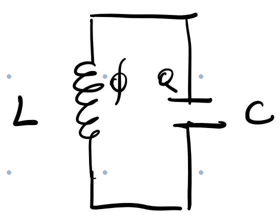
Superconducting qubits

The LC circuit as a harmonic oscillator

How can we make a circuit which can be used for quantum information processing? If we recall from any basic Electromagnetism course the parallel LC circuit will act in many ways like a mechanical harmonic oscillator. This suggests that an LC circuit might be able to be used for quantum information processing. However, we need to figure out if the electric circuit will also quantize as the mechanical system does and determine the relevant parameters of the system.

We consider a circuit involving an inductor (with inductance L) and and a capacitor (with capacity C)



$$E_C = \frac{1}{2} C V_C^2 \qquad E_L = \frac{1}{2} L I^2$$

ENERGY STORED IN THE CAPACITOR

VOLTAGE AT THE END OF THE CAPACITOR

CURRENT FLOWING IN THE INDUCTOR

VOLTAGE AT L

MAGNETIC FLUX $\phi = LI$ $\quad V_L = -L\frac{dI}{dt}$

TOTAL CHARGE $Q = CV_C$

$\Rightarrow$ CLASSICAL HAMILTONIAN $H_{CL} = E_C + E_L = \dfrac{Q^2}{2C} + \dfrac{\phi^2}{2L}$

Now we are stuck since we do not know the quantum operators for ϕ AND Q

LET'S GET INSPIRED BY THE MECHANICAL HARMONIC OSCILLATOR

$$H_{MEC} = \frac{p^2}{2m} + \frac{Kx^2}{2}$$

$\Rightarrow$ THINK OF Q AS p AND ϕ AS x

RECALL THE CANONICAL EQUATIONS

$$\frac{\partial H_m}{\partial p} = \dot{x} \quad \text{AND} \quad \frac{\partial H_m}{\partial x} = -\dot{p}$$

SIMILARLY $\frac{\partial H}{\partial \phi} = \frac{\phi}{L} = I = \dot{Q}$

$$\frac{\partial H}{\partial Q} = \frac{Q}{C} = V = -L\dot{I} = -\dot{\phi}$$

SAME AS MECHANICAL HARMONIC OSCILLATOR WITH Q = P AND X = ϕ ⇒ CAN DO QUANTIZATION

We may now write the quantum Hamiltonian of the LC circuit as

$$H_{LC} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\phi}^2}{2L}$$

WITH $\hat{Q} = i\sqrt{\frac{\hbar}{2Z_0}}\,(a^+ - a)$

$$\hat{\phi} = \sqrt{\frac{\hbar Z_0}{2}}\,(a^+ + a)$$

IN DOING THIS RECALL SECOND QUANTIZATION FOR MECHANICAL HARMONIC OSCILLATOR

$$\hat{a} = \frac{1}{\sqrt{2m\hbar\omega}}(im\omega x + p)$$

$$Z_0 = \sqrt{\frac{L}{C}}$$ IMPEDANCE (MASS EQUIVALENT)

WITH THIS $H_{LC} = \hbar\omega\,(a^+ a + \frac{1}{2})$ WITH $\omega = \frac{1}{\sqrt{LC}}$

	Classic Mechanical	Classic Electronic	Quantum Mechanical	Quantum Electronic
Displacement	x	Φ	$\hat{x}$	$\hat{\Phi}$
Flow	p	Q	$\hat{p} = -i\hbar\frac{d}{dx}$	$\hat{Q} = -i\hbar\frac{d}{d\Phi}$
Force Proportionality	m	C	m	C
Restoring Proportionality	k	$\frac{1}{L}$	k	$\frac{1}{L}$
Resonant Frequency	$\omega = \sqrt{\frac{k}{m}}$	$\omega = \frac{1}{\sqrt{LC}}$	$\omega = \sqrt{\frac{k}{m}}$	$\omega = \frac{1}{\sqrt{LC}}$
Commutation Relations	-	-	$[\hat{x}, \hat{p}] = i\hbar$	$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar$

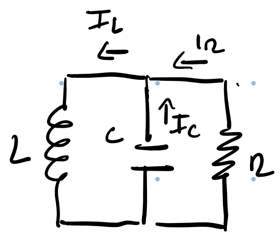
what would happen if we chose the variables dierently so that Q was x and P WAS ϕ?

Since these variables are canonical variables, then it should not be too surprising to realize that the analysis would have taken a slightly dierent path, but produced identical results.

$m \to L \quad K \to 1/C$ ETC. BUT YOU SHOULD GET THE SAME

NOTE: WE COULD HAVE USED $\hat{V}$ AND $\hat{I}$ $\quad \hat{V} = \frac{\hat{Q}}{C}$ AND $\hat{I} = \frac{\hat{\phi}}{L}$

In the circuit considered so far, we neglected any resistance. What would happen if we assume one? For simplicity let us put a resistor in parallel with C and L. Kirchoff's circuit laws tell us the net current at the junction is zero, and that the voltage across each element is equal.



$$I_L = I_R + I_C$$

$$I_R = \frac{V}{R}$$

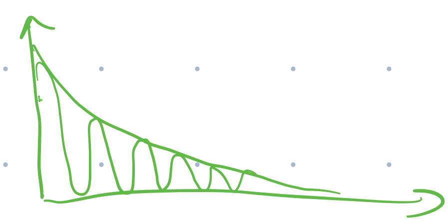
$$I_C = \dot{Q} = C\frac{dV}{dt}$$

$$V = -L\frac{dI_L}{dt}$$

$$\Rightarrow -I_C - I_R + I_L = 0 \quad \Rightarrow LC\frac{d^2 I_L}{dt^2} + \frac{L}{R}\frac{dI_L}{dt} + I_L = 0$$

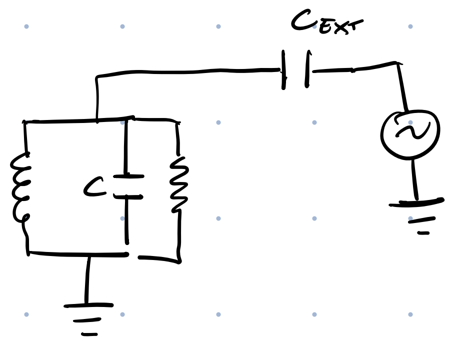
$$\Rightarrow I_L(t) = e^{-\beta t}\left[A_1 e^{t(\beta^2-\omega^2)^{1/2}} + A_2 e^{-t(\beta^2-\omega^2)^{1/2}}\right]$$

$\beta = 2RC$ $\longrightarrow$ DAMPED HARMONIC OSCILLATOR



The obvious consequence is that energy will dissipate over time. This may cause problems if, for example, we attempt to set our qubit into an excited state to be used later, but we wait too long and the energy has already dissipated. A simple solution is to decrease the resistance in the circuit to give a longer decay time. Superconductors have extremely small resistance, thus one way to increase our control over the circuit is to cool it down into the superconducting regime for the component material of the circuit.

