Image Processing for Physicists

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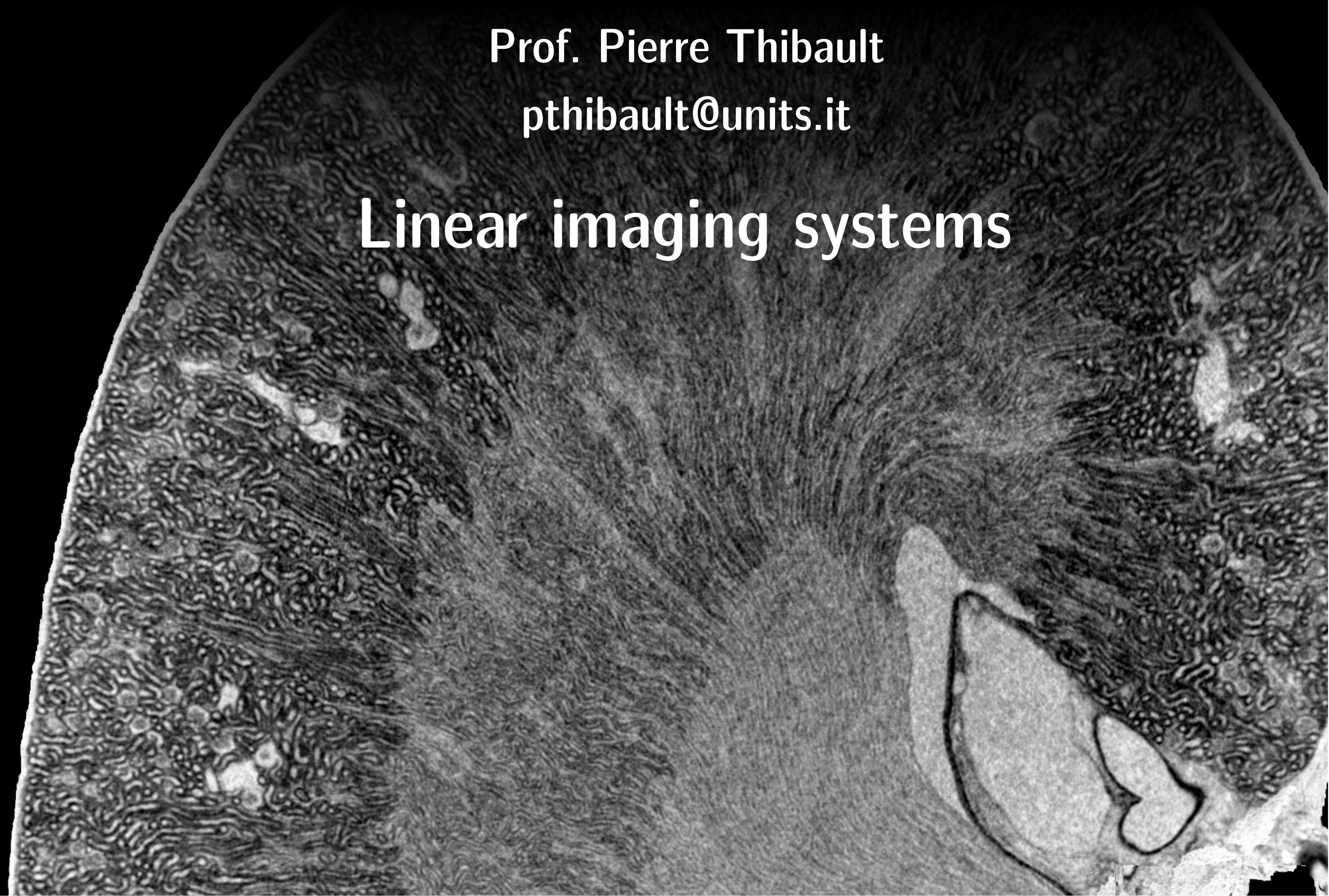
Linear imaging systems



Overview

- Definition of resolution
- Imaging systems:
  - Linear transfer model
  - Noise

Resolution

"the smallest detail that can be distinguished"

- No unique definition
  - Numerical aperture ← microscopy, photography, astronomy
  - Pixel size ← detector-limited
  - Other criteria (PSF, MTF) ← generalization
- What is "detail"?
- What is "distinguish"?

Resolution

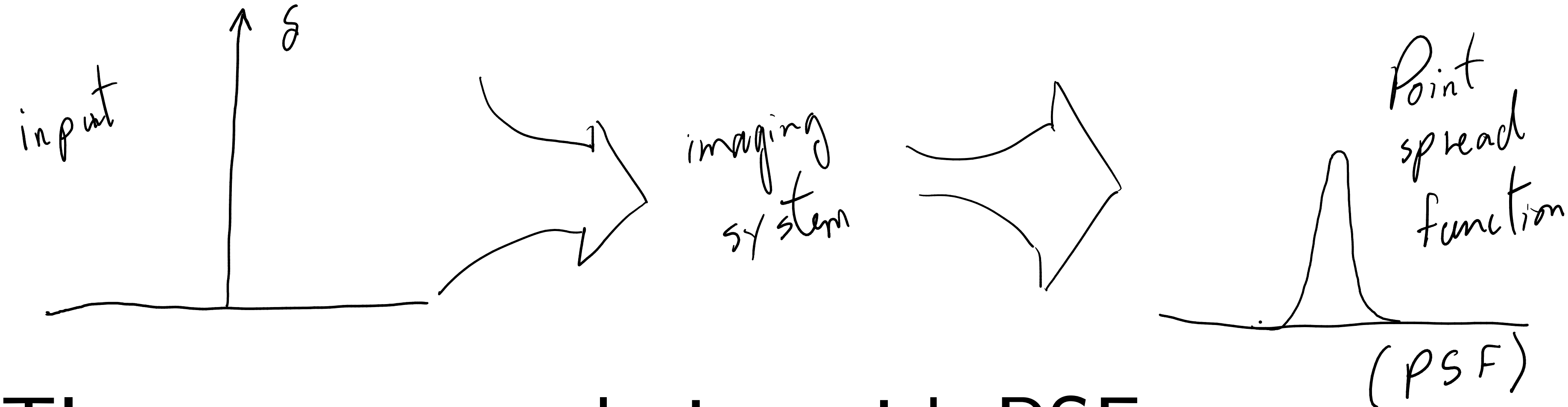
1280 x 1280

640 x 640

- **not** simply given by pixel size (i.e. sampling rate)
- light quality, optics quality, detector quality, algorithm quality, noise, ...

Linear translation-invariant systems

- Point spread function ("impulse response")



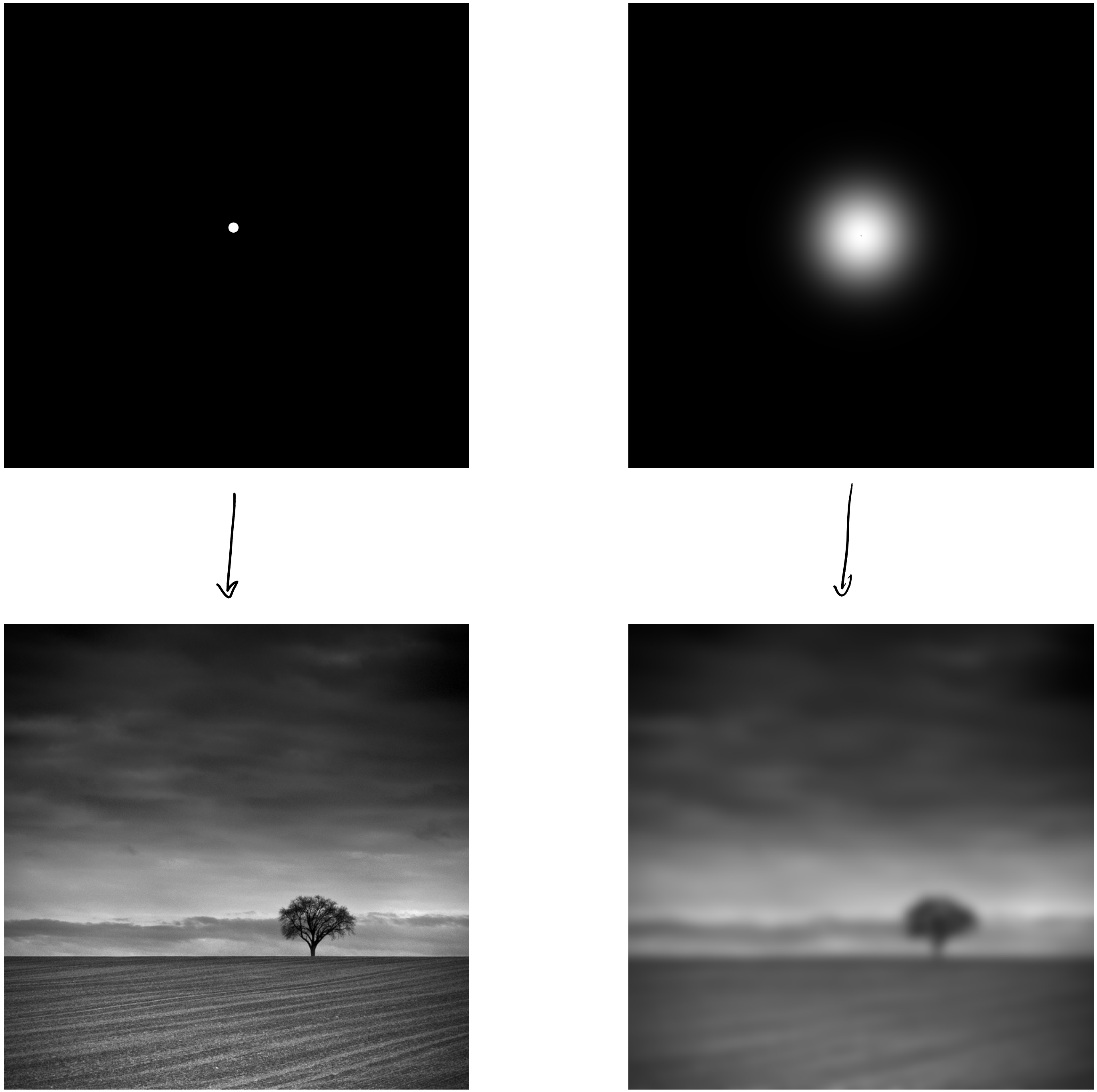
- LTI system: convolution with PSF

$$f(x,y) = \int dx'dy'\; f(x',y')\,\delta(x-x')\,\delta(y-y')$$

$\downarrow$ Imaging system

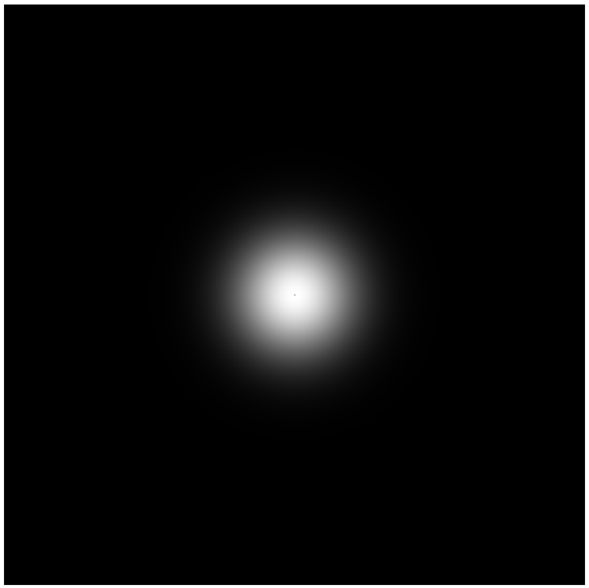
$$\int dx'dy'\; f(x',y')\,h(x-x',y-y') = f * h$$

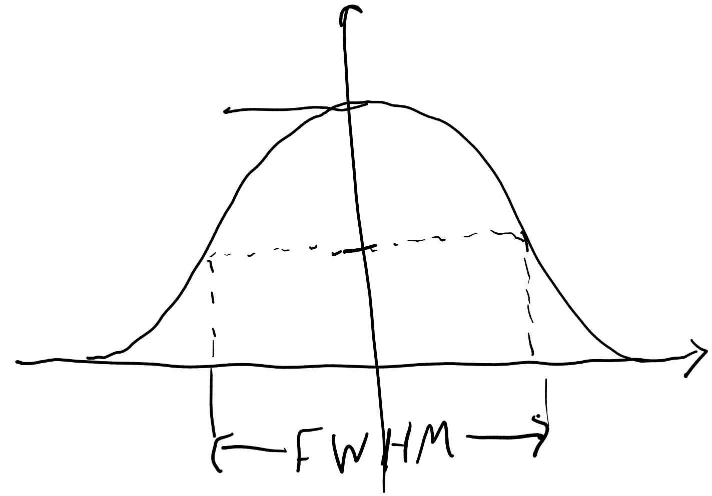
Point spread function



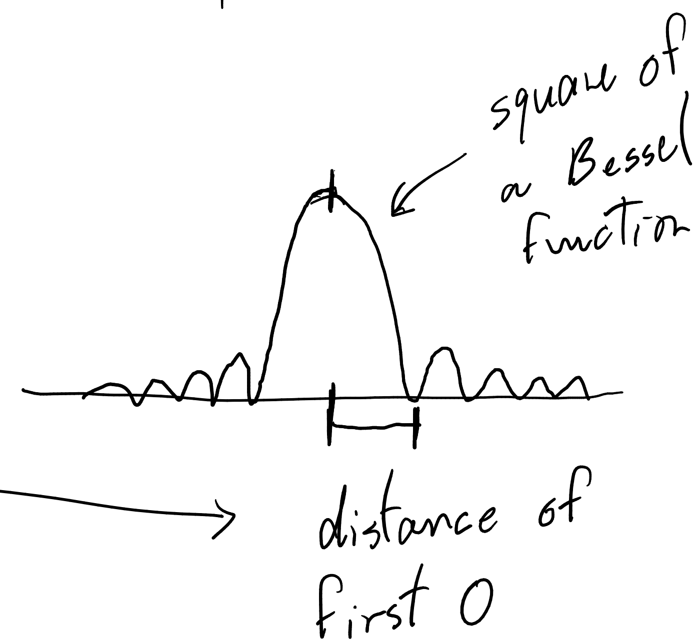
PSF and resolution

* Common way to describe resolution from PSF is "Full Width at Half Maximum" (FWHM)
(quantify)





* Another definition of resolution is Rayleigh criterion



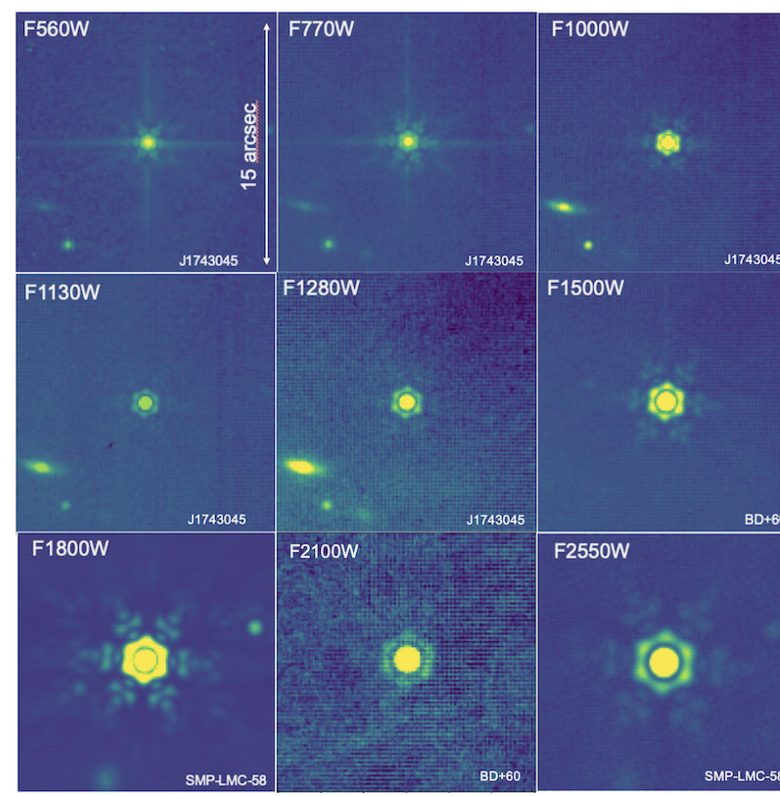
Measurement of the PSF

- Direct measurement from impulse

Most direct way: image a sharp point feature

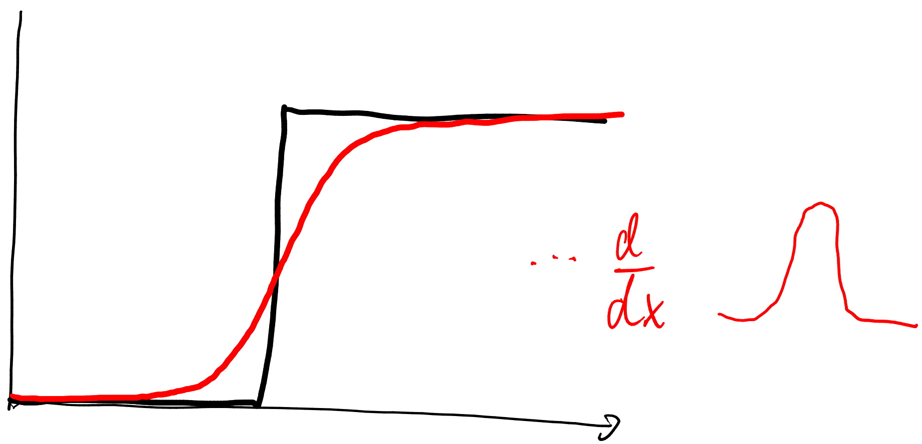
astronomy: just pick a star!

