Photonic Quantum Computing

The field is largely divided between discrete-variable (DV) and continuous-variable (CV) photonic quantum computation.

What is the Hilbert space of a photon? A photon is a single particle excitation of an optical field mode. In quantum field theory the Hilbert space is associated with orthonormal modes of the electromagnetic (EM) field

EACH MODE k HAS $a_k, a_k^\dagger$ $\quad [a_k, a_{k'}^\dagger] = \delta_{kk'}$

→ QUANTIZED ELECTROMAGNETIC FIELD

A single photon state can then be a single excitation of a single mode or a superposition of a single-photon state of many modes, for example

$$|1\rangle = \sum_k f_k a_k^\dagger |0\rangle$$

COMPLEX COEFF.

The Hilbert space is associated with the mode itself. The Hilbert space of each mode is mathematically equivalent to the Hilbert space of a simple harmonic oscillator. The total Hilbert space of the EM field is the tensor product Hilbert space of many modes.

Photonic quantum computation uses different physical properties of the quantized optical field. Some of these are discrete other are continuous

E.G. NUMBER OF COUNTED PHOTONS OR POLARIZATION

E.G. ELECTRIC FIELD QUADRATURES

DISCRETE: QUBIT $(d=2)$ OR QUDIT $(d>2)$

$\{|0\rangle, |1\rangle\}$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$\hookrightarrow \{|0\rangle, |1\rangle, \dots, |d-1\rangle\}$

$|\psi\rangle = c_0|0\rangle + \dots + c_{d-1}|d-1\rangle$

In dual-rail encoding, a single qubit is formed of a single photon excitation of any two optical field modes, say the two orthogonal polarisation of a single spatio-temporal mode. All possible (pure) polarisation states can be described as a superposition of this excitation over the two polarisation modes.

$$|0\rangle_L = a_H^\dagger \underbrace{|0\rangle_H}_{\text{HORIZONTAL POL}} \underbrace{|0\rangle_V}_{\text{VERTICAL POLARIZATION}} = a_H^\dagger |0,0\rangle_{HV} = |1,0\rangle_{HV} = |H\rangle$$

$$|1\rangle_L = a_V^\dagger |0,0\rangle_{HV} = |0,1\rangle_{HV} = |V\rangle$$

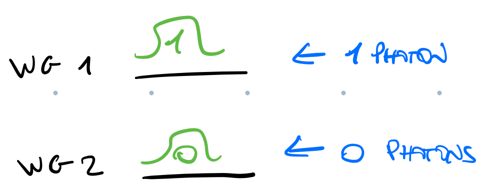
CONTINUOUS ENCODING → QUBIT DEFINED AS A SUPERPOSITION OF EIGENSTATES OF POSITION AND MOMENTUM OPERATORS

EXAMPLE GKP STATES BUT WE ARE NOT GOING TO SEE THEM

Number state encoding

Here we are going to focus only on discrete encoding. And in particular we are going to focus on number state qubit encoding. This is also one version of dual rail encoding that hopefully will become soon clearer. In particular we consider a simple physical qubit based on a single photon pulse of one of a pair of spatiotemporal modes.

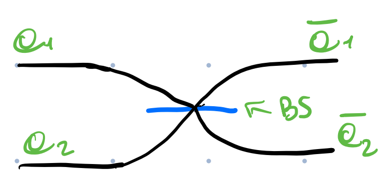
EG. TWO WAVEGUIDES



$$|0\rangle_L = |1\rangle_1 \otimes |0\rangle_2 = |10\rangle = a_1^\dagger |00\rangle \quad \leftarrow \text{PHOTON IN WAVEGUIDE 1}$$

$$|1\rangle_L = |0\rangle_1 \otimes |1\rangle_2 = |01\rangle = a_2^\dagger |00\rangle \quad \leftarrow \quad " \quad " \quad 2$$

A single qubit gate is implemented using a beam splitter



$a_1, a_2, \bar{a}_1, \bar{a}_2$ DESTROY OPERATORS OF THE VARIOUS MODES

$$U_{BS}(\theta,\varphi)=\begin{bmatrix}\cos\theta & ie^{-i\varphi}\sin\theta \\ ie^{-i\varphi}\sin\theta & \cos\theta\end{bmatrix}$$

$\eta = \cos^2\theta$ INTENSITY TRASMISSIVITY COEFF. OF EACH MODE

$$\begin{pmatrix}\bar{a}_1 \\ \bar{a}_2\end{pmatrix} = U_{BS}\begin{pmatrix}a_1 \\ a_2\end{pmatrix}$$

$$\bar{a}_1 = \cos\theta\, a_1 + ie^{-i\varphi}\sin\theta\, a_2$$
$$\bar{a}_2 = \cos\theta\, a_2 + ie^{-i\varphi}\sin\theta\, a_1$$