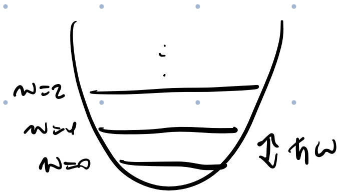
For our circuit to be useful we need to be able to couple it to the environment such that we can put energy in the system (qubit operations) or retrieve the energy (measure the qubit). This coupling adds increased parasitic effects on the harmonic oscillator including increased dissipation. Because we want our system to be somewhat isolated, a way of doing it is to couple to the external world capacitatively



CEXT CHANGES THE NET CAPACITANCE OF THE CIRCUIT

$$C_\Sigma = C + C_{EXT}$$

Another important aspect of controlling our system for quantum computation is knowledge and control of the state of the system. In particular, depending on the nature of the system, we know the probability of a certain energy level being occupied is dependent on temperature (Fermi-Dirac, Bose-Einstein or Maxwell-Boltzmann distribution functions). For T approaching 0K, the probability of the system to be in the ground state approaches 1. More over, we need to ensure that the gap between the ground state and the first excited state is much larger than the thermal energy of the system.

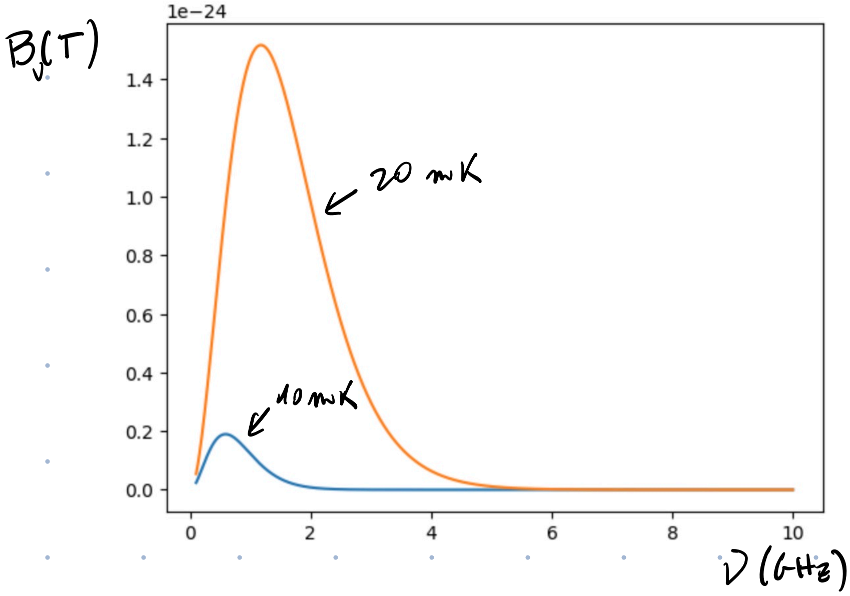


WE NEED THAT T IS LOW ENOUGH SUCH THAT

$$P(n=0) \simeq 1$$

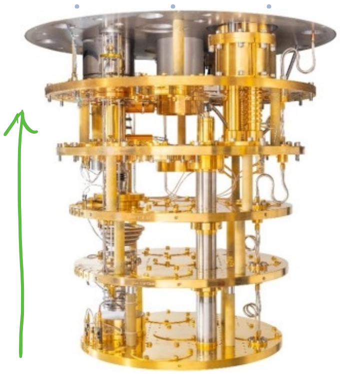
TYPICAL VALUES ARE $L = 1\,nH$, $C = 1\,pF$ $\Rightarrow$ $\omega = 2\pi \cdot 5\,GHz$

$\Rightarrow$ T SHOULD BE LOW SUCH THAT THE BLACKBODY SPECTRUM AT 5GHz IS NEGLIGIBLE



$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}$$

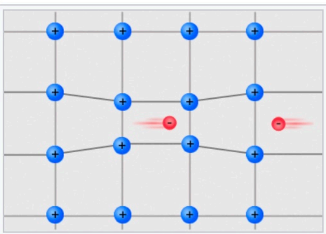
$\Rightarrow$ NEED TO COOL TO mK RANGE!! $\rightarrow$ DILUTION FRIDGES



TEMPERATURE

Notice that we have quantized the charge. And that the we want to work in the superconductive regime. So what is the charge in a superconductor?

COOPER PAIR $\rightarrow$ BOUND STATE BETWEEN TWO ELECTRONS



Schematic illustration of the Cooper pairing interaction in BCS superconductors

WIKIPEDIA

A PASSING ELECTRON "ATTRACTS" THE LATTICE CAUSING A SLIGHT RIPPLE TOWARDS ITS PATH

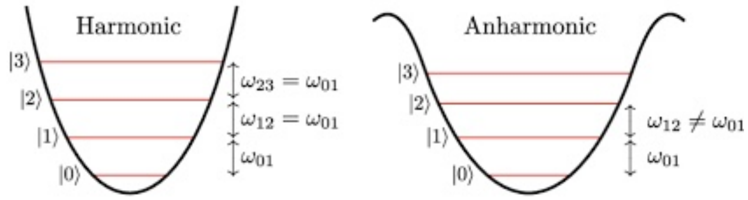
ANOTHER ELECTRON PASSING IN THE OPPOSITE DIRECTION IS ATTRACTED TO THAT DISPLACEMENT $\Rightarrow$ BOUND STATE

BINDING ENERGY $\approx$ 1 meV $\rightarrow$ BOSON WITH 0 SPIN AND CHARGE 2e

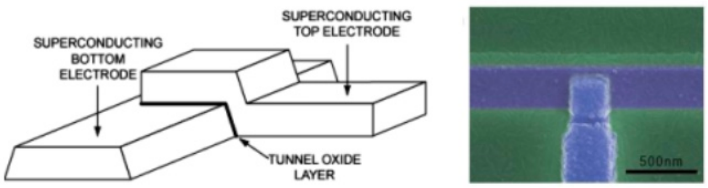
Nonlinear Oscillators (Josephson Junction)

We have now seen that LC circuit can be quantized to show discrete energy levels and that the charge is quantized (Cooper pairs). In theory we could use this to construct a qubit basis using any two levels of the system. We could also in principle create superposition states in the magnetic flux or the charge basis. However, all of the energy levels are separated by an identical amount of energy. When inducing a transition from the ground state to the first excited state, we may actually induce a transition from the first excited state to the second, or a transition from any two states in the system.

