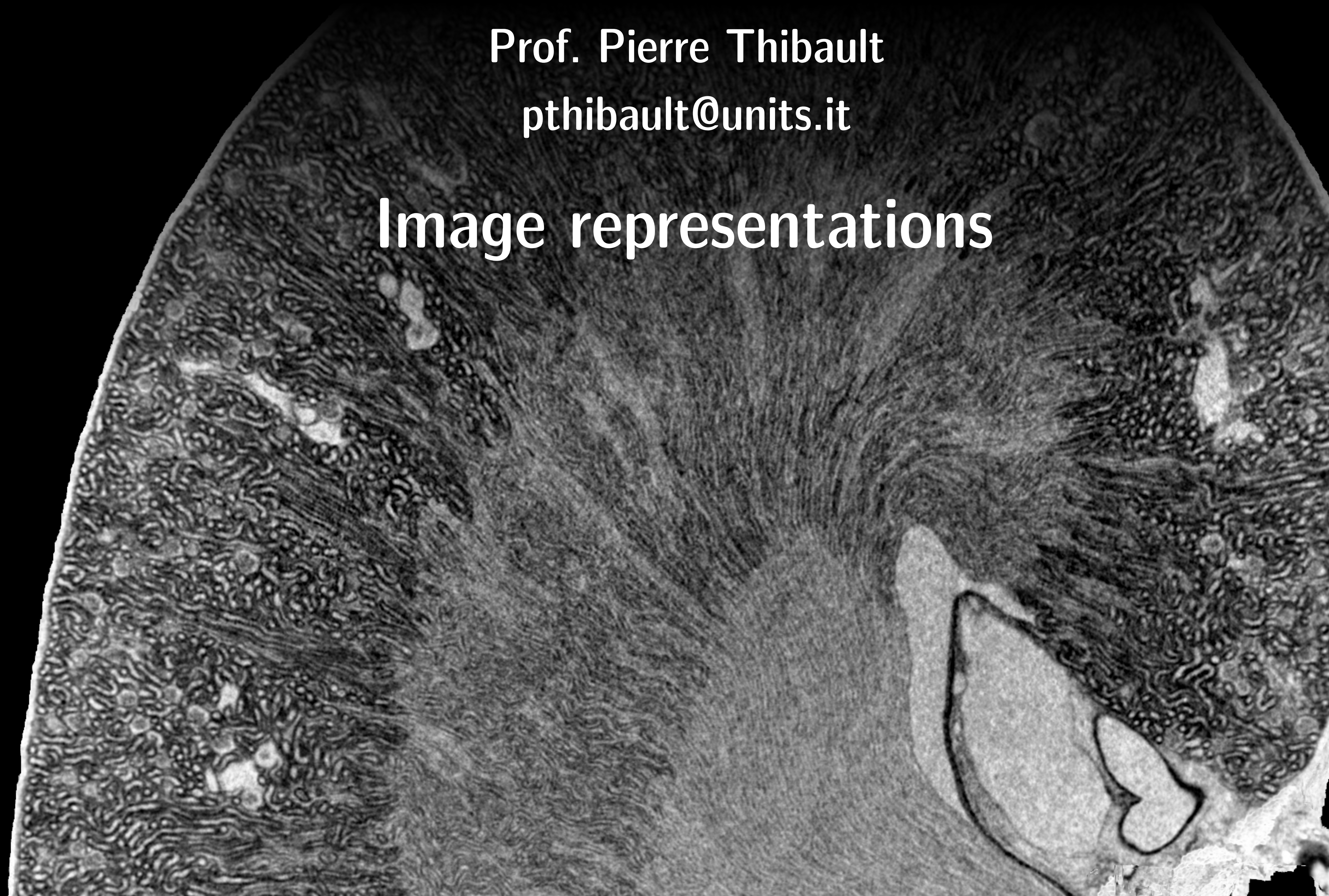
Image Processing for Physicists

Prof. Pierre Thibault
pthibault@units.it

Image representations



Overview

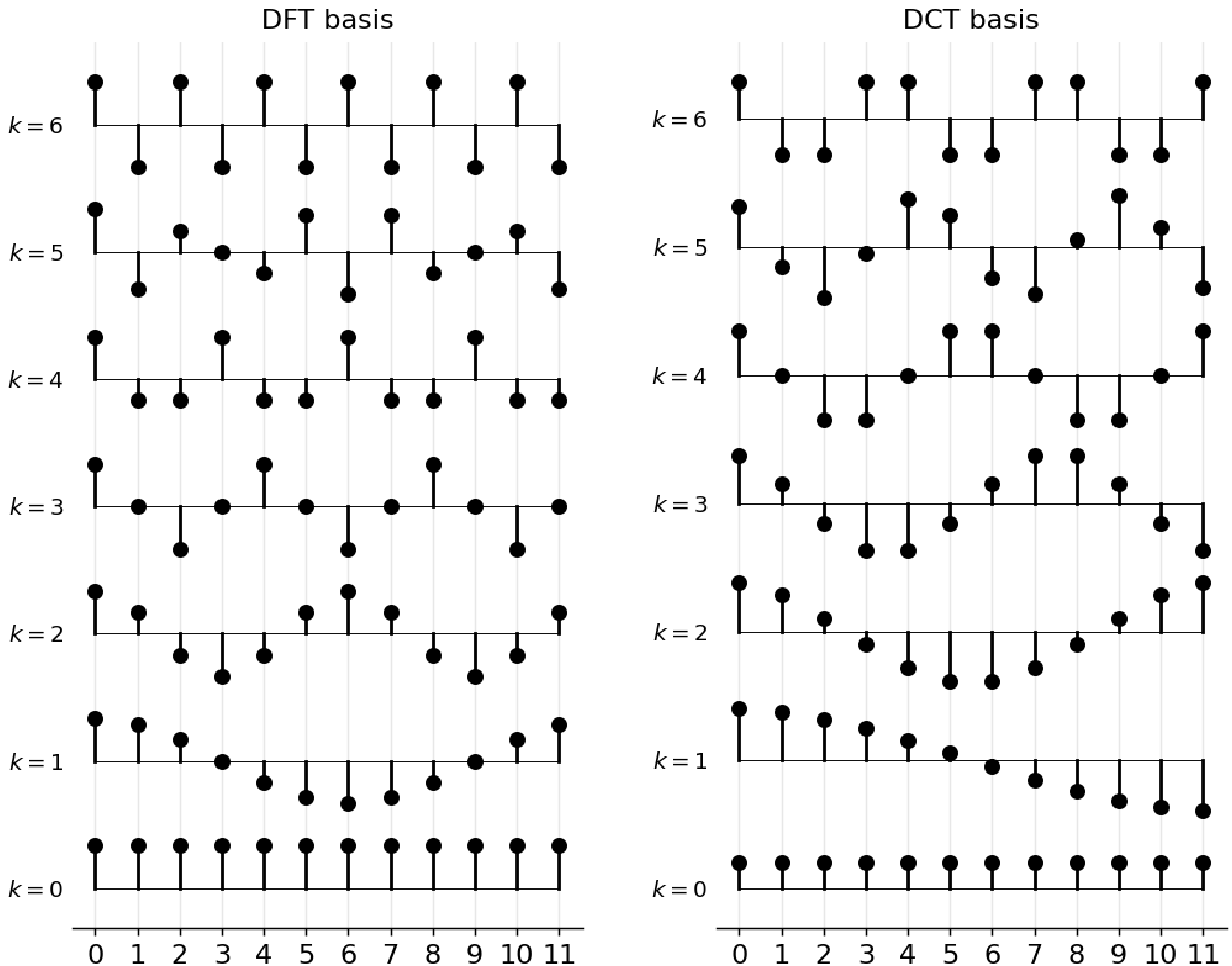
- The Discrete Fourier Transform as a change of basis
- Discrete Cosine Transform
- Windowed Fourier Transform
- Wavelet Transform
- (many others omitted!)

