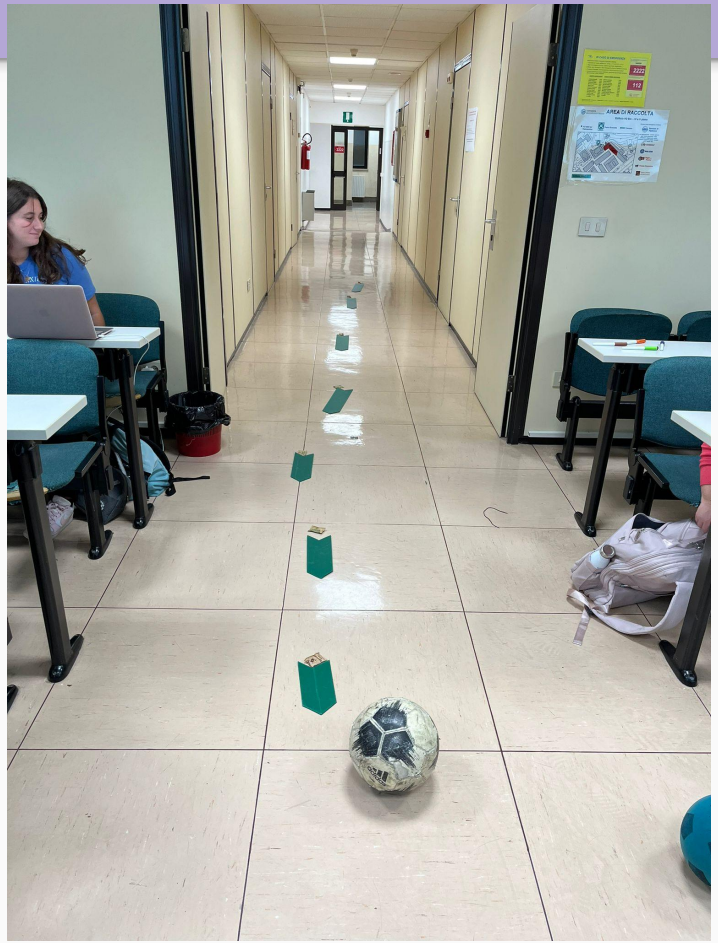
Video on Kinematics



Is there another way of teaching kinematics?



Let’s try …



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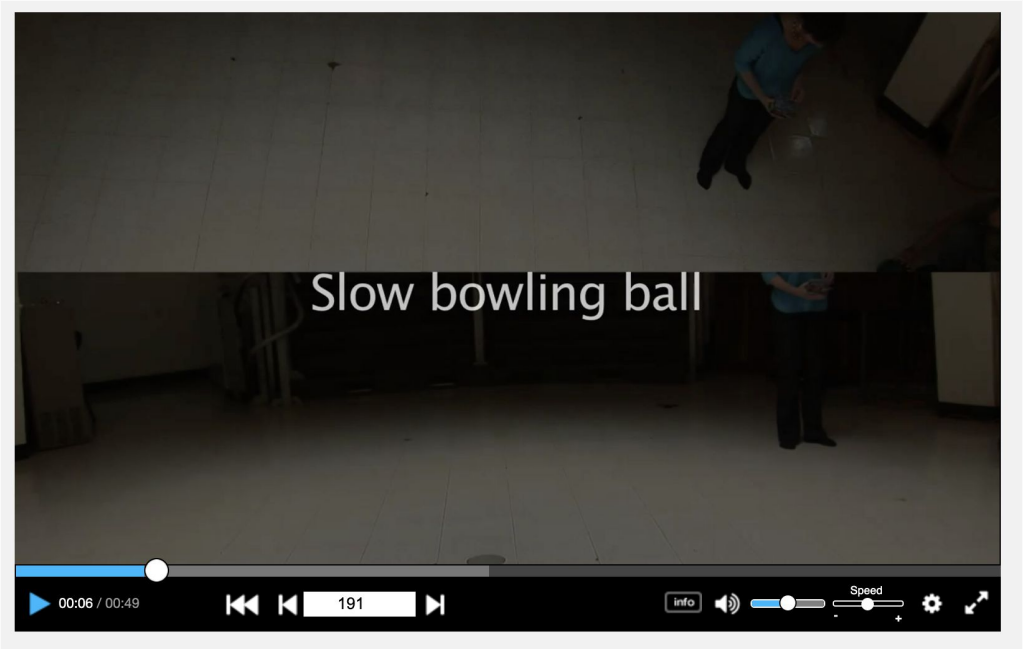


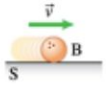
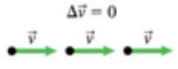
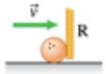
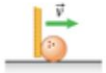
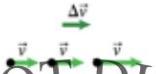
a. What patterns did you notice in the placement of the dots?

b. How can you use the distances between the dots to describe the motion of the bowling ball?

https://mediaplayer.pearsoncmg.com/assets/_frames.true/secs-experiment-video-1

Observational Experiment Table 3.1

How are motion and forces related?

Observational experiment	Analysis: Motion diagram
Experiment 1. A bowling ball B rolls on a very hard, smooth surface S without slowing down. 	
Experiment 2. A ruler R lightly pushes the rolling bowling ball opposite the ball's direction of motion. The ball continues to move in the same direction, but slows down. 	
Experiment 3. A ruler R lightly pushes the rolling bowling ball in the direction of its motion. The ball speeds up. 	

Pattern

- In all the experiments, the vertical forces add to zero and cancel each other. We consider only forces exerted on the ball in the horizontal direction.
- In the first experiment, the sum of the forces exerted on the ball is zero; the ball's velocity remains constant.
- In the second and third experiments, when the ruler pushes the ball, the velocity change arrow ($\Delta\vec{v}$ arrow) points in the same direction as the sum of the forces.

Summary: The $\Delta\vec{v}$ arrow in all experiments is in the same direction as the sum of the forces. Notice that there is no pattern relating the *direction* of the velocity $\vec{v}$ to the direction of the sum of the forces. In Experiment 2, the velocity and the sum of the forces are in opposite directions, but in Experiment 3, they are in the same direction.

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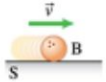
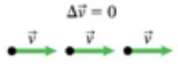
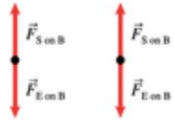
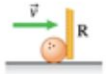
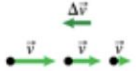
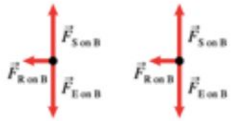
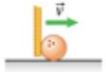
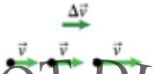
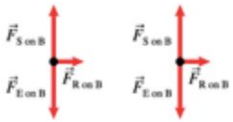
How are motion and forces related?

