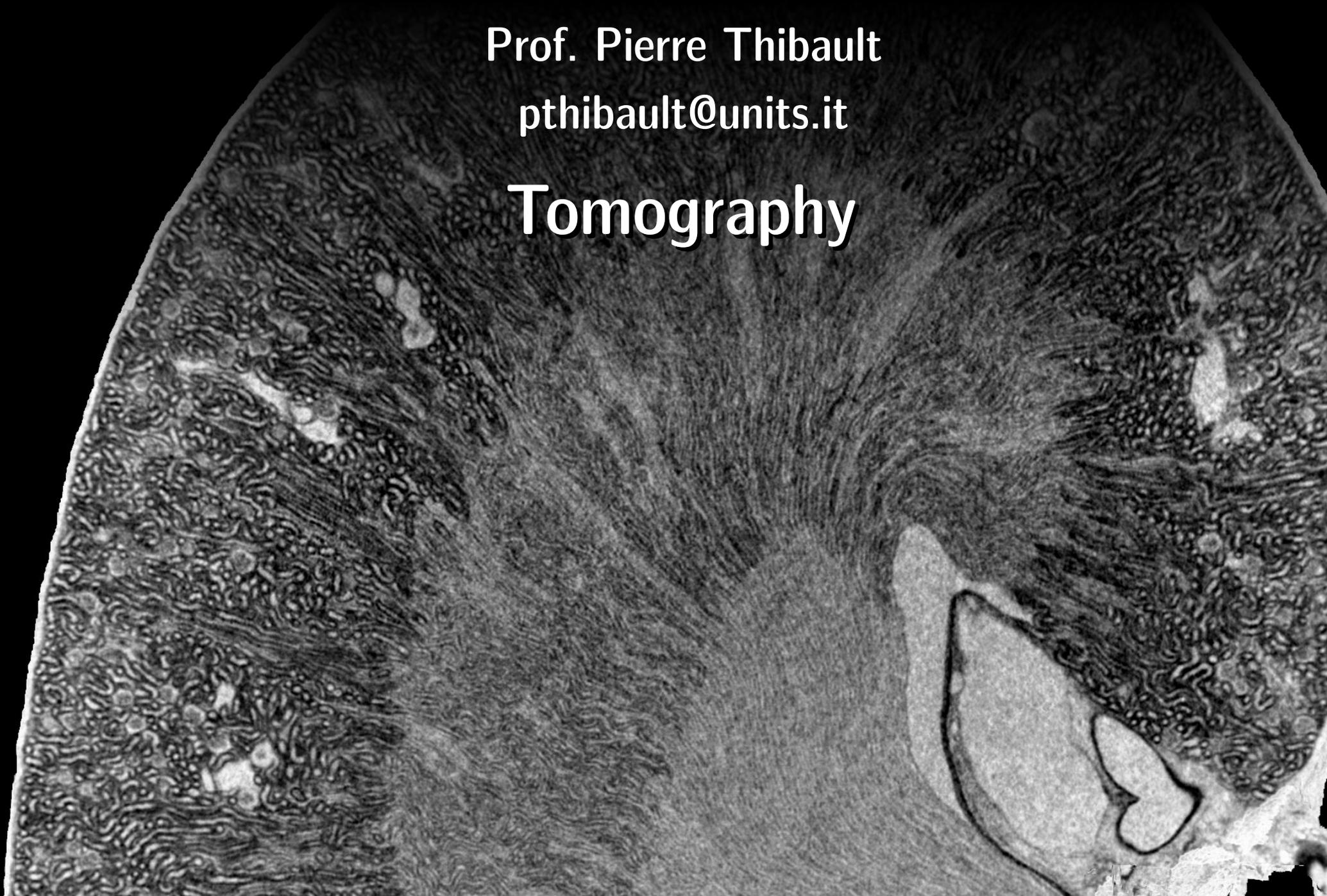# Image Processing for Physicists

Prof. Pierre Thibault
pthibault@units.it

Tomography
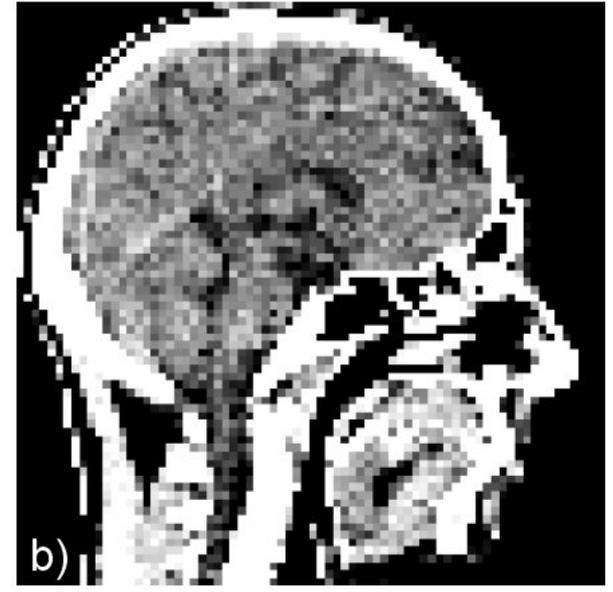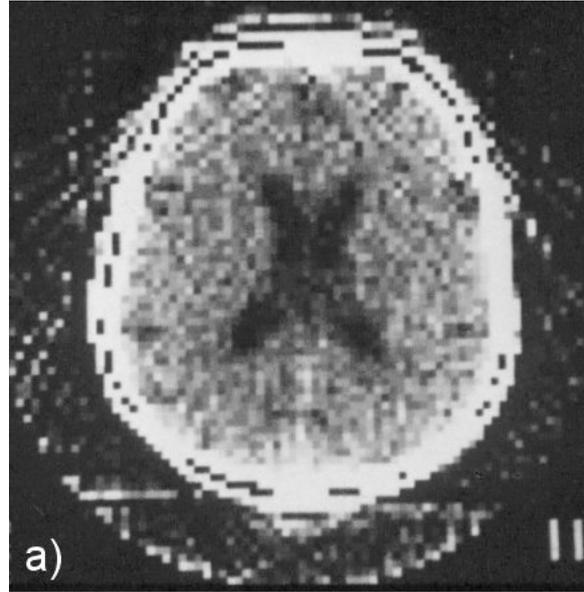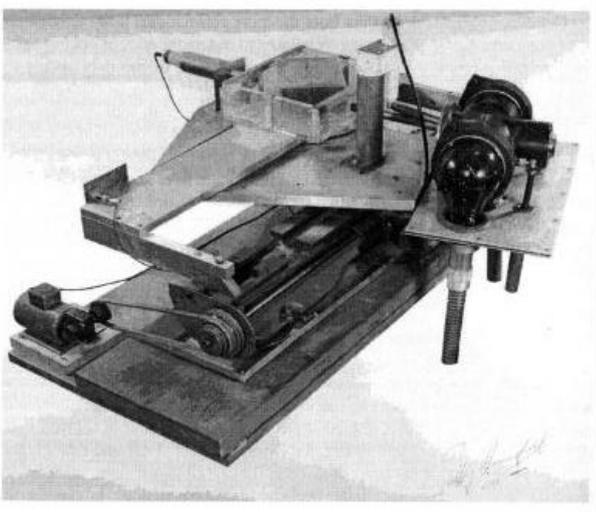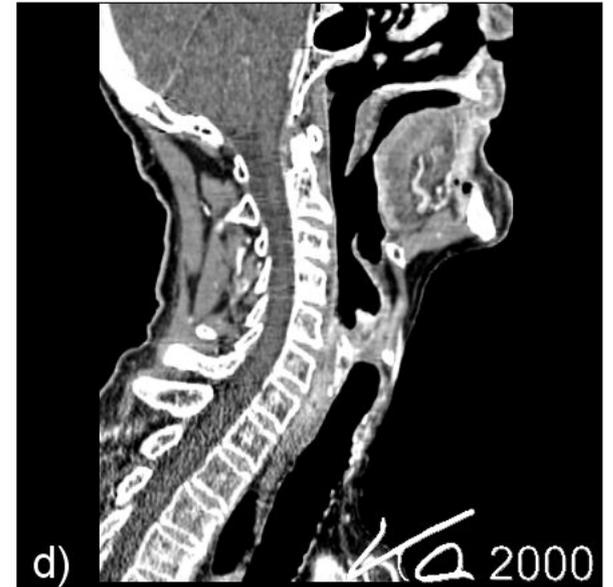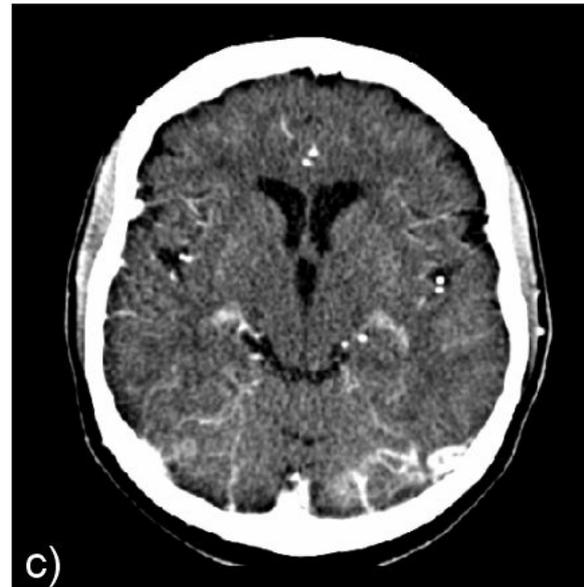
Overview

- Fundamentals of tomography

    - Physics & geometry

- Analytic formulation

    - Radon transform

    - Filtered back-projection

- Algebraic formulation

Examples of tomographic imaging
Computed (X-ray) Tomography (CT)

a)

b)

c)

d)