Image representations

Discrete Cosine Transform

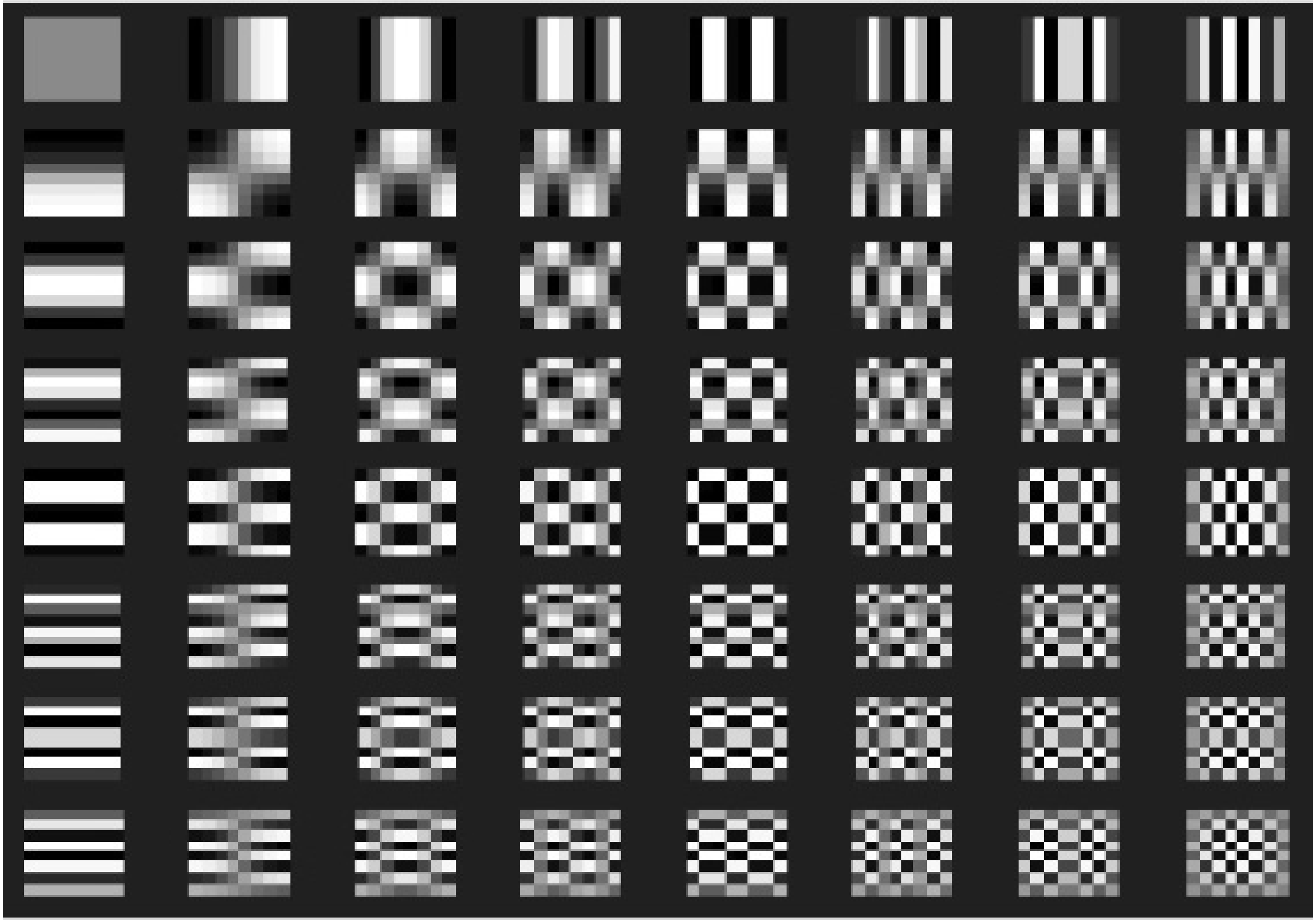
A variation on the theme of DFT

Discrete Cosine Transform



Discrete Cosine Transform

64 DCT basis vectors for 8x8 image



Discrete Cosine Transform

Image compression

1:1 bit rate

8:1 bit rate

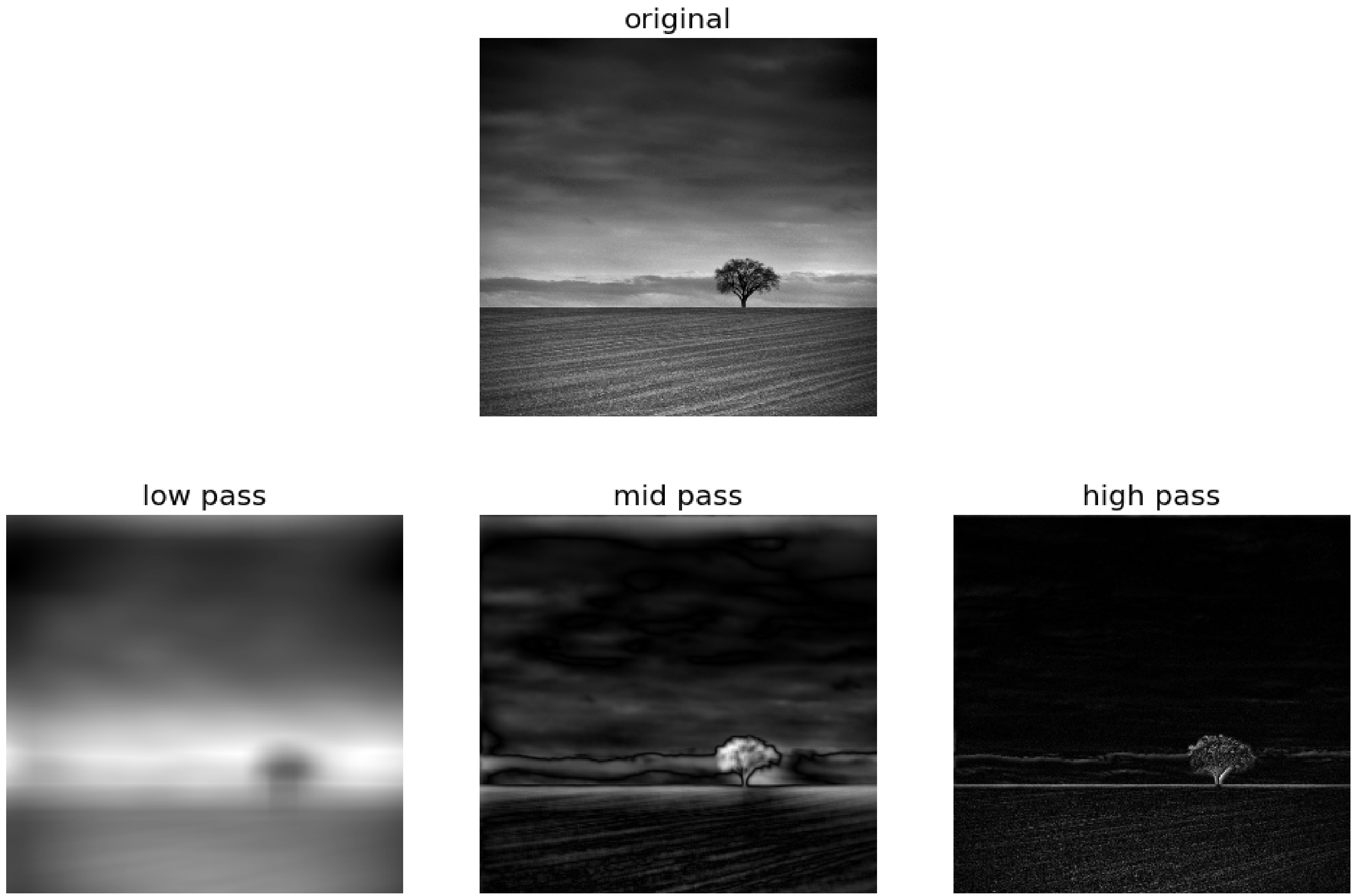
32:1 bit rate

128:1 bit rate

Historical overview

- 1822 Fourier: Fourier transform
- 1946 Gabor: “Gabor transform”, Short-time Fourier transform (STFT)
- 1974 Ahmed, Natarajan & Rao: Discrete Cosine Transform
- 1980s Morlet, Mallat, Daubechies, ... : Wavelets

