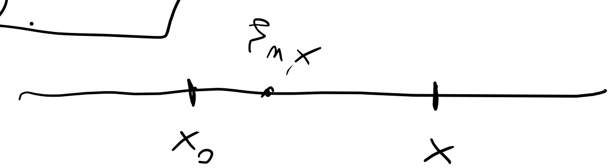
13 novembre

La volta scorsa ho terminato la dimostrazione del teorema sul resto di Lagrange scrivendo che

$$\boxed{\frac{R_n(x)}{(x-x_0)^{n+1}} = \frac{R_n^{(n+1)}(\xi_{n x})}{(n+1)!}} \qquad \left(R_n(x) = f(x) - P_n(x)\right)$$

$x_0 < \xi_{n x} < x$ $\qquad$ $x_0 < x$



$$R_n^{(n+1)}(\xi_{n x}) = f^{(n+1)}(\xi_{n x}) - \underbrace{P_n^{(n+1)}(\xi_{n x})}_{= 0}$$

perché $\deg P_n \leq n$

$$= f^{(n+1)}(\xi_{n x})$$

$$\frac{R_n(x)}{(x-x_0)^{n+1}} = \frac{f^{(n+1)}(\xi_{n x})}{(n+1)!}$$

$$R_n(x) = \frac{f^{(n+1)}(\xi_{n x})}{(n+1)!} (x-x_0)^{n+1}$$

Esercizio $f(x) = \cos x + \sin x$

Calcolare tutti i polinomi di McLaurin.

$$\cos x = \sum_{j=0}^{n} (-1)^j \frac{x^{2j}}{(2j)!} + o(x^{2n})$$

$$\sin x = \sum_{j=0}^{n} (-1)^j \frac{x^{2j+1}}{(2j+1)!} + o(x^{2n+1})$$

$$f(x) = \sin x + \cos x =$$

$$= \sum_{j=0}^{n} (-1)^j \frac{x^{2j+1}}{(2j+1)!} + \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!} + \overbrace{\underbrace{o(x^{2n+1})}_{o(x^{2n})} + o(x^{2n})}^{o(x^{2n})}$$

Cerco P_{2n}

$$o(x^{2n+1}) = o(x^{2n})$$

$$\lim_{x \to 0} \frac{o(x^{2n+1})}{x^{2n}} = \lim_{x \to 0} \frac{o(x^{2n+1})}{x^{2n+1}} \, x = 0 \cdot 0 = 0$$

$$= \sum_{j=0}^{n-1} (-1)^j \frac{x^{2j+1}}{(2j+1)!} + \underbrace{(-1)^n \frac{x^{2n+1}}{(2n+1)!}}_{= o(x^{2n})} + \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n})$$

$$= \underbrace{\sum_{j=0}^{n-1} (-1)^j \frac{x^{2j+1}}{(2j+1)!} + \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!}}_{P_{2n}(x)} + o(x^{2n})$$

$$f(x) = \sin x + \cos x =$$

$$= \sum_{j=0}^{n} (-1)^j \frac{x^{2j+1}}{(2j+1)!} + \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!} + \underbrace{\overbrace{o(x^{2n+1})}^{o(x^{2n})} + o(x^{2n})}_{o(x^{2n})}$$

$$\frac{o(x^{2n+1})}{}$$

$$P_{2n+1}(x)$$

$$\sin x = \sum_{j=0}^{n} (-1)^j \frac{x^{2j+1}}{(2j+1)!} + o(x^{2n+1})$$

$$\cos x = \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n})$$

$$= \sum_{k=0}^{n+1} (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n+2})$$

$$\sin x + \cos x = \sum_{j=0}^{n} (-1)^j \frac{x^{2j+1}}{(2j+1)!} + \sum_{k=0}^{n+1} (-1)^k \frac{x^{2k}}{(2k)!} + \underbrace{o(x^{2n+1}) + o(x^{2n+2})}_{o(x^{2n+1})}$$

$$= \underbrace{\sum_{j=0}^{n} (-1)^j \frac{x^{2j+1}}{(2j+1)!} + \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!}}_{P_{2n+1}} + \underbrace{(-1)^{n+1} \frac{x^{2n+2}}{(2n+2)!} + o(x^{2n+1})}_{o(x^{2n+1})}$$

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$$\lim_{x \to +\infty} \frac{x^2 \lg\left(\frac{6+x^2}{2+x^2}\right) - 4}{e^{\tan\left(\frac{3}{x}\right)} - 1 - \tan\left(\frac{3}{x}\right)} = L$$