Observational experiment	Analysis: Motion diagram	Analysis: Force diagrams for first and third positions
Experiment 1. A bowling ball B rolls on a very hard, smooth surface S without slowing down. 		
Experiment 2. A ruler R lightly pushes the rolling bowling ball opposite the ball's direction of motion. The ball continues to move in the same direction, but slows down. 		
Experiment 3. A ruler R lightly pushes the rolling bowling ball in the direction of its motion. The ball speeds up. 		

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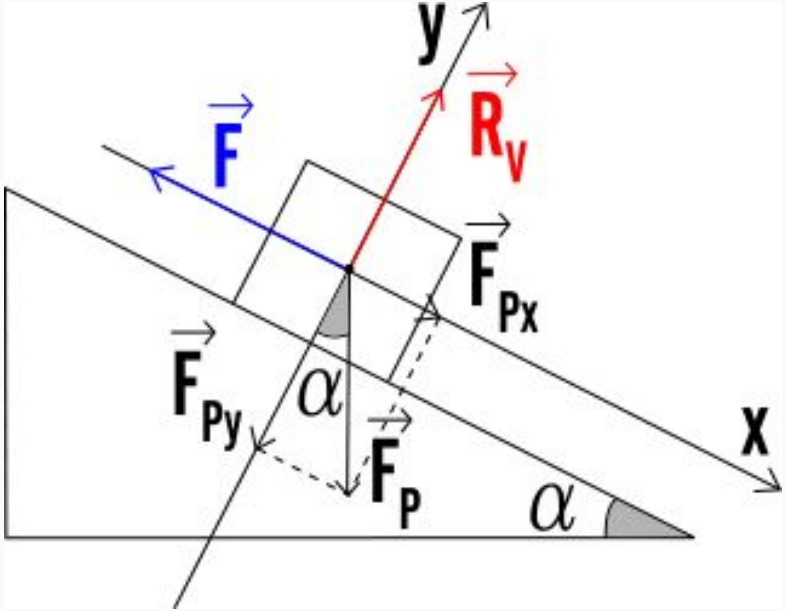
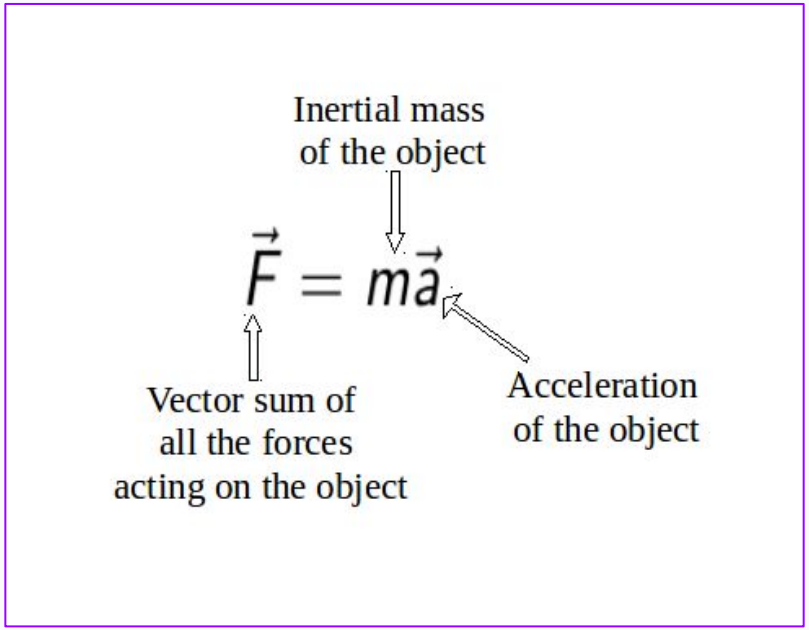
Key concepts in Dynamics

- The three laws of Dynamics
- The concept of acceleration
- The concept of linear momentum
- The vector nature of the force
- The observer system
- The inertial system