This will be a problem for any linear oscillator if we want to use them as 2 level systems. Ideally we need something that induces anharmonicity in the system



How can we do it? one option is to use the Josephson junction.This junction is nothing more than two superconducting materials sperate by an insulator



Josephson discovered that Cooper-pairs can tunnel through this insulating barrier and cause interesting effects. For us the most interesting one is that it will act like a non-linear inductor

$$I_S = I_c \sin \delta(t) = I_c \sin\left(2\pi \frac{\phi(t)}{\phi_0}\right)$$

MAGNETIC FLUX THROUGH JUNCTION

FLUX QUANTUM $\frac{h}{2e}$

CURRENT THROUGH THE JUNCTION

CRITICAL CURRENT (PHENOMENOLOGICAL PROPERTY)

$\delta_1 - \delta_2$ = DIFFERENCE IN PHASE ANGLE OF THE QUANTUM STATE OF THE SYSTEM ON EITHER SIDE OF THE JUNCTION

$\psi_2 = \sqrt{n_2}\, e^{i\delta_2}$ ← GINZBURG-LANDAU WAVE FUNCTION

$\psi_1 = \sqrt{n_1}\, e^{i\delta_1}$

$$V_S = \frac{\phi_0}{2\pi} \frac{d\delta}{dt} = \frac{d\phi}{dt}$$

VOLTAGE ACROSS JUNCTION

$$\Rightarrow \frac{dI_S}{dt} = I_c \cos\delta \frac{d\delta}{dt} \Rightarrow \frac{d\delta}{dt} = \frac{dI_S}{dt} \frac{1}{I_c \cos\delta}$$

$$\Rightarrow V_S = \frac{\phi_0}{2\pi I_c \cos\delta(t)} \frac{dI_S}{dt}$$

THIS LOOKS LIKE $V = -L \frac{dI}{dt}$ FOR THE INDUCTION

$$\Rightarrow \quad L_J = \underbrace{\frac{\phi_0}{2\pi I_c}}_{L_{J_0}} \frac{1}{\cos\delta(t)} = L_{J_0} \frac{1}{\cos\delta(t)} \longrightarrow$$

JOSEPHON INDUCTANCE

NOT LINEAR! δ CHANGES WITH I

LARGE IF $\delta \to \frac{\pi}{2}$

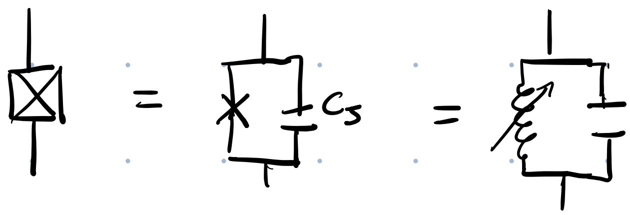
$L < 0 \quad \frac{\pi}{2} < \delta < \frac{3\pi}{2}$

THE ENERGY OF THIS NON LINEAR INDUCTOR IS

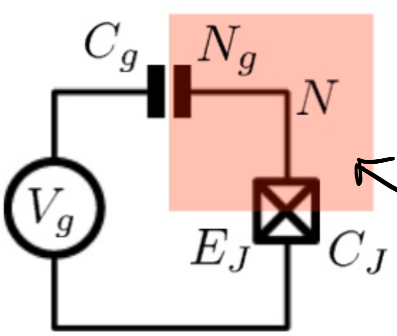
$$E_J = \int V I \, dt = \frac{I_c \phi_0}{2\pi} \int \sin\delta \frac{d\delta}{dt} dt = -\underbrace{\frac{I_c \phi_0}{2\pi}}_{E_{J_0}} \cos\delta$$

$$= -E_{J_0} \cos\delta$$

three dierent styles of depicting the Josephson Junction in a circuit. It is important to note that the Josephson Junction also has an internal capacitance which can be considered to be in parallel with the tunneling/inductive part ← MAKES SENSE BECAUSE THERE IS AN INSULATING GAP BETWEEN THE TWO SUPERCONDUCTORS

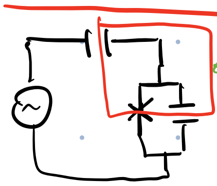


Cooper pair box



THE "ISLAND"

JOSEPHSON JUNCTION CONNECTED TO AN EXTERNAL VOLTAGE SOURCE THROUGH A GATE CAPACITOR



← THIS PART IS FULLY ISOLATED

This island is isolated from the outside world via the capacitors dielectric material and the Josephson Junction insulating gap. Initially, when there is no gate voltage, the island is charge neutral. As we turn on the voltage, charges will start accumulating on the outside of the capacitor. So, the charges in the island, will polarize, meaning that they will rearrange to cancel the external potential. So far no new charges have moved inside the island.

THIS IS THE NORMAL BEHAVIOR WITH CAPACITORS

There is however, a bridge onto the island across the Josephson Junction. As the island polarization increases, Cooper-pairs will being tunneling across the junction to try to re-neutralize the polarization. This causes an actual increase in the number of charges on the island. If we continue to increase the gate voltage, Cooper-pairs will continue to tunnel onto the island. If we remove the voltage, these extra Cooper-pair charges will tunnel back off of the island returning the system to its original state.

OF CHARGES ON C_g $\quad N_g = \frac{C_g V_g}{2e}$

OF CHARGES IN THE ISLAN $\quad N = \frac{Q}{2e}$

IDEA : QUBIT $\longrightarrow$ PRESENCE OR ABSENCE OF NEW CHARGES ON THE ISLAND

HAMILTONIAN $\quad \mathcal{H} = \mathcal{H}_{EL} + \mathcal{H}_{MAG}$

ELECTROSTATIC

MAGNETIC

$$\mathcal{H}_{EL} = \frac{Q^2}{2C_{TOT}}$$

ELECTRIC FIELDS STORED IN THE CAPACITORS C_S AND C_g

$$C_{TOT} = C_g + C_S$$ CAPACITORS IN PARALLEL

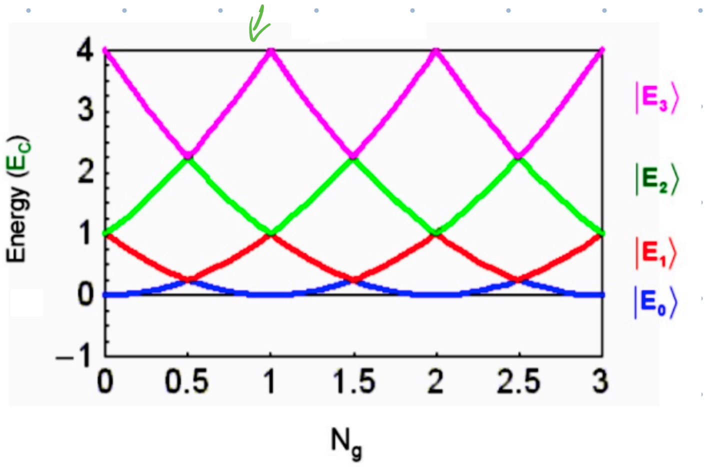
Our envisioned basis for the qubit is only related to the extra number of Cooper-pair charges on the island. Therefore the total charge we are concerned with is the dierence in Cooper-pair charges, which have tunneled onto the island, and the effective number of charges which have built up on the gate capacitor. $\Rightarrow Q_{TOT} = Q_{ISLAND} - Q_g = 2e\,(N - N_g)$

SAID DIFFERENTLY: WE ONLY CARE ABOUT THE NEW CHARGES THAT HAVE TUNNELED!

$$\Rightarrow H_{EL} = \frac{Q_{TOT}}{2\,C_{TOT}} = \frac{(2e)^2}{2\,C_{TOT}}(N-N_g)^2$$