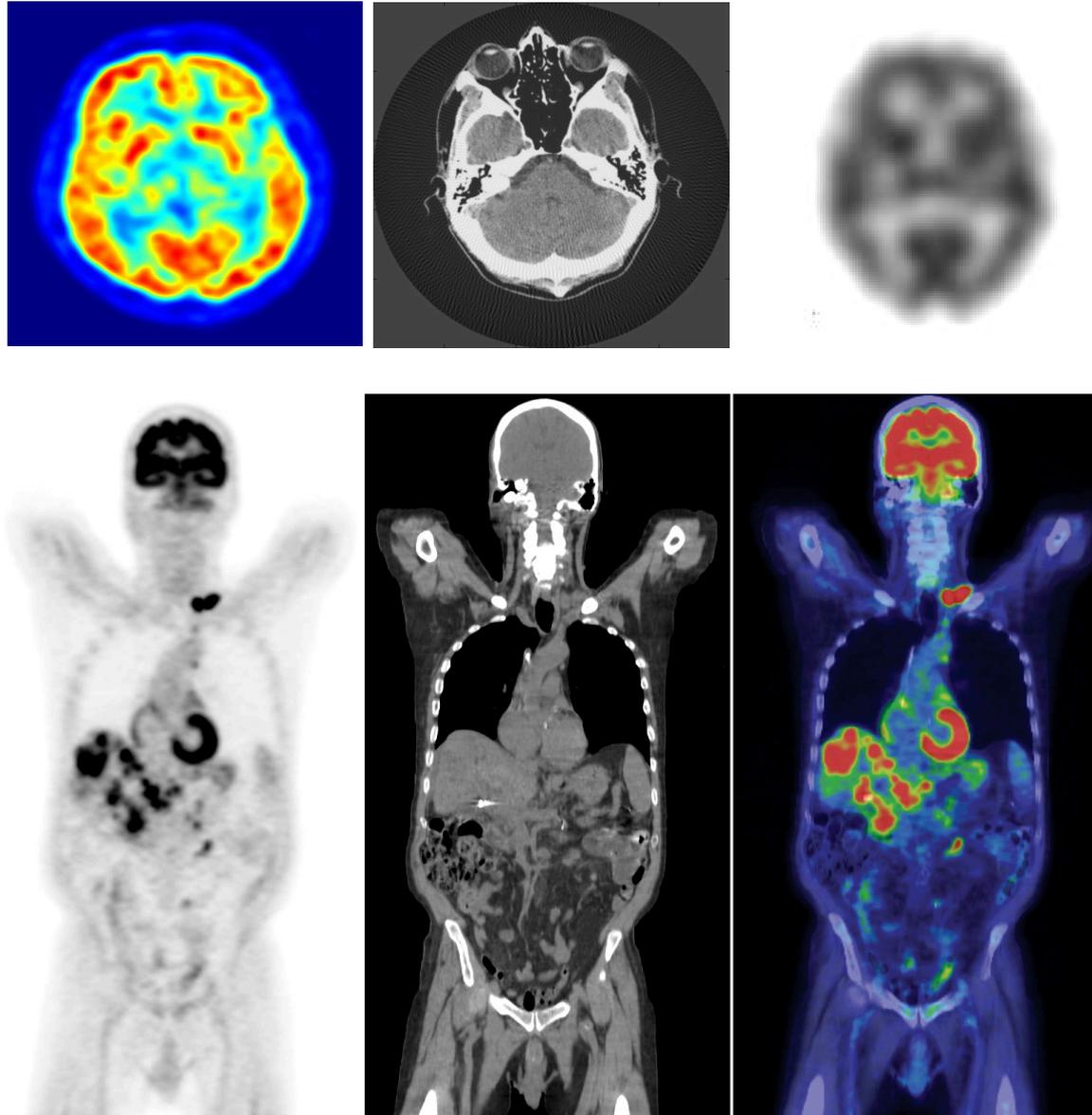1974, 80x80 pixels

2000, 512x512 pixels, spiral CT

source: W. Kalender, Publicis, 3rd ed. 2011

Tomography

Examples of tomographic imaging
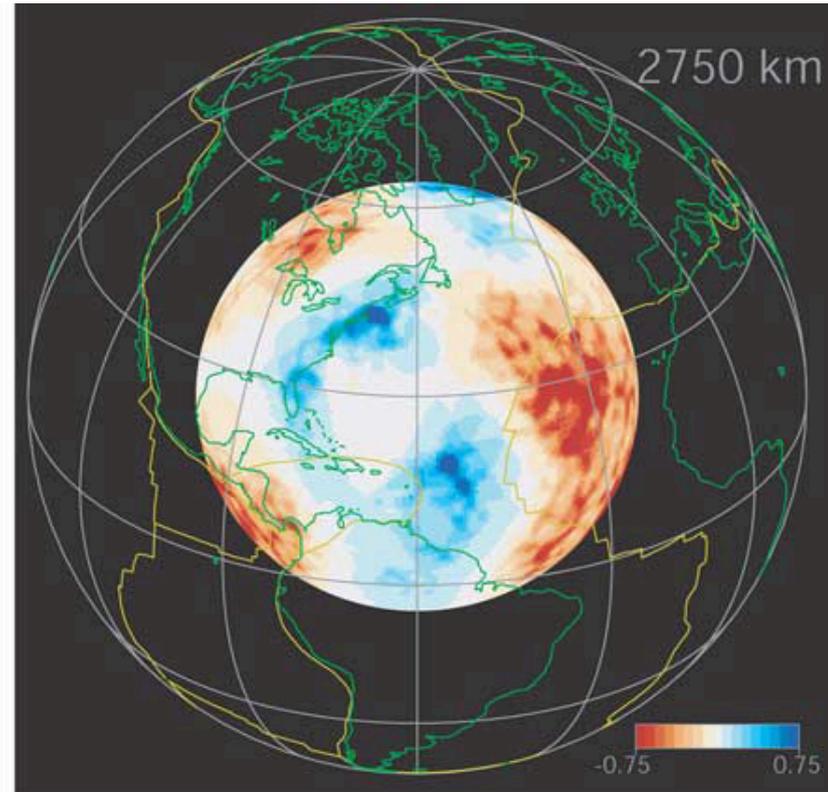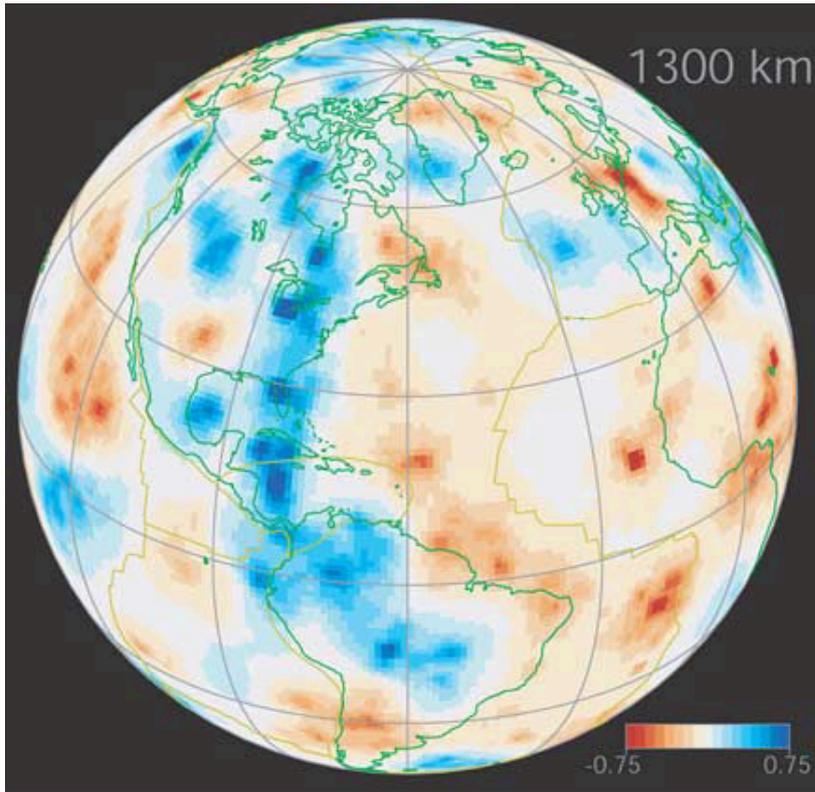
Positron emission tomography (PET) + CT

Single-Photon Emission Computed Tomography (SPECT)
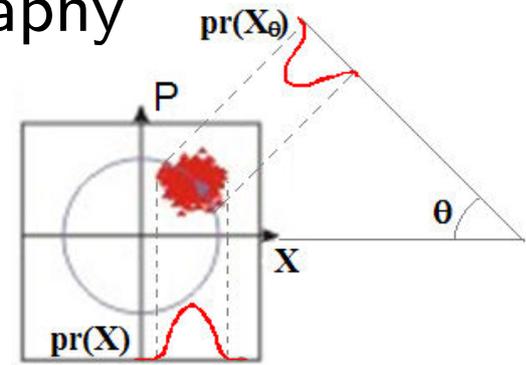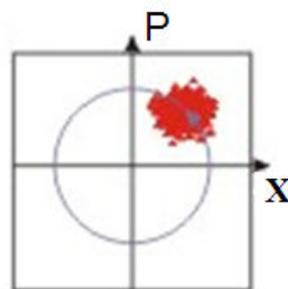

Tomography

Examples of tomographic imaging

Seismic tomography

source: Sambridge et al. G3 Vol.4 Nr.3 (2003)

Quantum state tomography
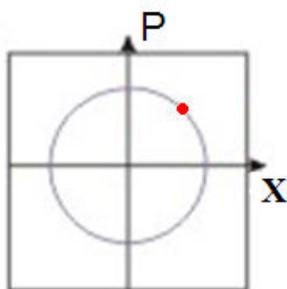
Tomography

Examples of tomographic imaging

Ultrasonography/tomography (US/UST)

Magnetic resonance imaging/tomography (MRI/MRT)
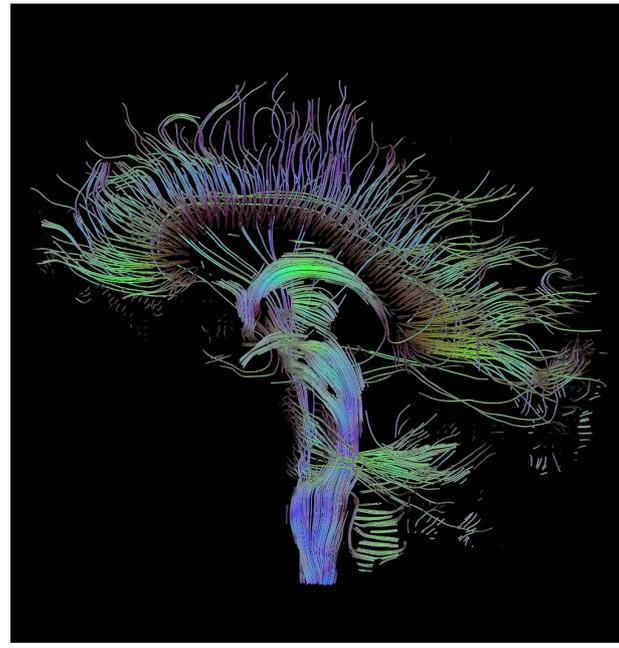
Reconstructions from projections

Reconstruction of volume from projections
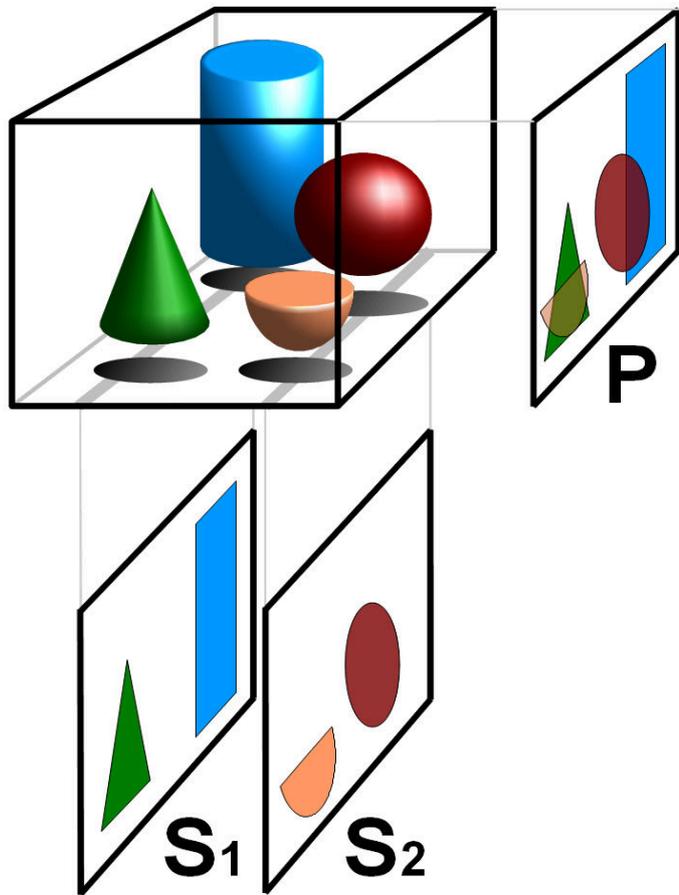
Digitization into voxels
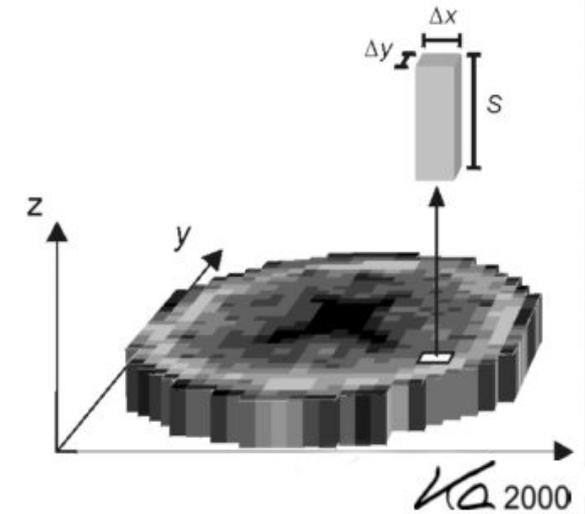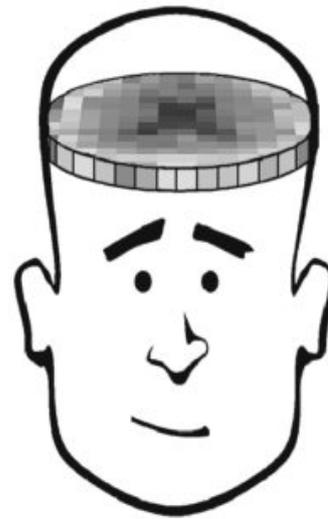
source: W. Kalender, Publicis, 3rd ed. 2011