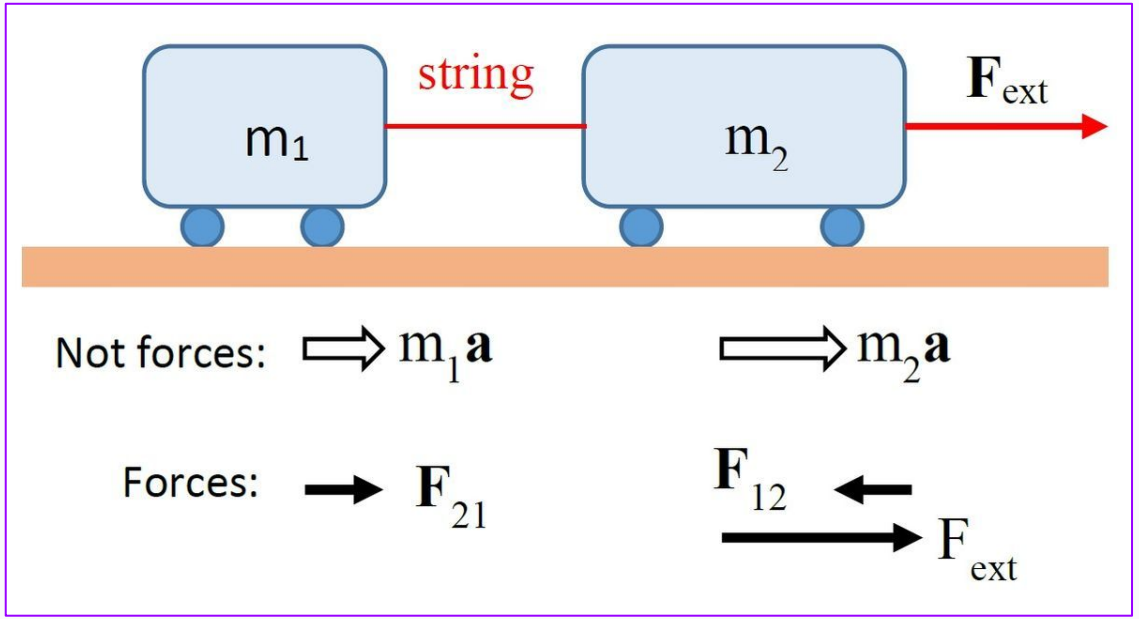
key concepts in Dynamics

- Newton’s second law



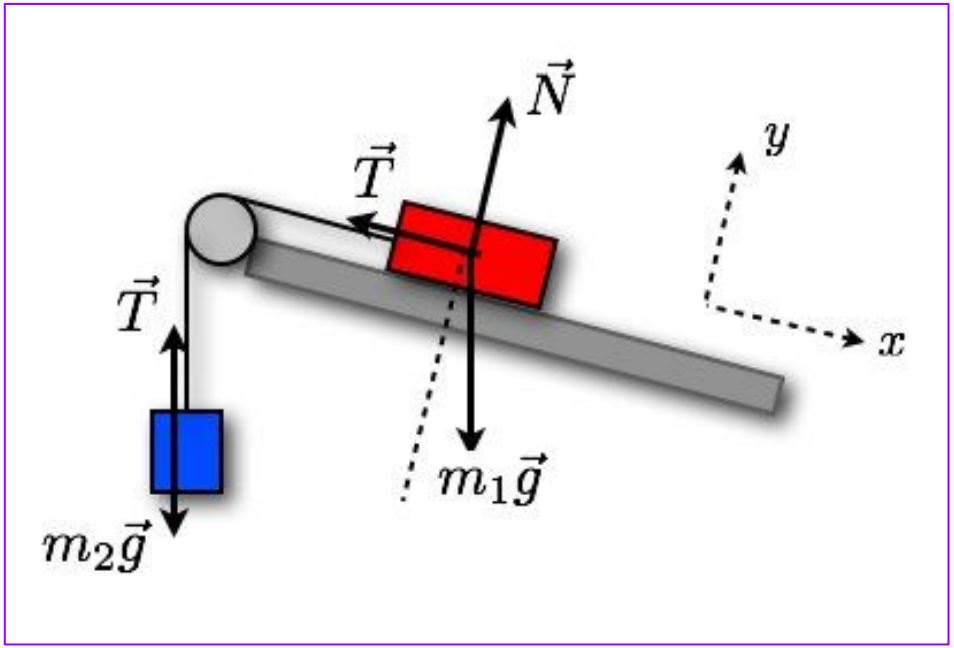
key concepts in Dynamics

- Newton's third law

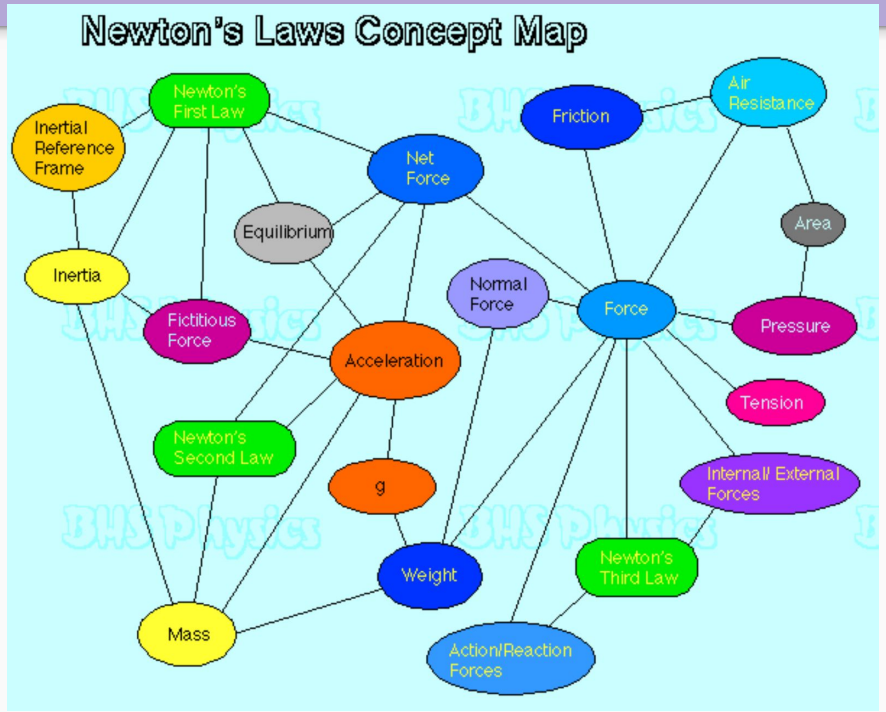


key concepts in Dynamics

- The Force …
- The free body approach
- “Force diagram”

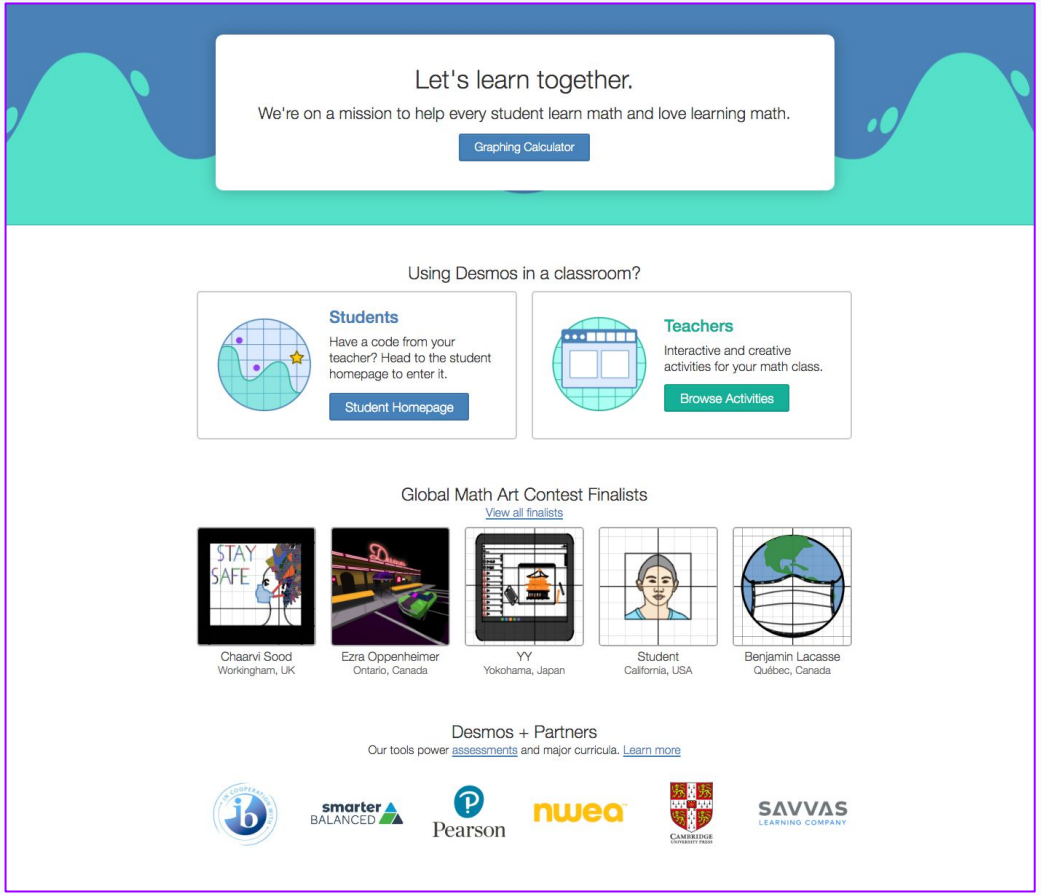


A concepts' map



Teacher Desmos: a useful tool …

https://teacher.desmos.com



Teacher Desmos: a useful tool …

What We Do

Desmos wants to build a world where every student learns math and loves learning math, where a student's access to the power and beauty of math doesn't depend on their place of birth, race, ethnicity, gender, or any other aspect of their identity.