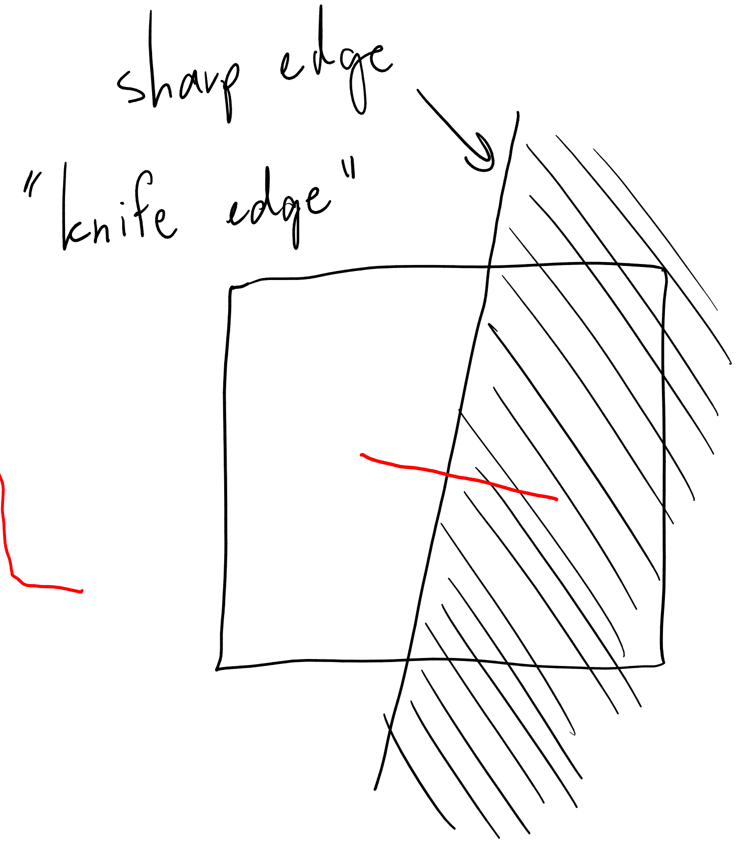
Example of measured PSF
(James Web Spatial Telescope)



Measurement of the PSF

- ~~Line~~ Edge-spread function



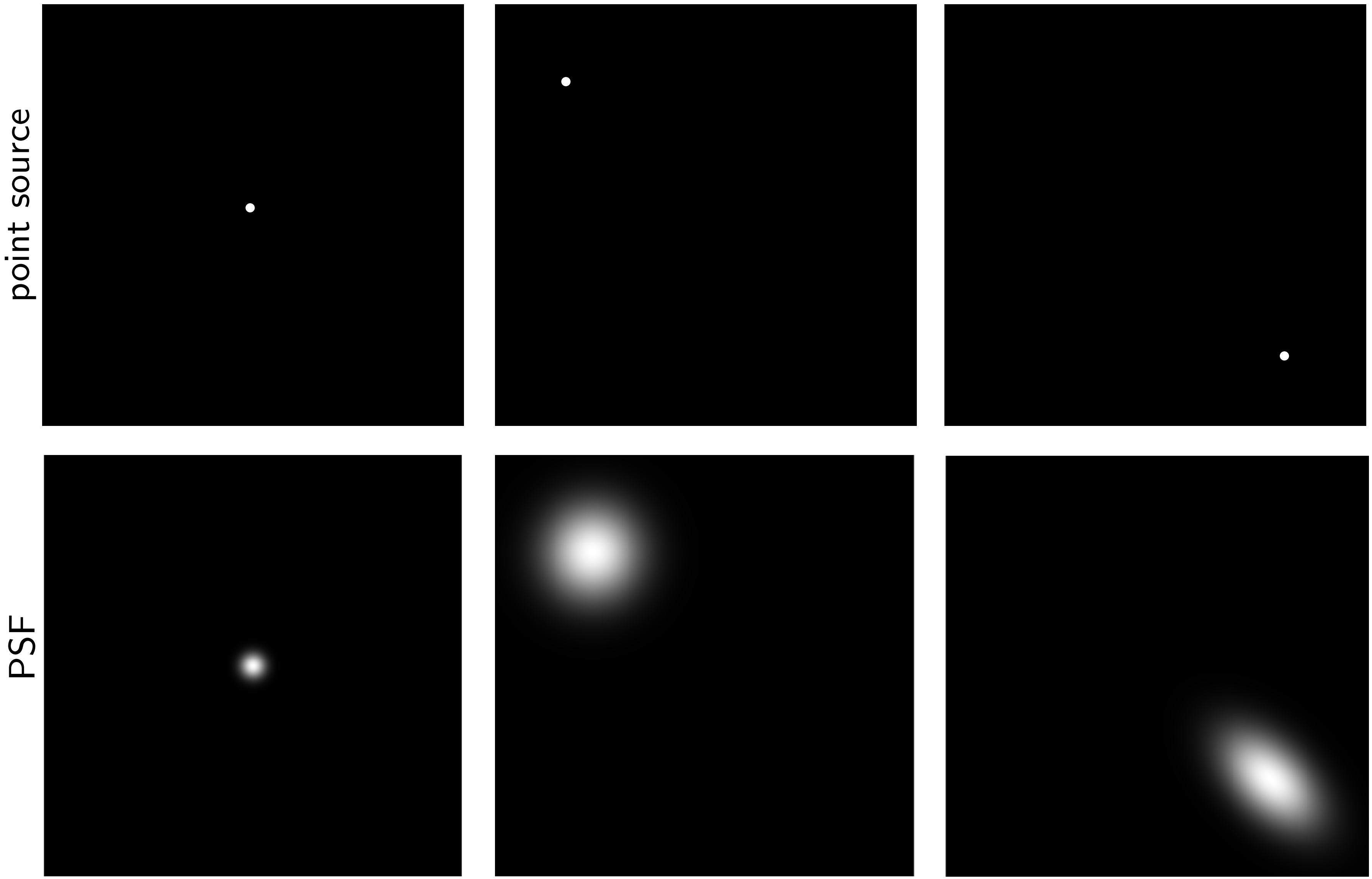


input: $H(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$

$$\frac{dH}{dx} = \delta(x)$$

PSF = derivative of edge-spread function

PSF and translation invariance



- Not translation invariant → PSF depends on position → not a convolution
- Useful to model system imperfections, lens aberrations, ...

The Fourier picture

LTI

$\mathcal{F}\{f * h\} = F(u) \cdot H(u)$ ← Fourier transform of PSF

||

"optical transfer function"

OTF

Consider a single spatial frequency u_0

$f(x) = A e^{2\pi i u_0 x}$ —Imaging system→ $H(u_0) A \, \delta(u - u_0)$

$F(u) = A \, \delta(u - u_0)$

A: original amplitude

↓

$H(u_0) A \, e^{2\pi i u_0 x}$

$H(u_0) A$: modulated amplitude

* pure oscillations of the form $e^{2\pi i u_0 x}$ are eigenfunctions of a LTI imaging system

Optical transfer function

Response of a system to an oscillating signal with well-defined frequency

$H(u) \rightarrow OTF = \mathcal{F}\{PSF\}$

↑ complex valued in general

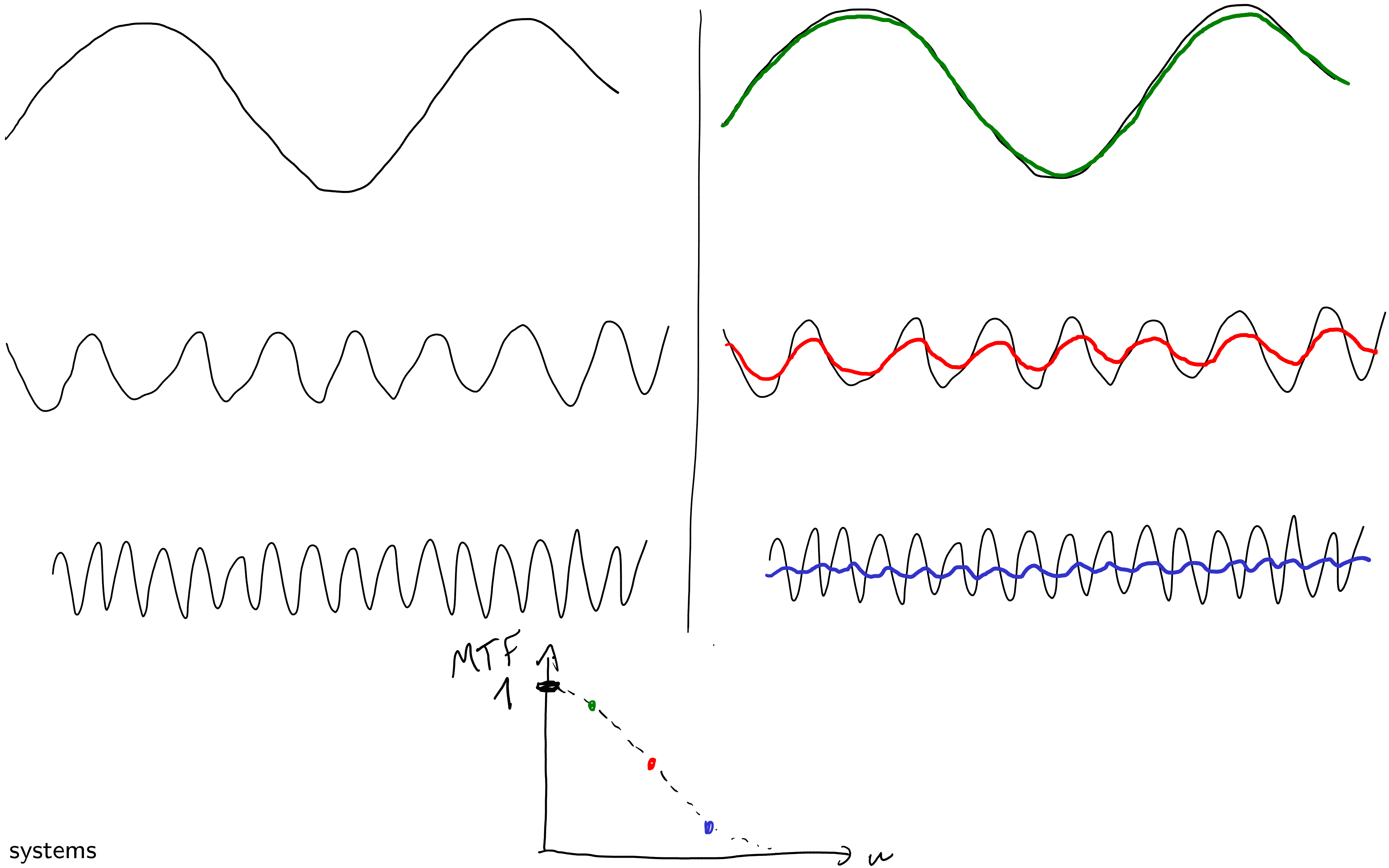
Amplitude $|OTF| = MTF$ "modulation transfer function"

Phase $\arg\{OTF\} = PTF$ phase transfer function

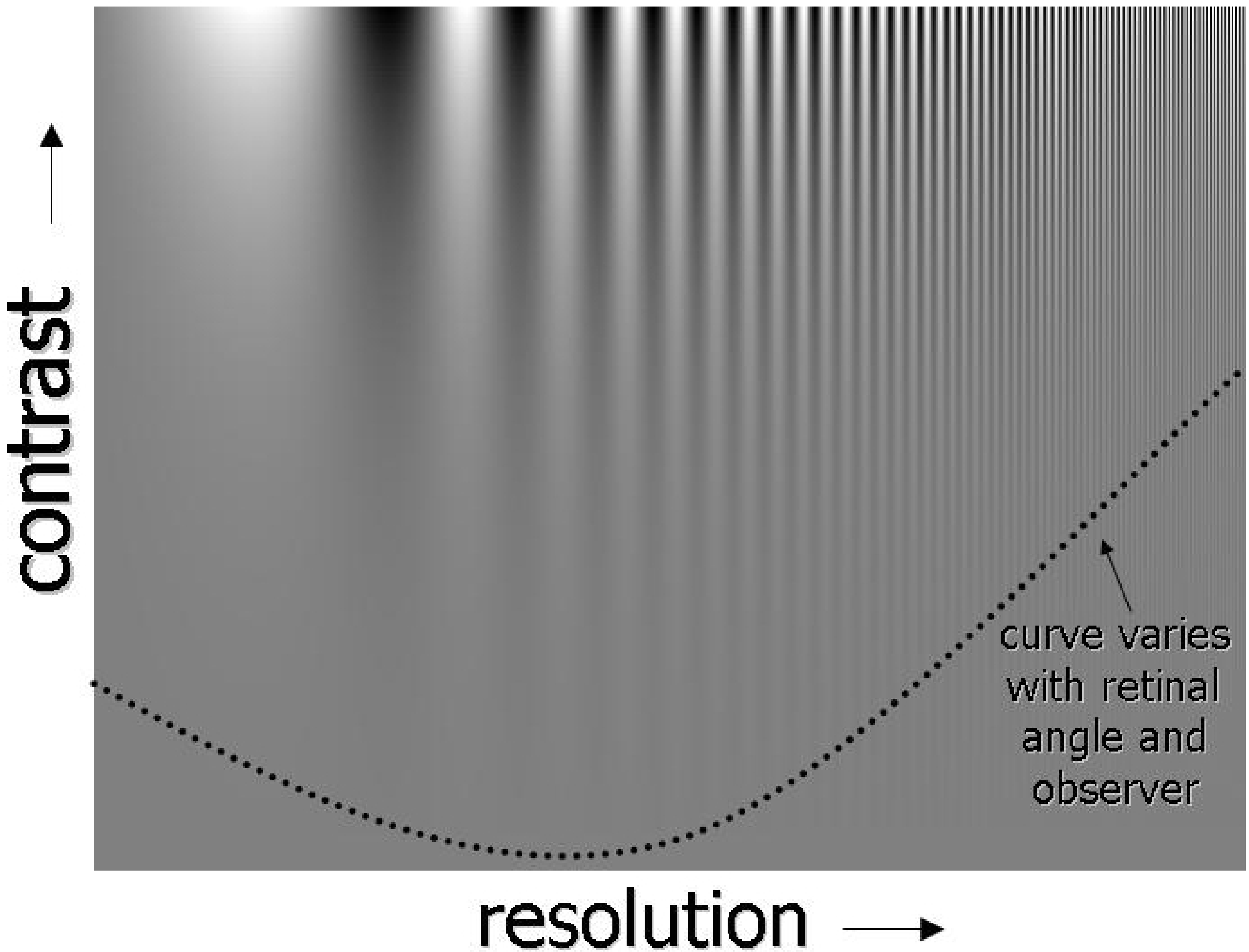
$$\mathcal{F}\{PSF\} = OTF = MTF\, e^{iPTF}$$

Modulation transfer function

Amplitude change of an oscillating signal for a given frequency



Eye MTF


contrast
curve varies with retinal angle and observer
resolution

Campbell-Robson curve

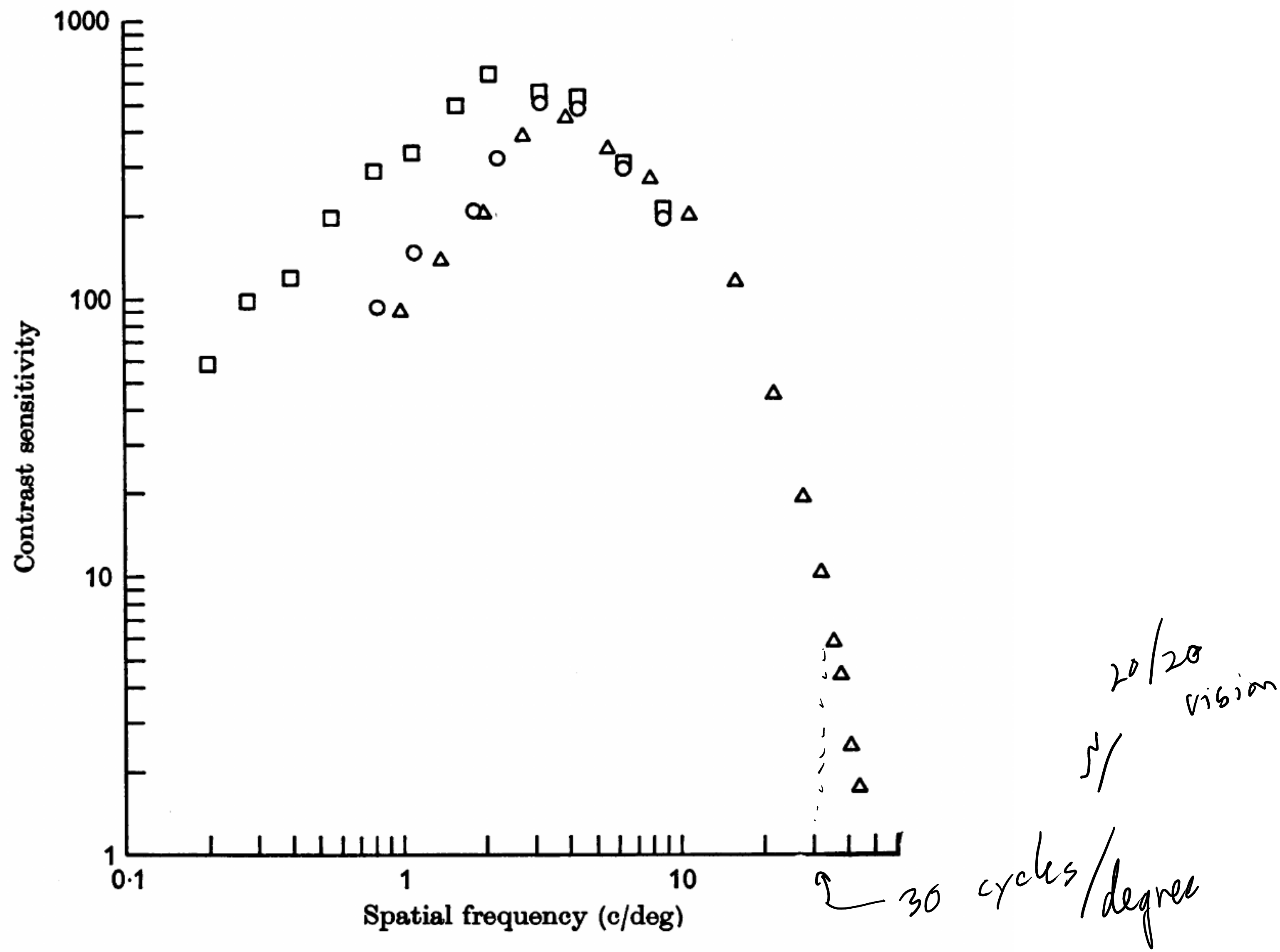
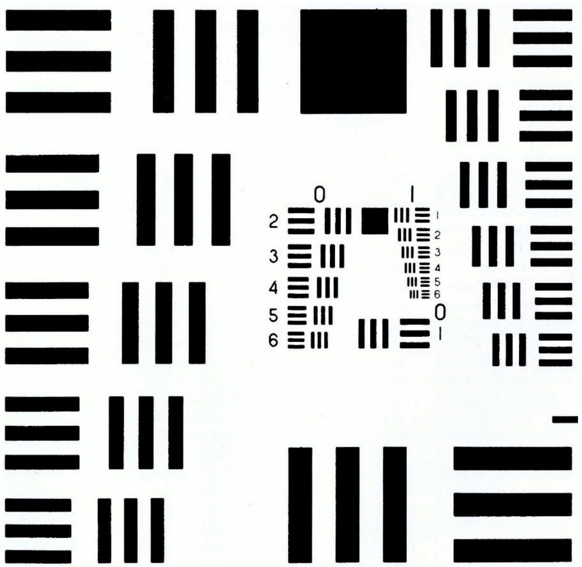
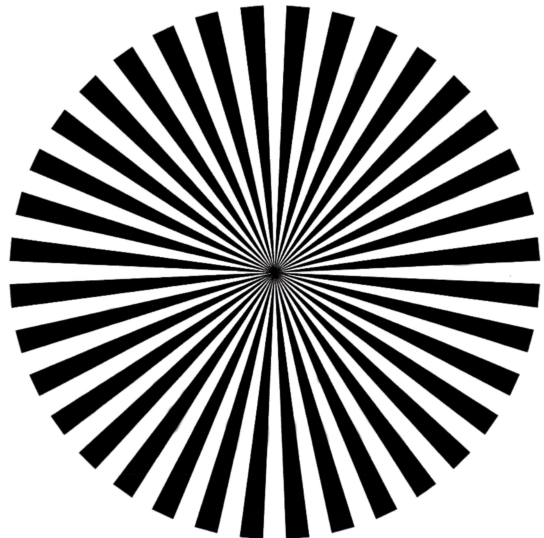


Fig. 2. Contrast sensitivity for sine-wave gratings. Subject F.W.C., luminance 500 cd/m². Viewing distance 285 cm and aperture 2° × 2°, △; viewing distance 57 cm, aperture 10° × 10°, □; viewing distance 57 cm, aperture 2° × 2°, ○.