Principles of X-ray CT

source: W. Kalender, Publicis, 3rd ed. 2011

Tomography
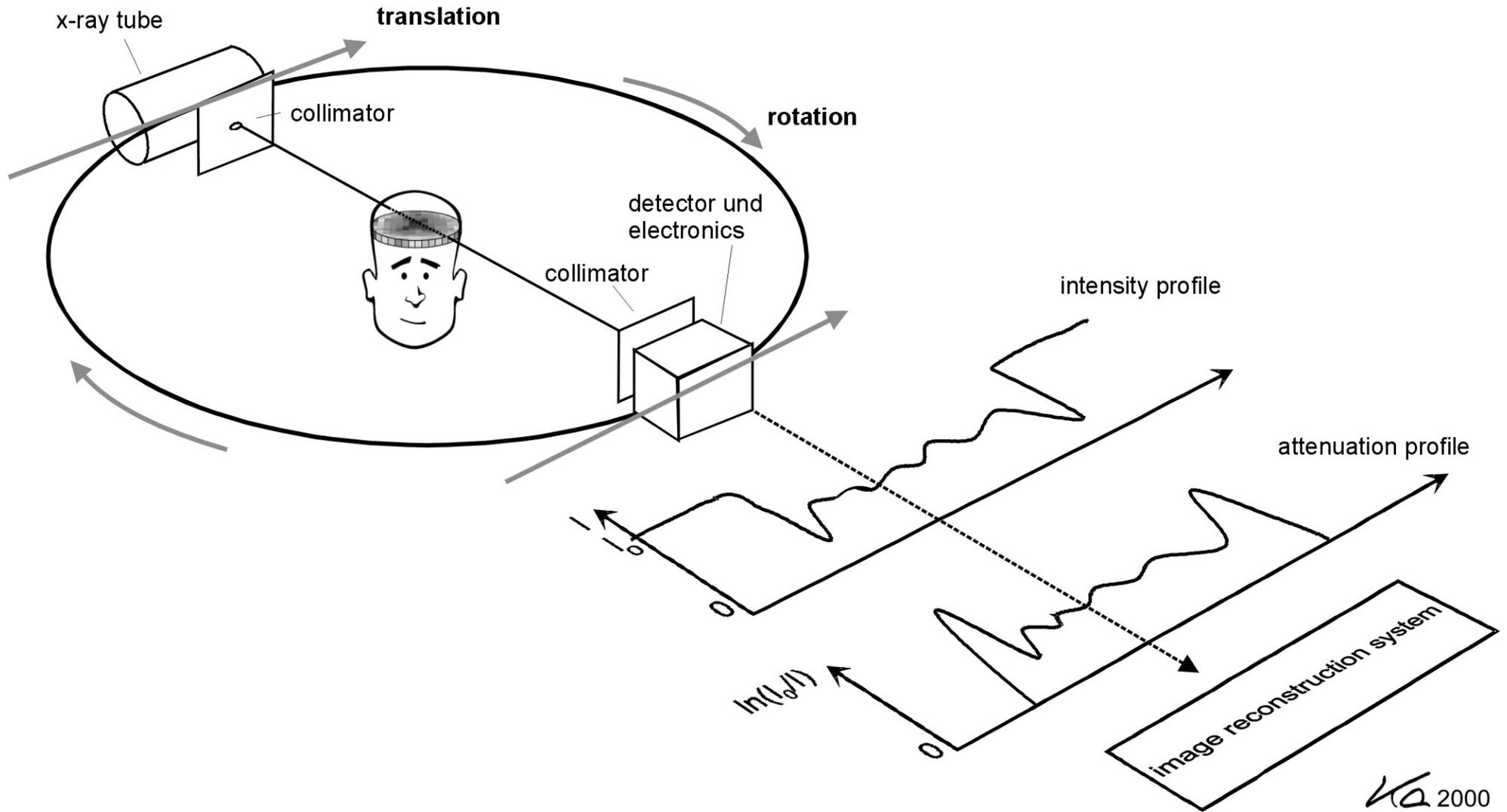
Beer-Lambert law

Radon transform

Rotated coordinate system

Radon transform
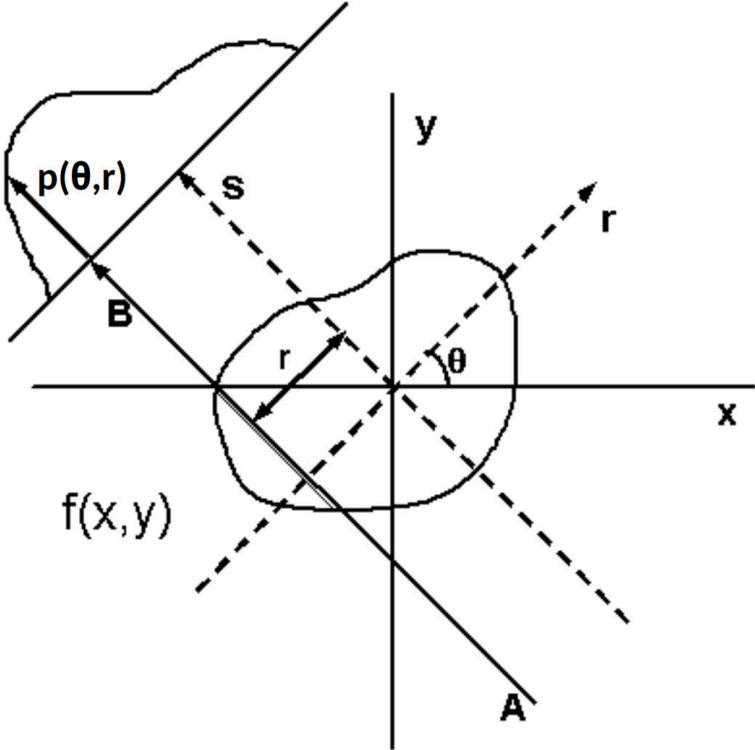
Sinogram
Representation of projection measured by a single detector line as a function of angle

The Fourier slice theorem

Fourier transform

projection

object

space domain

frequency domain
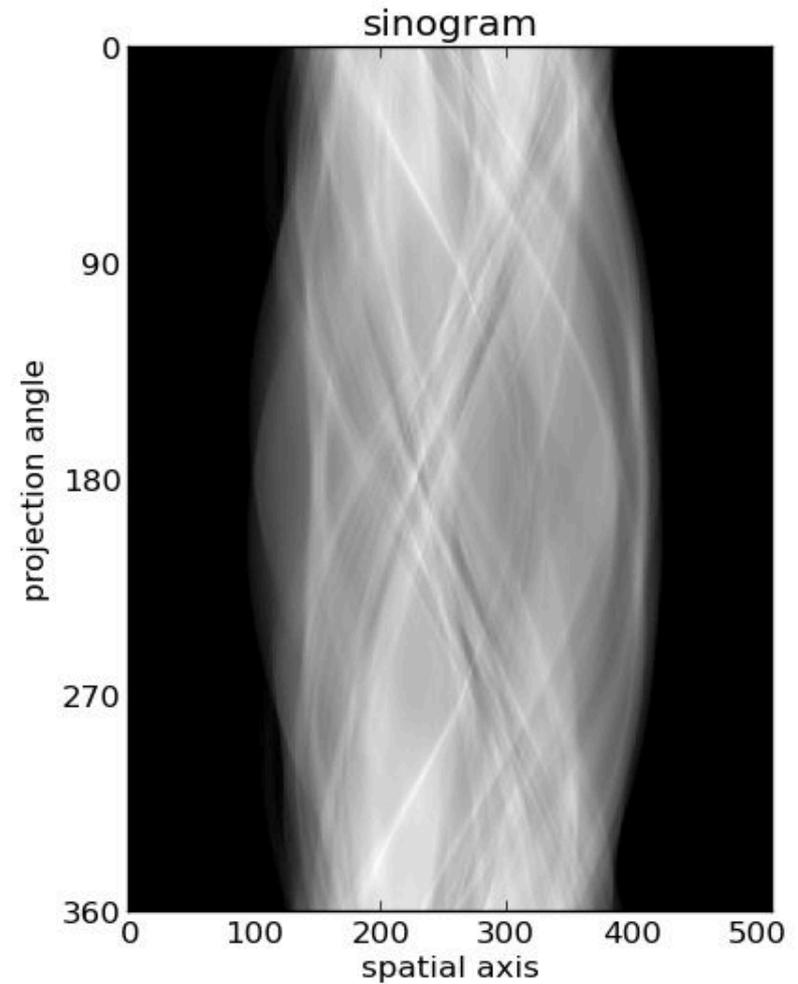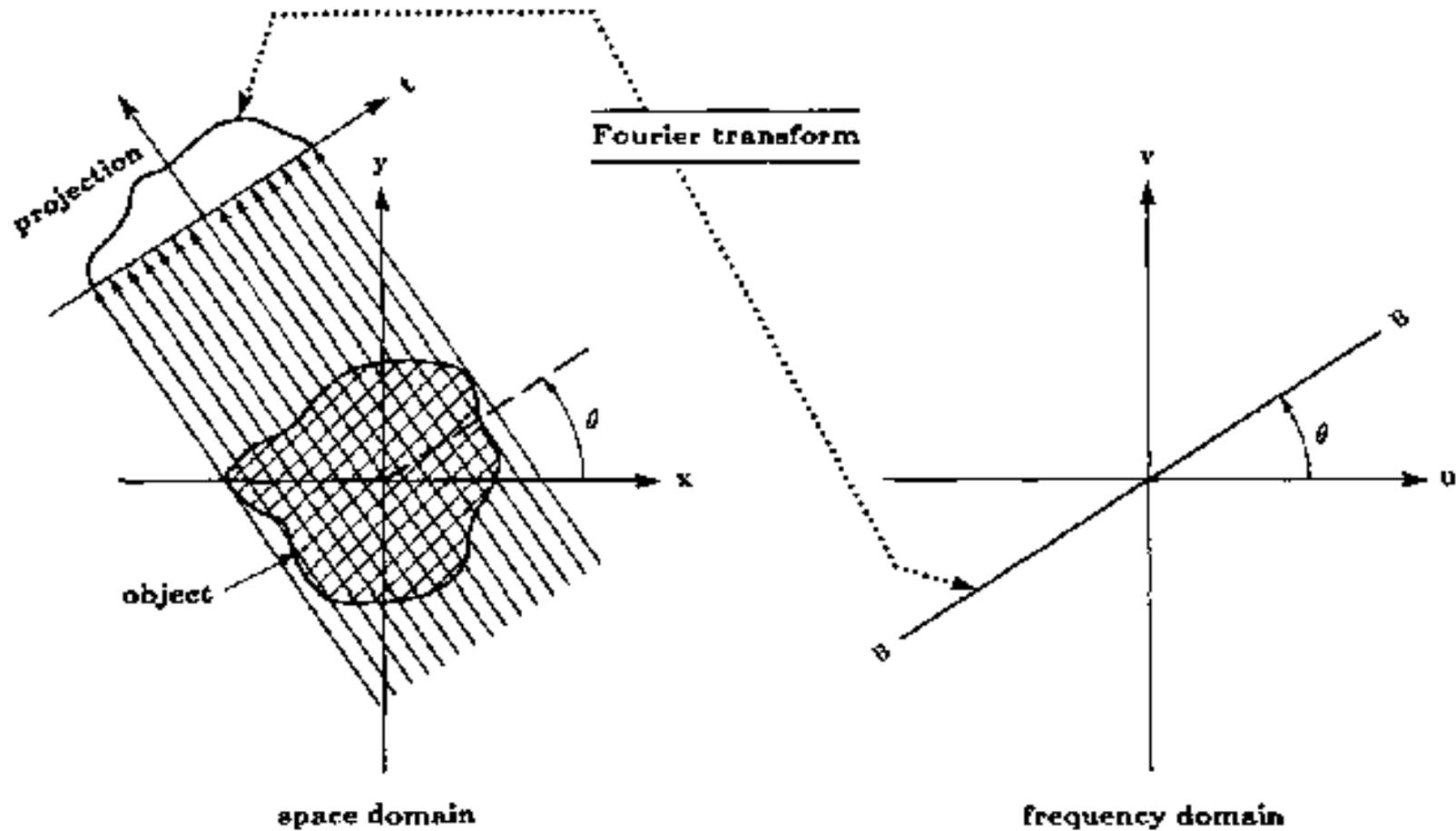
The Fourier slice theorem