Our free suite of math software tools, including the renowned Desmos Graphing Calculator and Scientific Calculator, are used annually by over 40 million teachers and students around the world. Our tools power core math curricula for many of the world's largest publishers, and our calculators are built into the majority of U.S. state-level assessments and digital college entrance exams. Desmos is also partnered with the Smarter Balanced Assessment Consortium, the International Baccalaureate MYP, and NWEA (view the full list of assessments here).

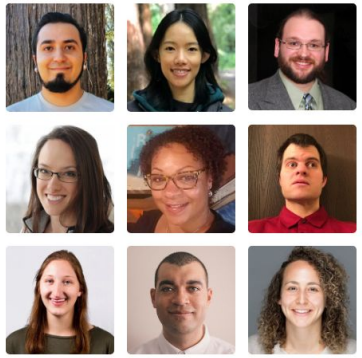
Our cutting-edge technology also powers our free digital classroom activities, thoughtfully designed by teachers for teachers to support and celebrate the different ways students come to know mathematics. These activities are guided by our pedagogical philosophy and open up a world of possibilities for students to explore concepts more deeply, collaborate with their peers on problem-solving, and apply knowledge creatively as mathematicians.

In 2020, we launched our core middle school math program, which pairs the open-source U.S. middle school curriculum from Illustrative Mathematics and Open Up Resources with Desmos' powerful technology, humanizing pedagogy, and intuitive design. We are working with an incredible cohort of schools and districts to implement and further improve this program over the years to come.

Come join us on our journey! Play with the calculator, check out our classroom activities, or find a career on our growing team. Stay in touch with us on Twitter, Facebook, Instagram, or email. Teachers and school leaders are also welcome to join our Educators Community.