Bandpass filtering

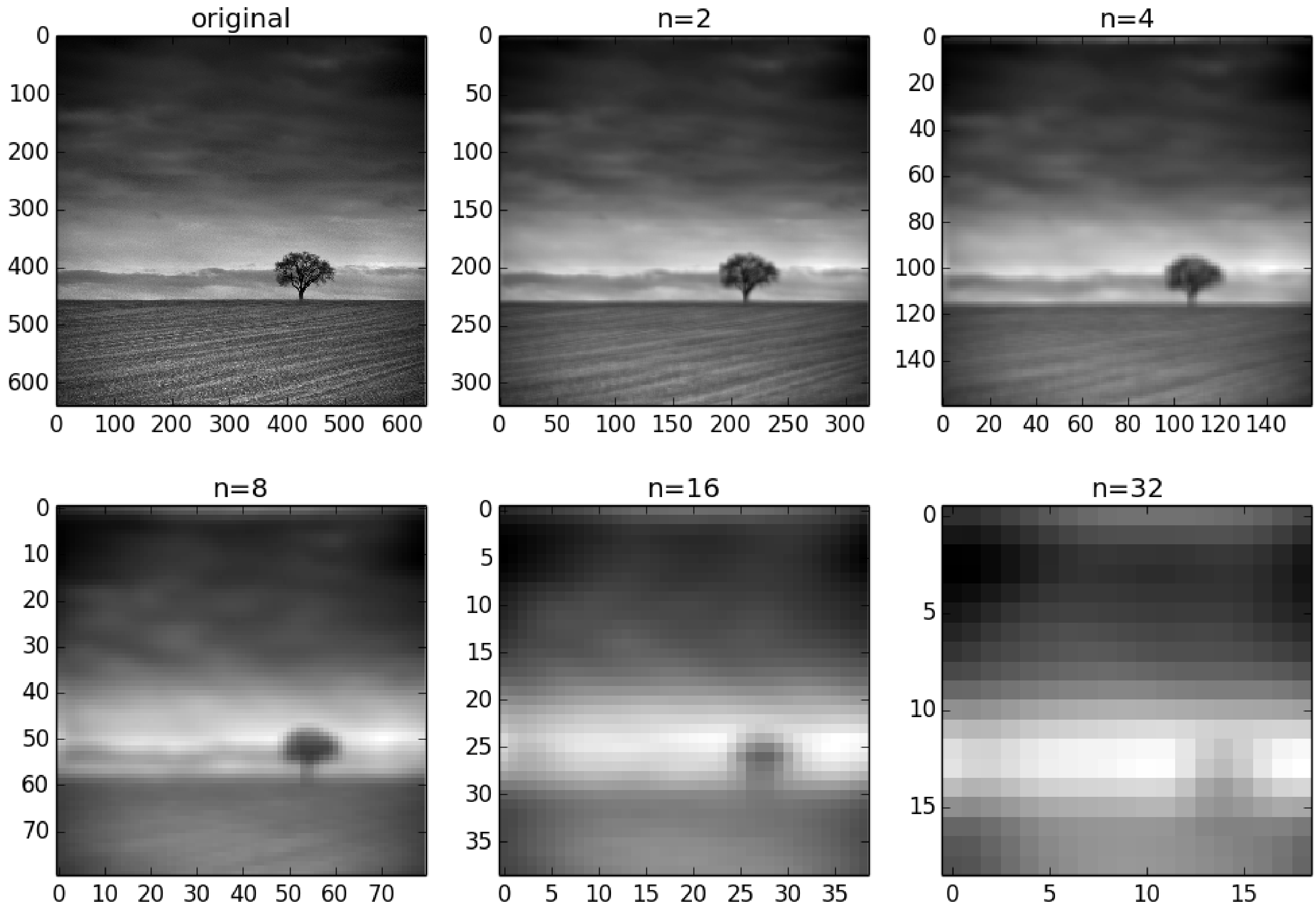


Don't need high spatial resolution

Need high spatial resolution

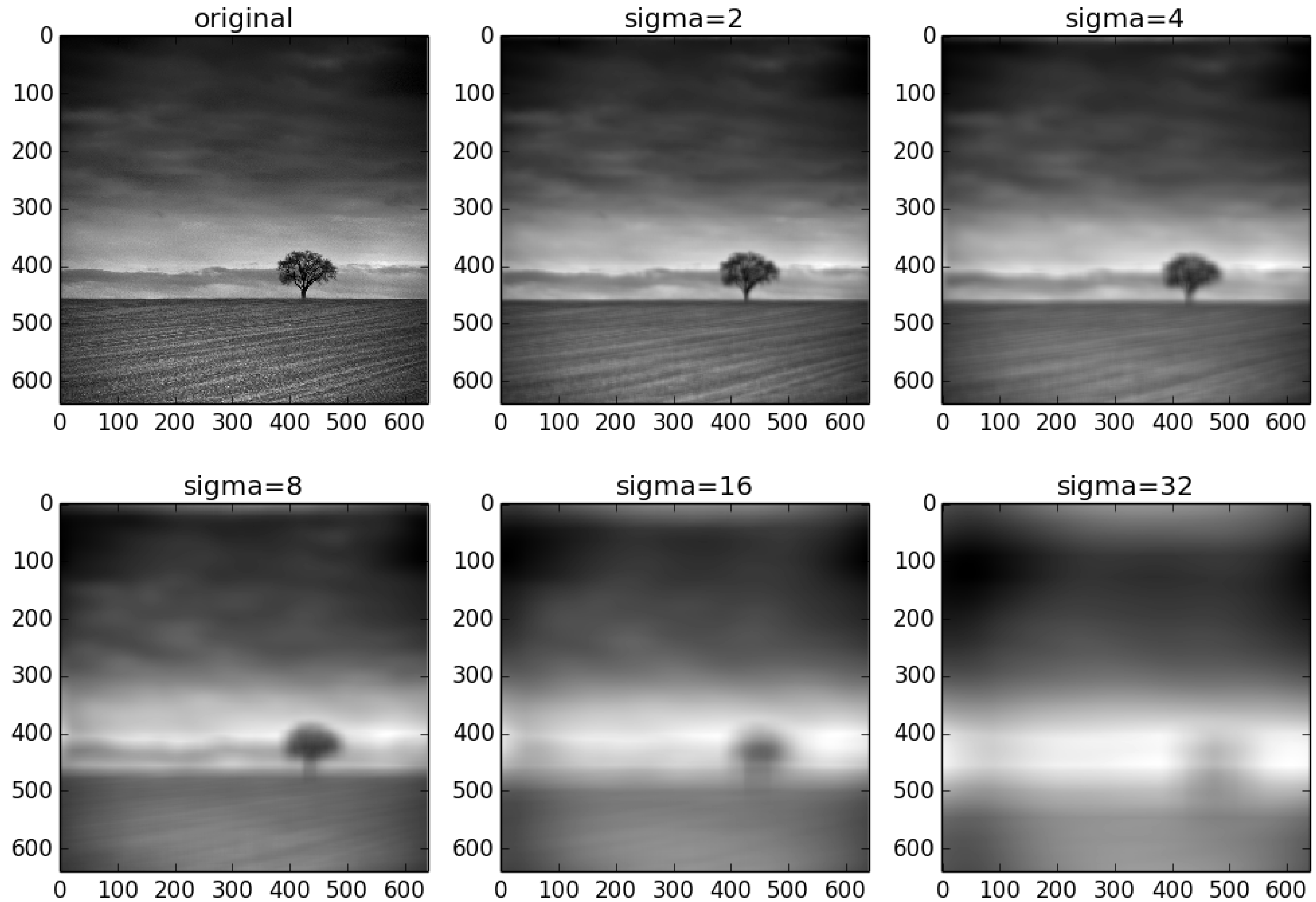
Multiresolution analysis

Subsampling (taking every n^{th} pixel) successively reduces high frequency content