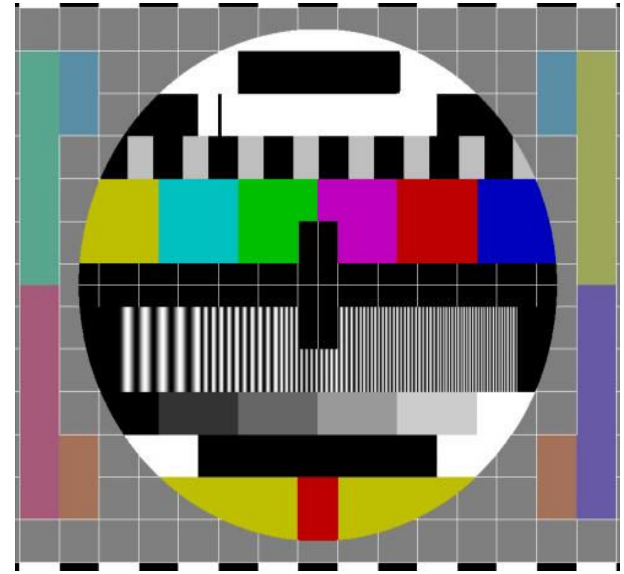
Measurement of MTF



USAF resolution target



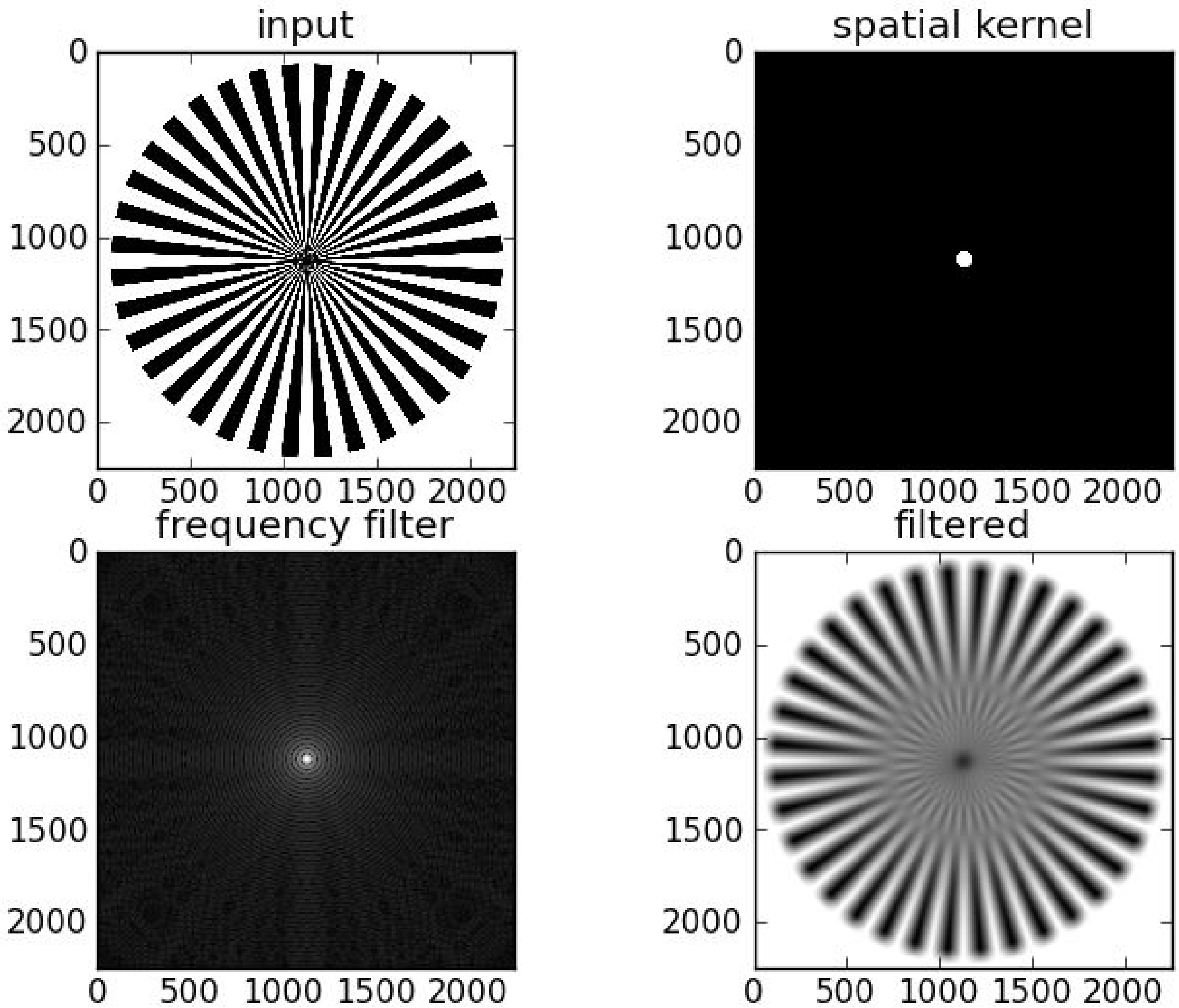
Siemens star



source: http://fotomagazin.de

Phase transfer function

describes how an oscillating signal changes in phase due to system



Phase transfer function

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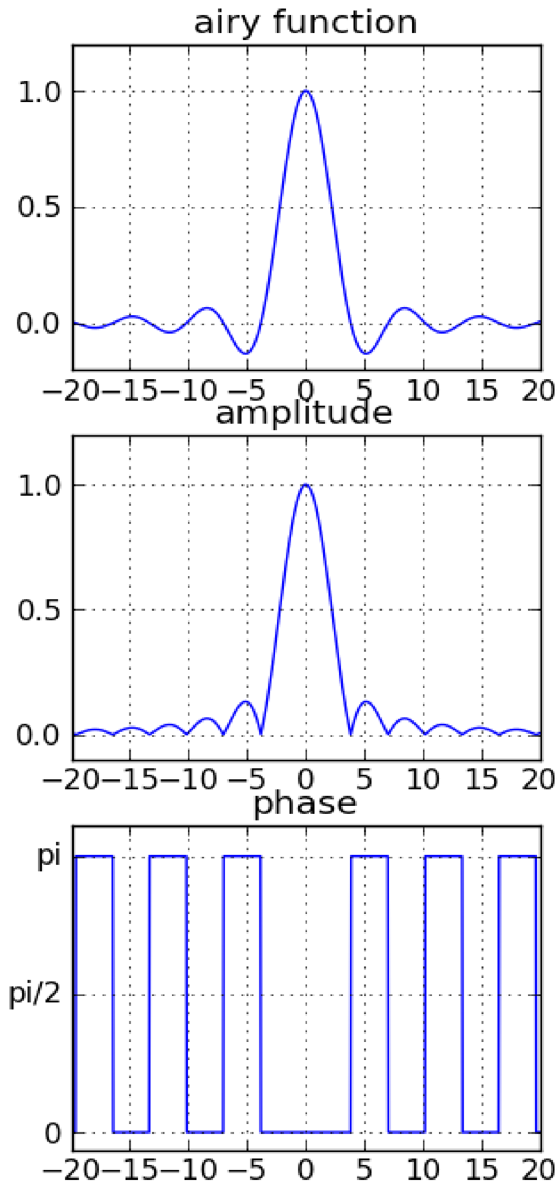
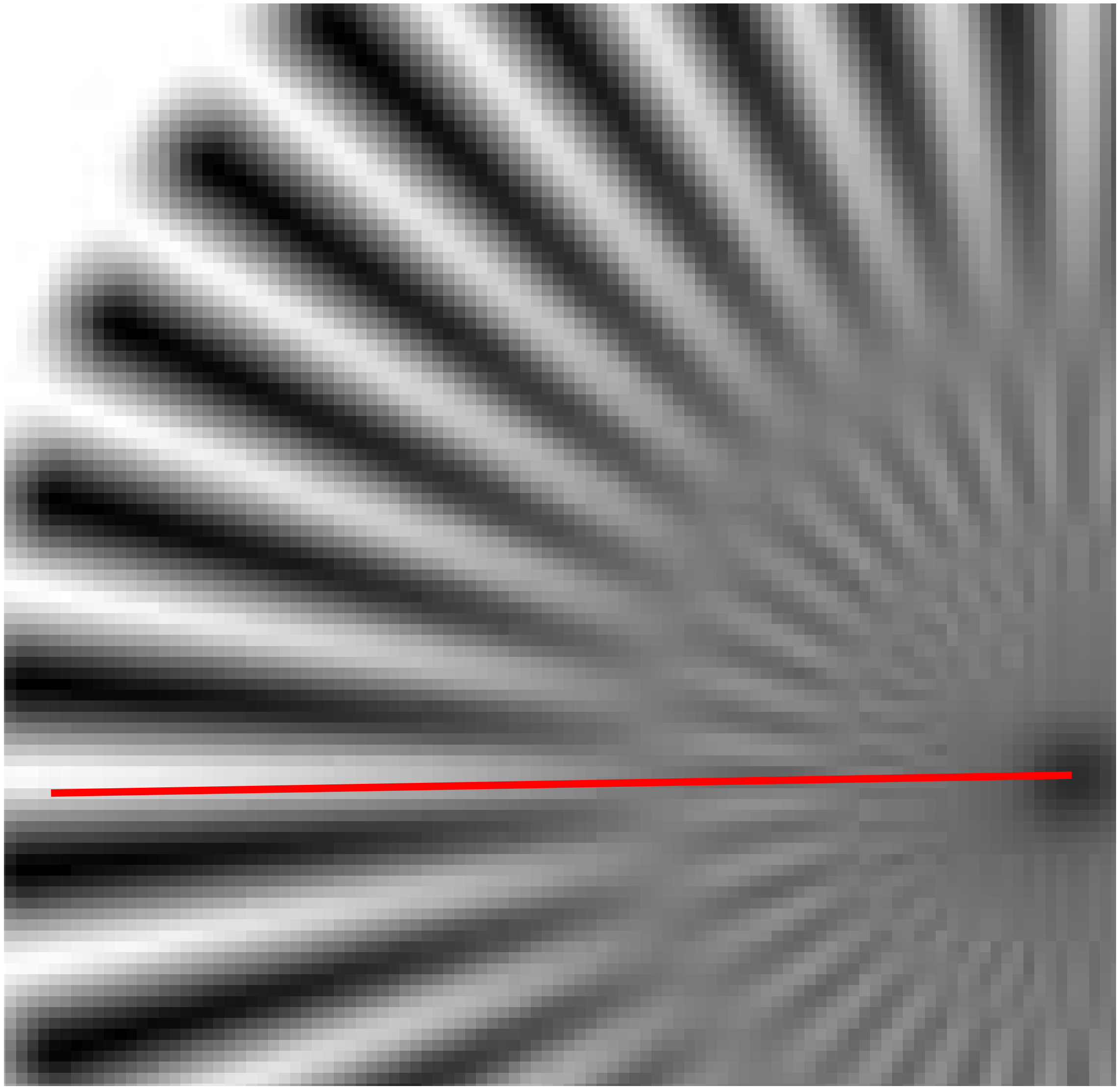
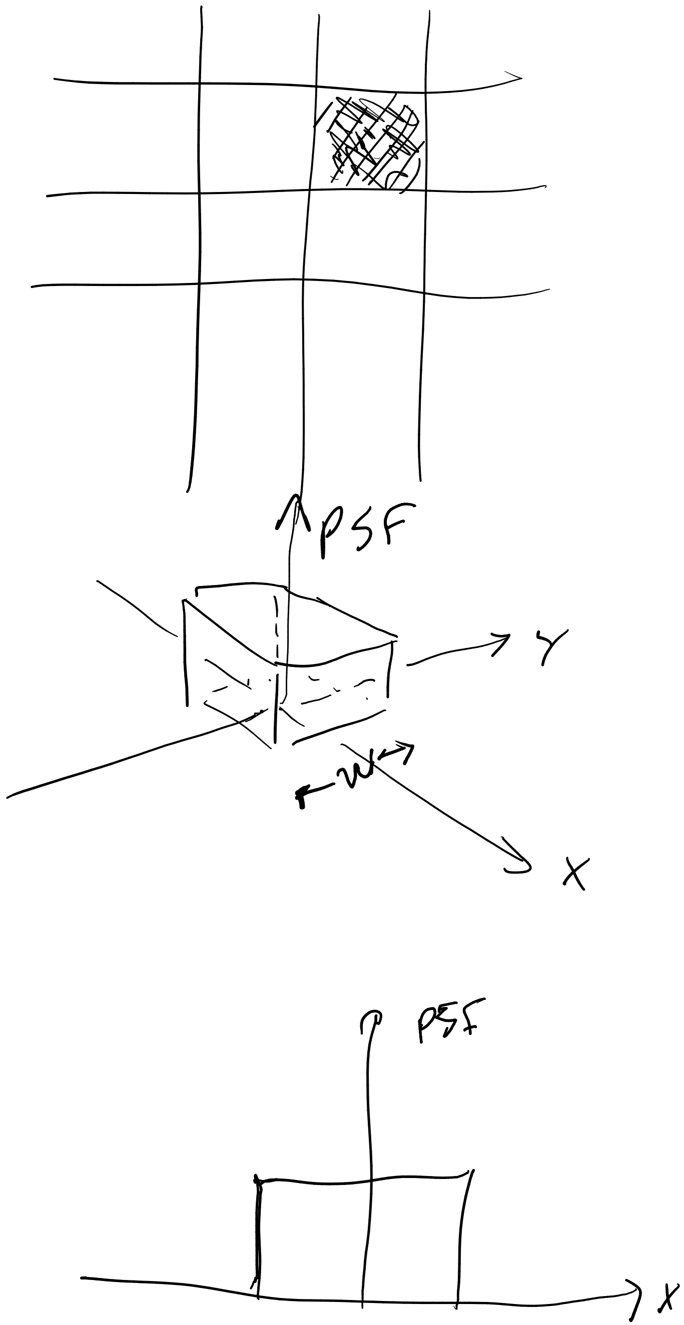
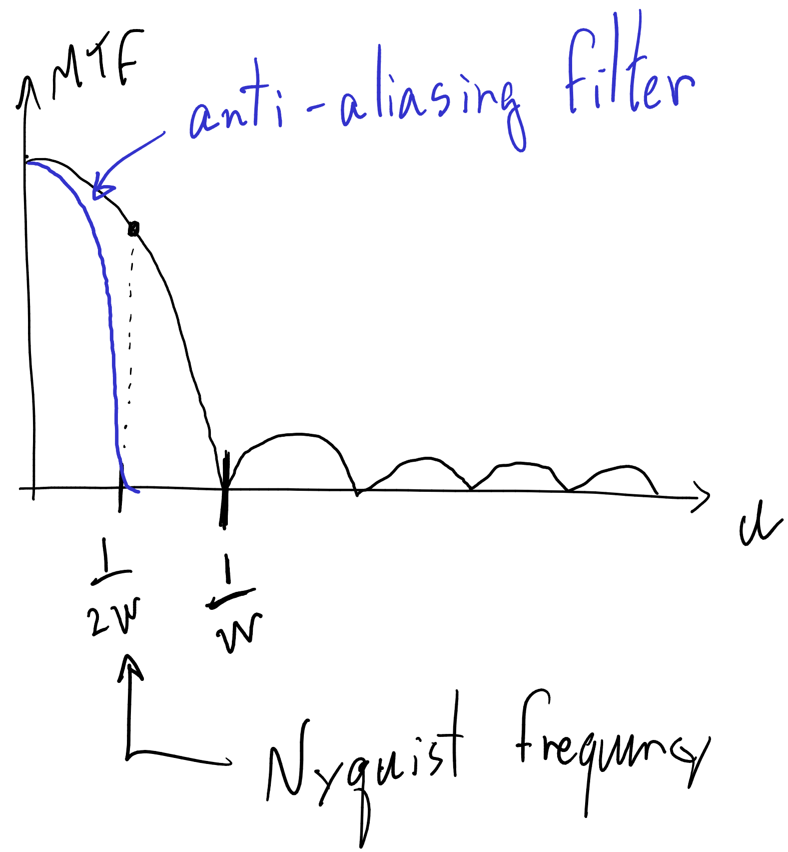




MTF of an ideal pixel

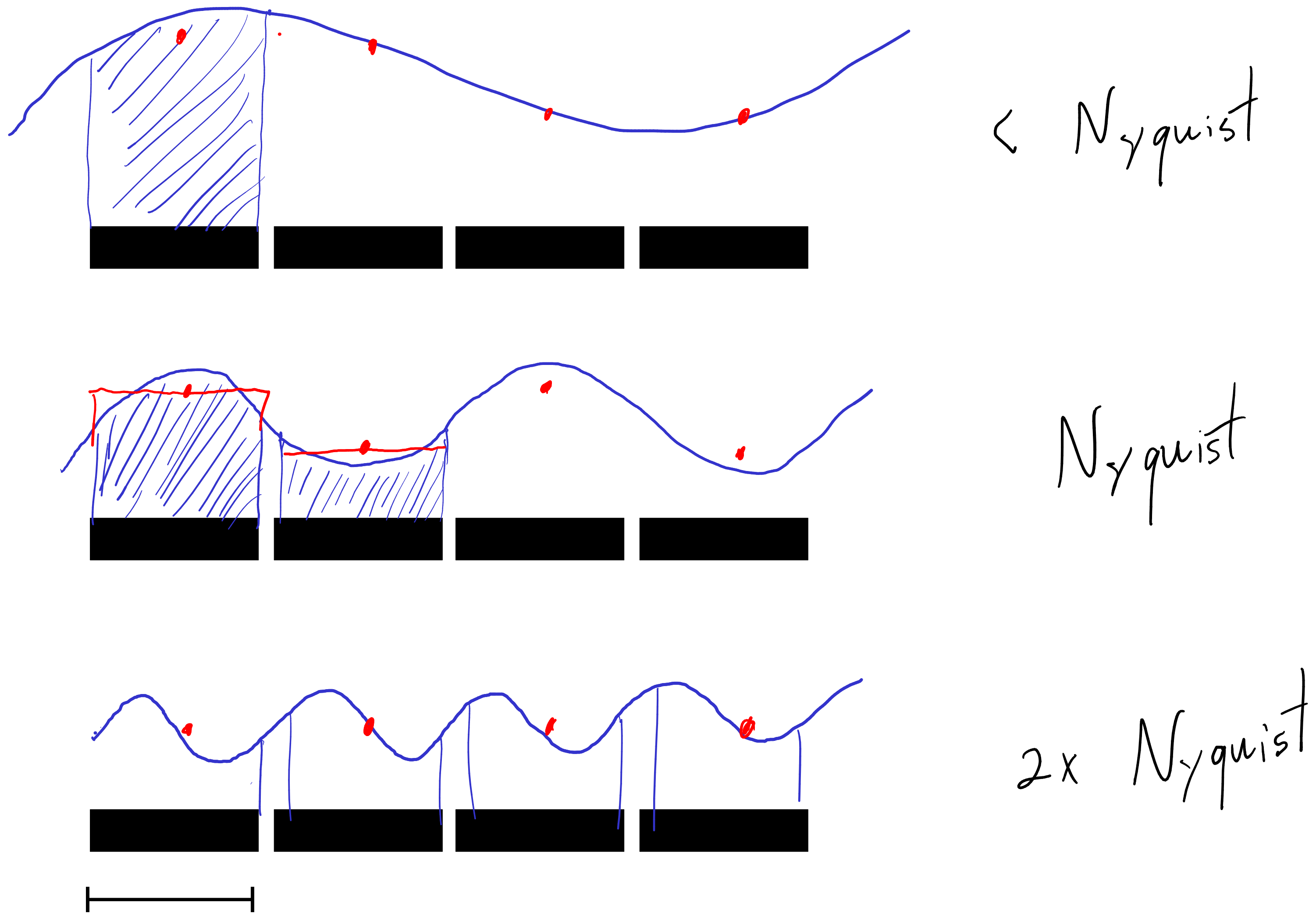
square





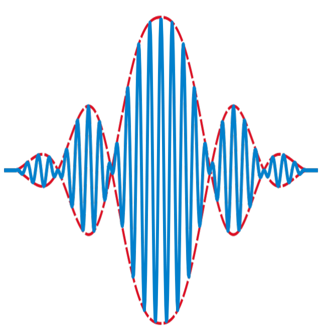
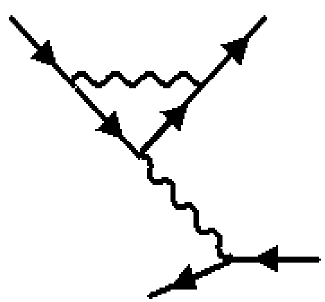
Pixel MTF