Who We Are

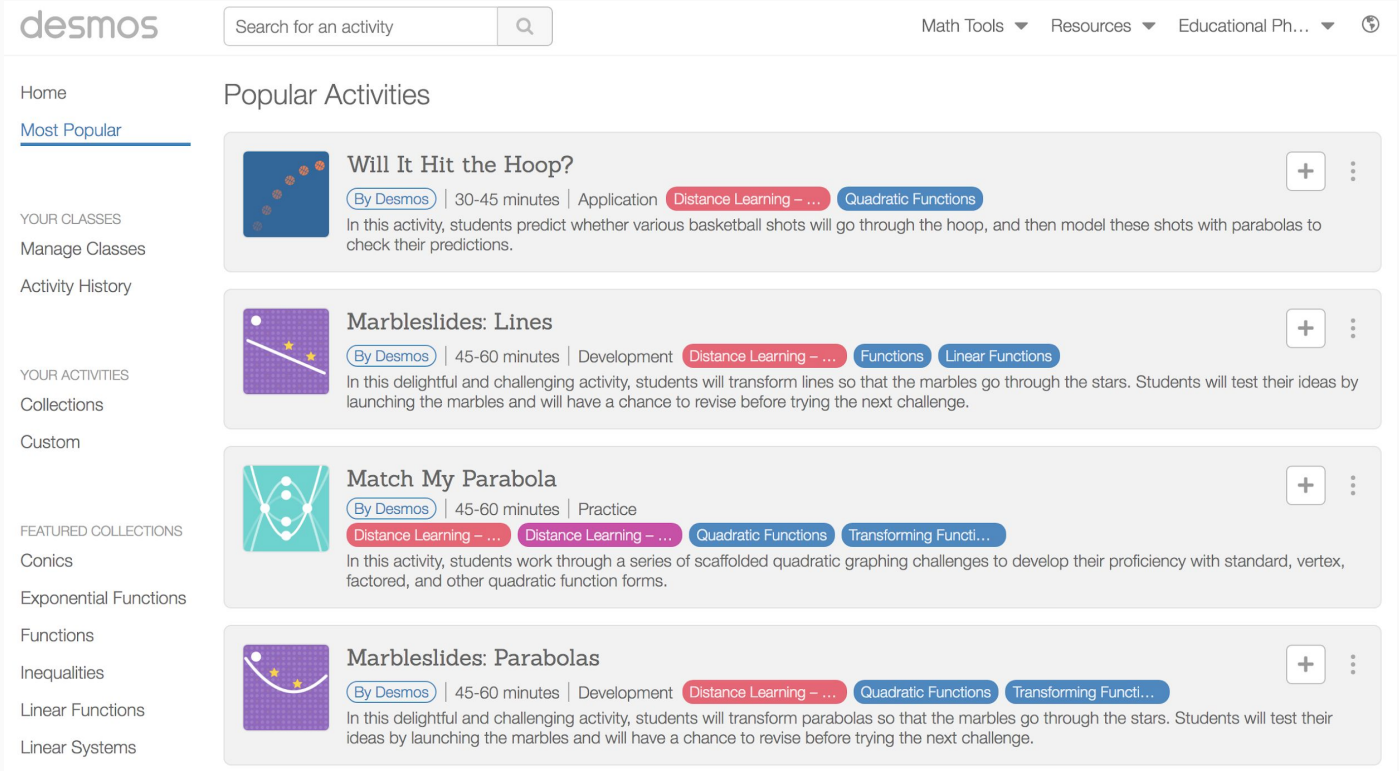
Meet the Team



We're a small but growing team.
Interested in joining? We're hiring! ›

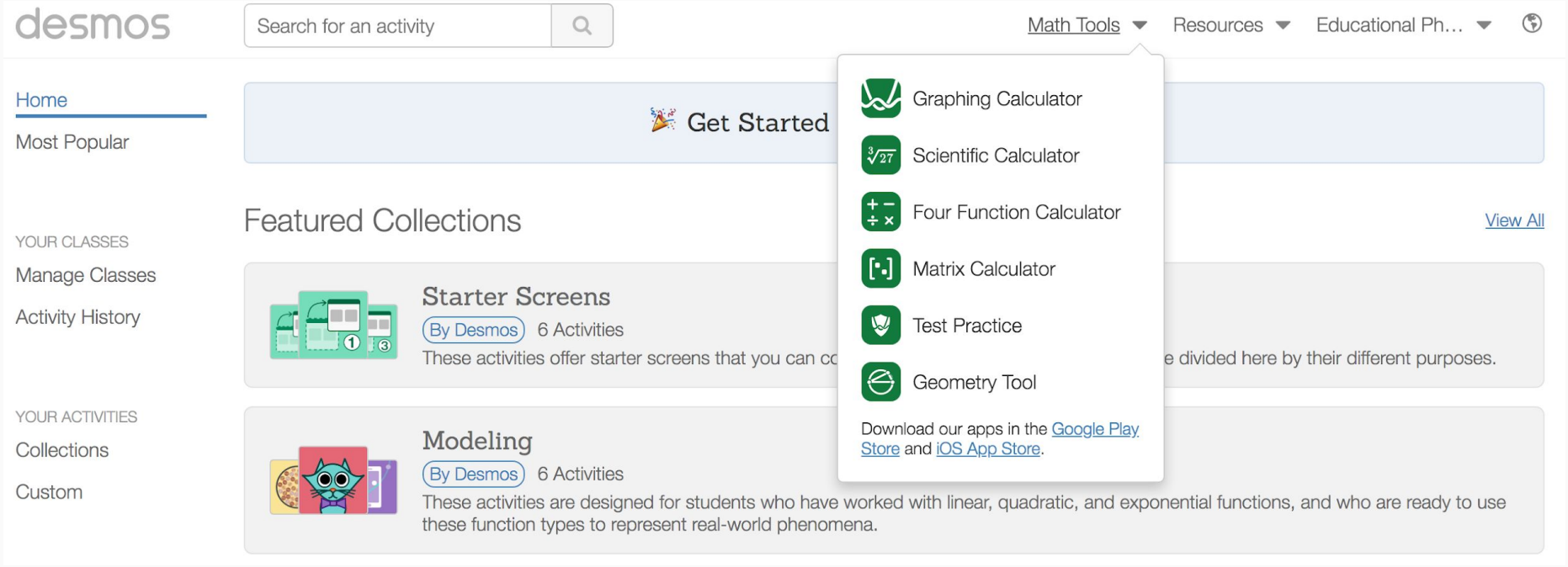
https://teacher.desmos.com

Teacher Desmos: a useful tool …



https://teacher.desmos.com

Teacher Desmos: a useful tool …



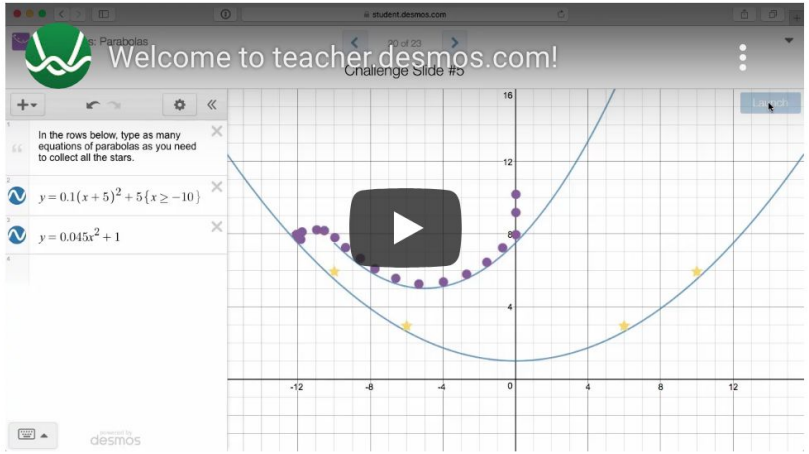
https://teacher.desmos.com

Teacher Desmos: a useful tool …

Learn Desmos: Classroom Activities

At Desmos, our mission is to help every student learn math and love learning math. With that in mind, we've assembled a collection of unique and engaging digital activities. And best of all? Everything you see at teacher.desmos.com is free. Get started with the video on the right, then dive deeper with the resources below.

Tips for Getting Started



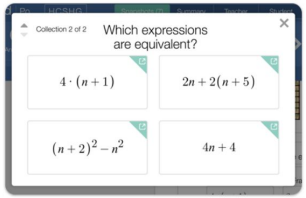
https://teacher.desmos.com

Teacher Desmos: a useful tool …

Next Steps

Facilitate Class Conversations

Learn how to facilitate Desmos activities like a pro with our **Classroom Conversation** features—Anonymize, Teacher Pacing, and Pause Class.

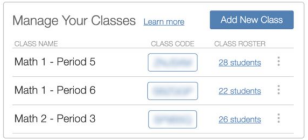


Select and Sequence Student Work

The Desmos teacher dashboard collects a *lot* of student ideas. Use our **Snapshots** tool to select and sequence those ideas as you orchestrate class discussions around student thinking.

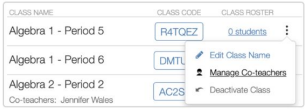
Send Feedback to Students

Leave **Written Feedback** on any student screen. Students will see your comment and develop their mathematical ideas further.



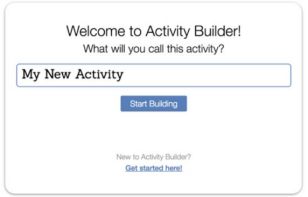
Manage Your Classes

Create classes to assign activities to the same group of students throughout a school term.



Add Co-teachers

Co-teaching is easy in Desmos when you and your co-teacher are both near the same computer. But when you're far from each other, you can *still* collaborate.