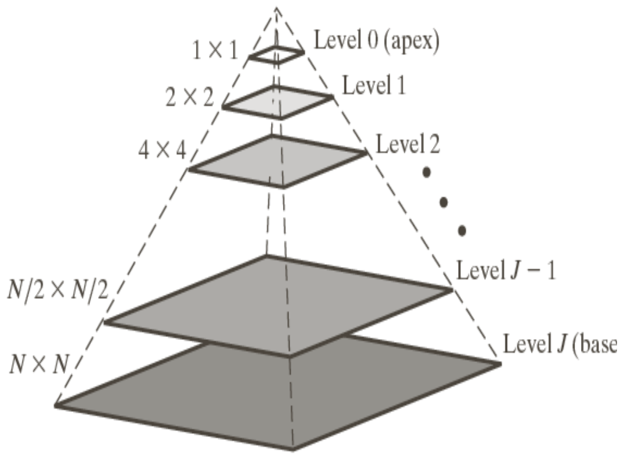
Multiresolution analysis

Multiple filtering with Gaussian filters, sigma determines resolution



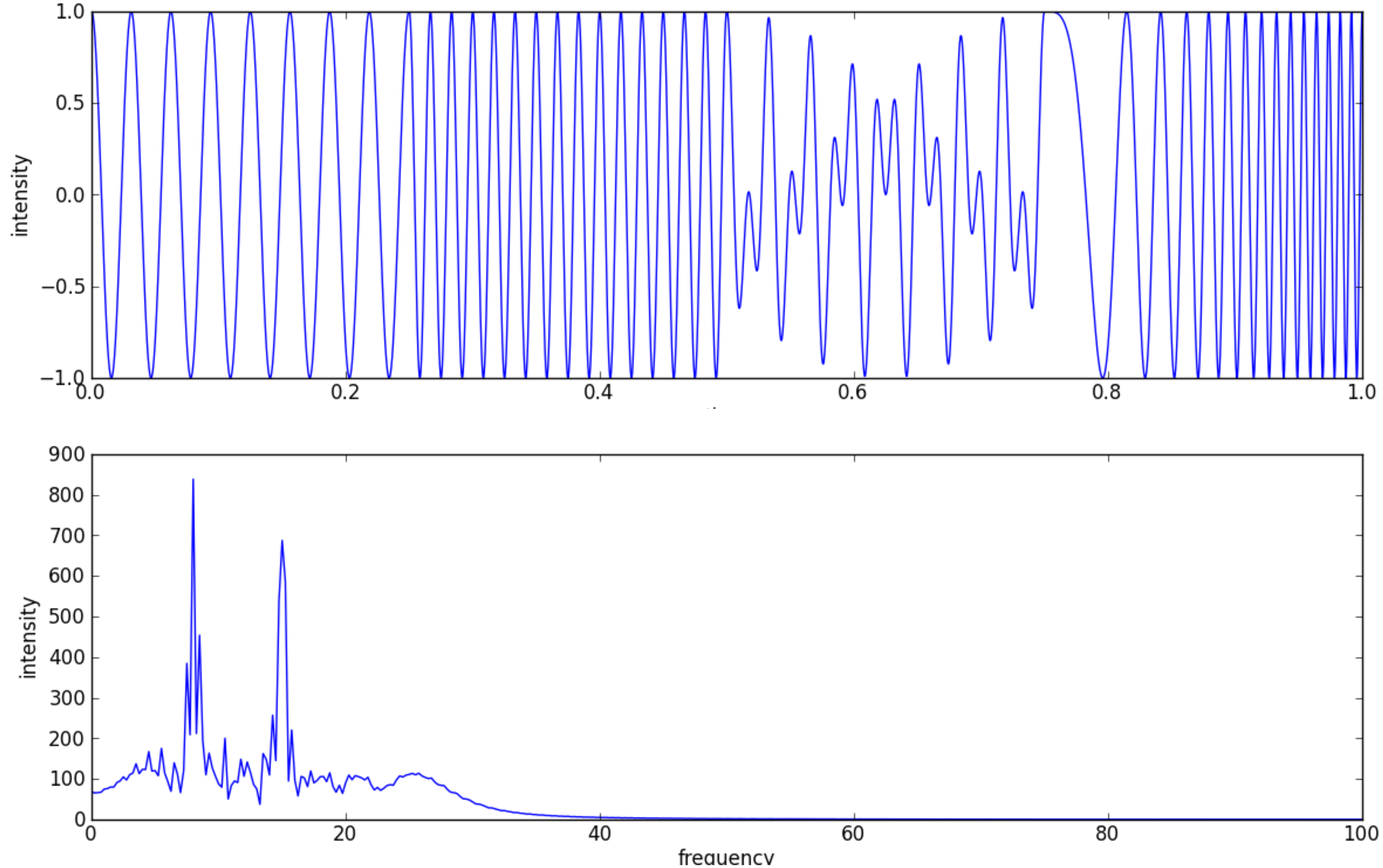
Pyramid representation

Scale-space representation, pyramidal representation



Stationary vs. non-stationary signals

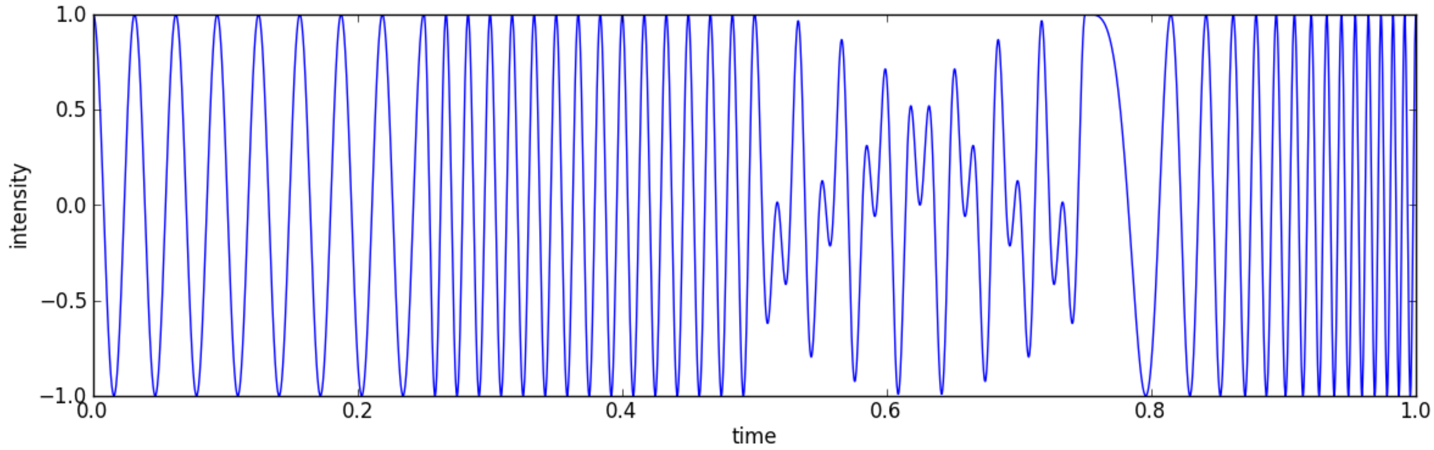
- Stationary signals: frequency doesn't change over time (spatially over the image)
- Non-stationary signals: frequency changes over time (spatially over the image)
- Examples of non-stationary signals: speech, most images



FT insufficient to localize the frequencies in our signal (image)