Frequency space sampling

Change of sampling grid from polar
to rectangular requires interpolation
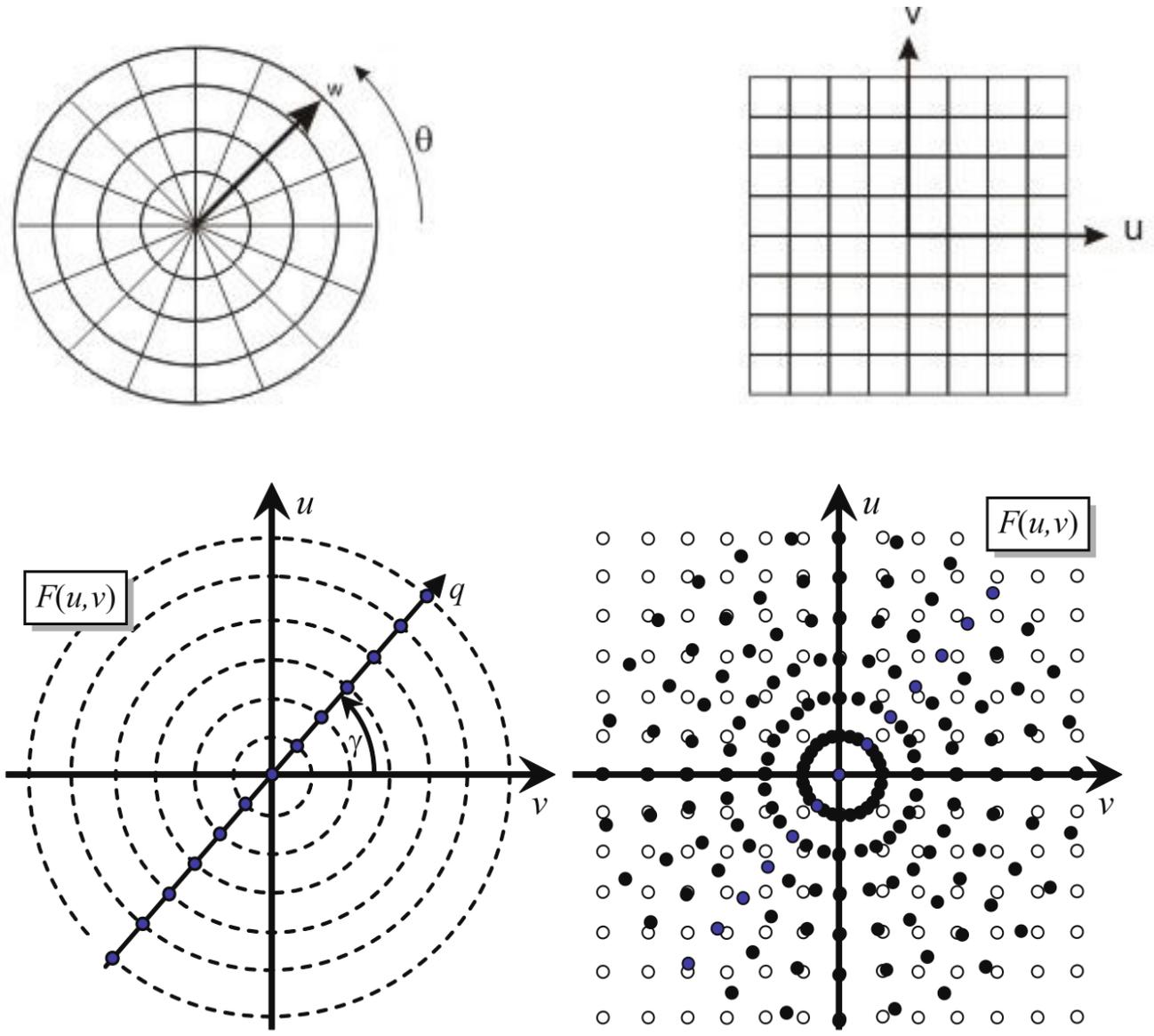
a

b

Filtered back-projection

frequency domain
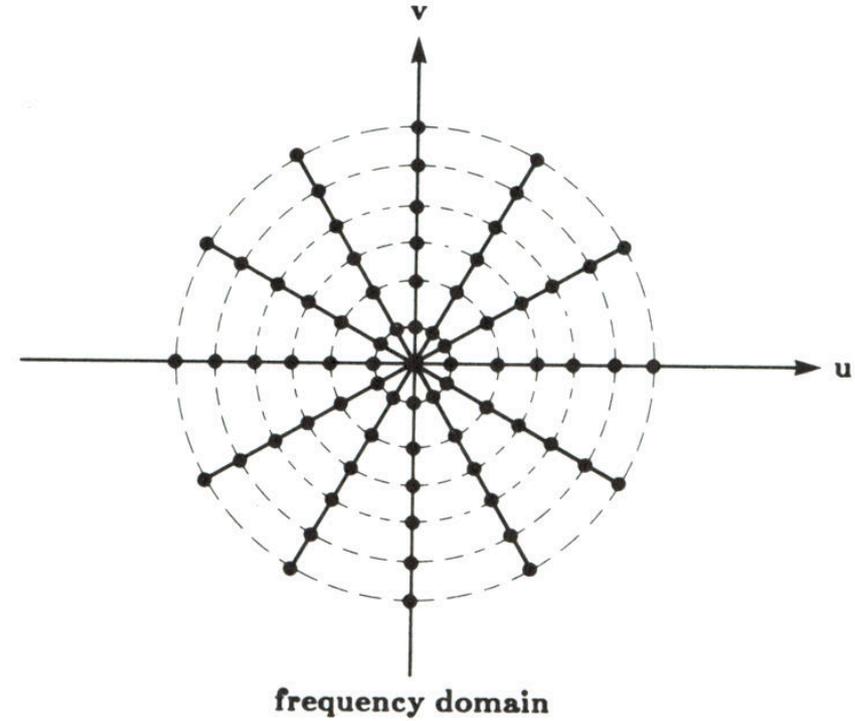
Filtered back-projection

Filtered back-projection

- Filter can be tuned to achieve image enhancement

- Trade-off between noise and sharpness
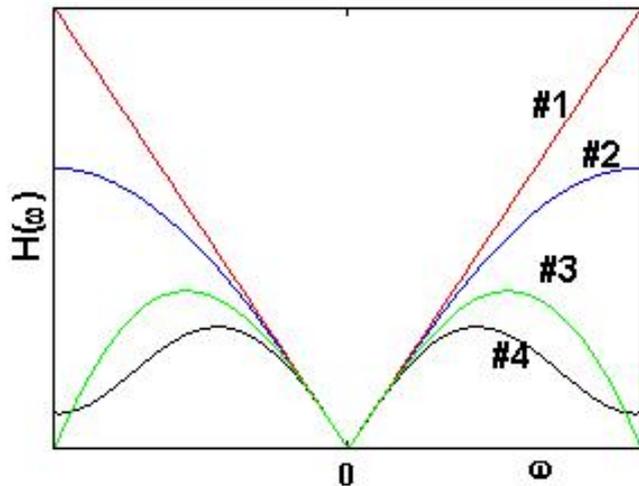
#1 ram-lak (ramp)

#2 Shepp-Logan

#3 cosine

#4 Hamming
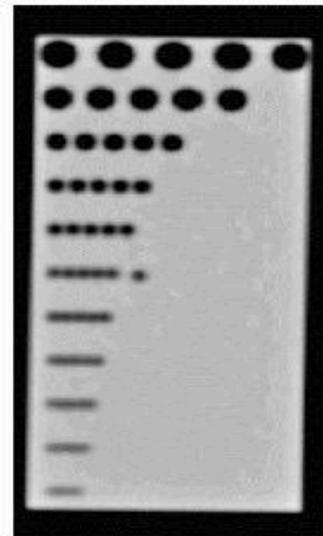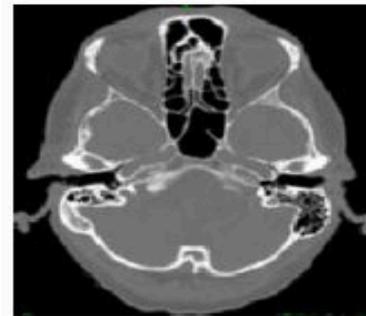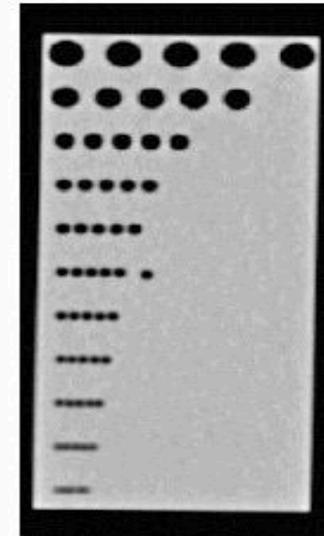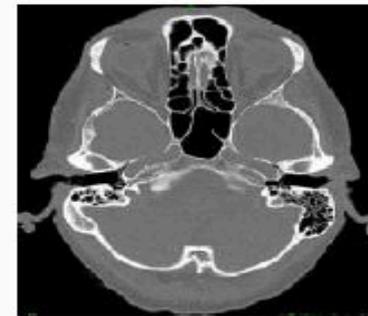
smoothing   standard   edge enhanced

Tomography

Medical CT

Photo from the Nobel Foundation archive.
Allan M. Cormack
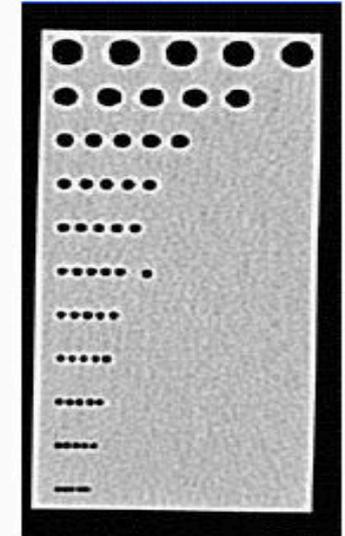
- Mathematical methods developed by Allan M. Cormack in the early 60s.