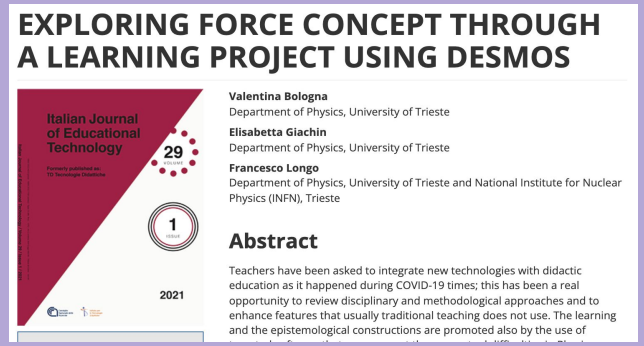
Create Custom Activities

Create your own Desmos-powered activities with our custom **Activity Builder**. Learn the ropes with our collection of video tutorials and editable sample activities.

https://teacher.desmos.com

LE FORZE

Percorso didattico per la classe seconda della scuola secondaria di primo grado



EXPLORING FORCE CONCEPT THROUGH A LEARNING PROJECT USING DESMOS

Italian Journal of Educational Technology

29

1

2021

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Department of Physics, University of Trieste and National Institute for Nuclear Physics (INFN), Trieste

Abstract

Teachers have been asked to integrate new technologies with didactic education as it happened during COVID-19 times; this has been a real opportunity to review disciplinary and methodological approaches and to enhance features that usually traditional teaching does not use. The learning and the epistemological constructions are promoted also by the use of

https://ijet.itd.cnr.it/article/view/1190

UNIVERSITÀ DEGLI STUDI DI TRIESTE

DIPARTIMENTO DI FISICA

Corso di Laurea Triennale in Fisica

Sviluppo di un percorso didattico innovativo sul concetto newtoniano di forza

Laureanda:
Elisabetta GIACHIN

Relatore:
Prof. Francesco LONGO

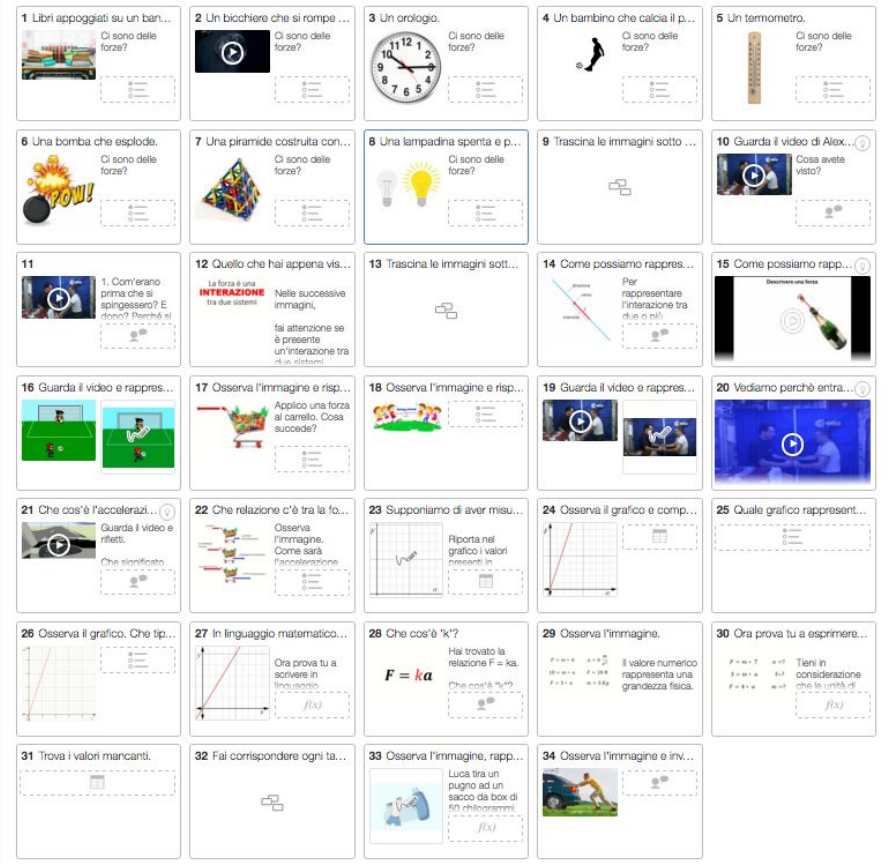
Correlatrice:
Prof.ssa Valentina BOLOGNA

ANNO ACCADEMICO 2019/2020

https://student.amplify.com/join/2FZXEN

LE FORZE

Percorso didattico per la classe seconda della scuola secondaria di primo grado



LE FORZE

Percorso didattico per la classe seconda della scuola secondaria di primo grado

Anonimia 59 studenti	Filtro	Pausa Cronologico	1 Libri app… Ci sono delle	2 Un bicch… Ci sono delle	3 Un orolo… Ci sono delle	4 Un bam… Ci sono delle	5 Un term… Ci sono delle	6 Una bo… Ci sono delle	7 Una pira… Ci sono delle	8 Una lam… Ci sono delle	9 Trascina …
Apollonius			×	•	•	•	•	•		×	•
Edray Goins			×	•	•	•	•	•	•	•	•
John Urschel			×	•	•	•	•	•	•	•	•
John Wallis			×	•	•	•	•	•	•	×	•
Blaise Pascal			×	•	•	•	•	•	•	•	•
Pierre-Simon Lap…			×	•	•	•	•	•	•	•	•
Jacques Hadamard			×	•	•	•	•	•	•	×	•
Pythagoras			×	•	•	•	•	•	•	×	•
Heisuke Hironaka			×							×	•
Mary Ellen Rudin			×	•	•	•	•	•	•	•	•
Giuseppe Peano			×	•	•	•	•	•	•	•	•
Eugenia Cheng			×	•	•	•	•	•	•	×	•
Émile Borel			×	•	•	•	•	•	•	•	•
Gladys West			×	•	•	•	•	•	•	•	•
Grigory Margulis			×	•	•	•	•	•	•	×	•

ANTEPRIMA DELLA SCHERMATA DELLO STUDENTE

Libri appoggiati su un banco di scuola.

Ci sono delle forze?

SI | NO

Misconception: what about net forces?

Un bicchiere che si rompe per effetto del suono.

Ci sono delle forze?

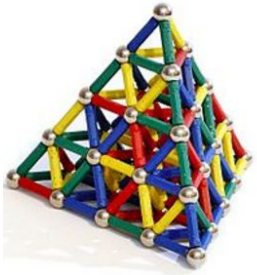
SI | NO

Un bambino che calcia il pallone.

Ci sono delle forze?

SI | NO

Una piramide costruita con dei magneti.



Ci sono delle forze?

SI

Una bomba che esplode.

Ci sono delle forze?

SI | NO

Un termometro.

Ci sono delle forze?

SI | NO

Una lampadina spenta e poi accesa.

Ci sono delle forze?

SI | NO

Un orologio.

Ci sono delle forze?

SI

Try to understand which are the prior/intuitive students' knowledge: What is a force?

Trascina le immagini sotto al cartellino corretto.

Esempi di NON Forza

Esempi di Forza

Collecting informations → create a disatisfaction because the student is not sure of his answers...

Guarda il video di Alexander e Pedro, due astronauti nello spazio, e rispondi.

Cosa avete visto?

