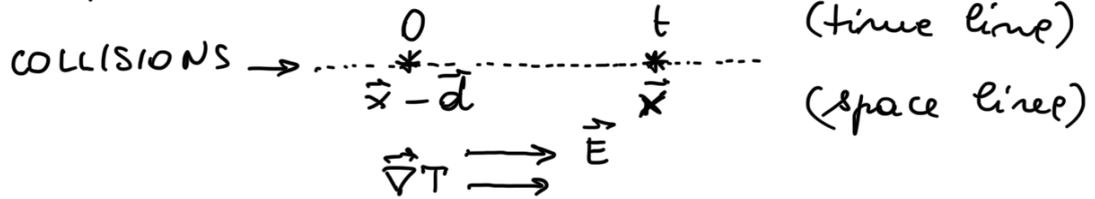


$t=0$: collision
 after t : another collision
 applied electric field \vec{E}

velocity:
 $v(x, t)$



a) $\langle \Delta \mathcal{E} \rangle$ lost in the II collision?

• Velocity immediately before the II collision:

$$v(\vec{x}, t^-) = v(\vec{x} - \vec{d}) - \frac{e\vec{E}t}{m}$$

↑ from $m\Delta v = F\Delta t$

• Velocity immediately after the II collision:

$$v(\vec{x}, t^+) = v(\vec{x})$$

• Energy loss per electron:

$$\begin{aligned}
 \Delta \mathcal{E} &= \frac{1}{2} m v^2(\vec{x}, t^+) - \frac{1}{2} m v^2(\vec{x}, t^-) = \\
 &= \frac{1}{2} m [v^2(\vec{x}) - v^2(\vec{x} - \vec{d})] + \frac{1}{2} \frac{(eEt)^2}{m} - \\
 &\quad - 2 \vec{v}(\vec{x} - \vec{d}) \cdot \frac{e\vec{E}t}{m}
 \end{aligned}$$

Averaging over all the e^- (i.e., over all the directions of $\vec{v}(\vec{x} - \vec{d})$) $\langle \vec{v} \cdot \vec{E} \rangle = 0$

$$\begin{aligned}
 \Rightarrow \langle \Delta \mathcal{E} \rangle &= \left\langle \frac{1}{2} m v^2(\vec{x}) - \frac{1}{2} m v^2(\vec{x} - \vec{d}) \right\rangle \\
 &\quad + \frac{(eE\tau)^2}{m} \quad \text{since } \langle t^2 \rangle = \frac{\tau^2}{2}
 \end{aligned}$$

whereas $\frac{1}{2} m v^2 = \mathcal{E}_{\text{thermal}}$, therefore:

$$\left\langle \frac{1}{2} m v^2(\vec{x}) - \frac{1}{2} m v^2(\vec{x} - \vec{d}) \right\rangle = \Delta \mathcal{E}_{\text{th}} = - \left\langle \frac{\partial \mathcal{E}}{\partial T} \vec{\nabla} T \cdot \vec{d} \right\rangle$$

$\langle \Delta \mathcal{E}_{\text{th}} \rangle$: need avg over all times.

$$\langle \vec{d} \rangle = \langle \vec{v} \rangle \langle t \rangle \quad (\text{avg displacement} = \text{avg velocity} \cdot \text{avg time})$$

avg velocity acquired thanks to \vec{E} in the avg time τ

$$\Rightarrow \langle \vec{d} \rangle = - \frac{e \vec{E} \tau}{m} \cdot \tau$$

$$\Rightarrow \langle \Delta \mathcal{E}_{th} \rangle = \left\langle \frac{\partial \mathcal{E}}{\partial T} \vec{\nabla} T \cdot \vec{E} \frac{e \tau^2}{m} \right\rangle$$

Thermal energy loss (density in space & time, i.e. local power density):

$$\langle \Delta \mathcal{E}_{th} \rangle \frac{1}{\tau} = \left\langle \frac{\partial \mathcal{E}}{\partial T} \vec{\nabla} T \right\rangle m \cdot \vec{E} \frac{e \tau}{m}$$

per unit
of time

= c_v
Specific heat

$$\boxed{\langle \Delta \mathcal{E}_{th} \rangle \frac{1}{\tau} = c_v \frac{e \tau}{m} \vec{\nabla} T \cdot \vec{E}}$$