Modulation transfer function of a single detector pixel



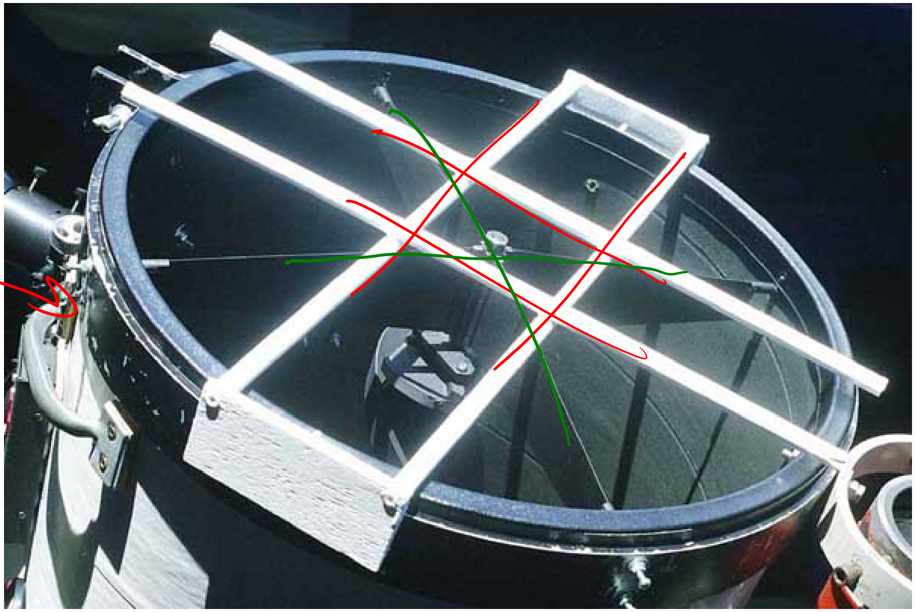
Imaging as a linear filter

$$\text{Output}(u) = \text{Input}(u) \underbrace{\times \text{MTF}_{\text{optics}}(u) \times \text{MTF}_{\text{detector}}(u) \left(\times \text{MTF}_{\text{algorithm}}(u) \times \ldots \right)}_{\text{effective MTF}}$$


PSF examples

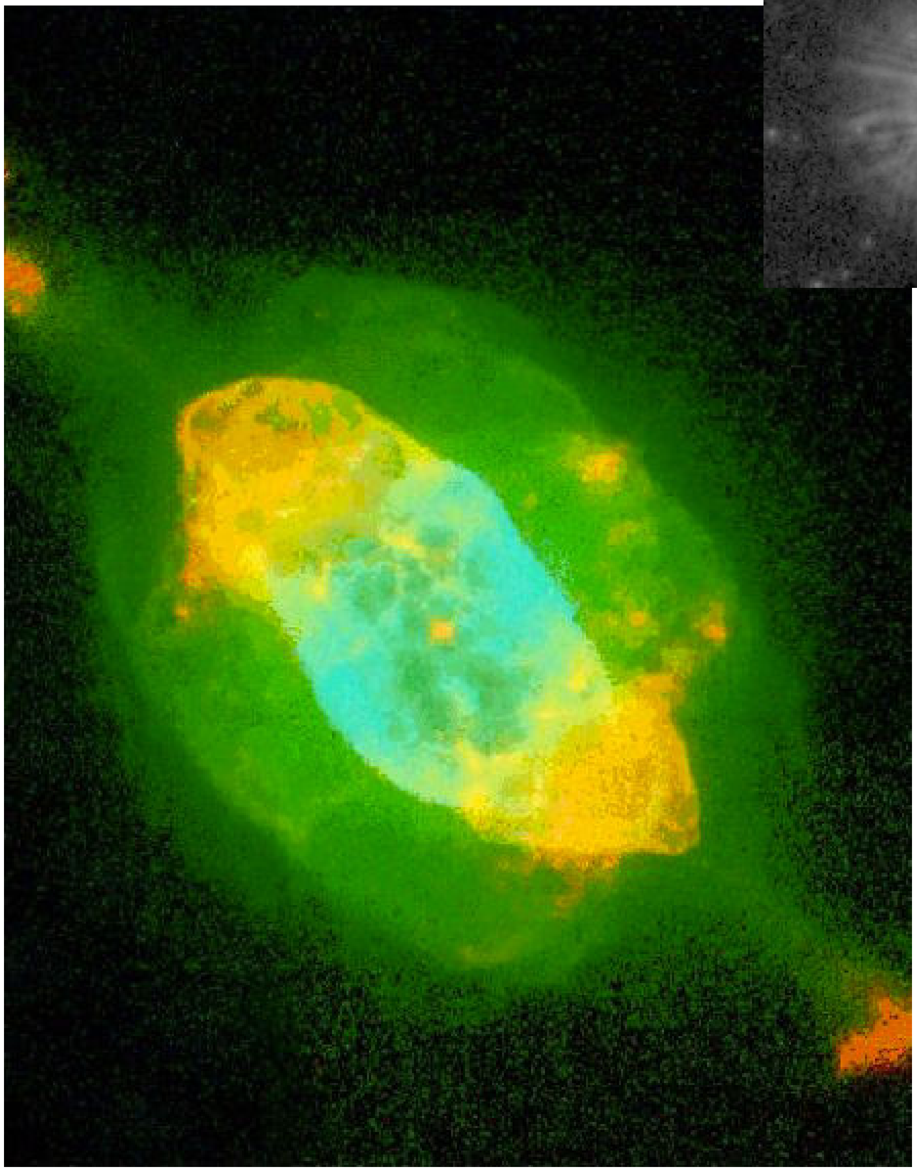
- isolated stars are essentially PSFs

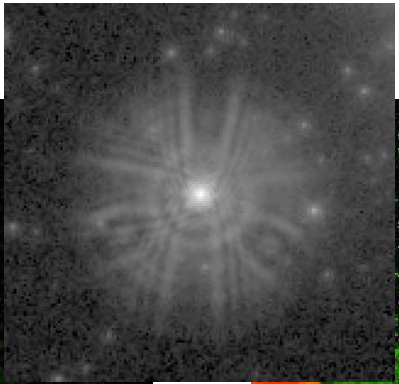


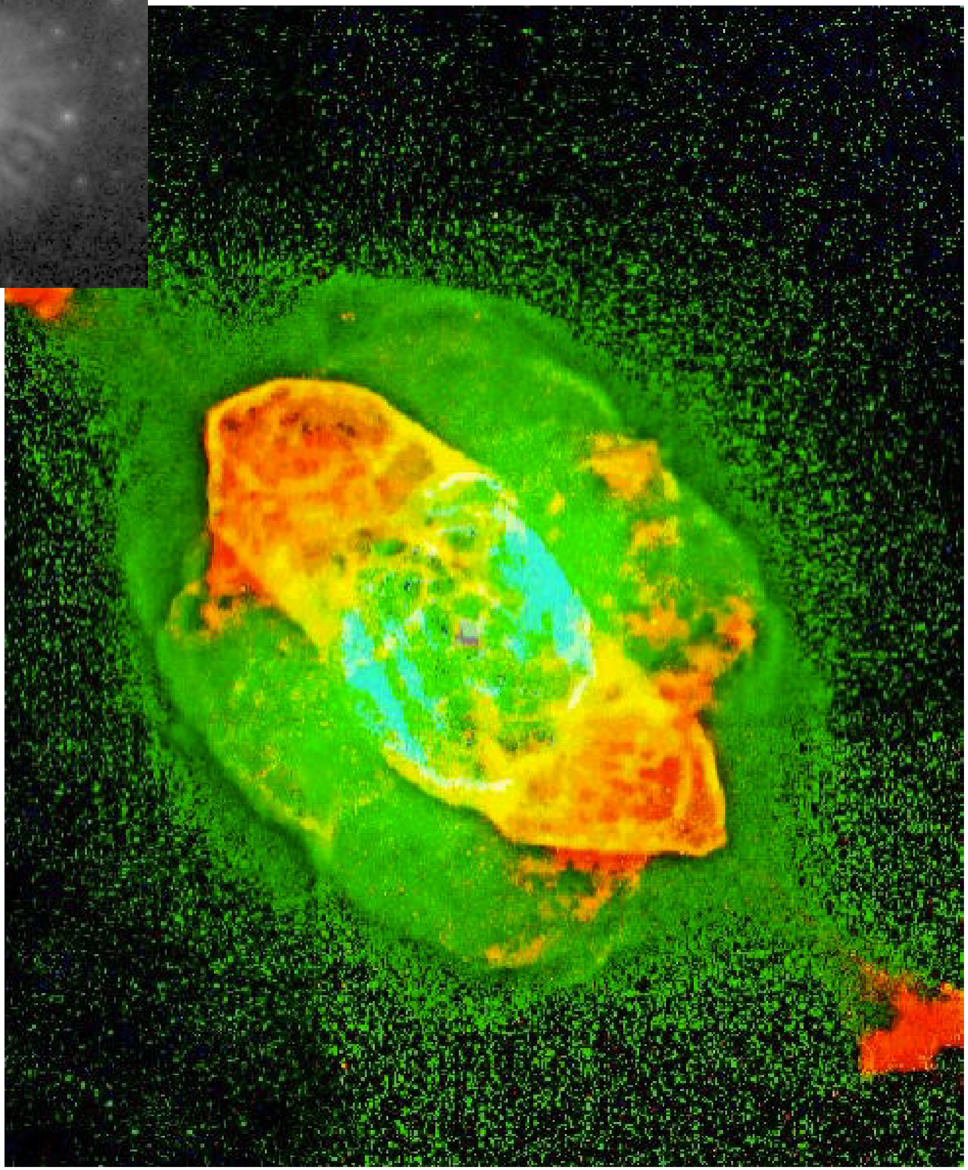
source: www.apod.nasa.gov

PSF examples

Hubble flawed mirror deconvolution (correction for spherical aberration)



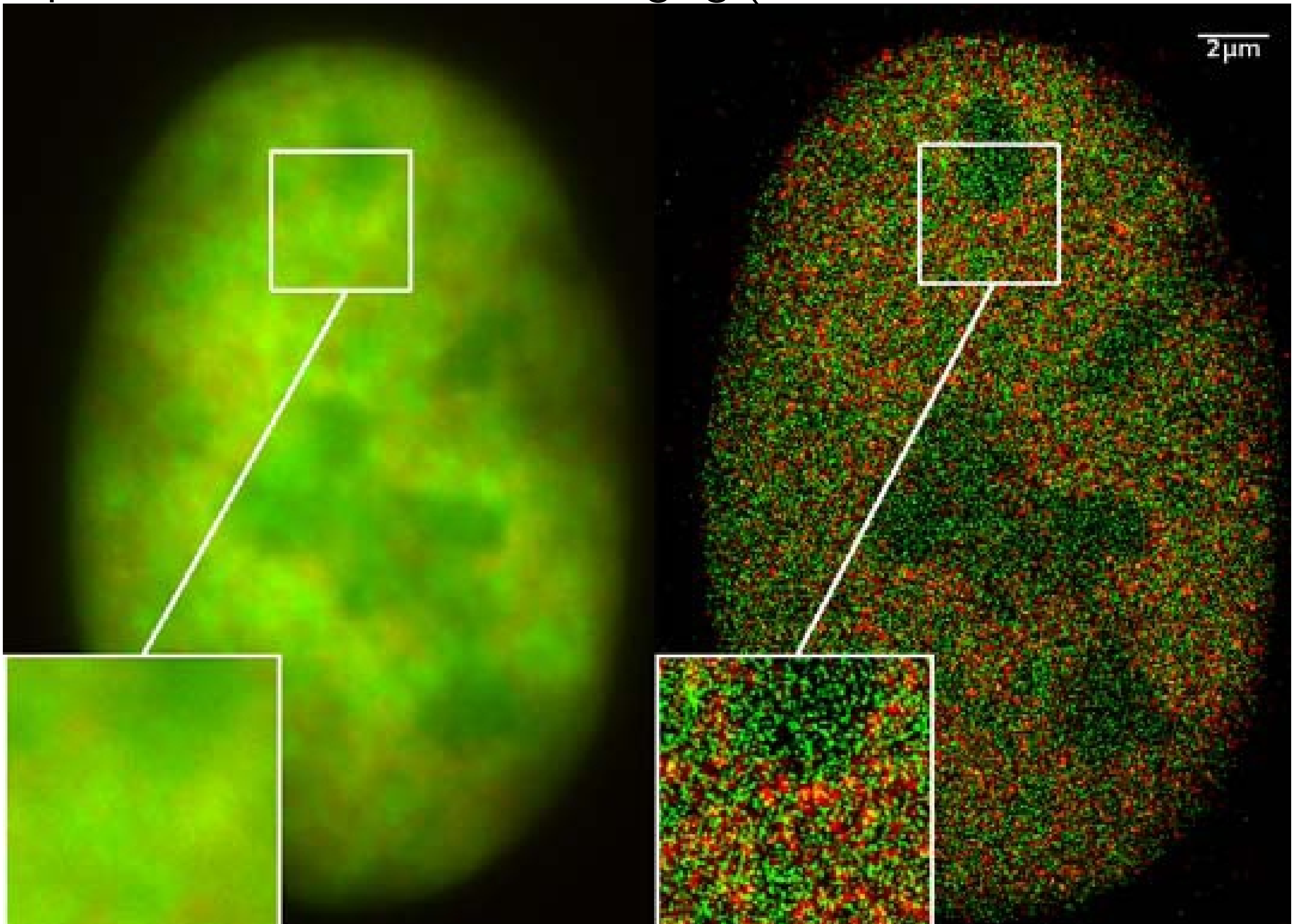




source: www.wikipedia.org

PSF examples

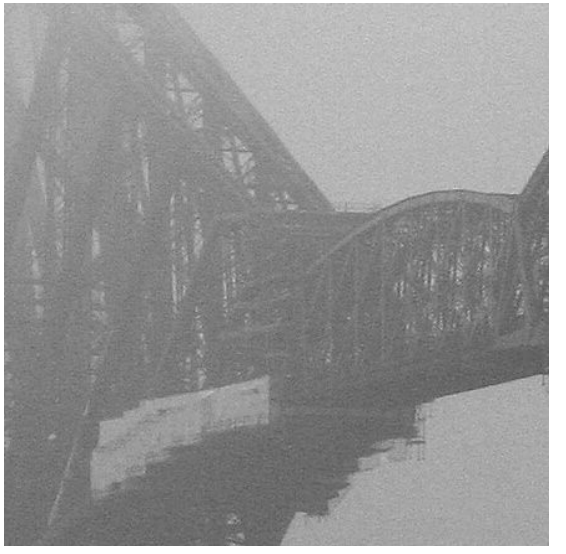
Super-resolution fluorescence imaging (STORM, STED, PALM, ...)

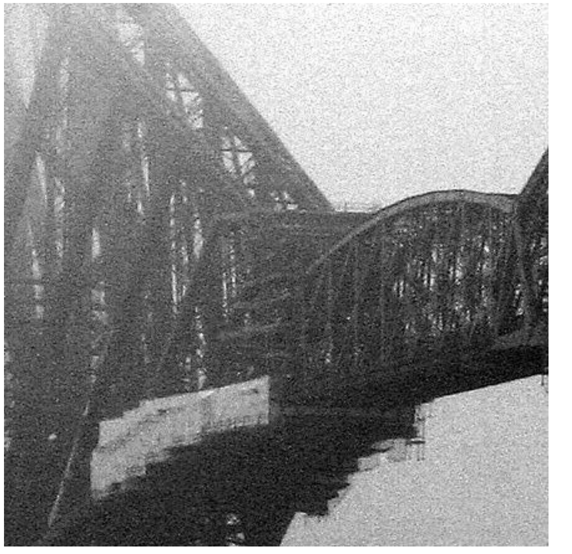


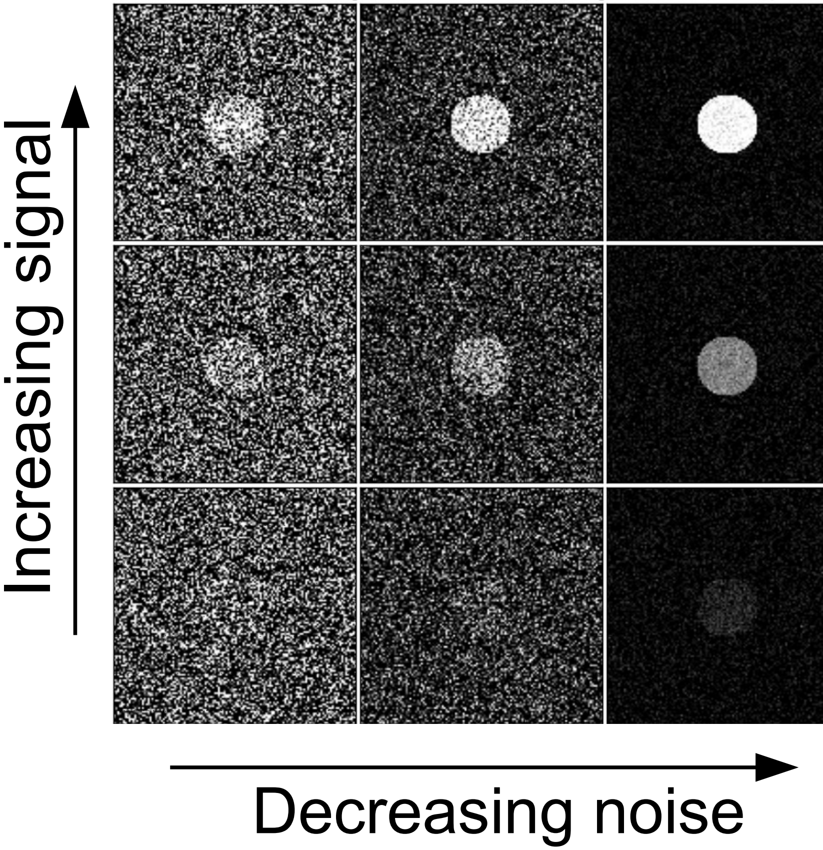
source: www.wikipedia.org

Contrast and noise

- Intensity operation: higher contrast, higher noise
- Contrast-to-noise remains constant