E_C CHARGING ENERGY

ELECTRIC PART



BAND-LIKE STRUCTURE

We have yet to account for the magnetic part of the Hamiltonian. As we found before, the Josephson Junction acts like an inductor and will be storing energy in its magnetic field. The energy in the junction will be $\quad H_{MAG} = -E_S \cos\delta$ (THE ONE WE DERIVED ABOVE)

OPERATOR

OPERATOR

$$\Rightarrow H = H_{EL} + H_{MAG} = E_C(\hat{N}-N_g)^2 - E_S \cos\delta \leftarrow \tfrac{1}{2}\left(e^{i\delta}+e^{-i\delta}\right)$$

NOW LET'S WRITE EVERYTHING IN TERMS OF N

THEY WORK AS x AND p

$$\hat{N} = \frac{\hat{Q}}{2e} \qquad \hat{\delta} = 4\pi e h \hat{\phi} \qquad \Rightarrow [\hat{\phi},\hat{Q}] = ih \Rightarrow [\hat{\delta},\hat{N}] = i$$

$\hat{N}$ IS THE OPERATOR THAT RETURNS THE NUMBER OF COOPER PAIRS IN THE ISLAND

$$\hat{N}|N\rangle = N|N\rangle \qquad \sum_N |N\rangle\langle N| = \mathbb{1}$$

$$|\delta\rangle = \sum_N e^{iN\delta}|N\rangle \leftarrow$$ EQUIVALENT TO THE POSITION REPRESENTATION AS A FUNCTION OF P

$$\psi_x = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{+\infty} \varphi(p)\, e^{ipx/\hbar}\, dp$$

VICEVERSA $\quad |N\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\delta'\, e^{-iN\delta'}|\delta'\rangle$

WE ALSO DEFINE

$$e^{i\delta} = \frac{1}{2\pi}\int_0^{2\pi} d\delta' \, e^{i\delta'} |\delta'\rangle\langle\delta'|$$

THIS IS OK BECAUSE

$$e^{i\delta}|\delta\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\delta' \, e^{i\delta'}|\delta'\rangle\langle\delta'|\delta\rangle = e^{i\delta}|\delta\rangle$$ AS WE NEED

$$\Rightarrow e^{i\delta}|N\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\delta' \, e^{i\delta'}|\delta'\rangle\langle\delta'|N\rangle =$$

$$= \frac{1}{2\pi}\int_0^{2\pi} d\delta' \, e^{i\delta'}|\delta'\rangle \sum_n e^{-iN\delta'}\langle n|N\rangle =$$

$$= \frac{1}{2\pi}\int_0^{2\pi} d\delta' \, e^{i\delta'}|\delta'\rangle e^{-iN\delta'} = \frac{1}{2\pi}\int_0^{2\pi} d\delta' \, e^{-i(N+1)\delta'}|\delta'\rangle$$

$$= |N-1\rangle$$

$$N = \frac{\mathbb{1}-\sigma_z}{2}$$

$$E_c\left(\frac{\mathbb{1}}{2}-\frac{\sigma_z}{2}-N_g\right)^2 \overbrace{\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)}^{\mathbb{1}}$$

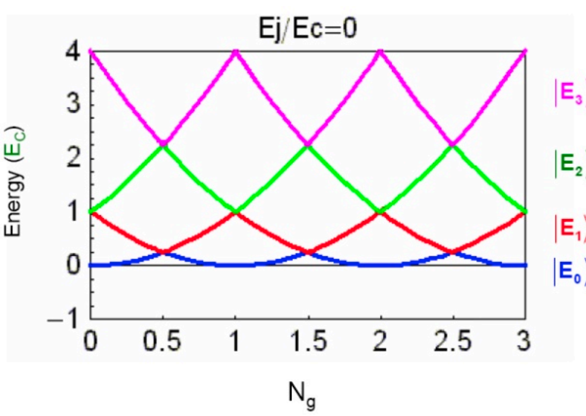
$$E_c\left(\left(\frac{\mathbb{1}}{2}-N_g\right)^2 + \frac{\sigma_z^2}{4} - 2\left(\frac{1}{2}-N_g\right)\sigma_z\right)$$

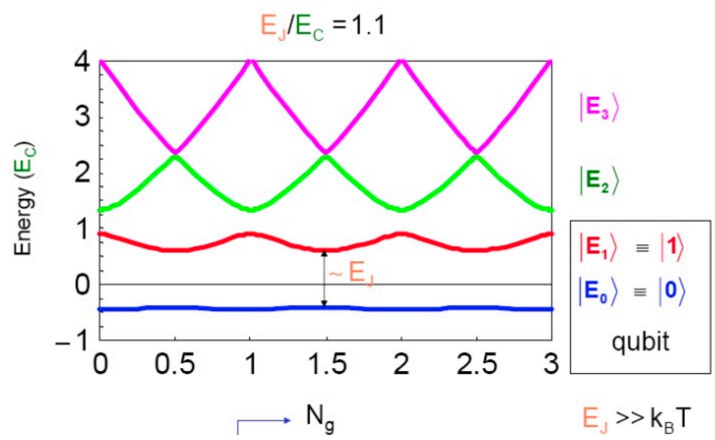
$$\Rightarrow e^{\pm i\delta}|N\rangle = |N\mp 1\rangle$$

With this we can now rewrite the hamiltonian in the basis of N

$$H = E_c(N-N_g)^2 - E_J\cos\delta = E_c(N-N_g)^2 - \frac{E_J}{2}\left(e^{i\delta}+e^{-i\delta}\right)$$

$$= \sum_N E_c(N-N_g)^2|N\rangle\langle N| - \frac{E_J}{2}\left(|N\rangle\langle N-1| + |N-1\rangle\langle N|\right)$$





the magnetic term of the Hamiltonian, which is proportional to the Josephson energy, efa ectively creates energy gaps at the degeneracy points. The gap size between the ground state and the first excited stated is approximately the Josephson Energy. However, the gap size decreases between increasing energy levels. This is exactly what we wanted since this will help us control transitions!