$L = ?$

$x \to +\infty$

$$\text{denom} = e^0 - 1 - 0 = 0$$

$$x^2 \lg\left(\frac{6+x^2}{2+x^2}\right) - 4 =$$

$$= x^2 \lg\left(\frac{1+\frac{6}{x^2}}{1+\frac{2}{x^2}}\right) - 4 = x^2 \lg\left(1+\frac{6}{x^2}\right) - x^2 \lg\left(1+\frac{2}{x^2}\right) - 4$$

$$= \lg \underbrace{\left(1+\frac{6}{x^2}\right)^{x^2}}_{\downarrow e^6} - \lg \underbrace{\left(1+\frac{2}{x^2}\right)^{x^2}}_{e^2} - 4 \xrightarrow{x\to\infty} \lg e^6 - \lg e^2 - 4$$

$$= 6 - 2 - 4 = 0$$

$x_0 \neq 0$

$y = x_0 z$

$$\lim_{y \to \infty} \left(1 + \frac{x_0}{y}\right)^y = e^{x_0} \qquad z = \frac{y}{x_0}$$

$$= \lim_{z \to \infty} \left(1 + \frac{1}{z}\right)^{z x_0} = \lim_{z \to \infty} \left(\left(1+\frac{1}{z}\right)^z\right)^{x_0} =$$

$$w = \left(1+\frac{1}{z}\right)^z \qquad = \lim_{w \to e} w^{x_0} = e^{x_0}$$

$$\lim_{x\to+\infty} \frac{x^2 \lg\left(\frac{6+x^2}{2+x^2}\right)-4}{\underbrace{e^{\tan\left(\frac{3}{x}\right)}-1-\tan\left(\frac{3}{x}\right)}_{\frac{9}{2x^2}}} = L \qquad \frac{0}{0}$$

$$\text{den} = e^{\tan\left(\frac{3}{x}\right)}-1-\tan\left(\frac{3}{x}\right) = \frac{\tan^2\left(\frac{3}{x}\right)}{2}+o\left(\tan^2\left(\frac{3}{x}\right)\right)$$

$$e^y-1-y = \cancel{1}+\cancel{y}+\frac{y^2}{2}+o(y^2)-\cancel{1}-\cancel{y}$$

$$= \frac{y^2}{2}+o(y^2)$$

$$= \frac{\tan^2\left(\frac{3}{x}\right)}{2} \cancel{(1+o(1))}$$

$$= \frac{9}{2x^2} \cancel{(1+o(1))}$$

$$\tan y = y(1+o(1))$$

$$\tan^2 y = y^2(1+o(1))^2 = y^2(1+o(1))$$

$$\lim_{x\to+\infty} \frac{\tan\left(\frac{3}{x}\right)}{\frac{3}{x}} = 1 \iff \lim_{y\to 0} \frac{\tan(y)}{y} = 1$$

$$\lim_{x\to+\infty} \frac{\tan^2\left(\frac{3}{x}\right)}{\left(\frac{3}{x}\right)^2} = 1$$

$$\left(\frac{3}{x}\right)^2 \frac{\tan^2\left(\frac{3}{x}\right)}{\left(\frac{3}{x}\right)^2} = \left(\frac{3}{x}\right)^2 \cancel{(1+o(1))}$$

$$\lim_{x\to+\infty} \frac{x^2 \lg\left(\frac{6+x^2}{2+x^2}\right)-4}{\underbrace{e^{\tan\left(\frac{3}{x}\right)}-1-\tan\left(\frac{3}{x}\right)}_{\frac{9}{2x^2}}} =$$

$$\text{Num} = x^2 \lg\left(\frac{6+x^2}{2+x^2}\right)-4 =$$

$$= x^2 \lg\left(\frac{6+x^2}{2+x^2}\,\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right)-4 =$$

$$= x^2 \lg\left(1+\frac{6}{x^2}\right) - x^2 \lg\left(1+\frac{2}{x^2}\right)-4$$

$$\lg(1+y) = y - \frac{y^2}{2} + o(y^2) = y + o(y)$$

$$x^2\left(\frac{6}{x^2} - \frac{6^2}{2x^4} + o(x^{-4})\right) - x^2\left(\frac{2}{x^2} - \frac{2^2}{2x^4} + o(x^{-4})\right)$$

$$-4$$

$$= \cancel{6-2-4} + \frac{1}{2x^2}\left(2^2-6^2\right) + o(x^{-2})$$

$$= -\frac{32}{2x^2} + o(x^{-2}) = -\frac{16}{x^2} + o(x^{-2}) = -\frac{16}{x^2}(1+o(1))$$

Il limite

$$\lim_{x\to+\infty} \frac{\text{Num}}{\text{den}} = \lim_{x\to+\infty} \frac{-\frac{16}{\cancel{x^2}}}{\frac{9}{2}\frac{1}{\cancel{x^2}}} = -\frac{32}{9}$$