- First clinically useful CT instrument developed by Godfrey Hounsfield in the early 70s.

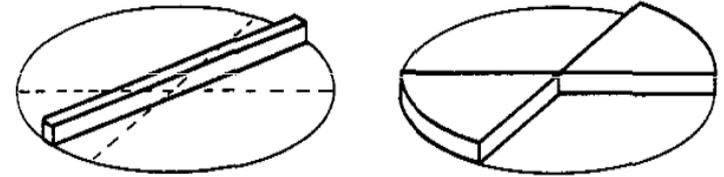- Cormack & Hounsfield were awarded the Nobel prize in 1979 "for the development of computer assisted tomography".
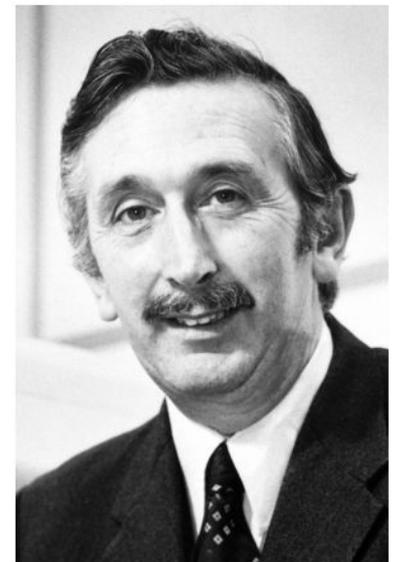
Photo from the Nobel Foundation archive.
Godfrey N. Hounsfield

Hounsfield units

Bone

Soft tissue

Lung

Bone

1000

800

600

400

250

200

Water   4
0   -4
Fat   -80
         -100

-200

-400

Lung   -550

-600

-800

         -950
Air   -990
-1000   -1000

80
70   Liver   70
60
50   50
40   Kidney
30
20   40
10
0   20

Medical CT
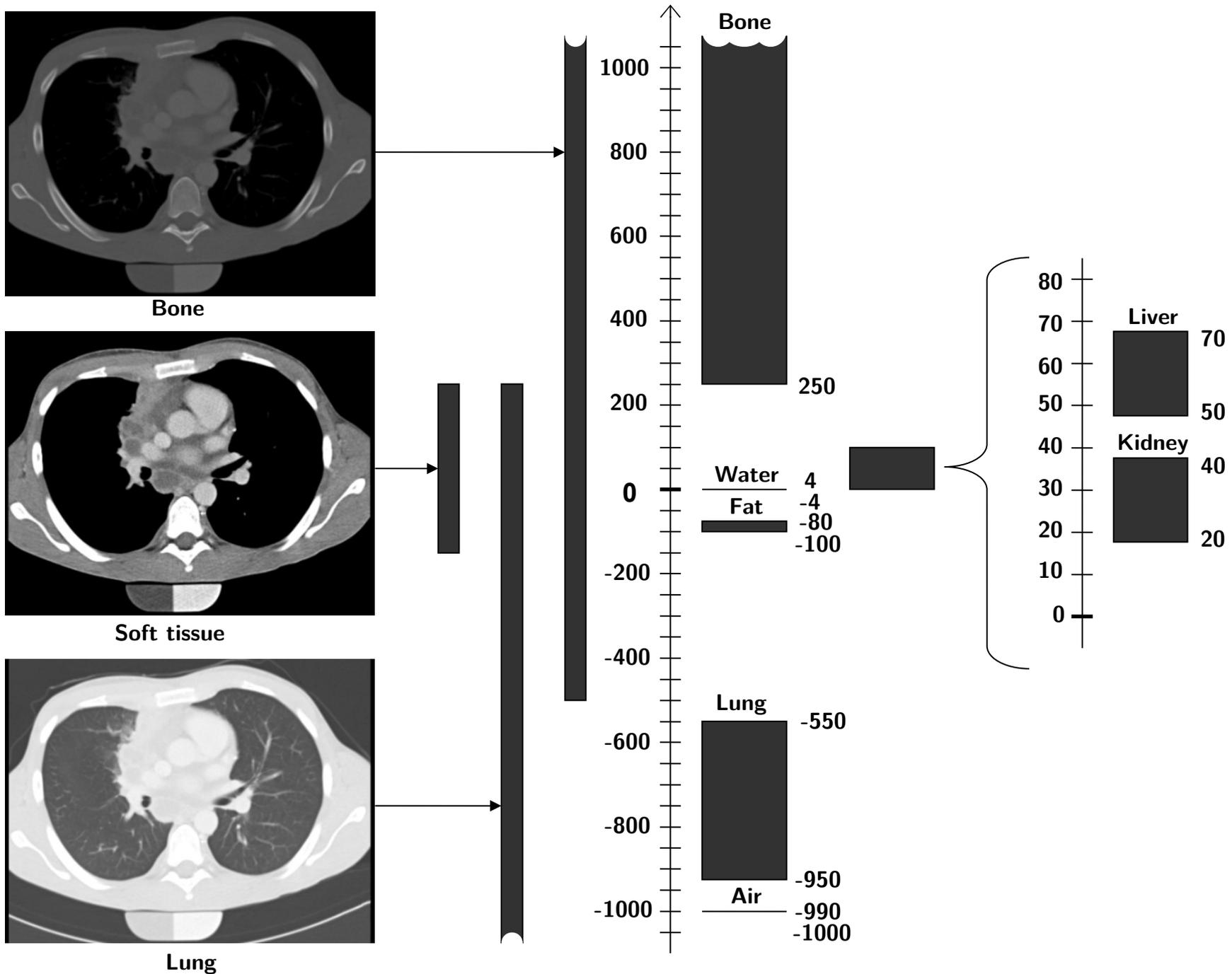
Geometries

pencil beam (1970)

1. translation

60. translation

> 24 h
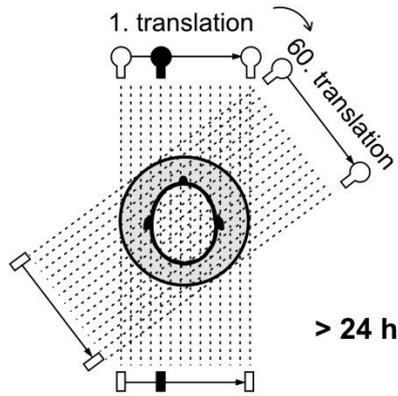
1st generation: translation / rotation

partial fan beam (1972)

1. translation

12. translation

300 s
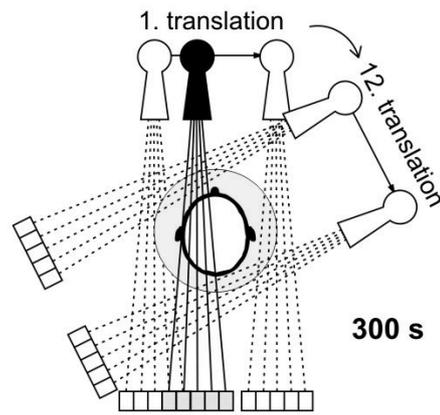
2nd generation: translation / rotation

fan beam (1976)

5 s

rotating detector arc
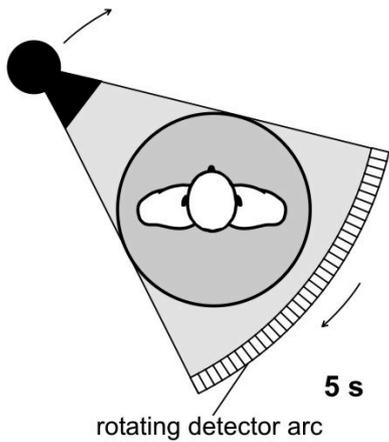
3rd generation: continuous rotation

fan beam (1978)
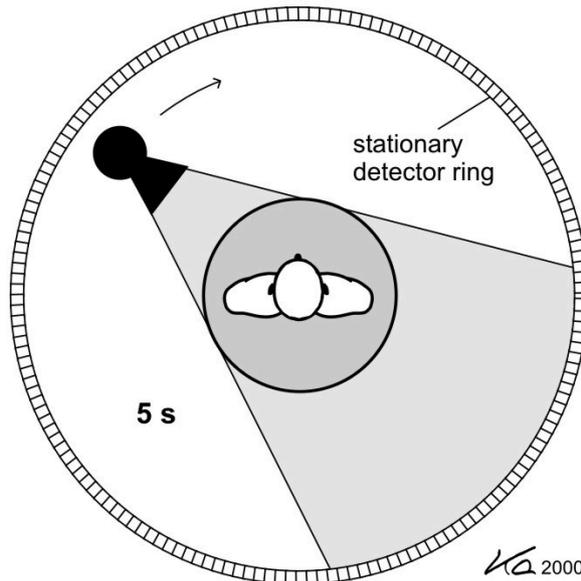
stationary detector ring