If we restrict the Hamiltonian to the first two levels we get the Hamiltonian for the qubit

$\hat{N} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \frac{\mathbb{1} - \sigma_z}{2}$

$$H = E_C \underbrace{\left(\frac{\mathbb{1}}{2} - \frac{\sigma_z}{2} - N_g\right)^2}_{\left(\frac{\mathbb{1}}{2} - N_g\right)^2 + \frac{\sigma_z^2}{4} - 2\left(\frac{\mathbb{1}}{2} - N_g\right)\sigma_z} \overbrace{(|0\rangle\langle 0| + |1\rangle\langle 1|)}^{\mathbb{1}} - \frac{E_S}{2}\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)$$

const, $\mathbb{1}$

$$= -\frac{E_C}{2}(1 - 2N_g)\sigma_z - \frac{E_S}{2}\sigma_x$$

IGNORE THE TERMS IN BLUE BECAUSE THEY ARE A COMMON OFFSET TO THE ENERGY

If we look at the plots above, we would like to work in the regime where $E_S \gg E_C$ because this will create a very flat energy gap. The question then becomes how to tune all the parameters

E_C, N_G, E_S

$N_g \propto V_g$, $E_C \propto \frac{1}{C_{TOT}}$ $\Rightarrow$ CAN REDUCE C_{TOT} TO REACH $E_S \gg E_C$

TUNING E_S $\quad E_S = I_C \cos\delta$

$\propto$ TO THE MAGNETIC FLUX THROUGH THE JUNCTION

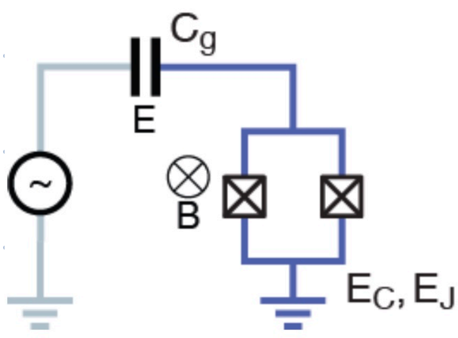
$\Rightarrow$ VERY HARD TO CONTROL BECAUSE IT REQUIRES HIGH MAGNETIC FIELDS

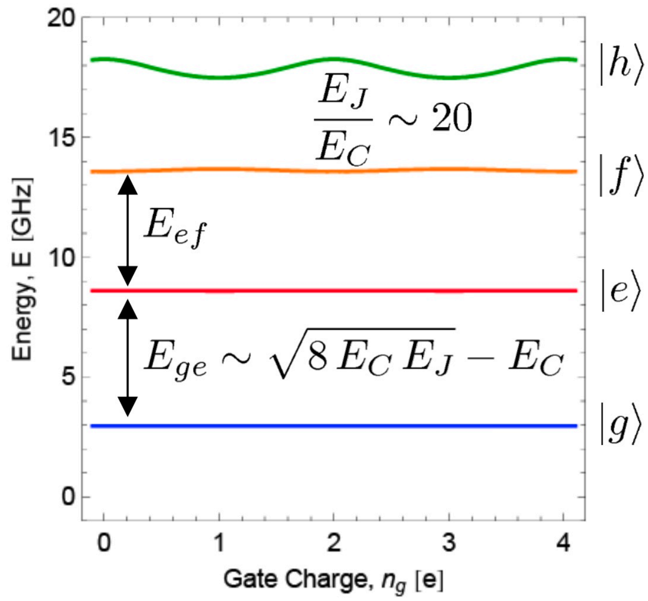
PROP. TO THE EASE OF TUNNELING

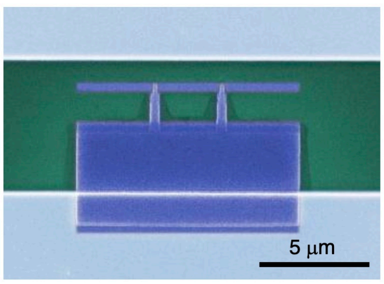
$\Rightarrow$ CONTROLLED BY FABRICATION

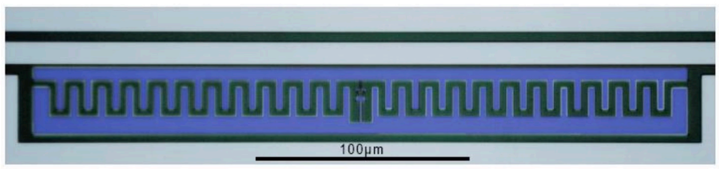
INCREASE I_C WITH THIN INSULATOR AND LARGE AREA

Without proof, we state that two Josephson junctions in parallel effectively modify the dependence of the cosine term in the Josephson energy—from the magnetic flux through each individual junction to the total magnetic flux through the loop formed by the junctions. Since the loop has a much larger area than the individual junctions, a much smaller magnetic field is required to control the cosine parameter. We refer to this type of circuit as a transmon.









FLUX THROUGH THE LOOP OF JUNCTIONS

$$H_{\text{TRANSMON}} = \frac{E_C}{2}(N - N_g)^2 - E_J \cos\left(\pi \frac{\phi_{\text{EXT}}}{\phi_0}\right)$$

Single qubit operations

We have seen that the qubit Hamiltonian is $H = -\frac{E_C}{2}(1-2N_g)\sigma_z - \frac{E_J}{2}\sigma_x$

As anticipated we can control the qubit by controlling the gate voltage. It follows that N_g can be written as a sum of a static component and a time dependent component.

$$N_g = N_{g,0} + N_g(t) \qquad N_{g,0} = 1/2 \text{ FOR SIMPLICITY}$$

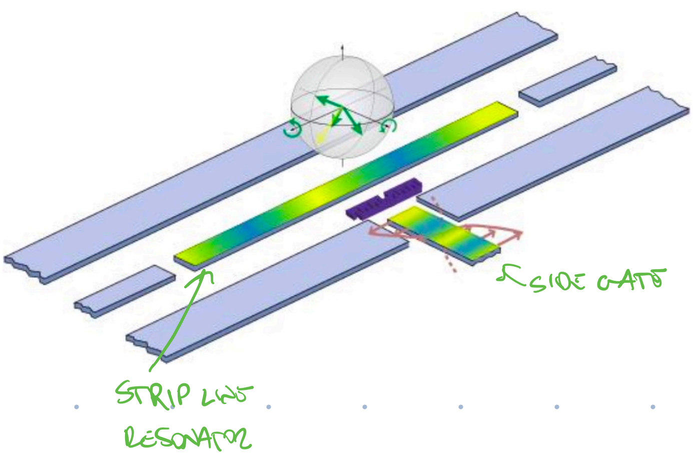
$$N_g(t) = A\cos(\omega_d t + \varphi)$$

ω_d NOT NECESSARILY EQUAL TO THE QUBIT FREQUENCY ω_q

performing a change of basis by rotating our frame by $\frac{\pi}{2}$ AROUND $\hat{y}$ $\Rightarrow$ $\sigma_z \rightarrow \sigma_x$, $\sigma_x \rightarrow -\sigma_z$

$$\Rightarrow H = \underbrace{\frac{E_C}{2} A}_{\tilde{A}} \cos(\omega_d t + \varphi)\sigma_z - E_J \sigma_x$$

$$\Rightarrow H = \tilde{A}\cos(\omega_d t + \varphi)\sigma_x + \frac{E_J}{2}\sigma_z$$

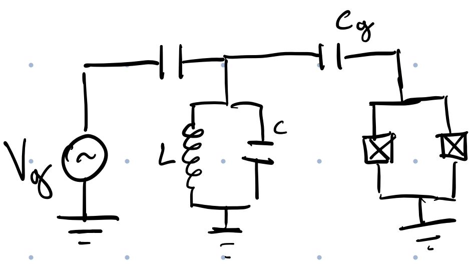


MICROWAVE SIGNAL APPLIED THROUGH THE RESONATOR OR THROUGH A SIDE GATE

Coupling the qubit to a resonator

The next step will be coupling the Cooper-Pair Box(Transmon) to a simple harmonic oscillating circuit.

