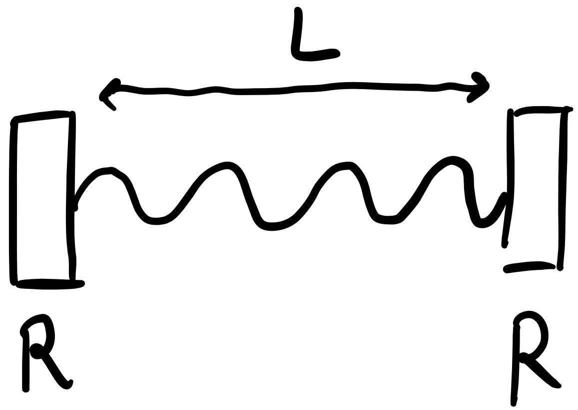
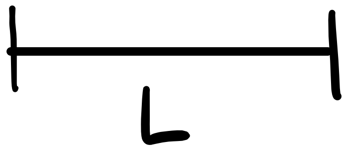
1D cavity



- $R = 1$



standing wave condition

$2L = n \cdot \lambda$ (nodi sugli specchi)

$n \in \mathbb{N}$

$$\rightarrow \lambda_n = \frac{2L}{n} \Rightarrow \nu_n = \frac{c}{\lambda_n} = n \cdot \frac{c}{2L}$$

$$\omega_n = 2\pi \nu_n$$

Free spectral range: $\Delta\omega = \omega_{n+1} - \omega_n = 2\pi \frac{c}{2L}$

- n: # of photons

loss rate: $\dot{n} = -T \cdot n \cdot \frac{1}{t_R}$

$\left[\begin{array}{l} t_R\text{: round trip time} \\ t_R = 2L/c \end{array} \right.$

$\rightarrow \dot{n} = -T \frac{c}{2L} \cdot n \Rightarrow$ exponential decay

$$n(t) = n(0)\, e^{-\frac{T \cdot c}{2L} t}$$

Decay of intensity lifetime: $\tau = \frac{2L}{c \cdot T}$

Decay of field lifetime: $\tau_F = 2 \cdot \tau$

(field is square root of intensity:

$$I \propto E^2 \Rightarrow e^{-t/\tau} = \left(e^{-t/\tau_F}\right)^2 = e^{-2t/\tau_F}$$

$$\Rightarrow \frac{1}{\tau} = \frac{2}{\tau_F} \Rightarrow \tau_F = 2\tau$$

- Classical field:

$$E(t) = e^{i\omega_c t} e^{-t/2\tau} \cdot \Theta(t) \qquad \Theta(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$PSD(\omega) = \left| FT\left[E(t)\right] \right|^2 =$$

$$= \left| \int_0^\infty dt\, e^{-i\omega t} e^{i\omega_c t} e^{-t/2\tau} \right|^2 =$$

$$= \left| \int_0^\infty dt\, e^{t\left(i(\omega_c - \omega) - \frac{1}{2\tau}\right)} \right|^2 =$$

$$= \left| \left[\frac{1}{i(\omega_c - \omega) - \frac{1}{2\tau}} e^{t\left(i(\omega_c - \omega) - \frac{1}{2\tau}\right)} \right]_0^\infty \right|^2 =$$

$= 0$ for $t = \infty$

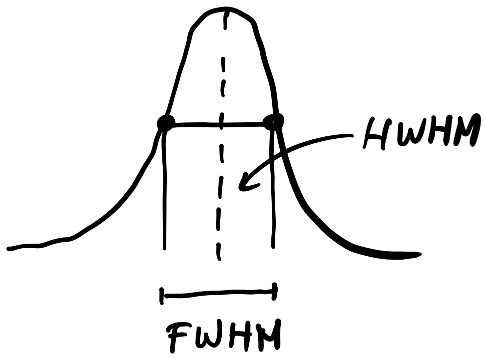
$= 1$ for $t = 0$

$$= \left| \frac{-1}{i(\omega_c - \omega) - \frac{1}{2\tau}} \right|^2$$

$$\boxed{\begin{aligned} &\frac{1}{x+iy} = \frac{x-iy}{(x+iy)(x-iy)} = \frac{x-iy}{x^2+y^2} \\ &\left|\frac{1}{x+iy}\right|^2 = \frac{(x-iy)(x+iy)}{(x^2+y^2)^2} = \frac{1}{x^2+y^2} \end{aligned}}$$

$$= \frac{1}{(\omega_c - \omega)^2 + \frac{1}{4\tau^2}}$$

Lorentzian lineshape!



$$\text{HWHM}: \quad \frac{1}{x^2 + \frac{1}{4\tau^2}} = \frac{1}{2}\,\frac{1}{\frac{1}{4\tau^2}}$$

$$\frac{1}{4\tau^2} = \frac{x^2}{2} + \frac{1}{8\tau^2}$$

$$\frac{x^2}{2} = \frac{1}{8\tau^2}$$

$$x^2 = \frac{1}{4\tau^2} \quad \longrightarrow \quad x_{HWHM} = \frac{1}{2\tau}$$

$$\text{FWHM}(\,\text{PSD}(\omega)\,) = \frac{1}{\tau} \qquad (!)$$

- $$Q_n = \frac{\omega_n}{\text{FWHM}} = \omega_n \cdot \tau = 2\pi n \frac{c}{2L} \cdot \frac{2L}{cT} = \frac{2\pi n}{T}$$

- $$f = \frac{\Delta\omega}{\text{FWHM}} = 2\pi \frac{\cancel{c}}{\cancel{2L}} \cdot \frac{\cancel{2L}}{\cancel{c}T} = \frac{2\pi}{T}$$

Where did we approximate $R \ll 1$?