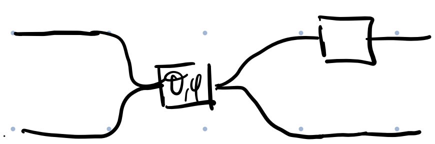
There can also be additional phase shifts on each rail. The unitary operator

$$U_P(\varphi) = e^{-i\varphi(a_1^\dagger a_1 - a_2^\dagger a_2)}$$

← RELATIVE PHASE SHIFT OF 2φ BETWEEN RAIL 1 AND 2

$$\Rightarrow \quad U_P(\varphi)|0\rangle_L = e^{-i\varphi}|0\rangle_L$$
$$U_P(\varphi)|1\rangle_L = e^{i\varphi}|1\rangle_L$$

A combination of the beamsplitter and relative-phase gates generate an arbitrary single qubit state.



THOSE GATES ARE CALLED "LINEAR OPTICAL GATES"
THEY PRESERVE THE TOTAL NUMBER OF PHOTONS

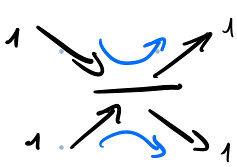
The question now becomes how to perform 2 qubit gates? It is not possible to do it deterministically.

The reason is that photons do not interact with each other. The only way in which photons can directly influence each other is via the bosonic symmetry relation.

Linear optical quantum computing exploits exactly this property, i.e., the bosonic commutation relation $[a, a^\dagger] = 1$

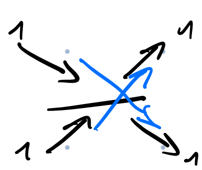
→ CONSIDER 50:50 BS $\Rightarrow \eta = \cos^2\theta = 1/2$ AND CONSIDER TWO PHOTONS ENTERING IT: $\Rightarrow |\psi\rangle = |1,1\rangle = a_1^\dagger a_2^\dagger |0\rangle$

In terms of quantum interfer- ence, there are two paths leading from the input state $|1,1\rangle_{in} \to |1,1\rangle_{OUT}$



BOTH PHOTONS REFLECTED

+



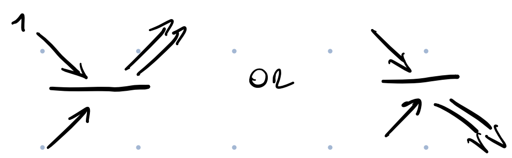
BOTH PHOTON TRANSMITTED

$$|1,1\rangle_{IN} \xrightarrow{TRANS} \cos^2\theta\, |1,1\rangle_{OUT}$$

$$|1,1\rangle_{IN} \xrightarrow{REF} -\sin^2\theta\, e^{i\varphi} e^{-i\varphi}\, |1,1\rangle_{OUT} \Rightarrow \cos^2\theta = \sin^2\theta = 1/2$$

SO THE TWO PATHS CANCEL OUT!

It follows that the two photons must exit the beam splitter from the same side



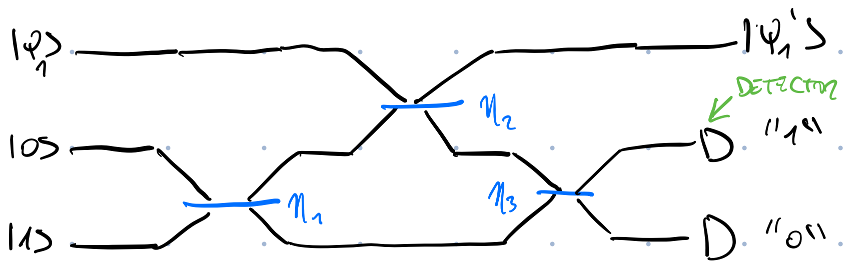
PHOTON BUNCHING — HONG-OU-MANDEL EFFECT

Two qubit gates are constructed making use of this property. A CZ gate can be constructed in linear optics using two nonlinear sign (NS) gates.

$$|\varphi_1\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle \longrightarrow |\varphi_1'\rangle = \alpha|0\rangle + \beta|1\rangle - \gamma|2\rangle$$

First of all we notice that to implement this gate we are moving outside of the dual-rail encoding scheme. What we said before is that we can use either 0 or 1 photon per rail. Here we are using up to two!

