Ex 1 Ch 1 AM

a) Probability of NO collision during the next t seconds:

$$P(t+dt) = \underbrace{P(t)}_{\text{probability of NO collision up to now } (t)} \cdot \underbrace{\left[1 - \frac{dt}{\tau}\right]}_{\text{probability of NO collision from } t \text{ until } t+dt}$$

product because events are independent

$$(*)\; P(t=0) = 1\,; \quad \Rightarrow \quad \boxed{P(t) = e^{-t/\tau}}$$

Similarly, backwords (consider always $dt>0$):

$$P(-t-dt) = P(-t)\left[1 - \frac{dt}{\tau}\right]$$

$$\Rightarrow \quad \boxed{P(-t) = e^{-t/\tau}}$$

(Note: (*) We suppose that the e^- picked at random had a collision at $t=0$)

b) Probability of NO collision until t and a collision between t and $t+dt$:

$\Rightarrow$ product of two probabilities:

$$\underbrace{e^{-t/\tau}}_{\text{from a)}} \cdot \underbrace{\frac{dt}{\tau}}_{\text{from the definition of } \tau}$$

c) and d)

Avg over all the e^- (i.e., over all times) of the time expected for the <u>next</u> collision:

$$\langle t\rangle = \int_0^\infty \underbrace{t' \, \underbrace{e^{-t'/\tau}\frac{dt'}{\tau}}_{\text{probability of a collision between } t' \text{ and } t'+dt'}}_{\text{weighted avg over time}} = \ldots = \tau$$

c) the same

e) avg time between last and next collision:
(t_1) (t_2)

$$\langle t_2 - t_1\rangle = \int_{-\infty}^{0} \frac{dt_1}{\tau}\, e^{t_1/\tau} \int_0^\infty \frac{dt_2}{\tau}\, e^{-t_2/\tau}(t_2-t_1)$$

$$= \ldots = 2\tau$$