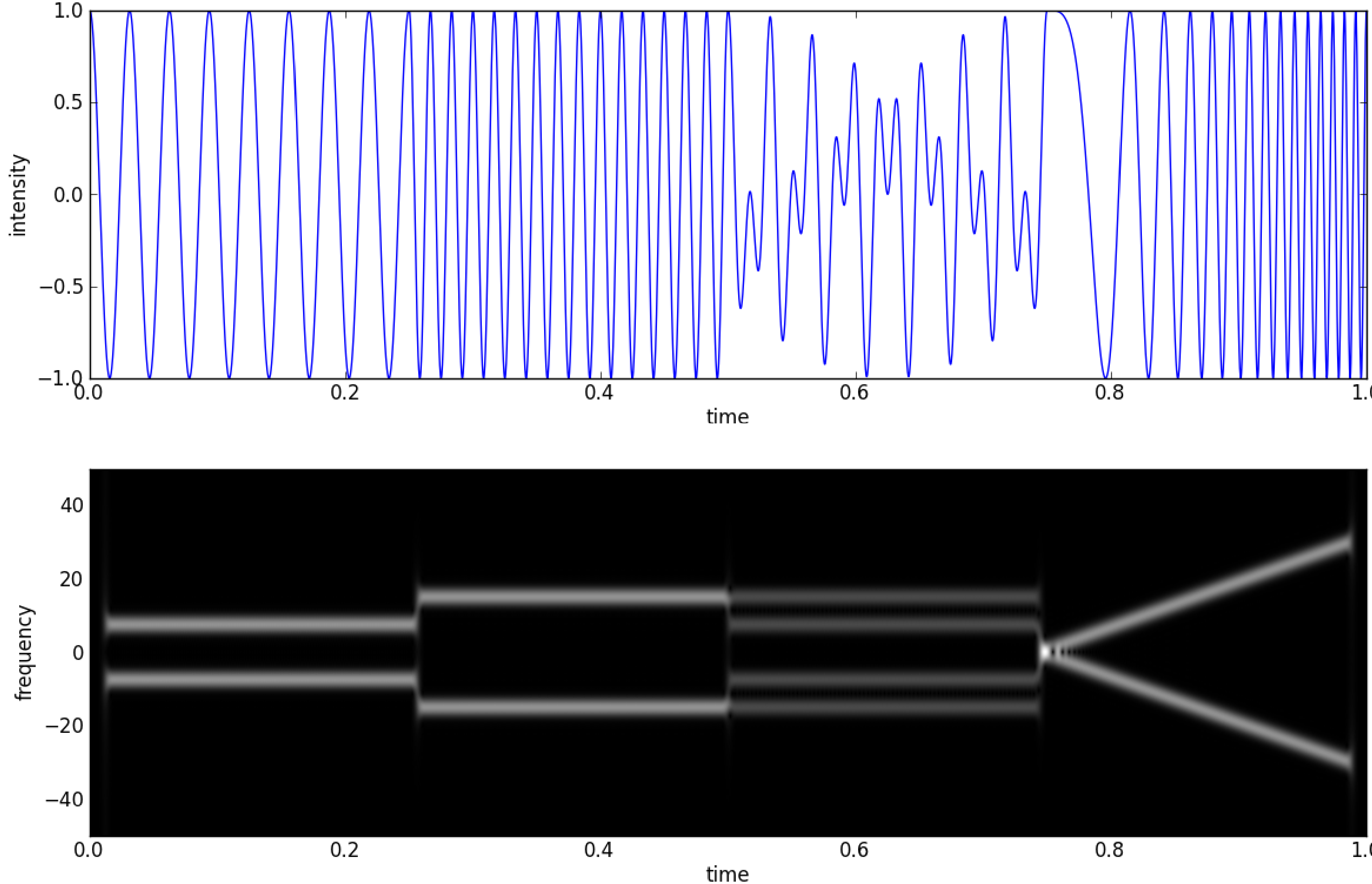
Windowed Fourier transform

- Windowed Fourier transform is part of the field of "time-frequency analysis"
- Also known as Short-time Fourier Transform (STFT)
- Time-frequency representations are used in many different contexts (Audio, image processing/optics, quantum mechanics)
- Idea: slice up signal into small parts, analyze each separately
  - Multiply with window function w (of width d) at position x0
  - Take Fourier transform of result
  - Slide window to new position
  - repeat