Random variables

- random variable, sample space

$X \qquad \Omega$

probability of measuring x: $p(x)$

$$p(\Omega) = 1$$

- probability density function

$$p(a < x < b) = \int_a^b \rho(x)\,dx$$

$\rho(x)$ ← probability density

$$\int_\Omega \rho(x)\,dx = 1$$

- expectation value

$$\langle f \rangle = \int_\Omega f(x)\,\rho(x)\,dx$$

mean: $\langle x \rangle = \int x\,\rho(x)\,dx$

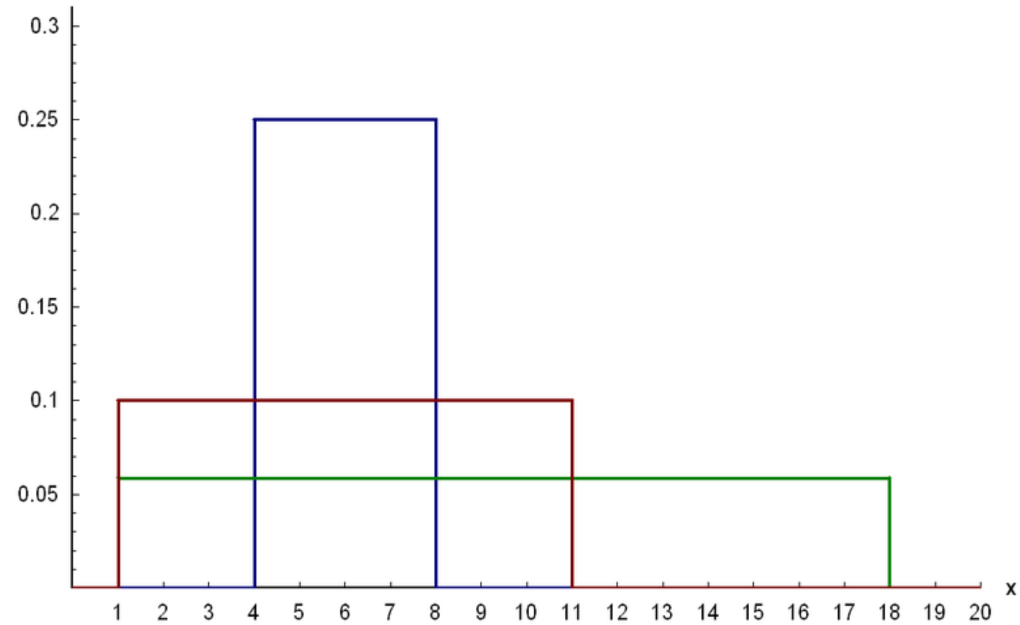
- variance

$$\mathrm{var}(x) = \langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

Uniform distribution

- probability density function

$$\rho(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$



- expectation value

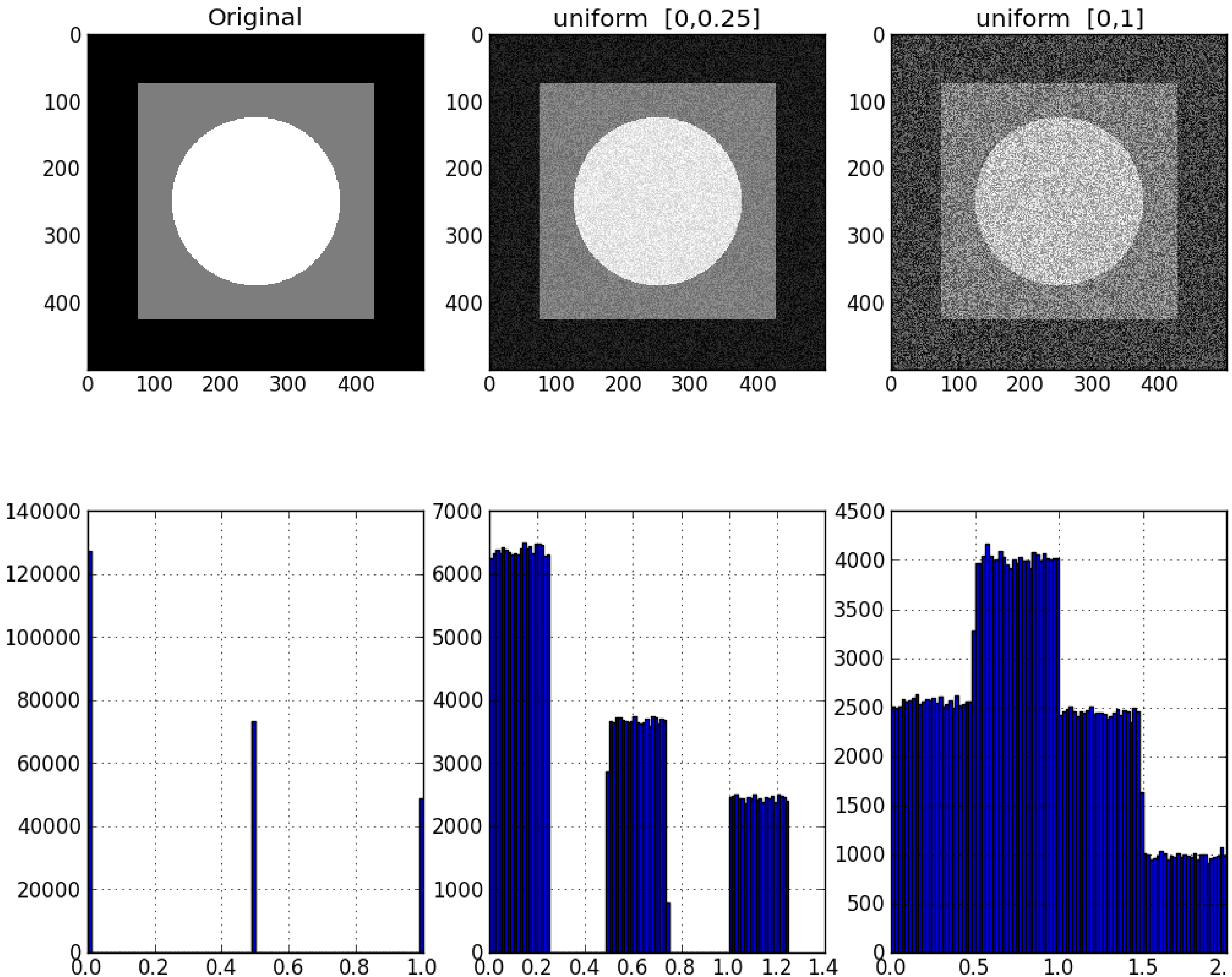
$$\langle x \rangle = \frac{1}{2}(a+b)$$

- variance

$$\mathrm{var}\, x = \frac{(b-a)^2}{12}$$

- occurrence → not very common in imaging, but useful to build other distribution

Uniform distribution

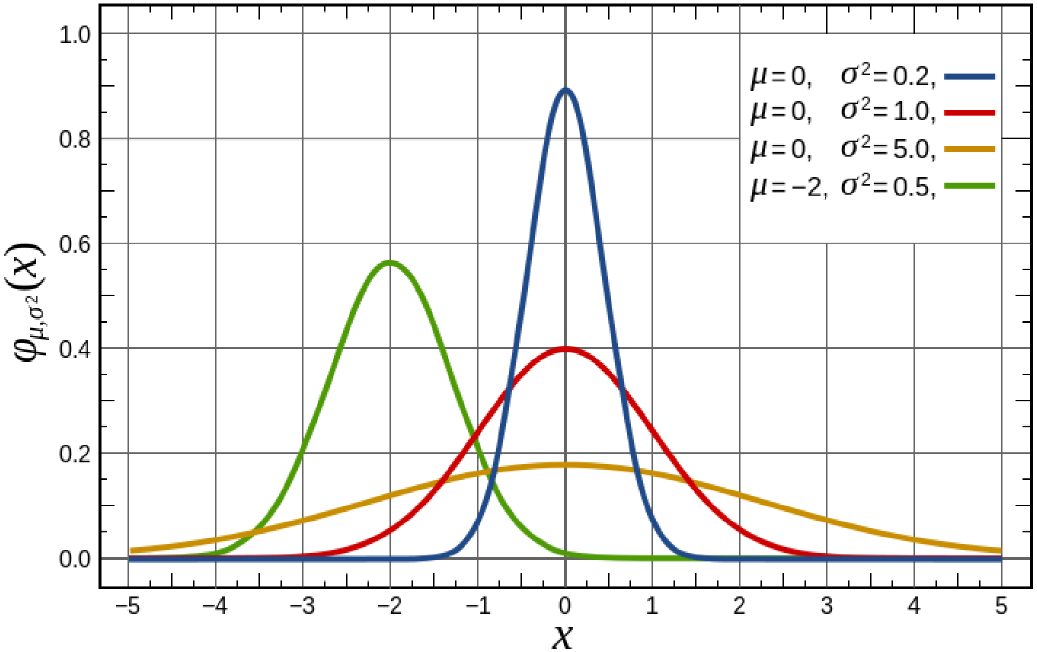


histogram

Gaussian distribution

- probability density function

$$\rho(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$



- expectation value

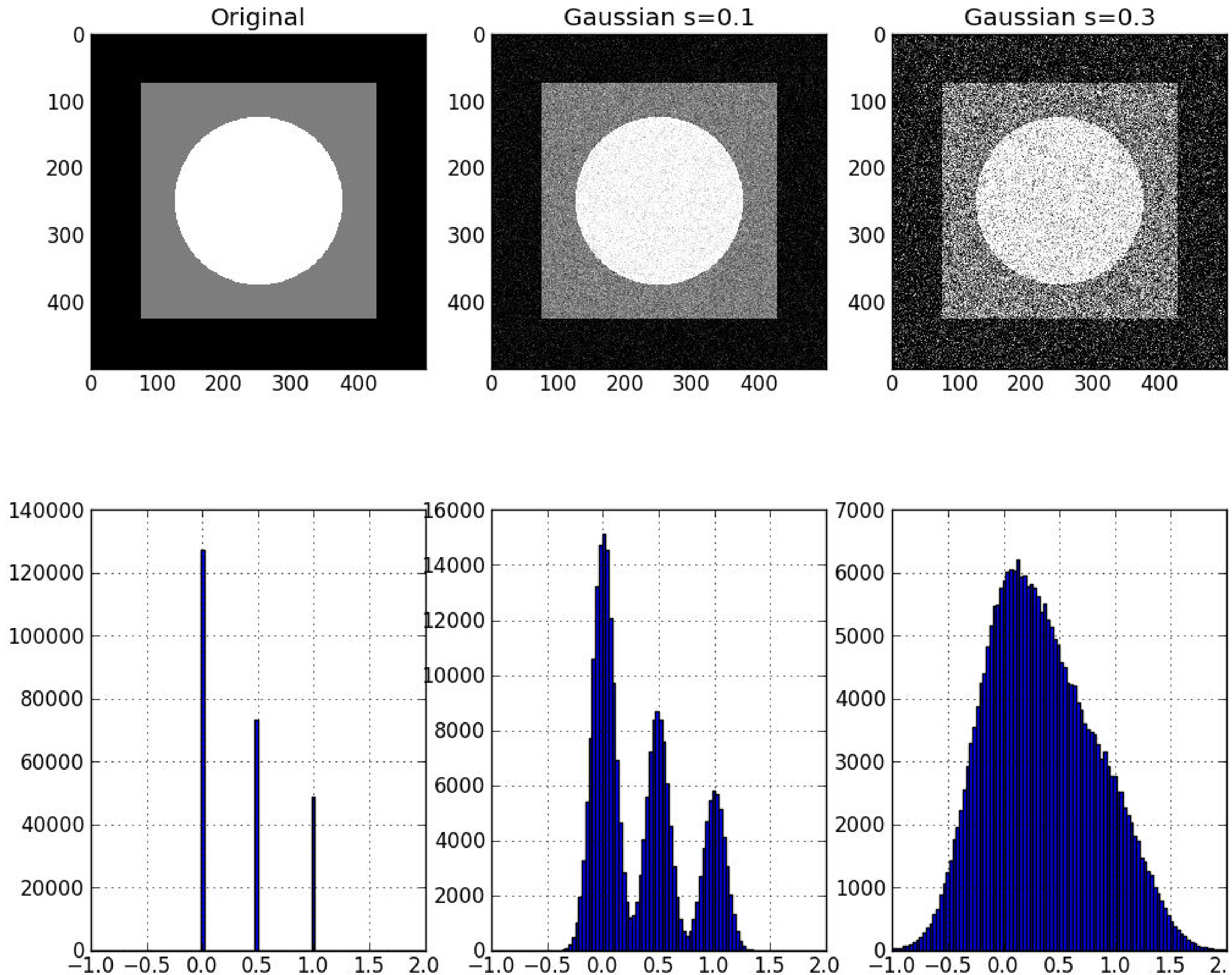
$$\langle x \rangle = \mu$$

- variance

$$\mathrm{Var}\, x = \sigma^2$$

- occurrence   very common

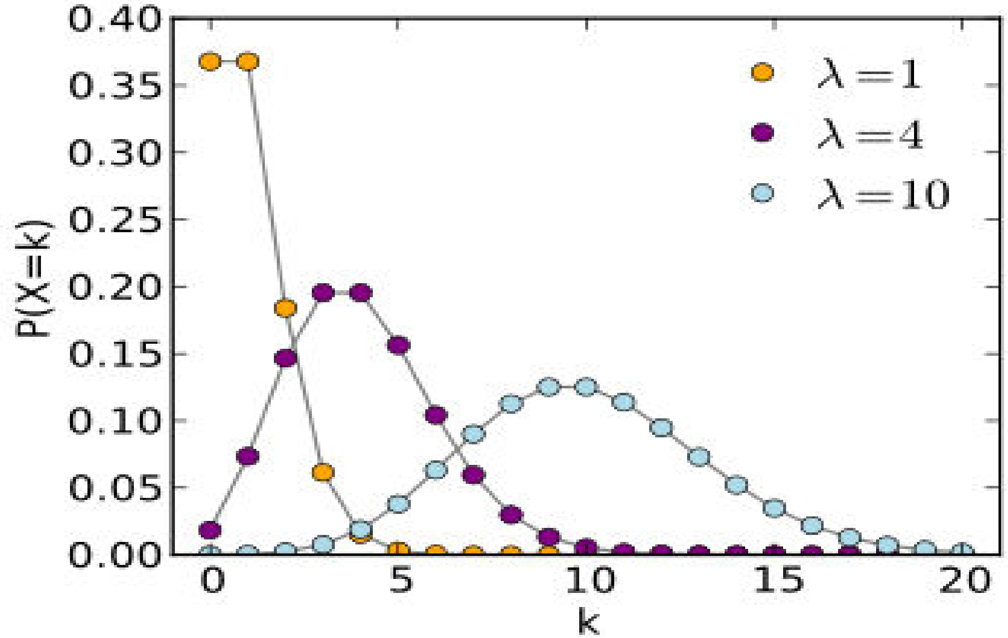
Gaussian distribution



Poisson distribution

- probability mass function

$$p(n) = \frac{1}{n!} \lambda^n e^{-\lambda}$$



- expectation value

$$\langle n \rangle = \lambda$$

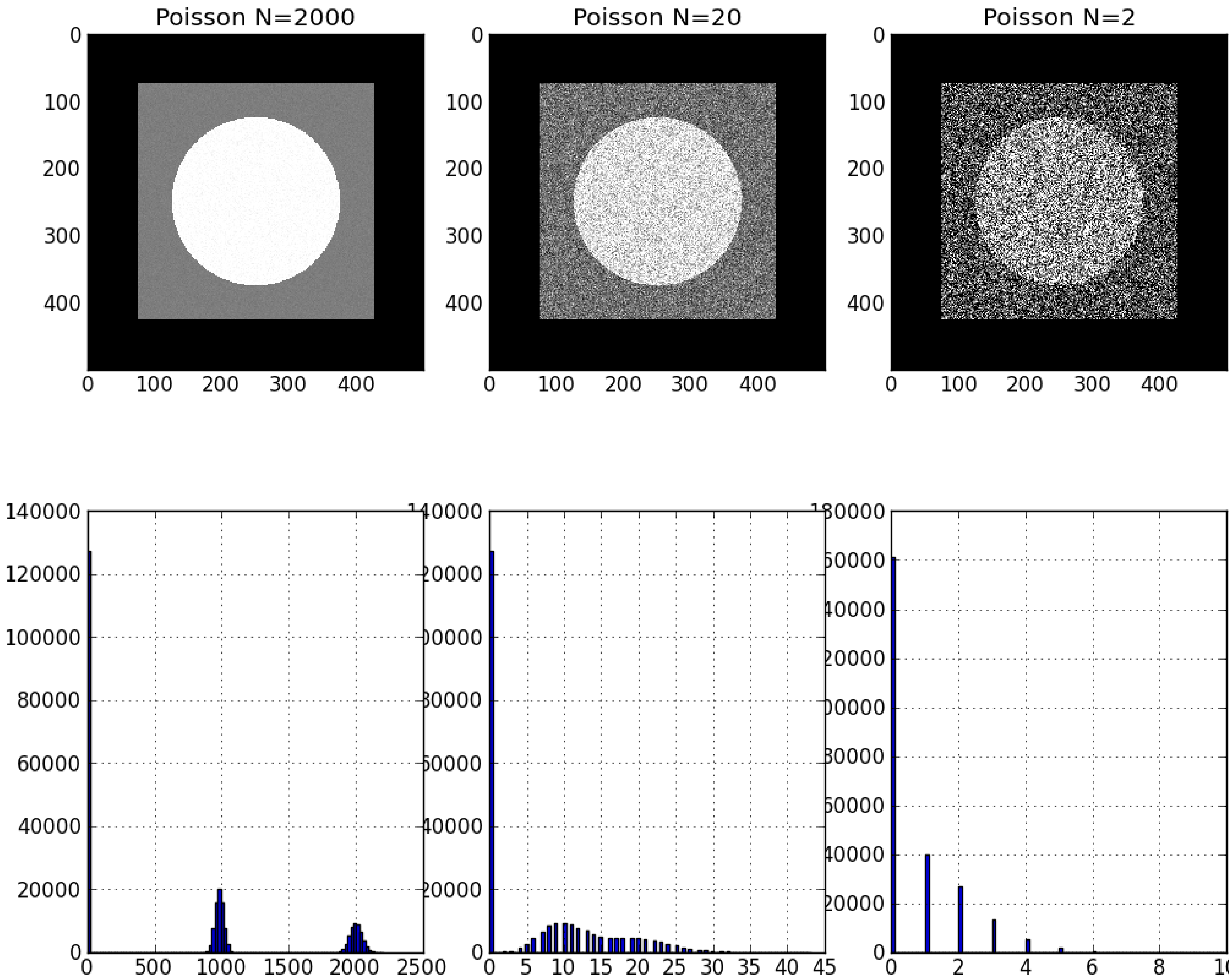
- variance

$$\text{var}\, n = \lambda$$

- occurrence   counting process (of independent events)

photons, electrons   "shot noise"

