$t=0$: collision
after t : another collision
applied electric field $\vec{E}$

velocity: $v(x,t)$

COLLISIONS $\rightarrow$ ....$\overset{0}{*}$..........$\overset{t}{*}$.... (time line)

$\vec{x}-\vec{d}$ $\qquad\qquad$ $\vec{x}$ $\qquad$ (space line)

$\vec{\nabla}T \longrightarrow \vec{E}$
$\vec{\nabla}T \longrightarrow$

a) $\langle \Delta \mathcal{E} \rangle$ lost in the $\underline{II}$ collision ?

- Velocity immediately <u>before</u> the II collision:

$$v(\vec{x},t^-) = v(\vec{x}-\vec{d}) - \frac{e\vec{E}t}{m}$$

$\uparrow$ from $m\Delta v = F\Delta t$

- Velocity immediately <u>after</u> the II collision:

$$v(\vec{x},t^+) = v(\vec{x})$$

- Energy loss <u>per electron</u>:

$$\Delta\mathcal{E} = \frac{1}{2} m v^2(\vec{x},t^+) - \frac{1}{2} m v^2(\vec{x},t^-) =$$

$$= \frac{1}{2} m \left[v^2(\vec{x}) - v^2(\vec{x}-\vec{d}) \right] + \frac{1}{2} \frac{(eEt)^2}{m} -$$

$$\cancel{- 2 \vec{v}(\vec{x}-\vec{d}) \cdot \frac{e\vec{E}t}{m}}$$

Averaging over all the e^- (i.e., over all the directions of $\vec{v}(\vec{x}-\vec{d})$) $\quad \langle \vec{v}\cdot\vec{E}\rangle = 0$

$$\Rightarrow \langle\Delta\mathcal{E}\rangle = \langle \frac{1}{2} m v^2(\vec{x}) - \frac{1}{2} m v^2(\vec{x}-\vec{d}) \rangle + \frac{(eE\tau)^2}{m} \quad \text{since } \langle t^2\rangle = \frac{\tau^2}{2}$$

whereas $\frac{1}{2} m v^2 = \mathcal{E}_{thermal}$, therefore:

$$\langle \frac{1}{2} m v^2(\vec{x}) - \frac{1}{2} m v^2(\vec{x}-\vec{d}) \rangle = \Delta\mathcal{E}_{th} = - \langle \frac{\partial \mathcal{E}}{\partial T} \vec{\nabla}T \cdot \vec{d} \rangle$$

$\langle \Delta\mathcal{E}_{th} \rangle$: need avg over all times.

$\langle \vec{d} \rangle = \underbrace{\langle \vec{v} \rangle}\underbrace{\langle t \rangle}_{=\tau}$ (avg displacement = avg velocity · avg time)

avg velocity acquired thanks to $\vec{E}$ in the avg time τ

$\Rightarrow \langle \vec{d} \rangle = -\frac{e\vec{E}\tau}{m} \cdot \tau$

$\Rightarrow \langle \Delta \mathcal{E}_{th} \rangle = \langle \frac{\partial \mathcal{E}}{\partial T} \vec{\nabla} T \cdot \vec{E} \, \frac{e\tau^2}{m} \rangle$

Thermal energy loss (density in space & time, i.e. local power density):

e^- density

$\langle \Delta \mathcal{E}_{th} \rangle \frac{n}{\tau} = \underbrace{\langle \frac{\partial \mathcal{E}}{\partial T} \vec{\nabla} T \rangle n}_{= c_v} \cdot \vec{E} \, \frac{e\tau}{m}$

per unit of time

Specific heat

$$\boxed{\langle \Delta \mathcal{E}_{th} \rangle \frac{n}{\tau} = c_v \frac{e\tau}{m} \vec{\nabla} T \cdot \vec{E}}$$