The motivation for doing this comes from Cavity Quantum Electrodynamics. In this way, greater control and readout of the state of the atom is possible.



$$\otimes B \quad H = \underbrace{\hbar \overset{\text{RESONATOR}}{\omega_r} \left(a^+ a + \tfrac{1}{2}\right)}_{\text{LC CIRCUIT}} + \underbrace{\frac{E_C}{2}\left(1 - 2N_g\right)\sigma_z - \frac{E_S}{2}\sigma_x}_{\text{2-LEVEL CIRCUIT}}$$

$$\Rightarrow H = H_r + H_{\text{INTERACTION}} + H_{\text{QUBIT}}$$

The second term is proportional to Ng that is related to the coupling between the LC circuit and the transmon qubit, while the third term we have seen to be responsible to the splitting of the energy bands (and thus to the definition of the qubit hamiltonian).

This system is similar to that of a cavity QED system, where we want to exchange information between the resonator (the optical cavity) and a qubit. This system is governed by the Jaynes-Cumming hamiltonian.

$$H_{JC} = \hbar \omega_c \left(a^+ a + \tfrac{1}{2}\right) + \underbrace{\frac{\hbar \omega_a}{2}}_{\text{2-LEVEL ATOM}} \sigma_z + \frac{\hbar g}{2}\underbrace{\left(a\sigma_+ + a^+\sigma_-\right)}_{\text{INTERACTION HAMILTONIAN}}$$

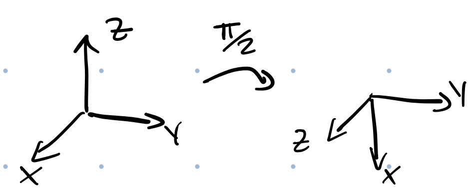
OPTICAL CAVITY
QUANTUM HARMONIC OSCILLATOR

2-LEVEL ATOM

INTERACTION HAMILTONIAN
ATOM ABSORBS A PHOTON ($\hat{a}$)
AND FLIPS THE SPIN ($\hat{\sigma}_+$)
AND VICEVERSA

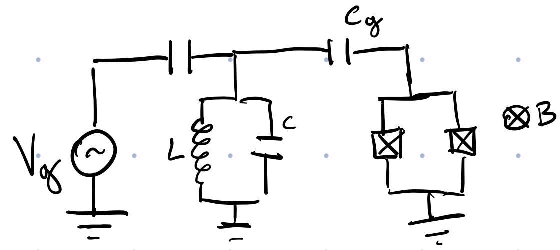
To turn our transmon hamiltonian into the JC hamiltonian we can see that we can make a rotation of $\frac{\pi}{2}$ about the y axis on the qubit to obtain $\sigma_z \to \sigma_x$, $\sigma_x \to -\sigma_z$

Imagine rotating the full Bloch sphere by $\pi/2$ AROUND Y



$$\Rightarrow H = \hbar\omega_r(a^\dagger a + \tfrac{1}{2}) + E_C(1-2N_g)\sigma_x + \frac{E_J}{2}\sigma_z$$

Good, so the qubit part and the resonator part are the same in both hamiltonians and we just need to manipulate the interaction part. Recall that N_g is the number of charges built up on the gate capacitor. This is a continuous value (set by Vg) but also contains a quantized part (the charges that come from the quantized LC circuit)



$$\Rightarrow N_g = \underset{\substack{\text{CLASSICAL} \\ \text{CONTINUOUS} \\ \text{CONTROLLED BY} \\ V_g}}{N_g^{CL}} + \underset{\text{DEPENDS ON LC}}{N_g^{qm}}$$

WITH V_g WE CAN SET $N_g^{CL} = 1/2$ FOR SIMPLICITY

$$\Rightarrow H = \hbar\omega_r(a^\dagger a + \tfrac{1}{2}) + E_C\big(\underbrace{1 - 2(N_g^{CL}}_{=0 \text{ IF } N_g^{CL} = 1/2} + N_g^{qm})\big)\sigma_x + \frac{E_J}{2}\sigma_z$$

$$= \hbar\omega_r(a^\dagger a + \tfrac{1}{2}) + E_C N_g^{qm}\sigma_x + \frac{\underbrace{E_J}_{\hbar\omega_q}}{2}\sigma_z$$

Now remember the definitions of the charge and voltages for an LC circuit and its quantization (all the things we saw at the beginning) $2e N_g^{qm} = Q = C_g V \Rightarrow N_g^{qm} = \frac{C_g}{2e} V$

Compared to the quantization we did at the beginning where we assumed Q like p and ϕ like x so that

$$\hat{Q} = i\sqrt{\frac{\hbar}{2Z_0}}\,(a^\dagger - a)$$

we now choose to do the opposite and choose Q like x and ϕ like p (at the beginning we saw that we can always choose as long as we are consistent) ← THIS MEANS THAT I HAVE TO FOLLOW THIS CONVENTION

ALSO FOR $\hbar\omega_r(a^\dagger a + 1/2)$ TO ENSURE THAT $a, a^\dagger$ ARE THE SAME OPERATORS

$$\hat{Q} = \sqrt{\frac{\hbar}{2Z_c}}\,(a^\dagger + a) \quad \text{WITH } Z_c = \sqrt{L/C}$$

RESONATOR C

← BEFORE IT WAS $\frac{1}{\sqrt{LC}}$

$$\Rightarrow V = \frac{\hat{Q}}{C} = \sqrt{\frac{\hbar}{2C^2\sqrt{L/C}}}\,(a^\dagger + a) = \sqrt{\frac{\hbar}{2C\sqrt{LC}}}\,(a^\dagger + a) = \sqrt{\frac{\hbar\omega_r}{2C}}\,(a^\dagger + a)$$

GOING BACK TO THE HAMILTONIAN (THE INTERACTION TERM)

$$H_{INT} = E_C N_g \sigma_x = E_C \frac{C_g}{2e}\hat{V}\sigma_x = E_C \frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}\,(a^\dagger + a)\sigma_x$$

BUT $\sigma_x = \sigma_+ + \sigma_-$

$$\Rightarrow (a^\dagger + a)\sigma_x = \underline{a^\dagger\sigma_+} + a^\dagger\sigma_- + a\sigma_+ + \underline{a\sigma_-}$$

THESE TWO TERMS CANCEL WITH THE ROTATING WAVE APPROXIMATION

WHEN YOU GO IN THE INTERACTION PICTURE THEY GO AS

$$a^\dagger\sigma_+ \rightarrow a^\dagger\sigma_+ e^{i(\omega_r + \omega_A)t} \Rightarrow \text{VERY FAST!}$$

$$a\sigma_- \rightarrow a\sigma_- e^{-i(\omega_r + \omega_A)t}$$

$$\Rightarrow H_{INT} = \underbrace{E_C \frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}}_{=\frac{\hbar g}{2}}\,(a^\dagger\sigma_- + a\sigma_+)$$

Qubit measurement

There are a number of reasons why it is useful to couple a qubit to a resonator. One of these is to do quantum non demolition (QND) measurements