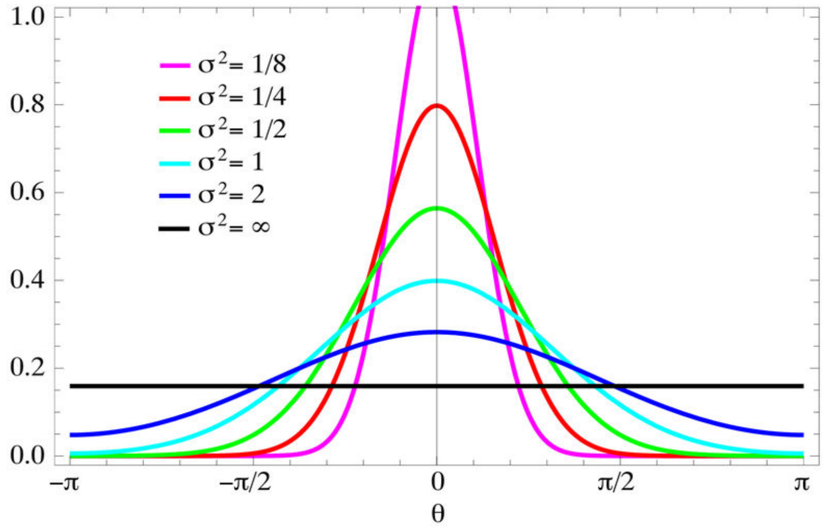
Poisson distribution



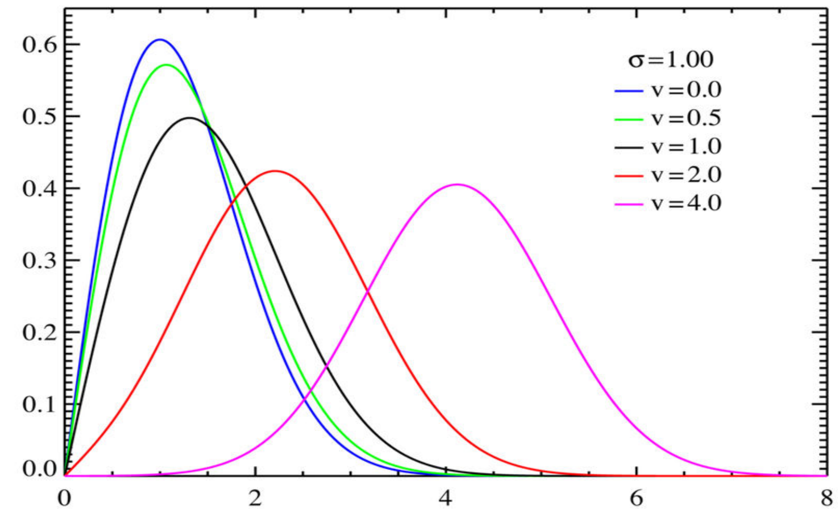
Poisson distribution

Many other distributions

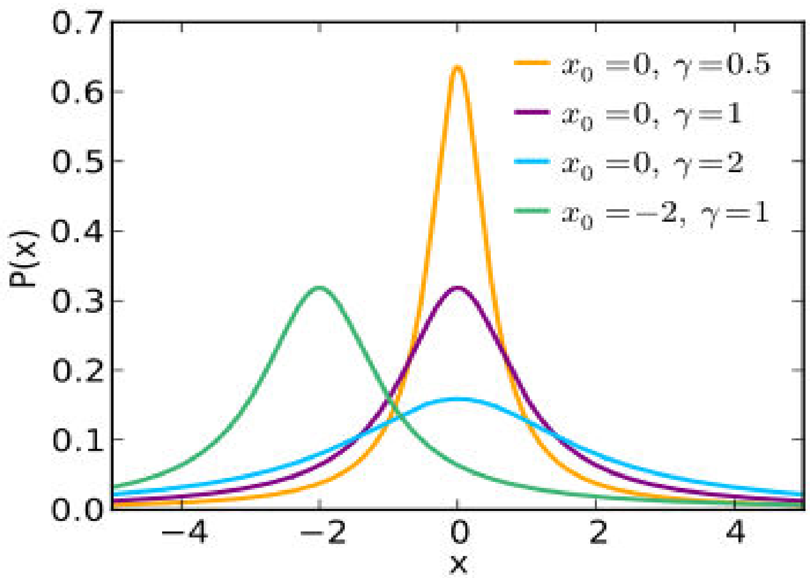
Wrapped normal distribution



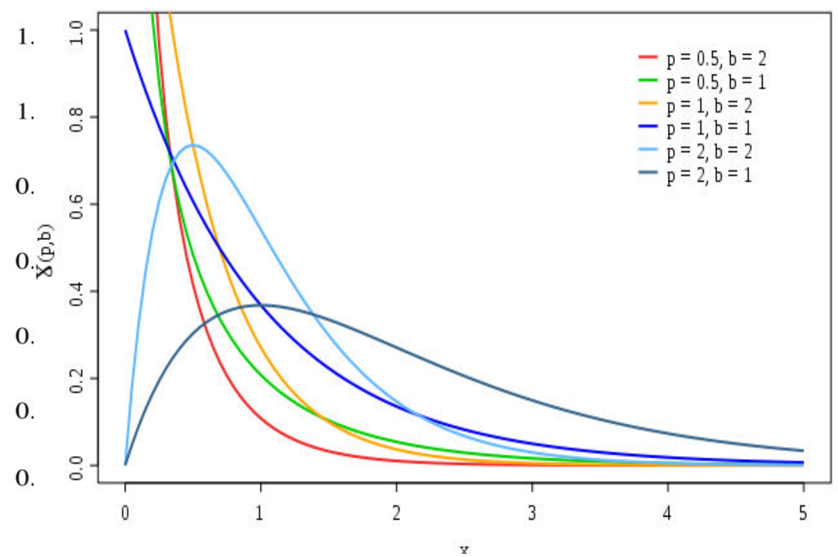
Rice distribution



Lorentz distribution



Gamma distribution

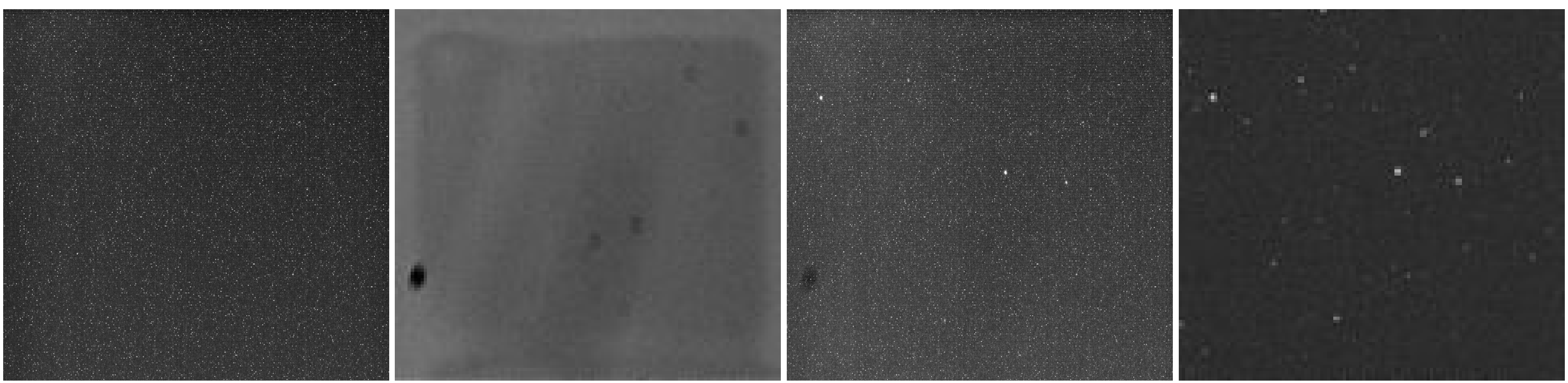


Detector noise (CCD)

- Various sources:
  - shot noise (photon statistics, Poisson)
  - dark current (thermal electronic fluctuations in semiconductor, Poisson)
  - readout noise (fluctuations during amplification and digitization, Gauss)
  - many other imperfections ...
- dark frame measures detector noise, hot pixels, dead pixels
- bright frame measures gain differences and imperfections (dust, etc)

"flat field"

dark frame | *bright frame* | *raw image* | *calibrated image*



source: H. Raab, Johannes-Kepler-Observatory, Linz

Correlation & Convolution

* Convolution: $f * g = \int_{-\infty}^{\infty} f(x')\, g(x-x')\, dx'$

Fourier: $\mathcal{F}\{f * g\} = F \cdot G$

* Correlation: $f \circledast g = \int_{-\infty}^{\infty} f^*(x')\, g(x+x')\, dx'$

$$f \circledast g = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} F^*(u)\, e^{-2\pi i u x'}\, du \int_{-\infty}^{\infty} G(u')\, e^{2\pi i u'(x+x')}\, du'$$

$$= \iint du\, du'\, F^*(u)\, G(u')\, e^{2\pi i u' x} \underbrace{\int_{-\infty}^{\infty} dx'\, e^{2\pi i x'(u-u')}}_{\delta(u-u')}$$

$$\int_{-\infty}^{\infty} du\, F^*(u)\, G(u)\, e^{2\pi i u x} = \mathcal{F}^{-1}\{F^* G\}$$

Noise power spectrum

- power spectrum of pure noise image

$$NPS = \langle | \mathcal{F}\{ n(x,y) \} |^2 \rangle$$

ensemble average

random variable (multivariate)

$$N(u,v) = \mathcal{F}\{ n(x,y) \}$$

- connection to auto-correlation

$$|N(u,v)|^2 = N^* N$$

$$\mathcal{F}^{-1}\{ NPS \} = \mathcal{F}^{-1}\{ \langle N^* N \rangle \} = \underbrace{\langle n \circledast n \rangle}_{\text{noise autocorrelation}}$$

Measuring NPS

1) measure multiple realizations of random variable n

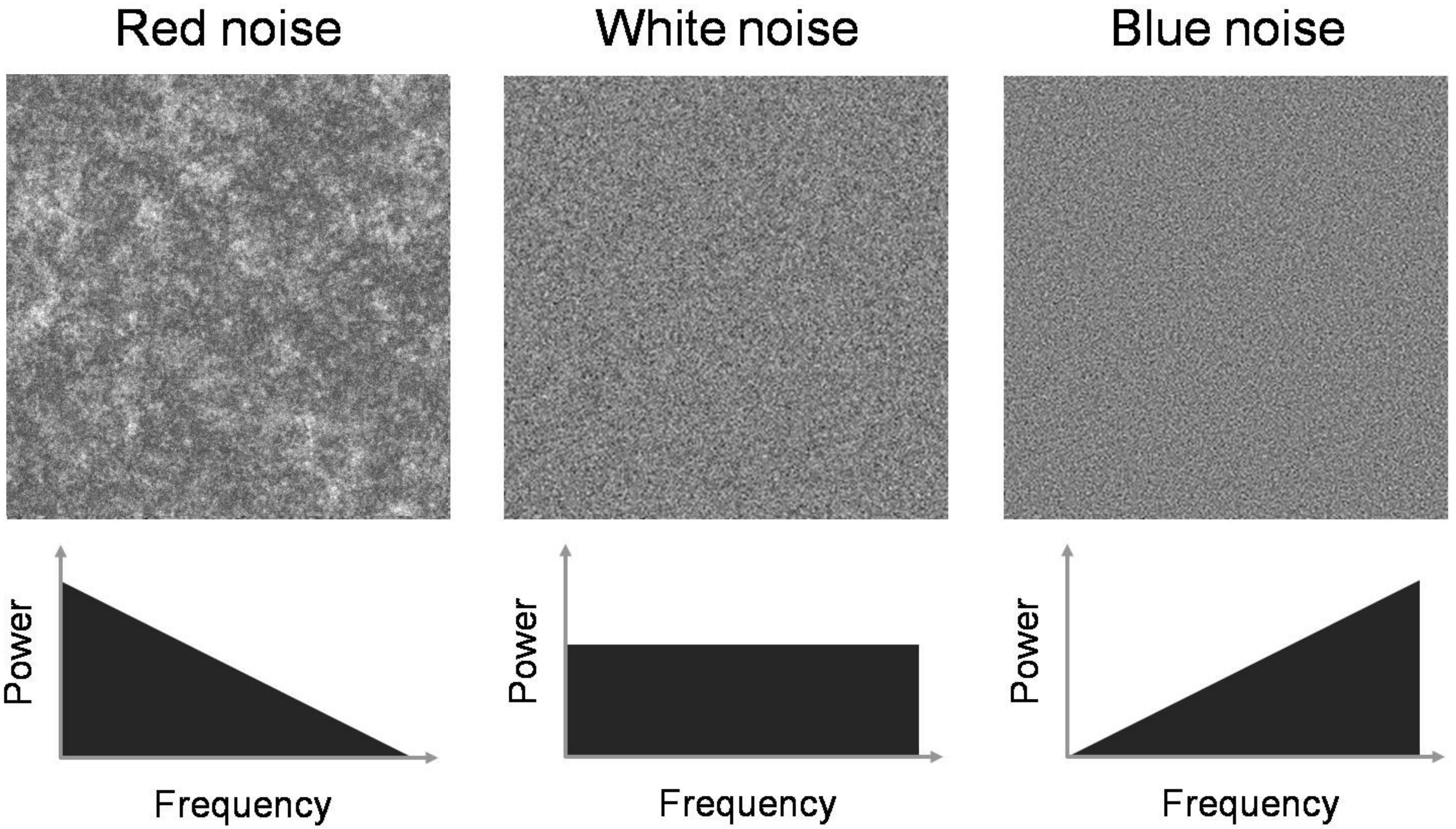
$$\{n_i\}$$

2) $N_i(u,v) = \mathcal{F}\{n_i(x,y)\}$

3) $\langle |N(u,v)|^2 \rangle = \frac{1}{M} \sum_i |N_i(u,v)|^2 = NPS$

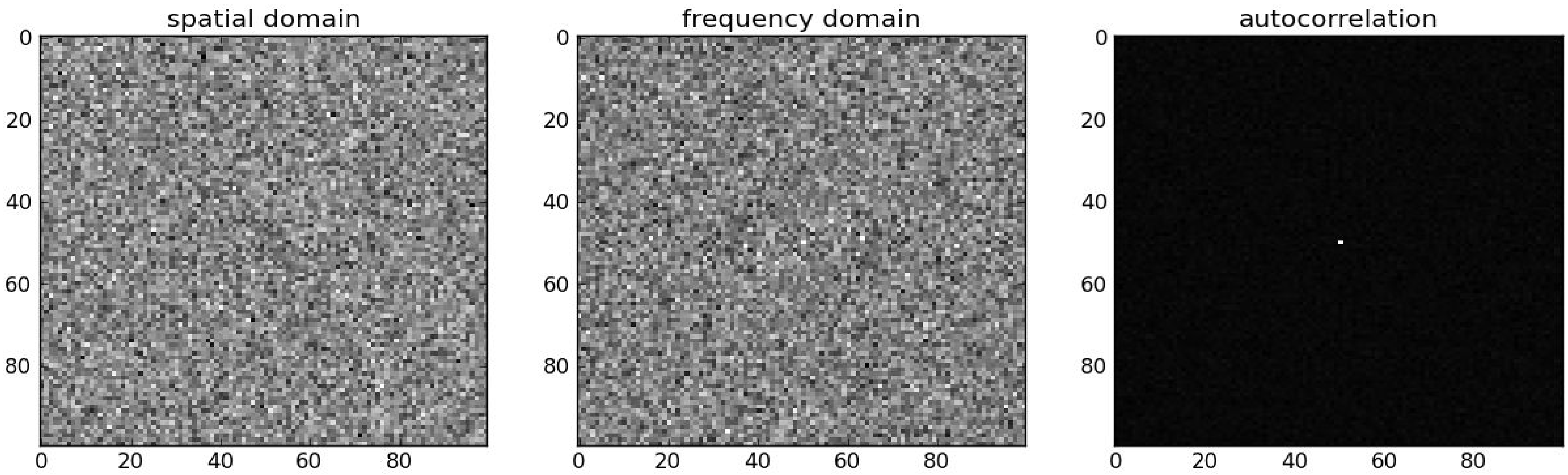
4) $\mathcal{F}^{-1}\{NPS\}$: estimate of noise autocorrelation

Noise power spectrum



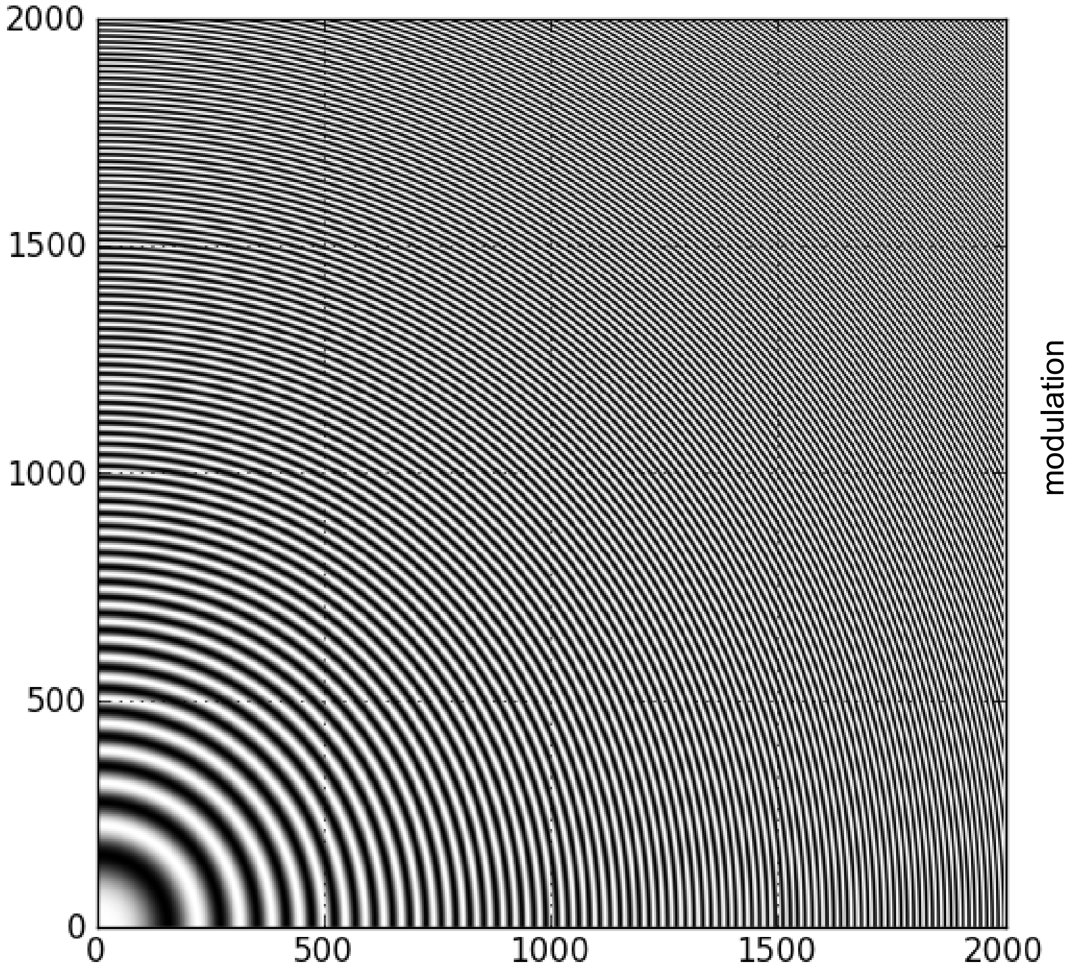
source: http://scien.stanford.edu/pages/labsite/2008/psych221/projects/08/AdamWang/project_report.htm