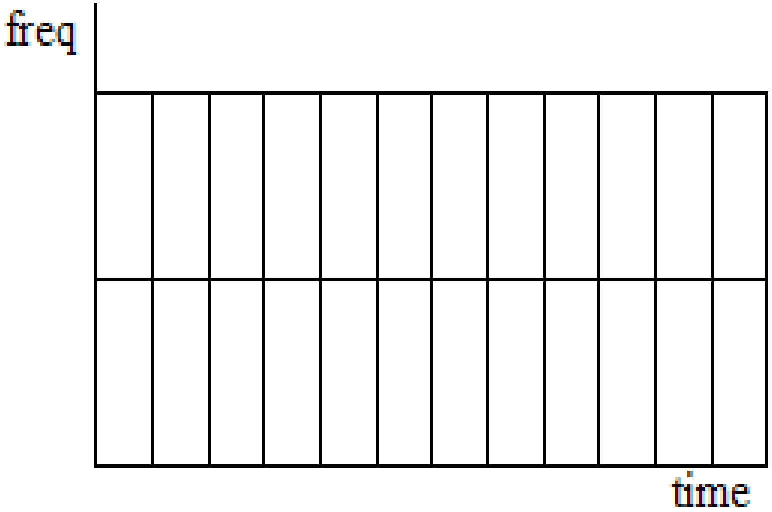
Analogy to audio signals

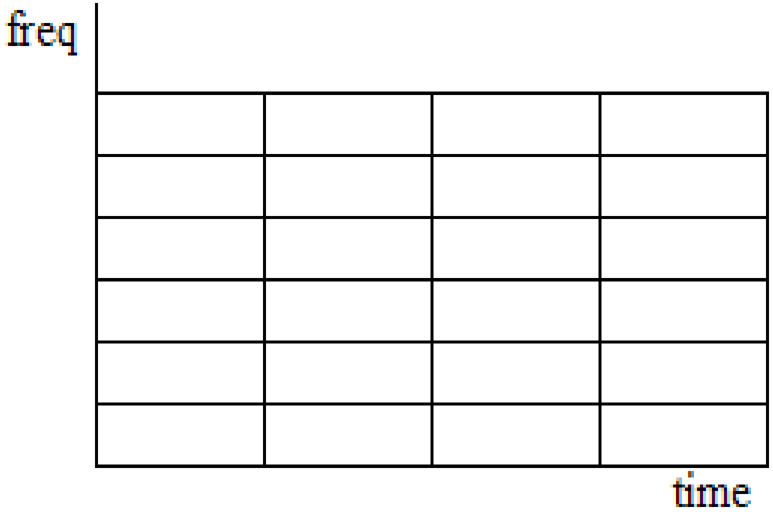
Spectrogram



Uncertainty relation

- Finite area in the time-frequency plane



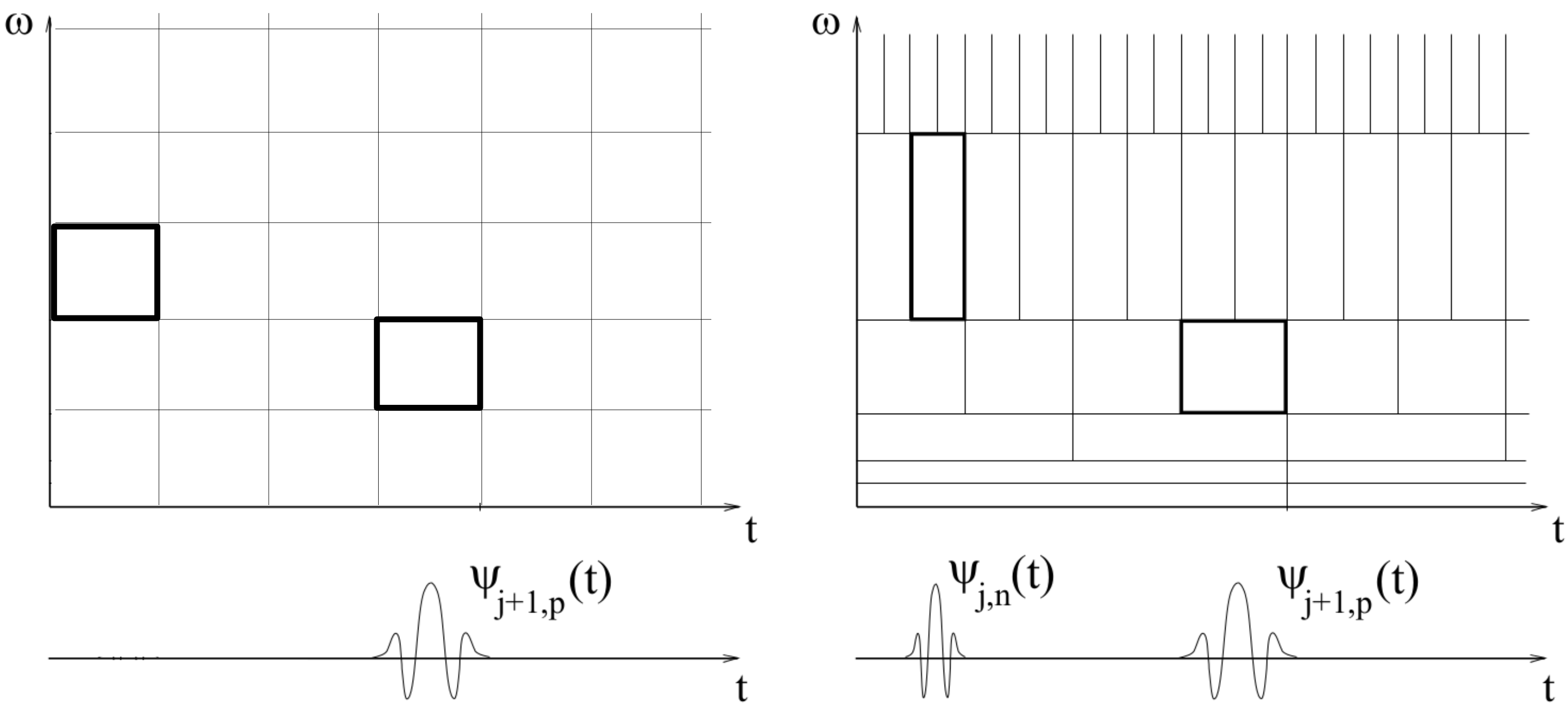


- This is limitation of WFT and hence development of **wavelets**

Continuous wavelet transform (WT)

- Parameters: translation and scaling

- Analyze signal at <u>different scales</u> instead of <u>different frequencies</u>