Condividi con la classe

1. Com'erano prima che si spingessero? E dopo? Perché si sono mossi?

2. Cos'è cambiato dopo la spinta? Ha cambiato posizione solo uno dei due o si sono spostati entrambi?

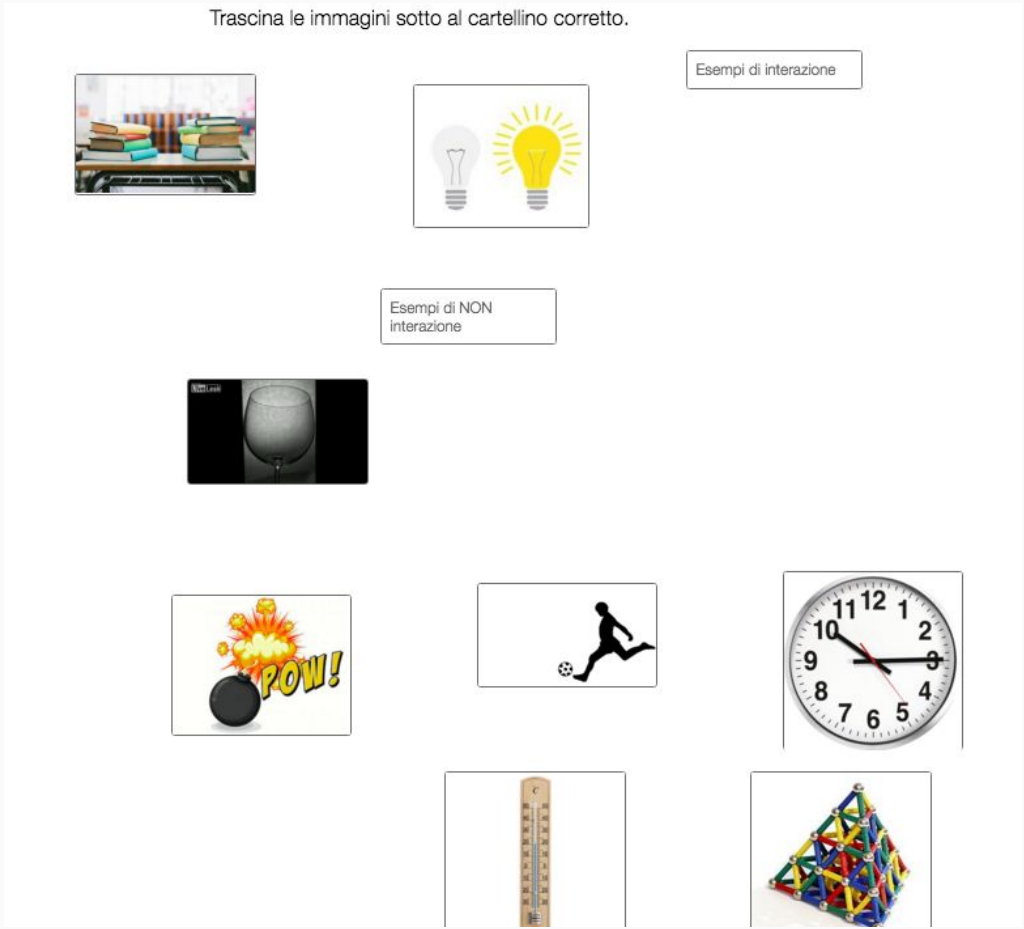
Condividi con la classe

Quello che hai appena visto è un esempio di interazione.

La forza è una
INTERAZIONE
tra due sistemi

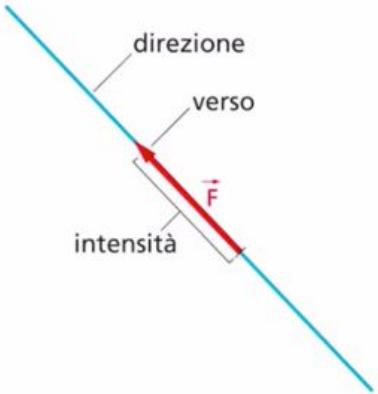
Nelle successive immagini,

fai attenzione se è presente un'interazione tra due sistemi.



The same images but another request: not forces but interactions

Come possiamo rappresentare la forza?



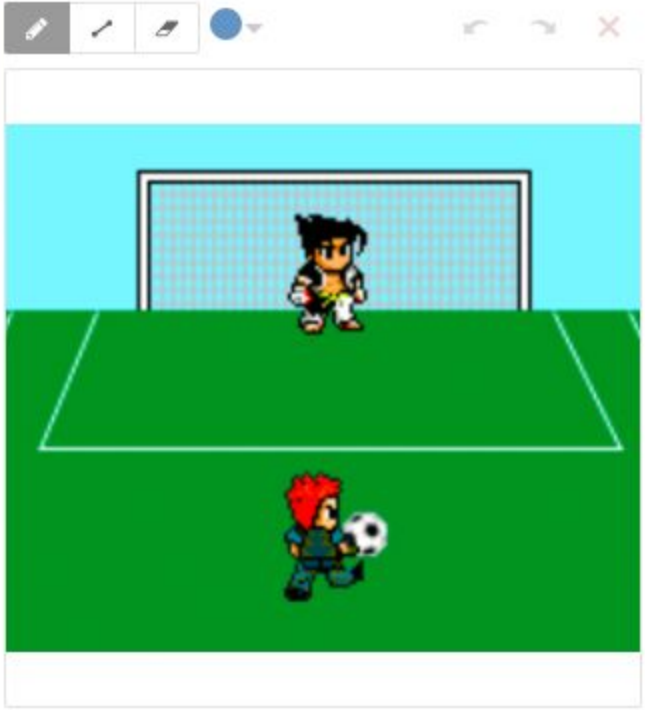
Per rappresentare l'interazione tra due o più oggetti, scegliamo di utilizzare una FRECCIA.

Osserva l'immagine. Che cosa caratterizza la freccia?

Condividi con la classe

Come possiamo rappresentare la forza?

Guarda il video e rappresenta con una freccia la forza applicata sul pallone.



Osserva l'immagine e rispondi.

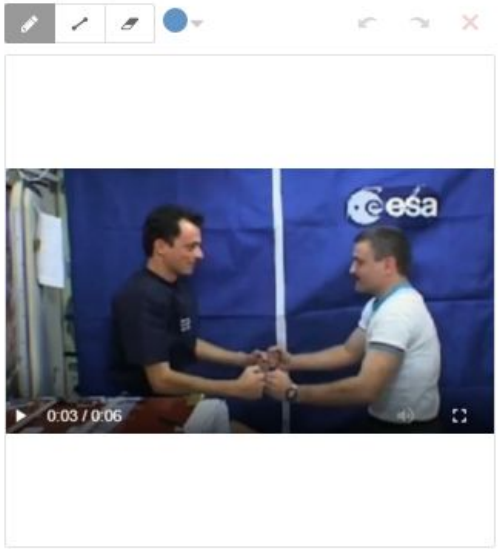
Applico una forza al carrello. Cosa succede?