5 s

4th generation: continuous rotation

x-ray tube

detector

detector electronics

electrical power   projection data

source: Kalender, Publicis, 3rd ed. 2011

Tomography

CT systems

X Ray Tube

Cooling Unit

Filter
Collimator

Detection System

Medical CT
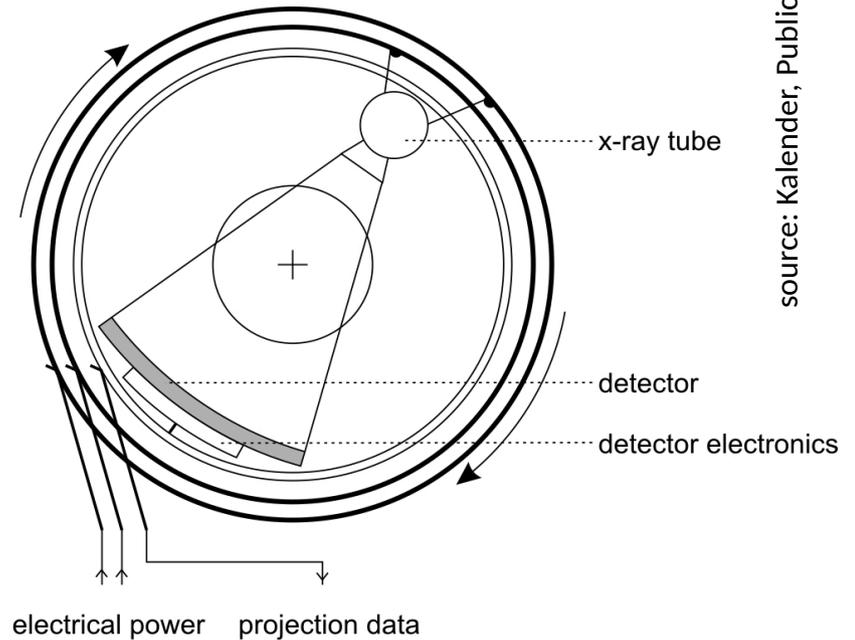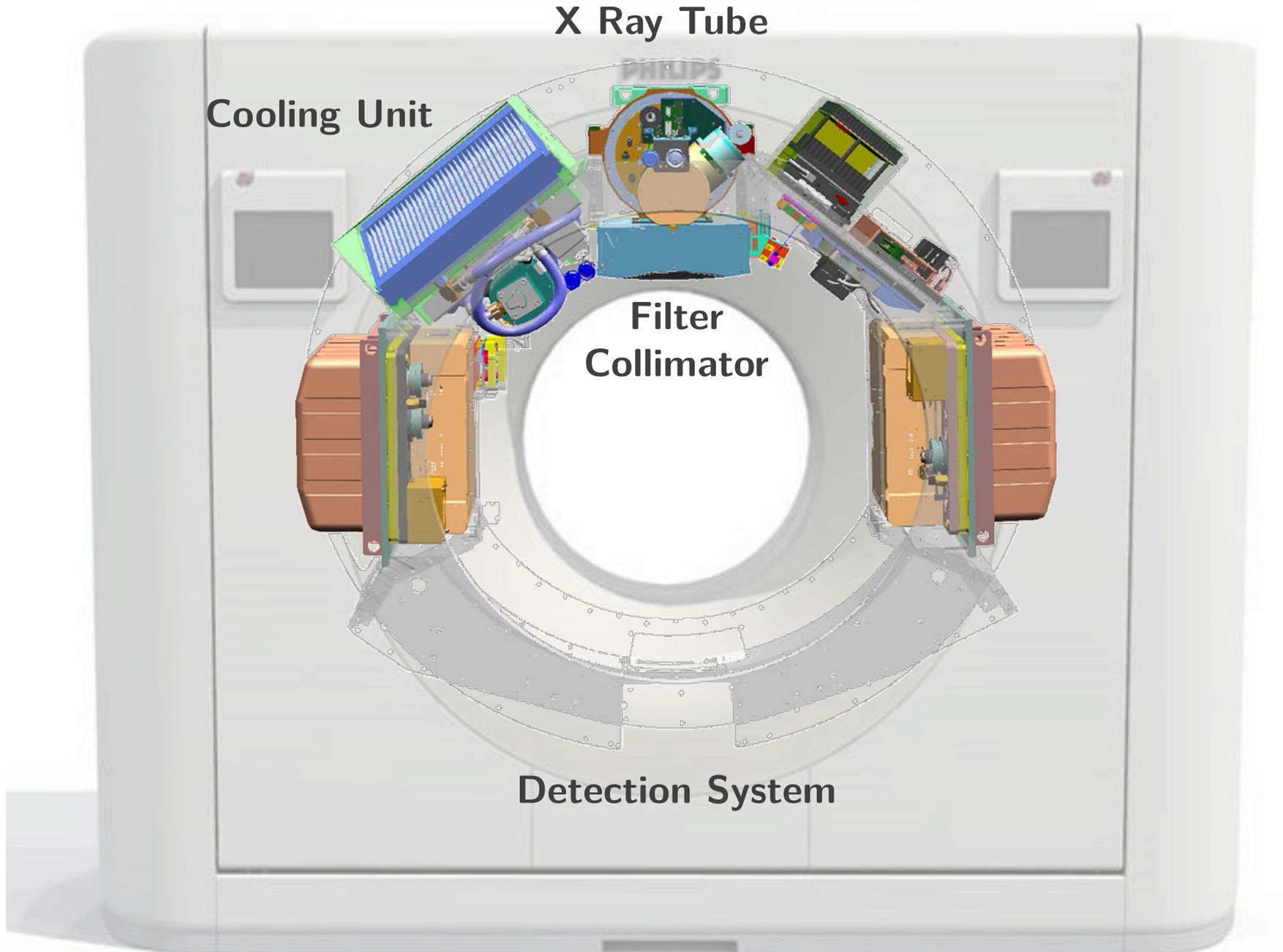
CT systems

- 3rd generation scanners:

  – 1 X-ray source

  – 1D detector

  – *Fan beam* geometry

  – Total scan time less
    than 5 second.

X-ray
tube

Fan
beam

Detector array
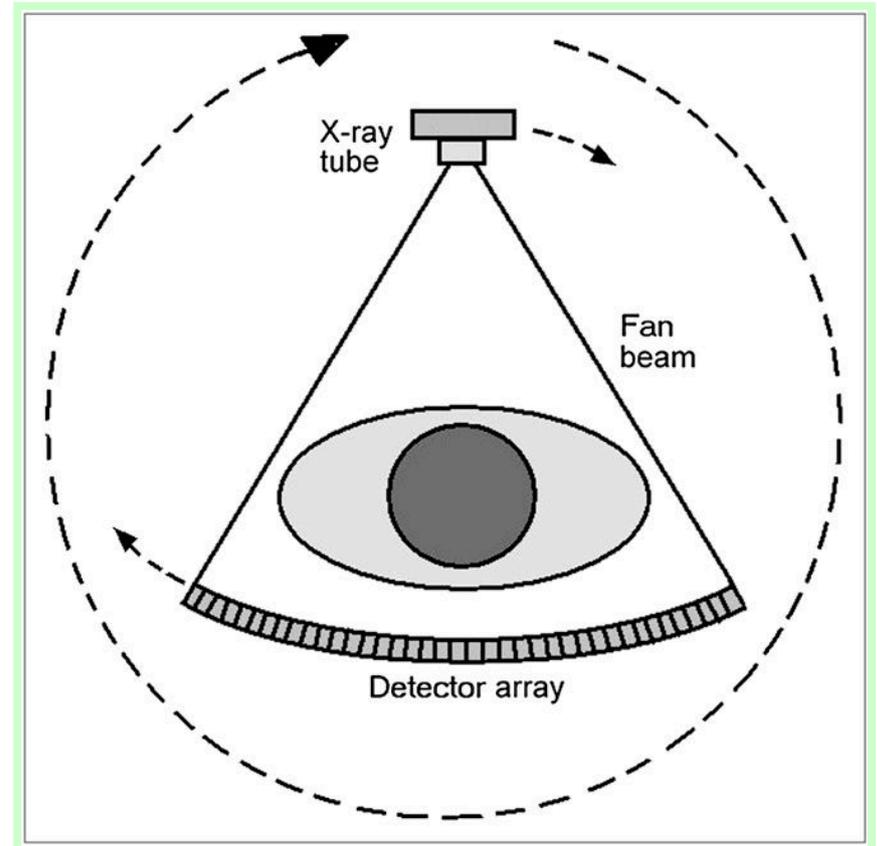
CT systems

- Recent scanners: 2D detectors (cone beam)
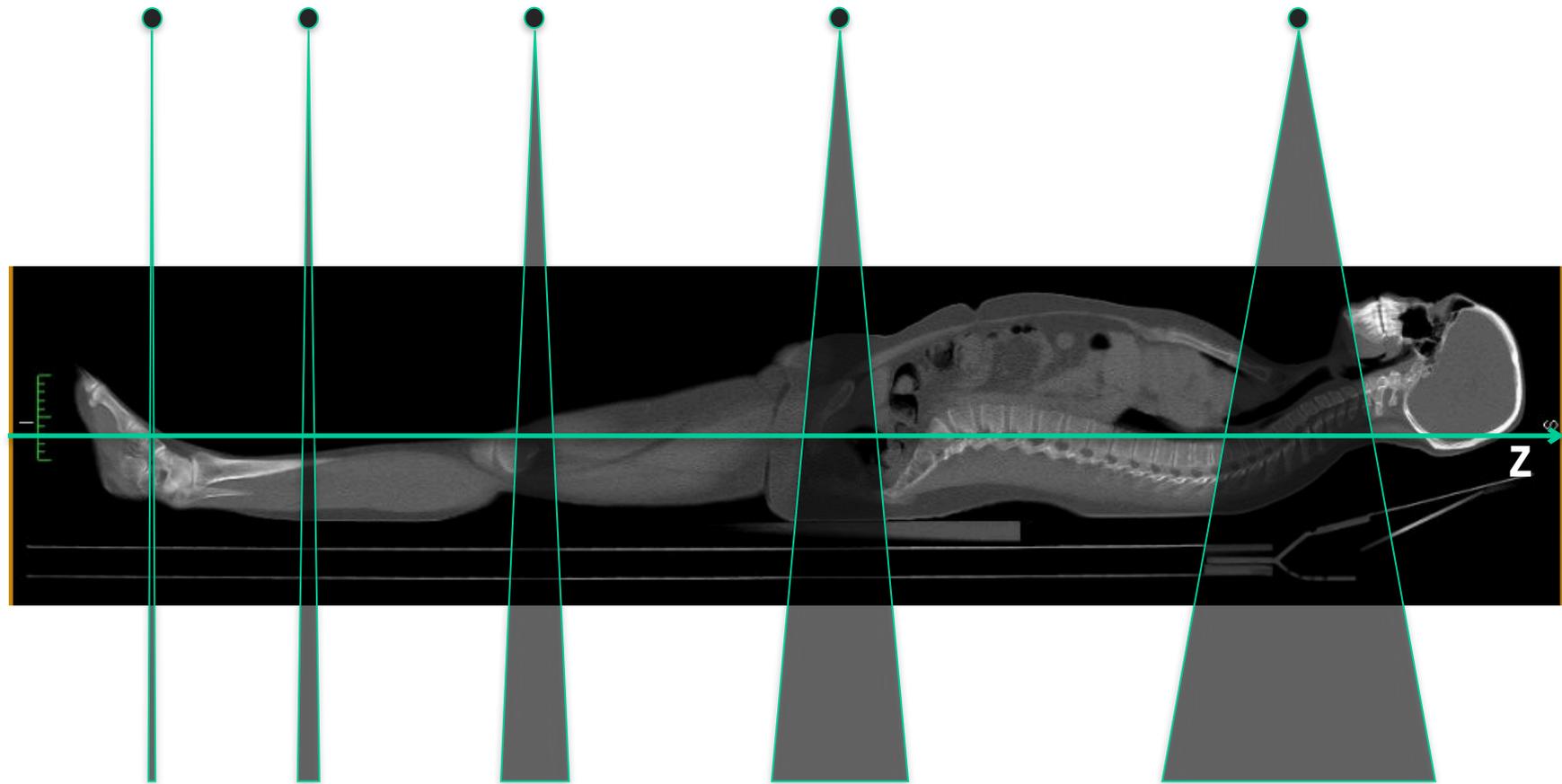
NxT	1x5mm	4x1mm	16x0.75mm	64x0.6mm	320x0.5mm
t_{rot}	0.75s	0.5s	0.42s	0.33s	0.27s
year	1995	1998	2001	2004	2008