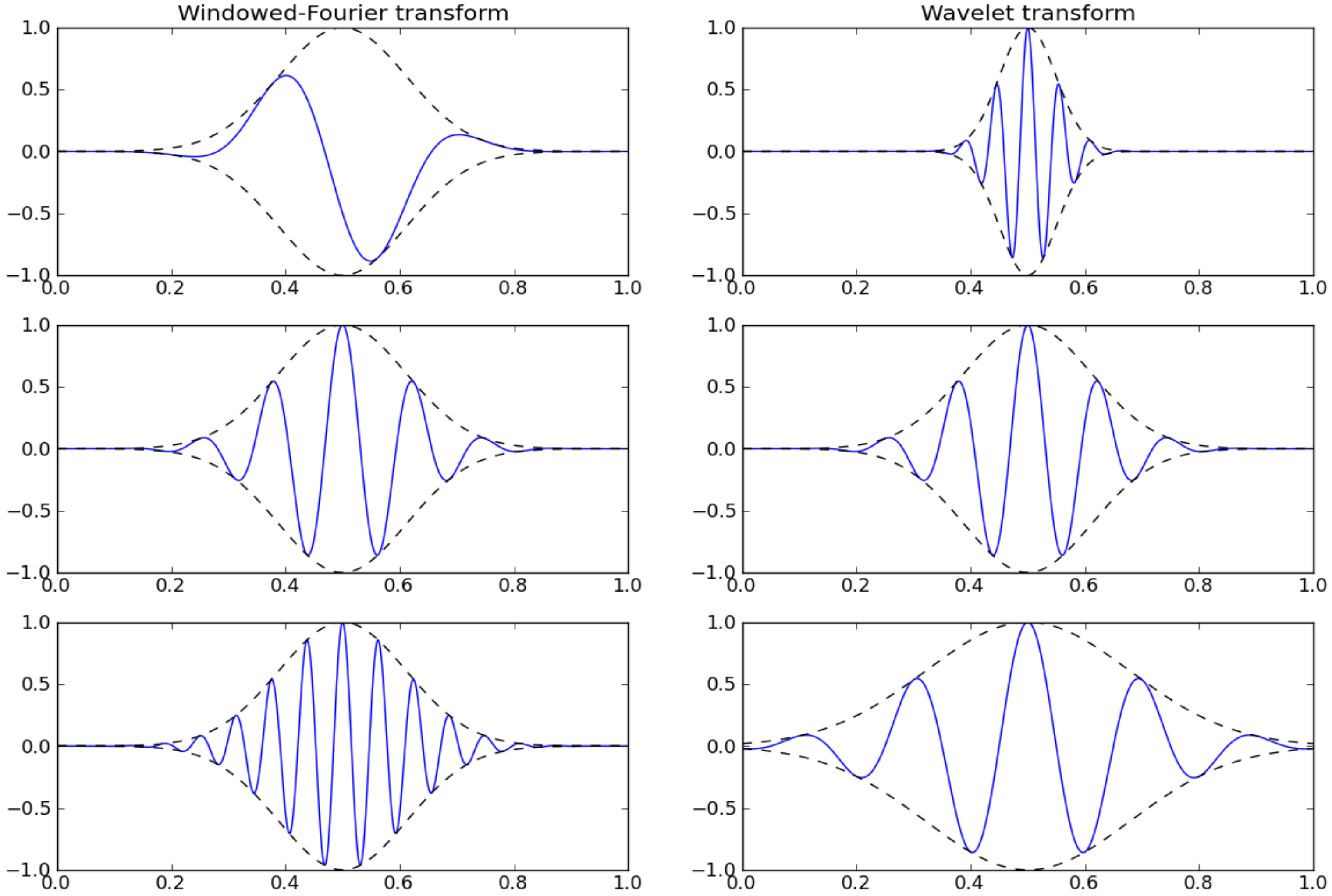


Source: Mallat, “A wavelet tour of signal processing”

WFT vs WT

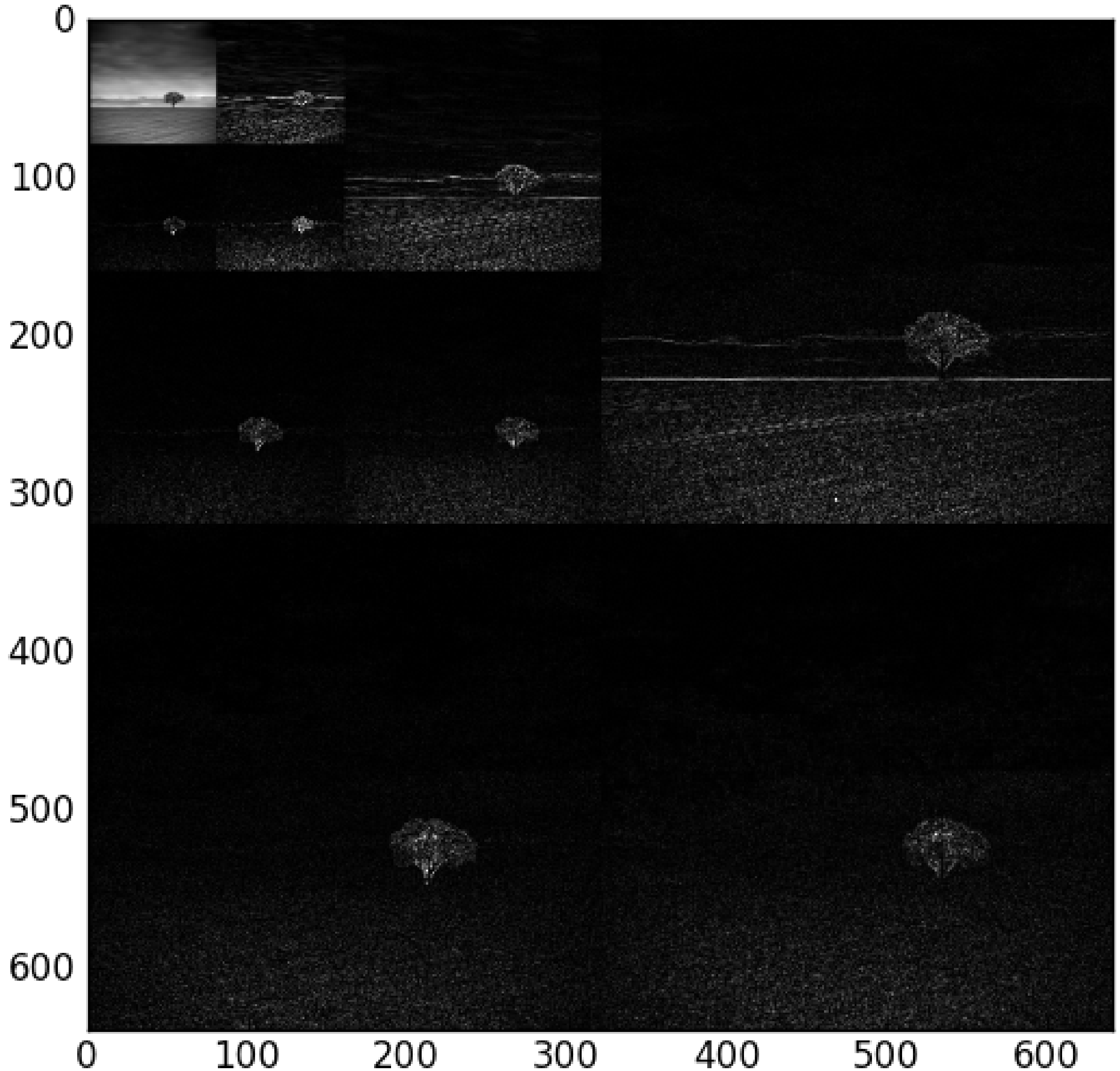
WFT - keep window width constant
- change modulation

Wavelet - keep shape constant
- change scale



Discrete Wavelet decomposition of image

- Perform each DWT, collect and tile all coefficients
- Here: 3 level decomposition



Summary

- Images can be represented by different basis functions.
- Fourier basis: localized in frequency, delocalized in real space.
- Windowed Fourier Transform: localized – to some extent – in both spaces
- Wavelet analysis decomposes a signal in position and scale (instead of position and frequency as for WFT).
- Sparse representations are representations in which the image content is represented by a few relevant coefficients, while the other pixels are close to zero
- Sparse representations have advantages for compression, denoising, ...