- ○ Il carrello si sposta verso destra.
- ○ Il carrello rimane fermo.
- ○ Il carrello si sposta verso sinistra.

Osserva l'immagine e rispondi.

- ◯ Le due squadre esercitano due forze uguali e contrarie.
- ◯ La squadra di destra esercita una forza maggiore.
- ◯ La squadra di sinistra esercita una forza minore.

Guarda il video e rappresenta con due frecce di colore diverso le due forze esercitate da Alexander e Pedro.



Vediamo perchè entrambi si muovono.

Che cos'è l'accelerazione?

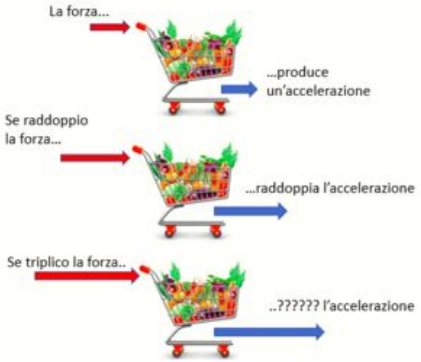
Guarda il video e rifletti.

Che significato ha la tabella del video? La macchina parte da ferma e per raggiungere la velocità di 40 m/s, ad ogni istante varia la sua velocità.

Da che cosa è causata l'accelerazione?

Condividi con la classe

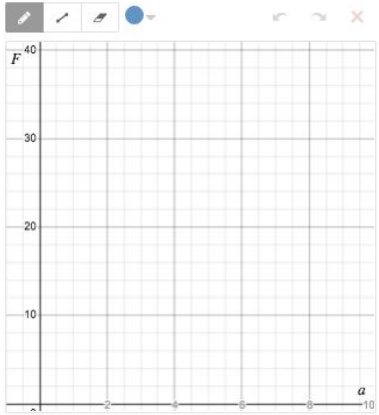
Che relazione c'è tra la forza e l'accelerazione?



Osserva l'immagine. Come sarà l'accelerazione quando triplico la forza?

- L'accelerazione si dimezza.
- L'accelerazione triplica.
- L'accelerazione diminuisce.

Supponiamo di aver misurato i valori di forza e accelerazione applicati al carrello.

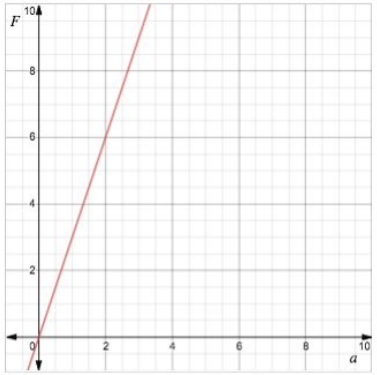


Riporta nel grafico i valori presenti in tabella.

Unisci i punti rappresentati nel grafico con una linea.

a (m/x^2)	F (N)
0	0
1	10
2	20
3	30
4	40

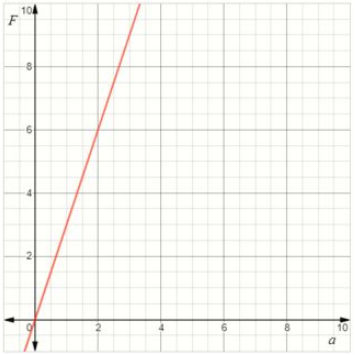
Osserva il grafico e completa la tabella.



a (m/s^2)	F(N)
0	
1	
	6
3	

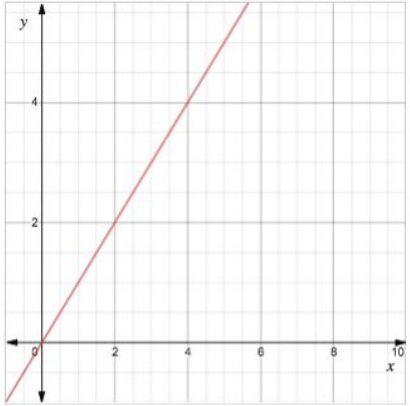
Quale grafico rappresenta meglio la relazione tra forza e accelerazione?

Osserva il grafico. Che tipo di proporzionalità c'è tra forza e accelerazione?



- Proporzionalità diretta (aumenta l'accelerazione, aumenta la forza)
- Proporzionalità inversa (aumenta l'accelerazione, diminuisce la forza)

In linguaggio matematico la proporzionalità diretta si esprime con la relazione y=kx



Ora prova tu a scrivere in linguaggio matematico la relazione che c'è tra la forza e l'accelerazione.

Condividi con la classe

Che cos'è 'k'?

Hai trovato la relazione F = ka.

Che cos'è "k"? La MASSA (m).

Riscrivi la relazione tra forza e accelerazione tenendo conto di cos'è k.

$F = ka$

Condividi con la classe

Osserva l'immagine.

Il valore numerico rappresenta una grandezza fisica.

$F = m * 6$ $\quad a = 6\ \frac{m}{s^2}$

$10 = m * a$ $\quad F = 10\ N$

$F = 3 * a$ $\quad m = 3\ Kg$

Ora prova tu a esprimere il valore numerico della grandezza fisica.

$F = m * 7$ $\quad a = ?$

$3 = m * a$ $\quad F = ?$

$F = 4 * a$ $\quad m = ?$

Tieni in considerazione che le unità di misura sono:

Newton (N) per la forza,

chilogrammo (Kg) per la massa,

metro al secondo quadrato (m/s^2) per l'accelerazione.

Condividi con la classe

Trova i valori mancanti.

F(N)	m (kg)	a (m/s^2)
10		5
6	3	
	1	10
4		2
	6	4
12	4	
1		1

Fai corrispondere ogni tabella alla rispettiva rappresentazione grafica e in linguaggio matematico

F=1a

F=5a

Osserva l'immagine, rappresenta le forze e rispondi.

Luca tira un pugno ad un sacco da box di 50 chilogrammi. Sapendo che il sacco si sposta con un'accelerazione di 5 m/s^2, che forza dovrà applicare Luca per farlo muovere?

Condividi con la classe

Osserva l'immagine e inventa il testo del problema.

Condividi con la classe

Knowledge pieces useful for problem solving and Jeopardy problems