Medical CT

CT systems
Anti-scatter

1st Generation

3rd Generation

Small Z coverage

Large Z coverage

Scatter

Antiscatter septa

Z

Z

Scatter

Z

Z
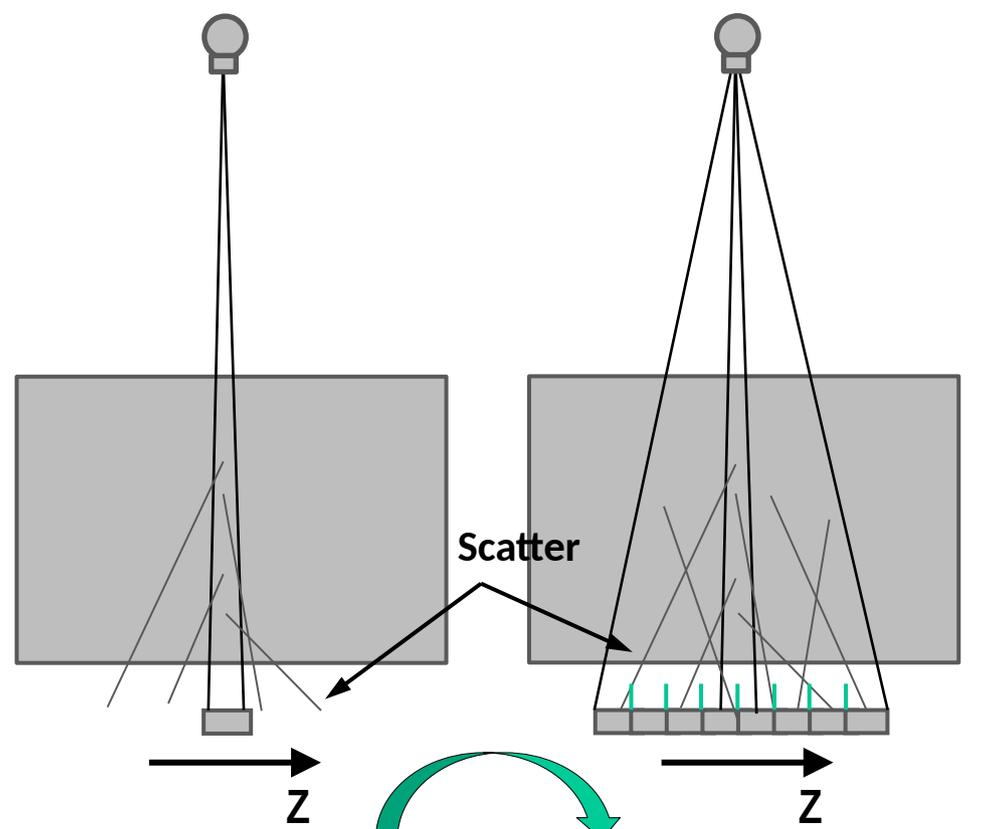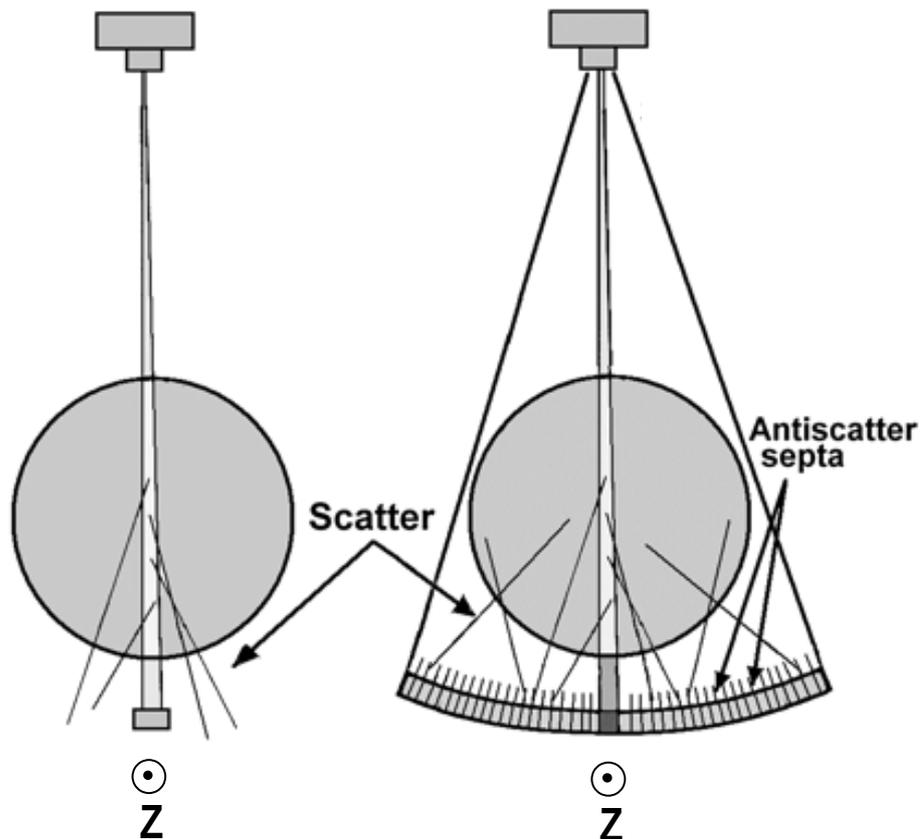
1D anti-scatter grid

2D anti-scatter grid

Z

Medical CT

CT systems
Helical CT

slice volume

reconstruction plane

data interpolation

slice thickness

helical data acquisition path
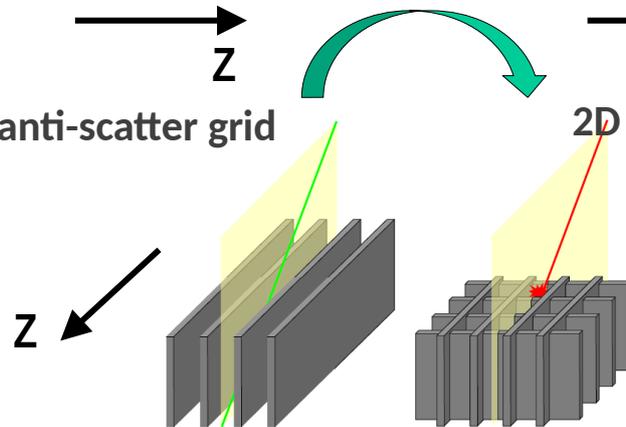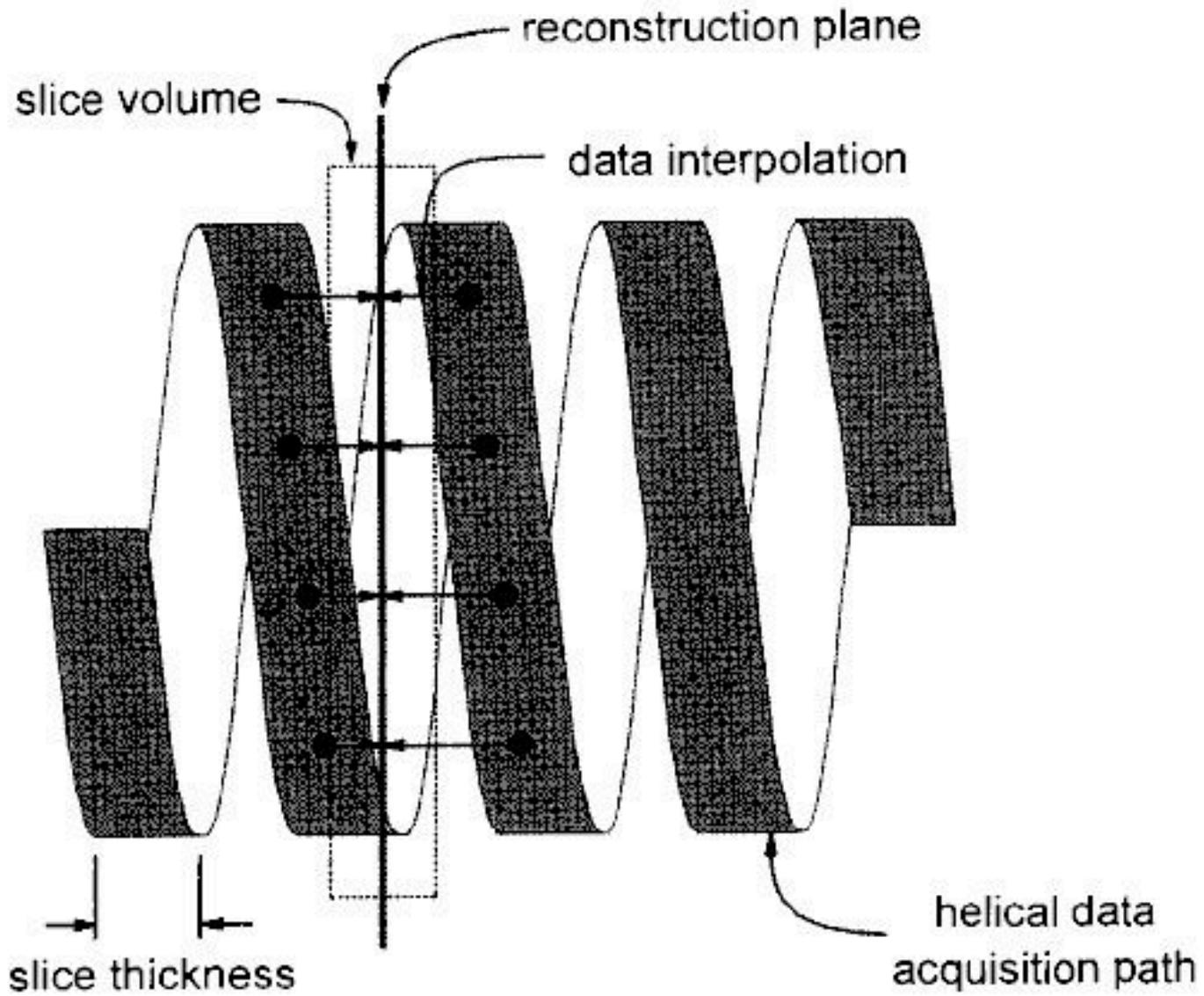
Algebraic formulation

Tomography can be formulated as a set of linear equations
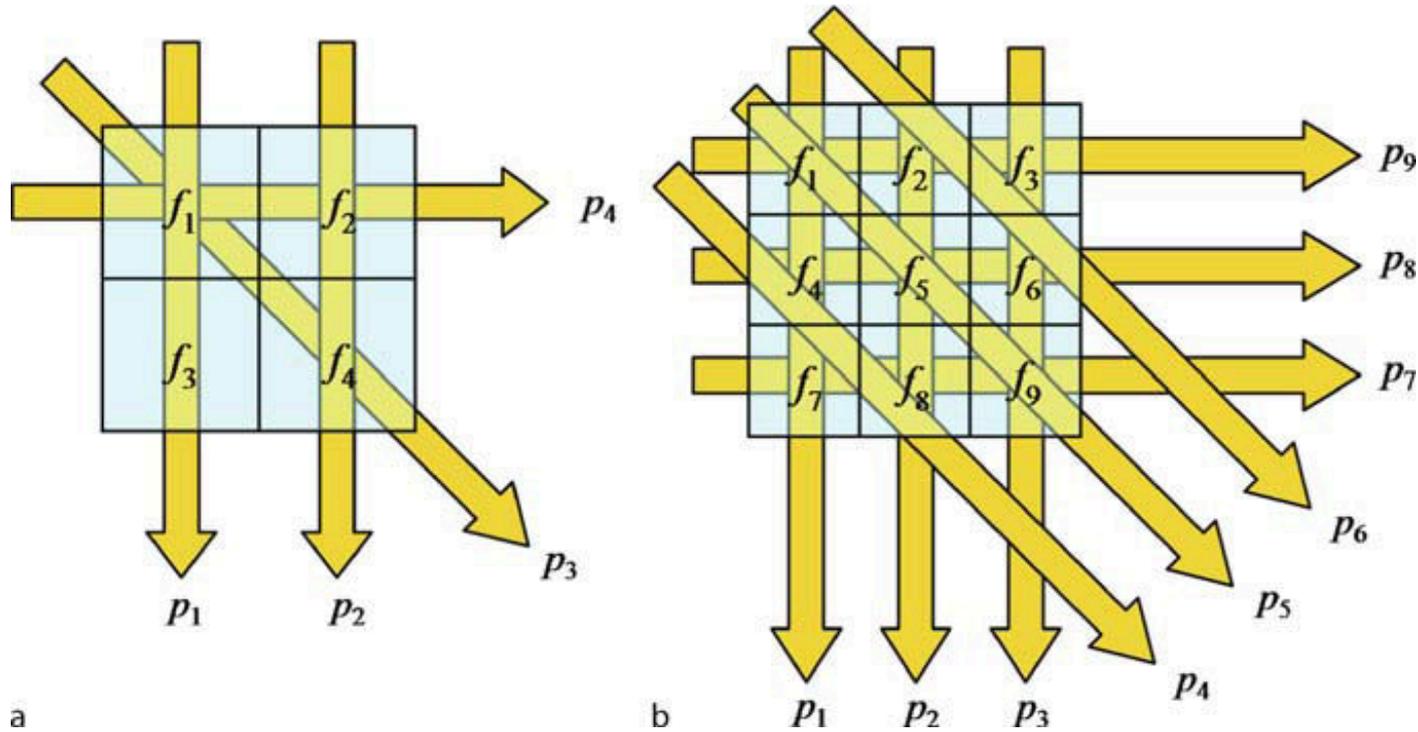
source: Buzug, Springer, 1st ed. 2008