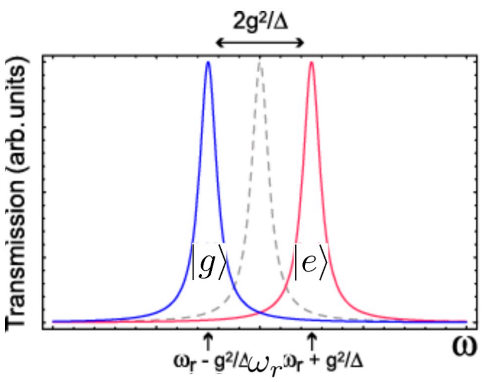
We will work in the dispersive limit $\quad |\Delta| = |\omega_q - \omega_r| \gg g$

IN THIS LIMIT THE COUPLING IS A PERTURBATION AND ONE CAN SHOW THAT

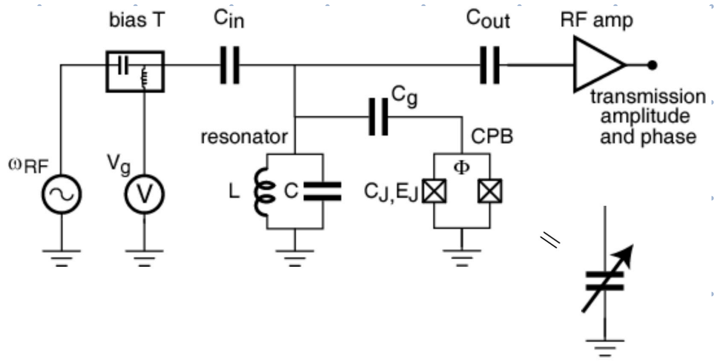
$$H \simeq \hbar \left(\omega_r + \frac{g^2}{\Delta}\sigma_z\right) a^\dagger a + \frac{1}{2}\hbar\left(\omega_q + \frac{g^2}{\Delta}\right)\sigma_z$$

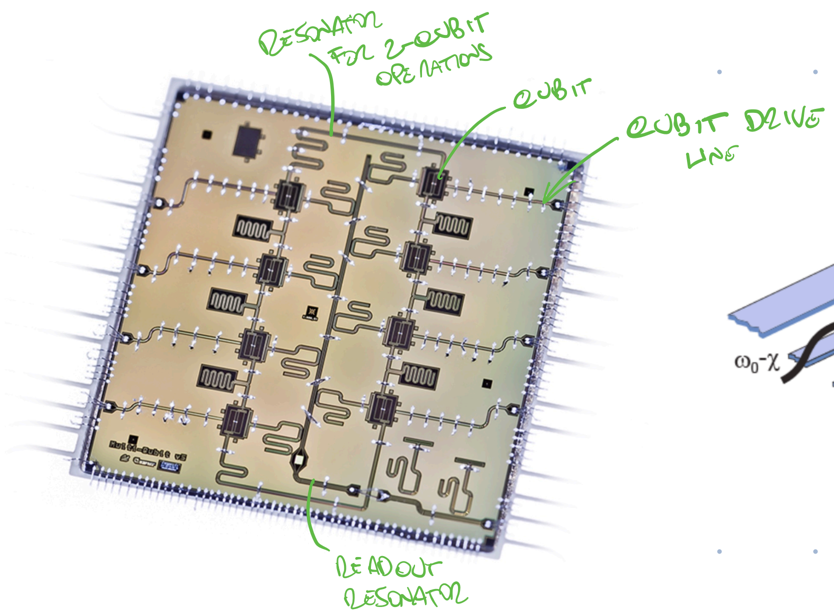
$H_{INT} \simeq \frac{\hbar g}{\Delta}\left(a\sigma_+ + a^\dagger \sigma_-\right)$ THEN GO IN ROTATING FRAME WRT TO H_{INT} AND EXPAND TO SECOND ORDER.

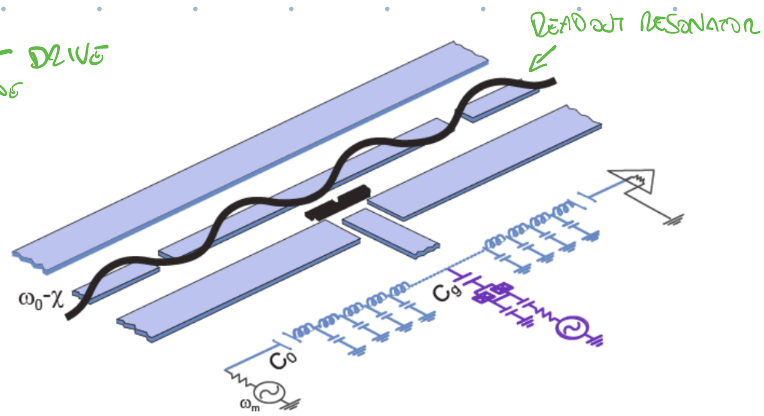
$U_{INT} = \exp\left(\frac{g}{\Delta}\left(a\sigma_+ + a^\dagger\sigma_-\right)\right)$