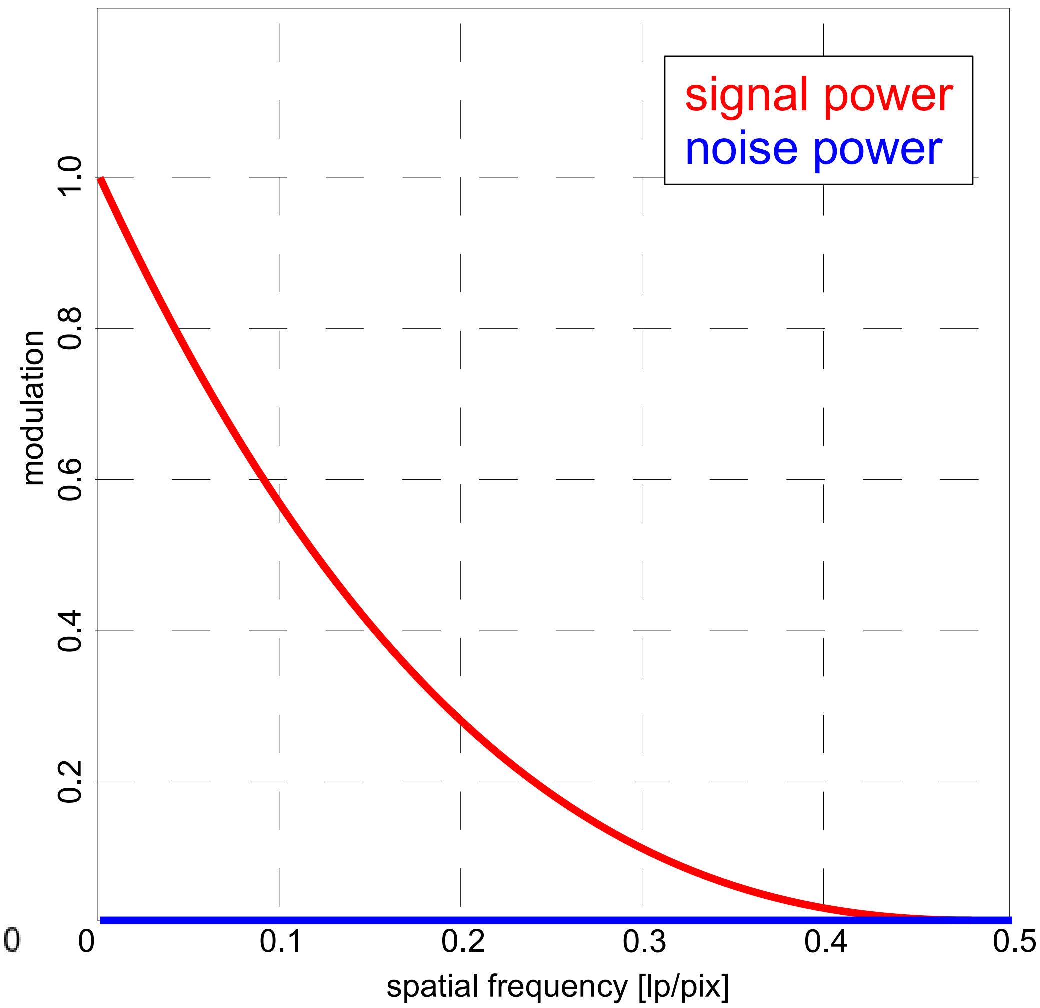
White noise



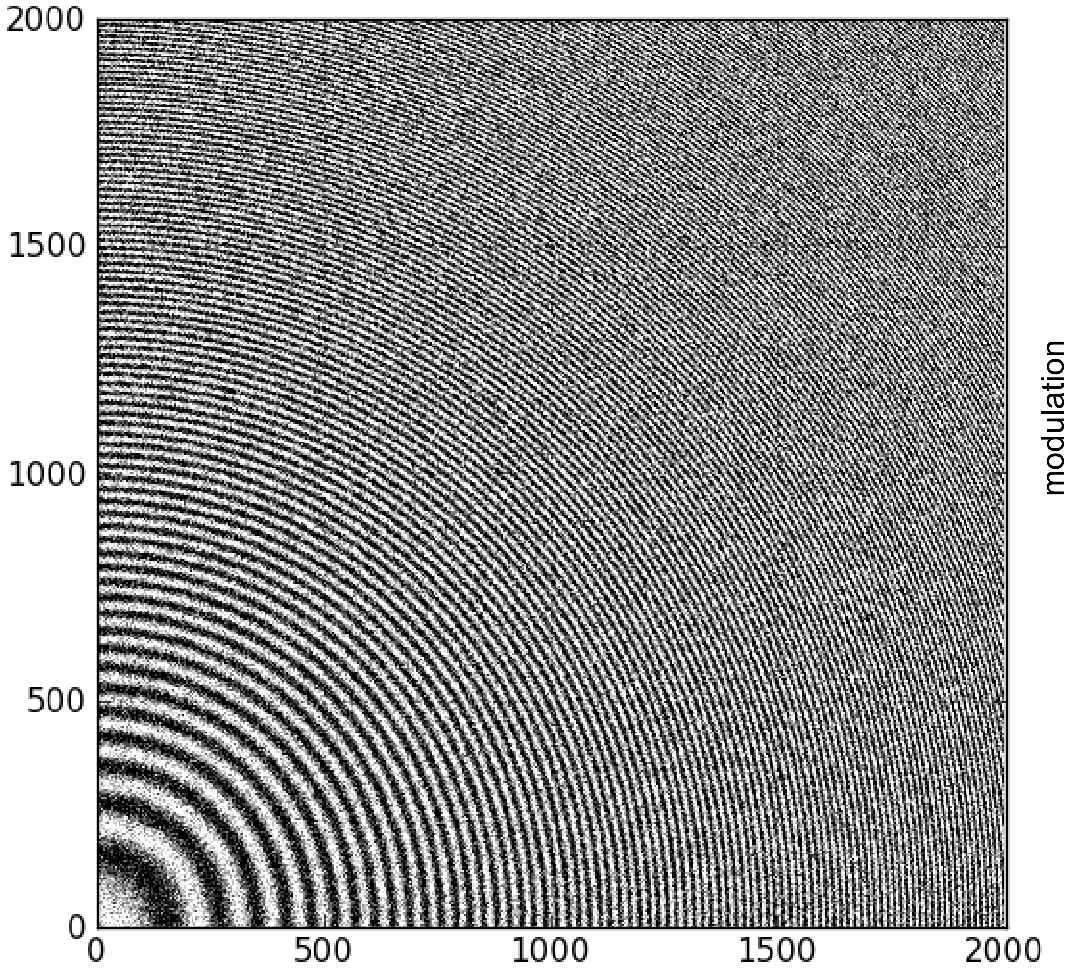
- white noise in spatial domain equals white noise in frequency domain
- white noise is perfectly uncorrelated
- all other types of noise are correlated to some degree
- white noise is an idealization