How can an NS gate be done? It is immediately clear that we cannot make the NS gate with a regular phase shifter, because only the state $|2\rangle$ picks up a phase. A linear optical phase shifter would also pick up a phase i or $-i$ on $|1\rangle$. How- ever, it is possible to perform the NS-gate probabilistically using projective measurements. The fact that two NS gates can be used to create a CZ gate was first realized by Knill, Laflamme, and Milburn (2001). Their probabilistic NS gate is a 3-port device, including two ancillary modes, the output of which is measured with perfect photon-number discriminating detectors

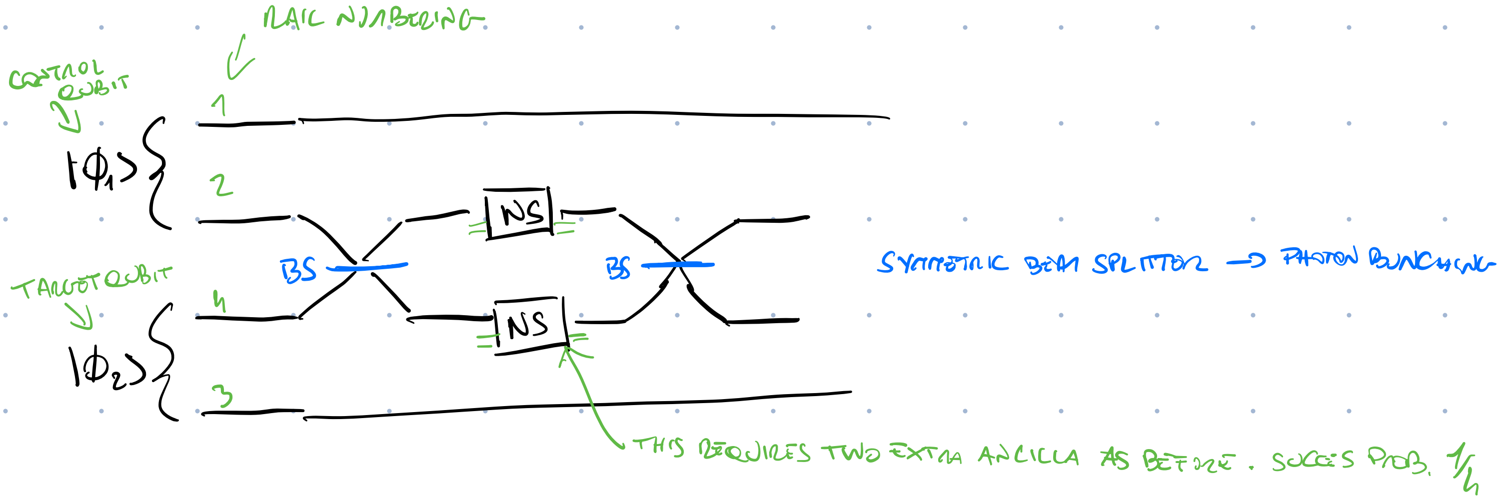


DESIRED MEASUREMENT OUTCOME THAT CORRECTLY IMPLEMENTS THE NS GATE

$$\eta_1 = \eta_3 = \cos^2\theta = \frac{1}{4-2\sqrt{2}} \qquad \eta_2 = 3-2\sqrt{2}$$

It was shown that the success probability of such a scheme is 1/4.

To realize a CZ gate then one can implement the following circuit



$$|\phi_1\rangle \otimes |\phi_2\rangle = (\alpha\underbrace{|01\rangle}_{|1\rangle_L} + \beta\underbrace{|10\rangle}_{|0\rangle_L})(\gamma|01\rangle + \delta|10\rangle) =$$

$$= \alpha\gamma|0101\rangle + \alpha\delta|0110\rangle + \beta\gamma|1001\rangle + \beta\delta|1010\rangle$$

$$\xrightarrow[BS]{} \alpha\gamma(|0200\rangle + |0002\rangle) + \alpha\delta|0110\rangle + \beta\gamma|1001\rangle + \beta\delta|1010\rangle$$

$$\xrightarrow[NS]{} -\alpha\gamma(|0200\rangle + |0002\rangle) + \alpha\delta|0110\rangle + \beta\gamma|1001\rangle + \beta\delta|1010\rangle$$

$$\xrightarrow[BS]{} -\alpha\gamma|0101\rangle + \alpha\delta|0110\rangle + \beta\gamma|1001\rangle + \beta\delta|1010\rangle$$

THIS IS NOT SEPARABLE AND IF $\alpha = \beta = \gamma = \delta = \frac{1}{\sqrt{2}}$ THEN MAXIMALLY ENTANGLED!

THE SUCCESS PROBABILITY IS $\frac{1}{16}$!

$$\frac{1}{2}\left(-|0101\rangle + |0110\rangle + |1001\rangle + |1010\rangle\right) = \frac{1}{2}\left(-|11\rangle_L + |10\rangle_L + |01\rangle_L + |00\rangle_L\right)$$

PERFECT C-Z GATE

If you immagine concatenating several gates like this one, then it is immediately clear that the probability of successful quantum computation becomes exponentially small.

For this reason alternative approaches have been developed. These are based either on continuous variables (GKP states) or, using discrete variables measurement-based quantum computing (also known as one-way quantum computing) that uses cluster states (entangled states of many qubits) to perform operations. Both approaches are extremely interesting and related to techniques of quantum error correction, but we do not have the time to go through them.