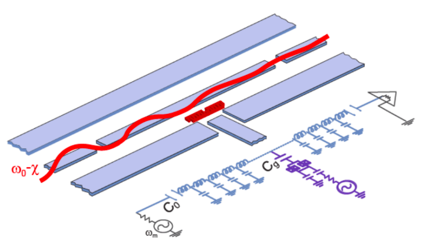


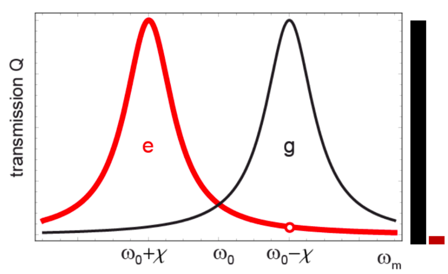
IDEA: SIT ON ONE OF THE PEAKS AND MEASURE THE TRANSMITTED AMPLITUDE



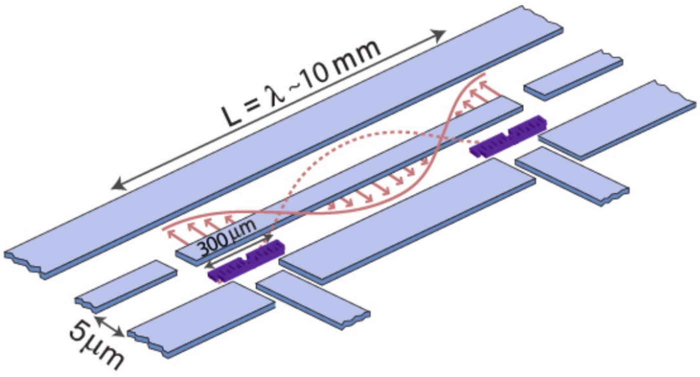








TWO QUBIT GATES



STRIPLINE RESONATOR USED AS A "QUANTUM BUS" TO CREATE ENTANGLED STATES

MULTIPLE QUBITS COUPLED TO A RESONATOR ⟶ TAVIS-CUMMING HAMILTONIAN

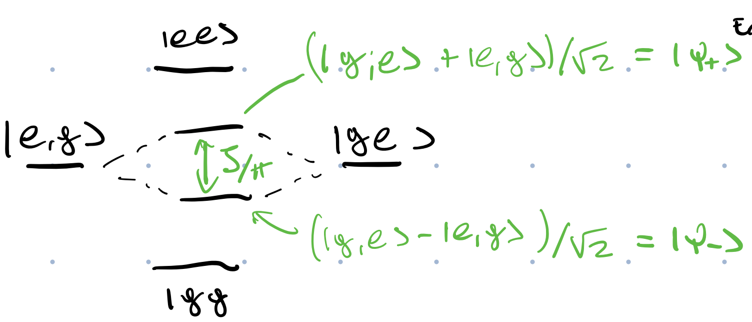
$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2}\right) + \sum \frac{\hbar\,\omega_{q,i}}{2}\sigma_z + \sum_i \hbar g_i \left(a^\dagger \sigma_-^i + a\,\sigma_+^i\right)$$

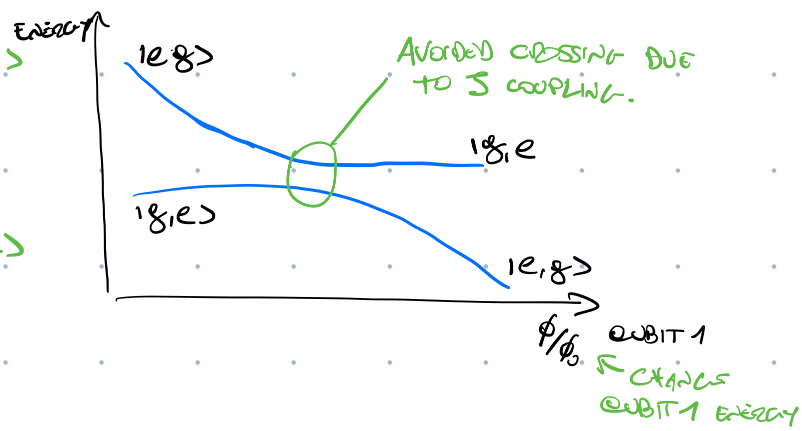
IN THE DISPERSIVE LIMIT ⇒ $\Delta = |\omega_{q,i} - \omega_r| \gg g_i$ AND APPLYING SECOND ORDER PERTURBATION THEORY WE GET

$$H = \hbar\omega_r \left(a^\dagger a + 1/2\right) + \hbar \sum_i \frac{\omega_{q,i} + \chi_i}{2}\sigma_z^i + \hbar J \underbrace{\left(\sigma_+^1 \sigma_-^2 + \sigma_-^1 \sigma_+^2\right)}$$

FLIP-FLOP INTERACTION MEDIATED BY VIRTUAL PHOTONS

$$J = \frac{g_1 g_2}{2}\left(\frac{1}{\Delta_1} + \frac{1}{\Delta_2}\right)$$

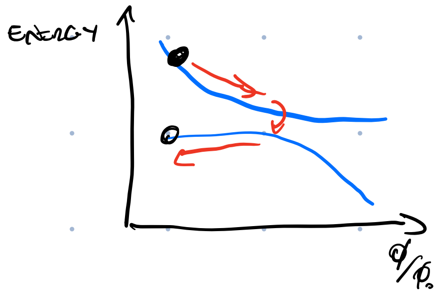




IDEA: KEEP QUBIT 2 FIXED AND PREPARE $|e,g\rangle$, THEN CHANGE THE ENERGY OF QUBIT 1 UNTIL YOU REACH THE AVOIDED CROSSING. AT THIS POINT $|e,g\rangle$ IS NOT AN EIGENSTATE OF THE SYSTEM ⟶ $|\psi_\pm\rangle$ ARE THE REAL EIGENSTATES

⇒ $|eg\rangle$ WILL EVOLVE IN TIME $\quad |\psi(t)\rangle = \alpha(t)\,|\psi_+\rangle + \beta(t)\,|\psi_-\rangle$

THE INTERACTION CAN THEN BE LEFT ON UNTIL YOU CREATE $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|eg\rangle - i|g,e\rangle\right)$



$\Rightarrow$ AT THE END DETUNE QUBIT 1 AGAIN TO SWITCH OFF THE INTERACTION

MATHEMATICALLY: $H_{INT} = \hbar J\left(\sigma_+^1\sigma_-^2 + \sigma_-^1\sigma_+^2\right) = \hbar J \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\sigma_x \oplus \emptyset_2$

$$U = e^{-\frac{i}{\hbar}Ht} = e^{-iJt\sigma_x} \oplus \underbrace{e^{-i\emptyset_2 t}}_{\mathbb{1}} = \left(\mathbb{1}\cos Jt - i\sigma_x \sin Jt\right) \oplus \mathbb{1}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(Jt) & -i\sin(Jt) & 0 \\ 0 & -i\sin(Jt) & \cos(Jt) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = |gg\rangle\langle gg| + |ee\rangle\langle ee| + \cos(Jt)\left[|eg\rangle\langle ge| + |ge\rangle\langle eg|\right] - i\sin(Jt)\left[|eg\rangle\langle ge| + |ge\rangle\langle eg|\right]$$

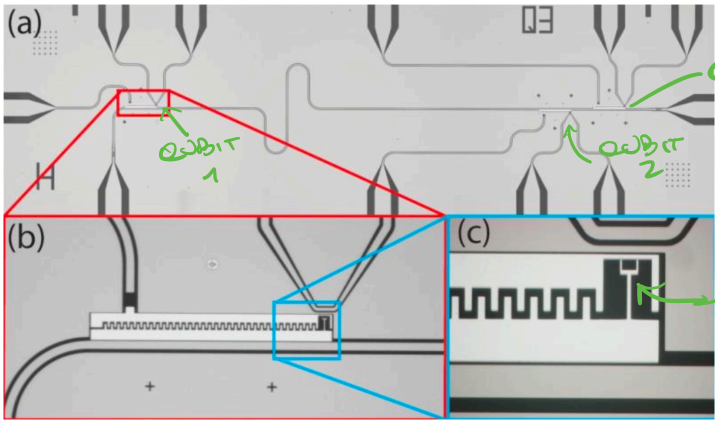
IF WE START WITH $|eg\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

$$U|e,g\rangle = \cos(Jt)\,|eg\rangle - i\sin(Jt)\,|ge\rangle$$

$\Rightarrow$ FOR $t = \frac{\pi}{2J}$ $\quad |e,g\rangle \to |g,e\rangle$ SWAP

FOR $t = \frac{\pi}{4J}$ $\quad |e,g\rangle \to \frac{1}{\sqrt{2}}\left(|eg\rangle - i|g,e\rangle\right)$ $\quad \sqrt{i\text{SWAP}}$

MAXIMALLY ENTANGLED STATE



QUBIT 3

JUNCTIONS OF THE TRANSMON