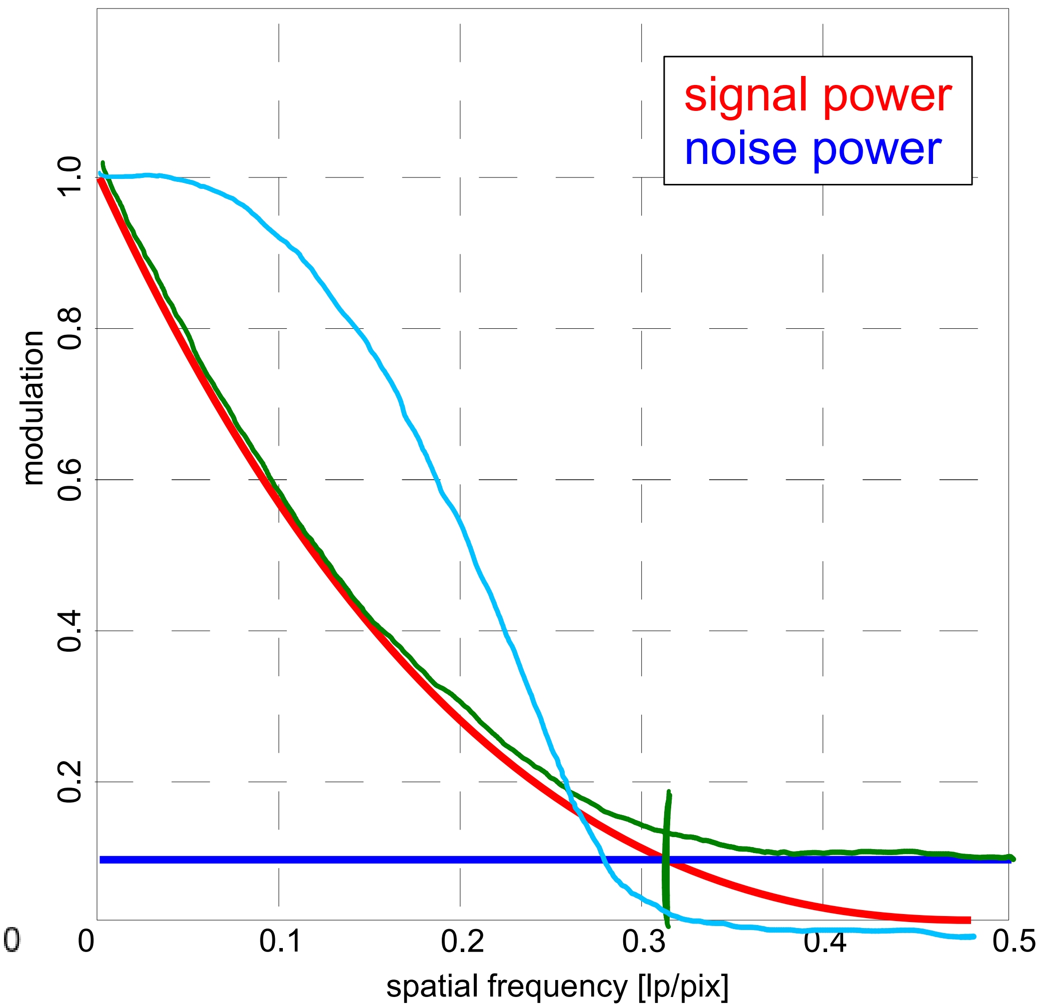
Signal power vs. noise power



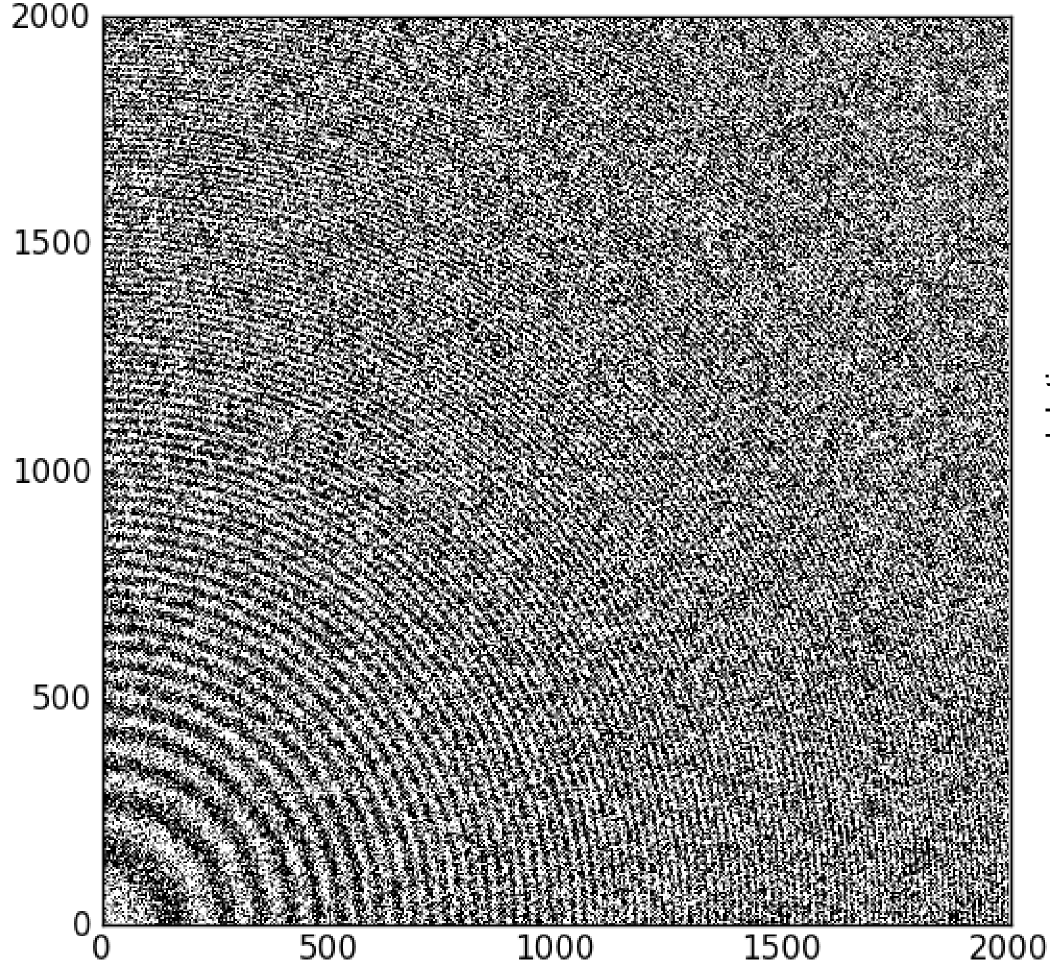


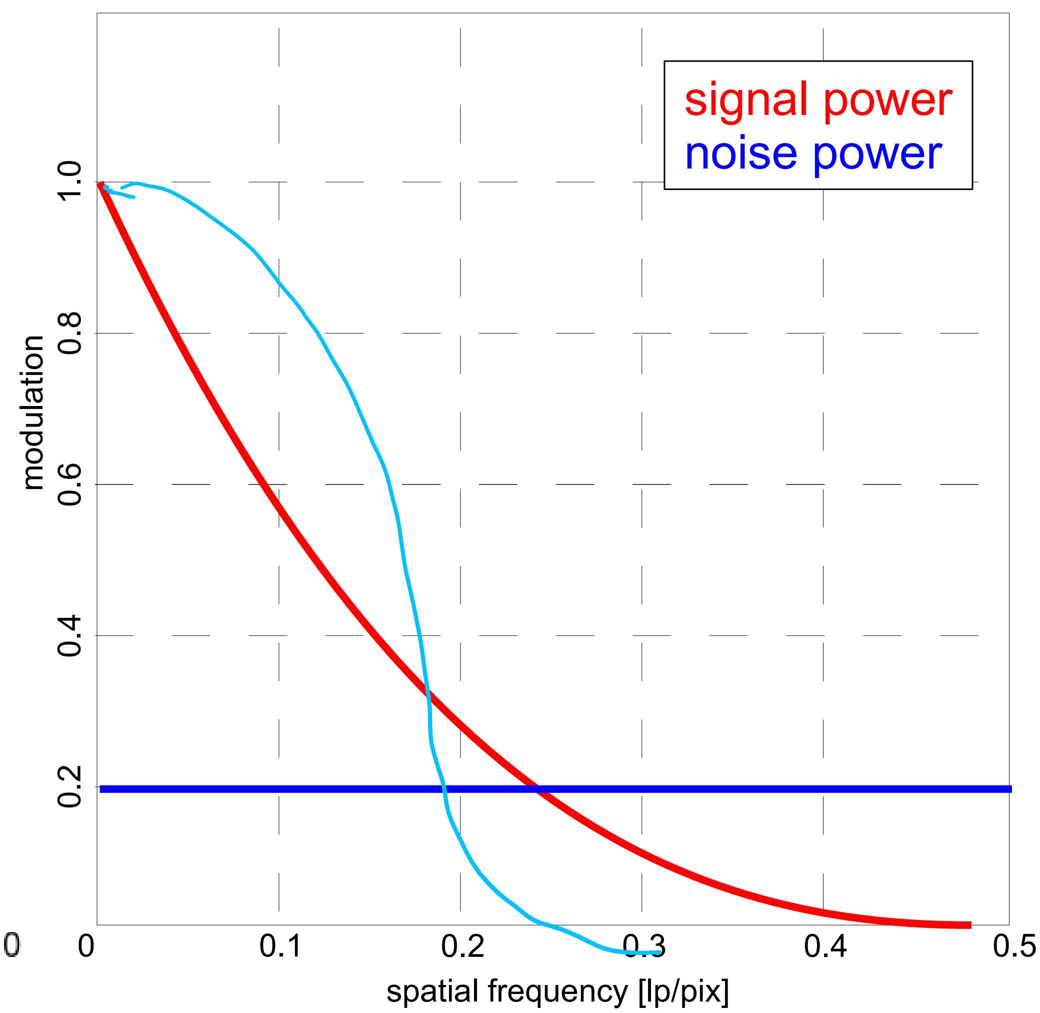
Signal power vs. noise power



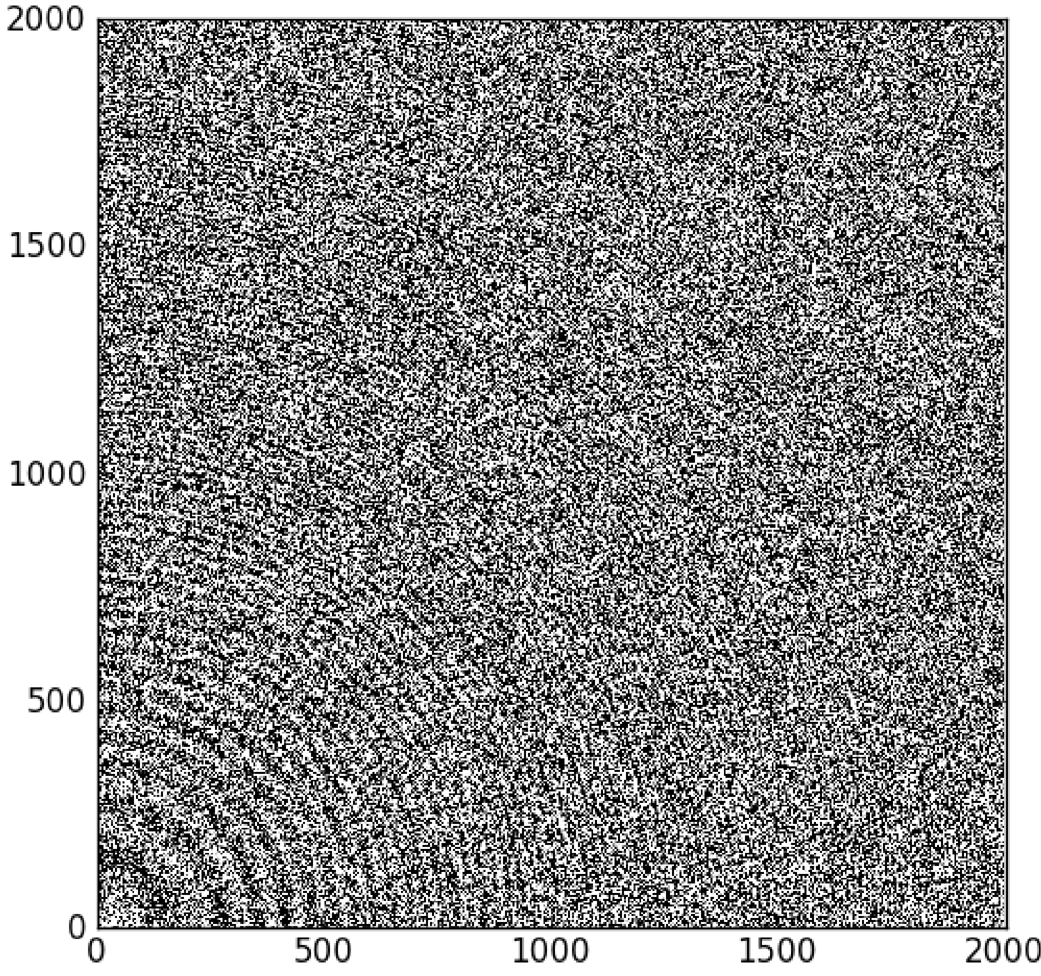


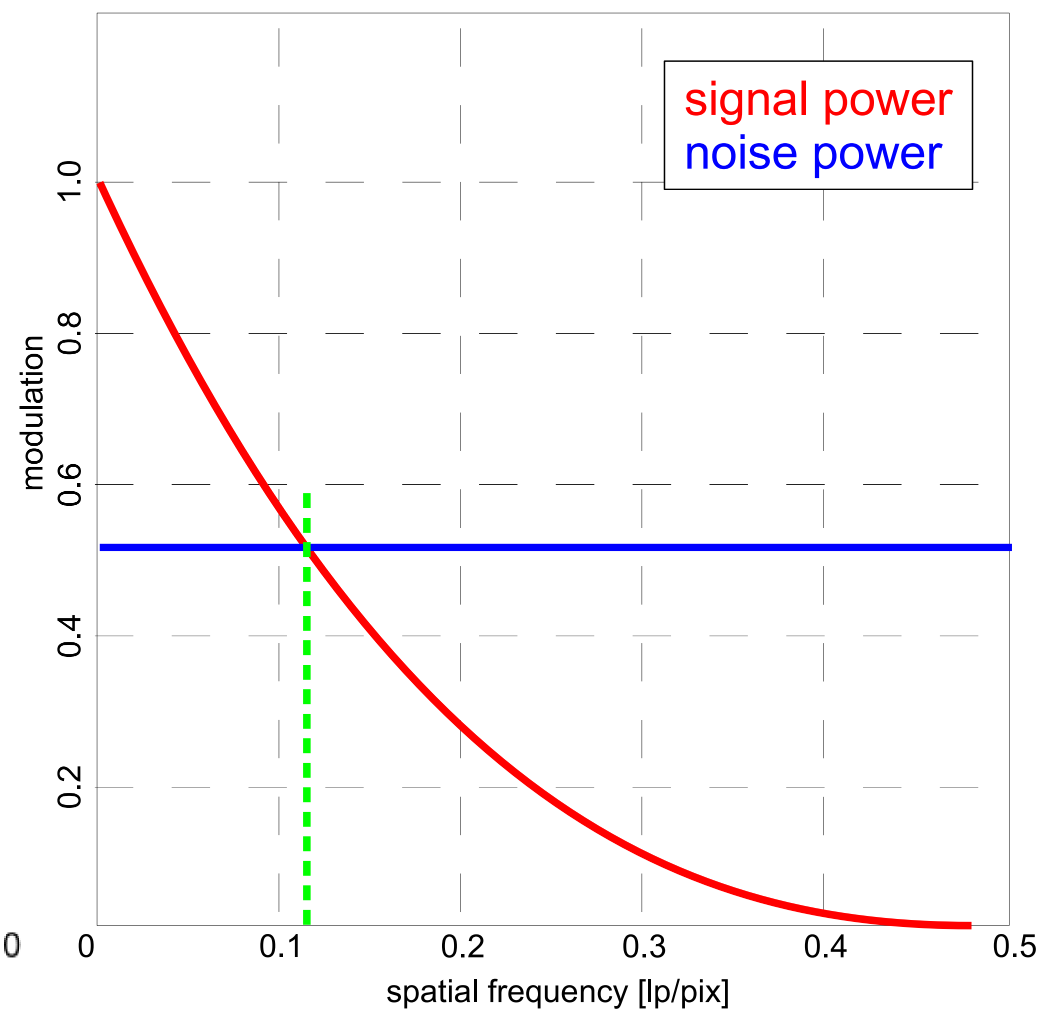
Signal power vs. noise power





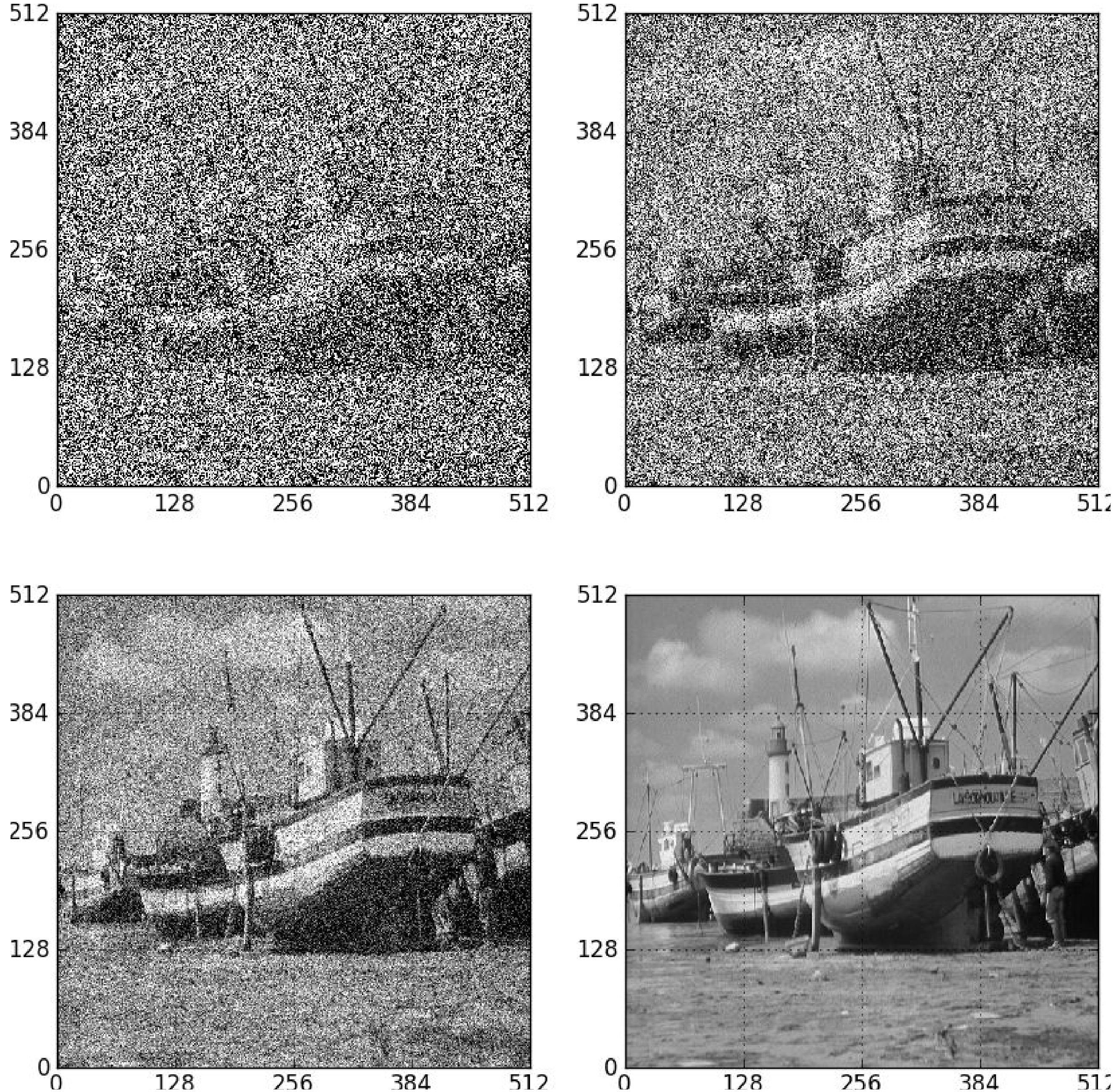
Signal power vs. noise power





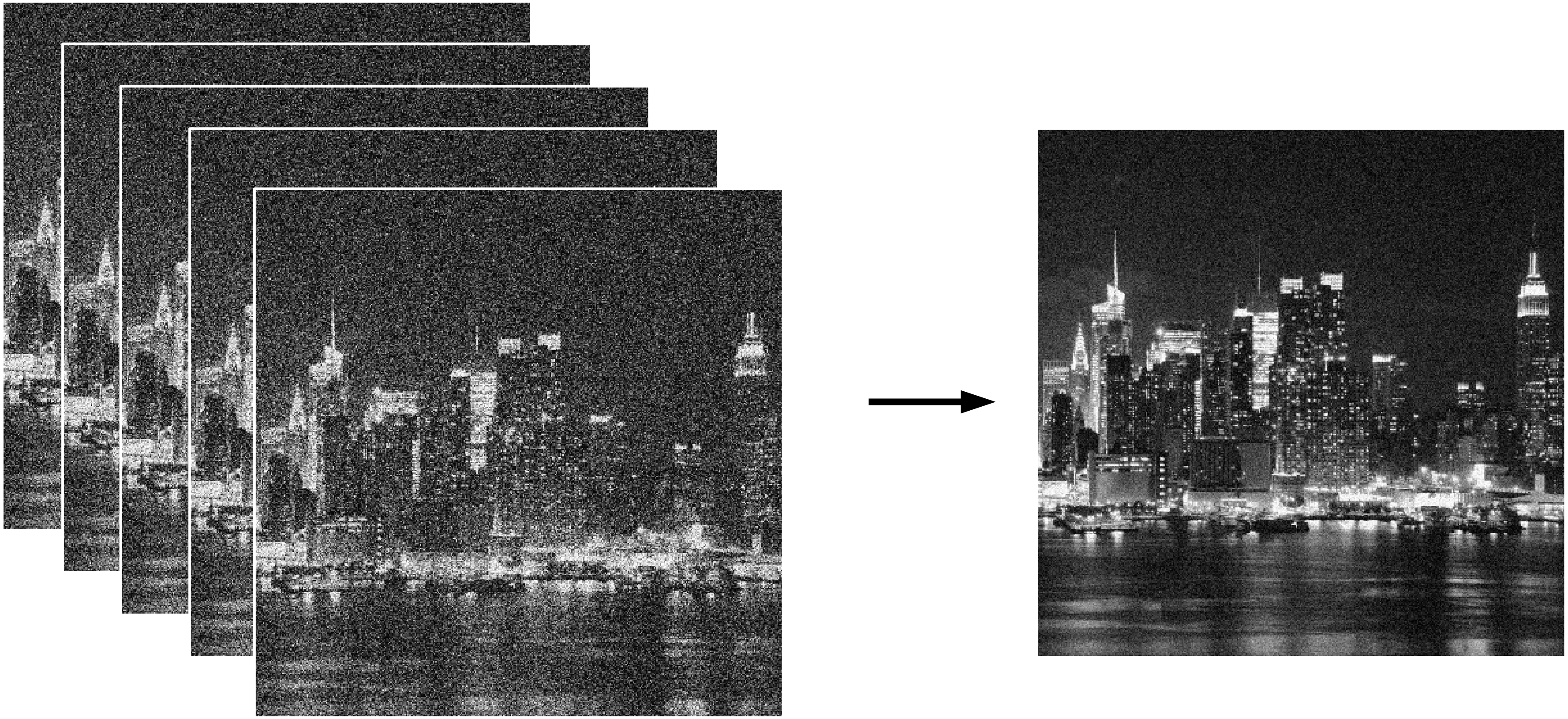
- Noise power exceeds signal power for high frequencies
- Small scale image details are lost in noise first

Signal power vs. noise power



Noise reduction by averaging

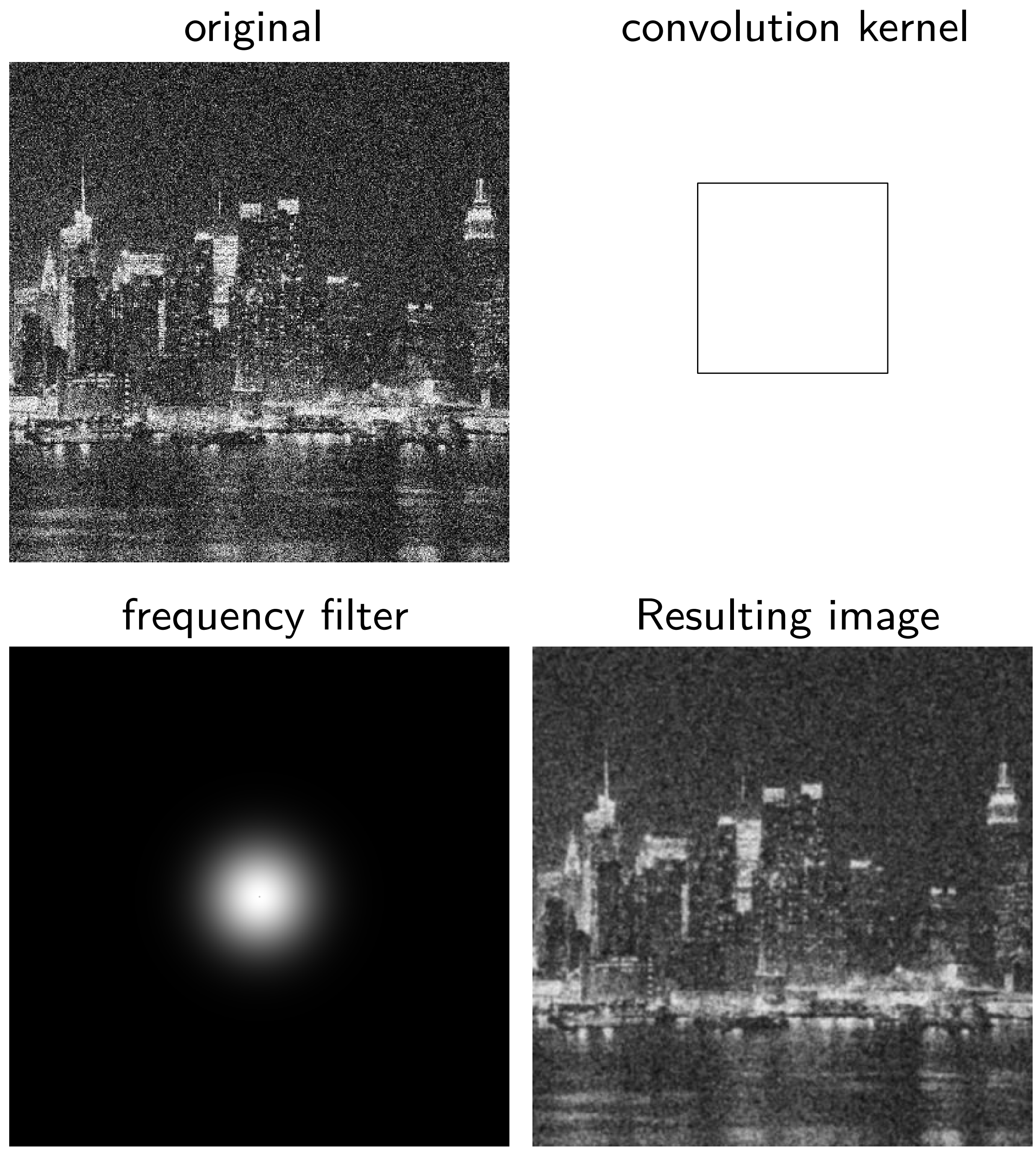
- Average multiple images



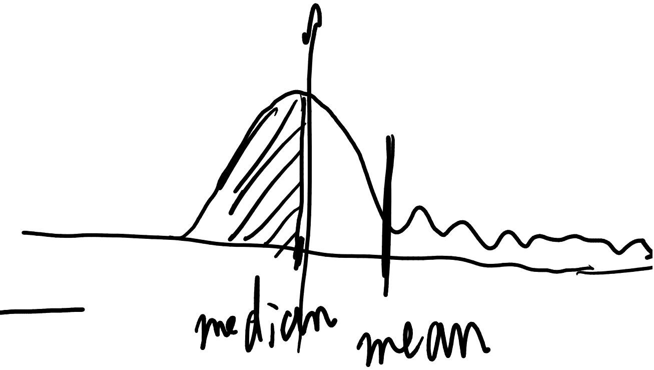
- requirement: additive noise, zero mean

Denoising by linear filtering

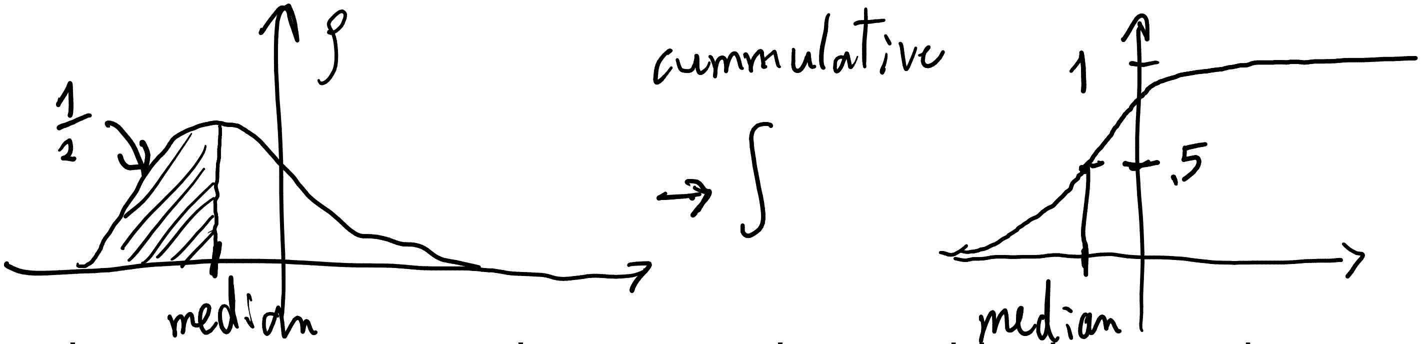
- use spatial convolution or frequency filtering to reduce noise
- noise reduction possible, but at cost of sharpness
- trade-off between noise reduction and resolution
- need fancier methods



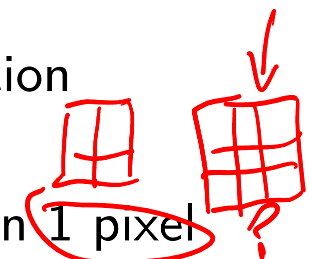
Median filtering



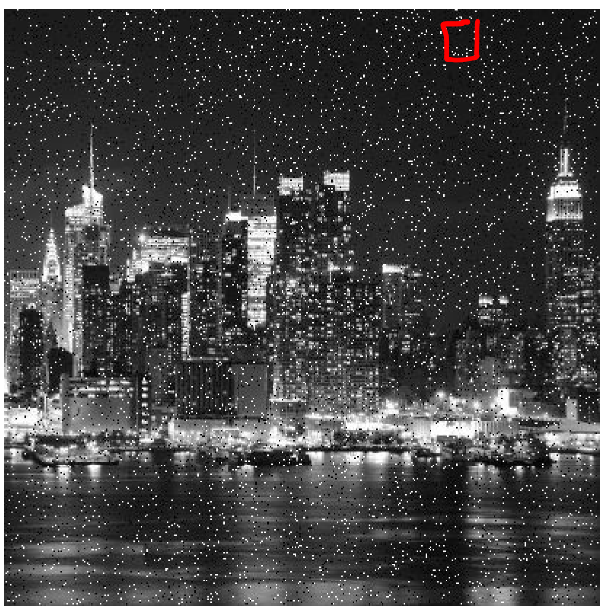
- Use median as estimator for fat tail distributions



- less sensitive to outliers in pixel ensemble, better edge preservation

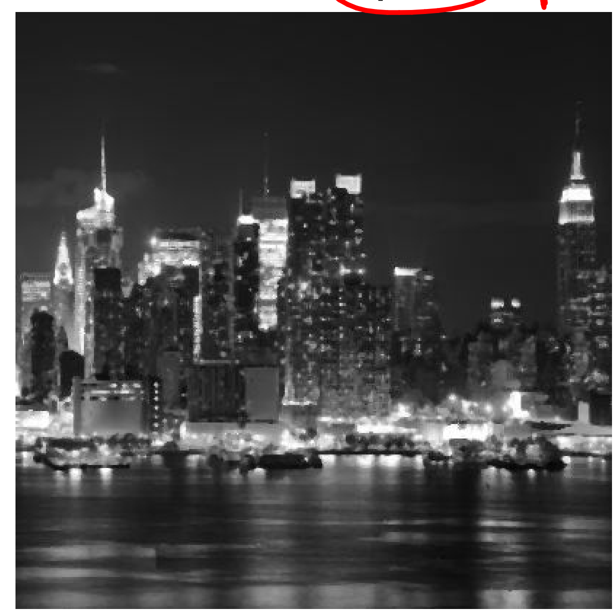


Salt and pepper noise



Gauss sigma=1 pixel

Median 1 pixel

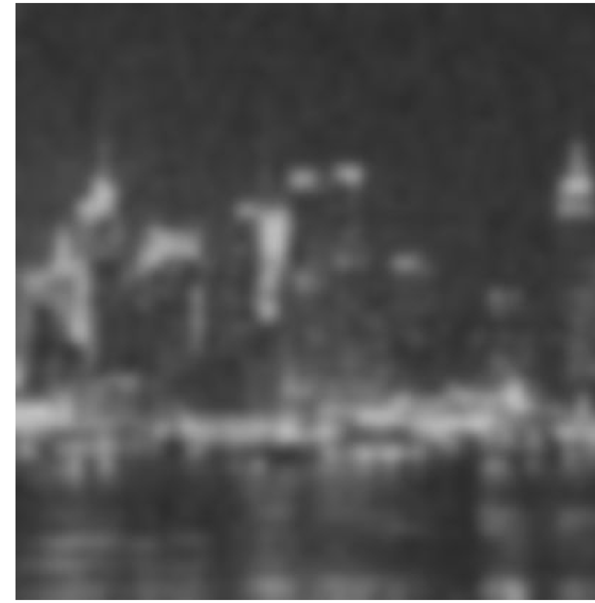


Median filtering

1x Gauss

2x Gauss

5x Gauss



1x Median

2x Median

5x Median

edge-preserving to some extent

Common abbreviations

Abbreviation	Name	Definition
IRF	Impulse response function	Linear operator map of delta function
PSF	Point spread function	Image of point object (optical IRF)
OTF	Optical transfer function	Fourier transform of PSF
PTF	Phase transfer function	Phase part of OTF
MTF	Modulation transfer function	Amplitude of OTF
CTF	Contrast transfer function	MTF for non-sinusoidal objects
PDF	Probability density function	Probability distribution for a given random variable
SPS	Signal power spectrum	Amplitude squared of signal F.T.
NPS	Noise power spectrum	Amplitude squared of noise F.T.
SNR	Signal to noise ratio	Mean signal / mean noise
CNR	Contrast to noise ratio	Mean contrast / mean noise