Weighting coefficients

Weighting measures:

- Logic

- Area

- Path length

- Distance to pixel center
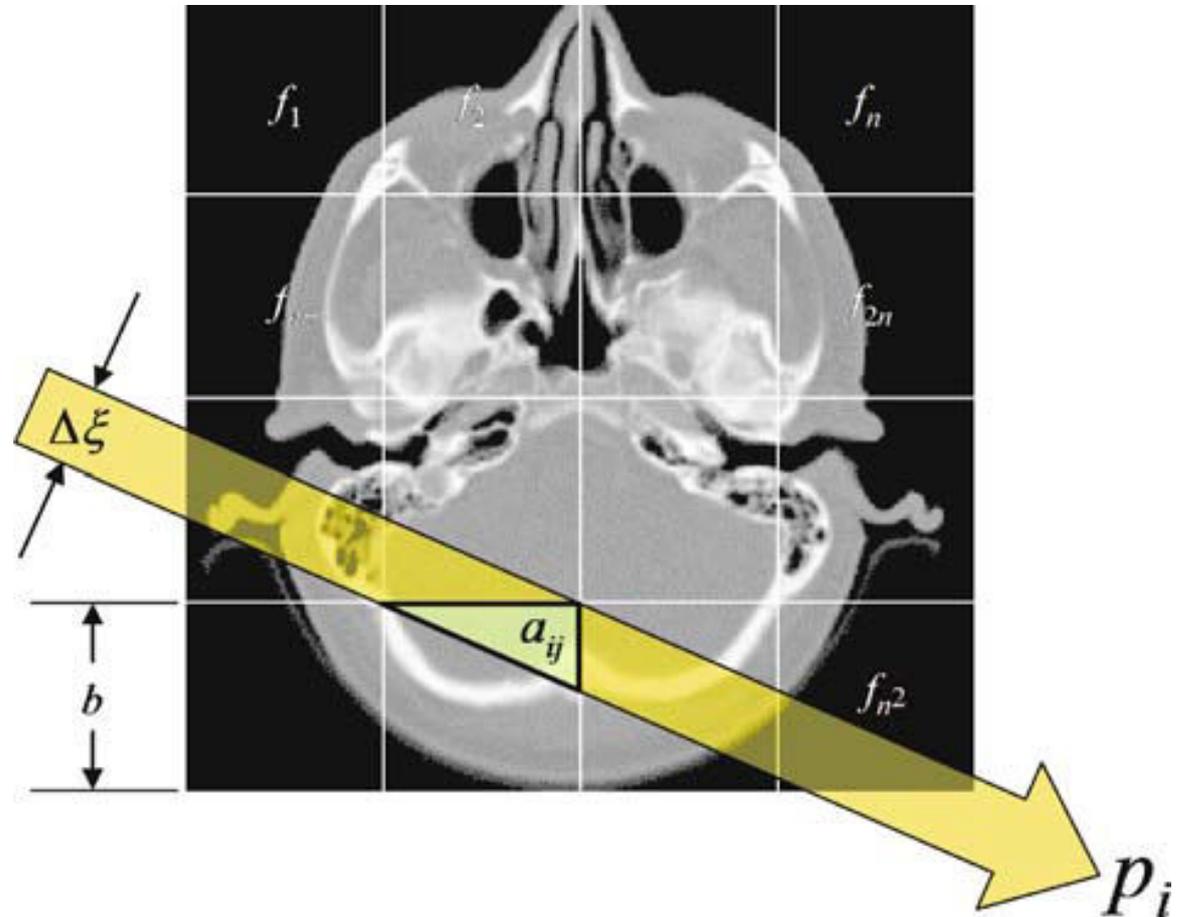
Differences in calculation effort, smoothness, noise sensitivity, …

System Matrix
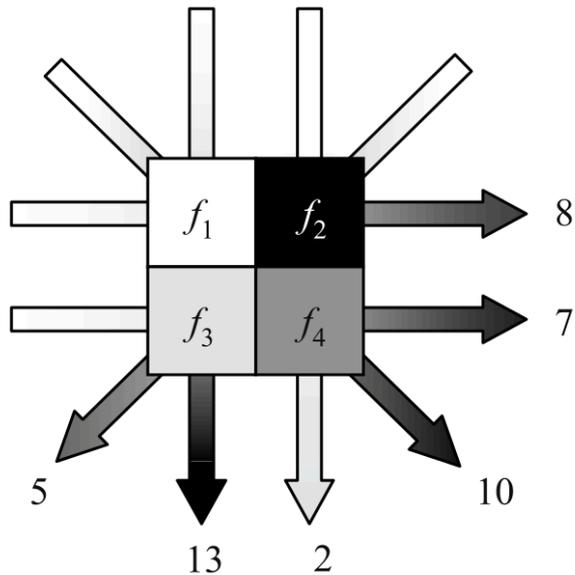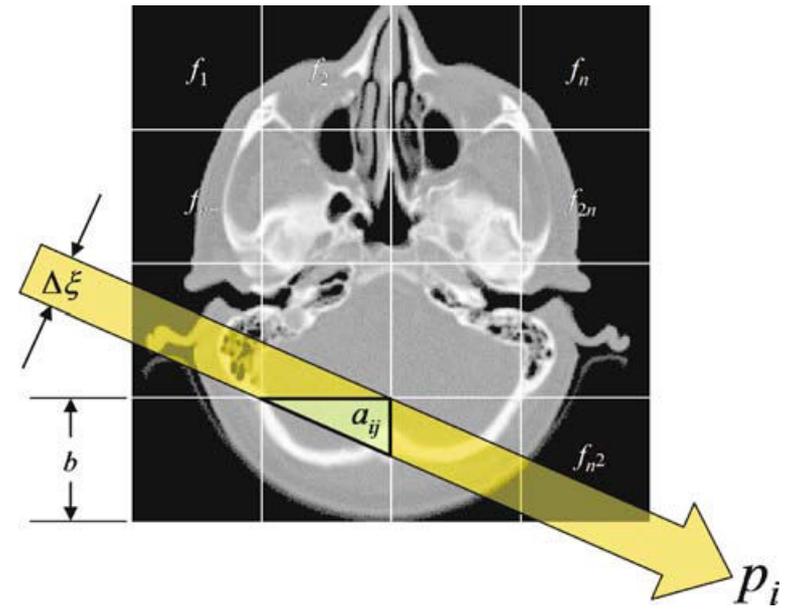
$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} 5 \\ 13 \\ 2 \\ 10 \\ 7 \\ 8 \end{pmatrix}$$

source: Buzug, Springer, 1st ed. 2008

Tomography

Matrix (pseudo)-inversion

Tomographic reconstruction = linear system inversion

Iterative methods:

- ART   Algebraic reconstruction technique
- SART   Simultaneous algebraic reconstruction technique
- SIRT   Simultaneous iterative reconstruction technique
- MART   Multiplicative algebraic reconstruction technique
- MLEM   Maximum likelihood expectation maximization
- OSEM   Ordered subset expectation maximization
- ... and many, many more

FBP vs algebraic methods

Filtered backprojection 100% dose
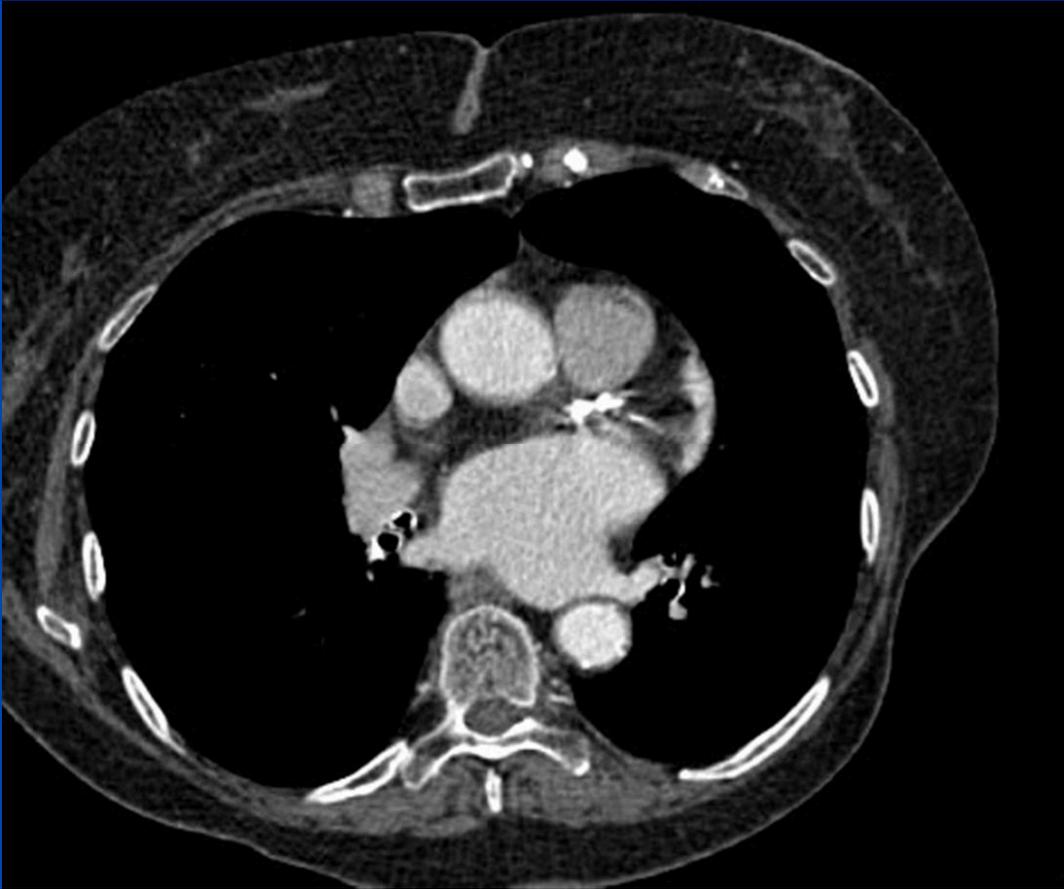
iterative 40% dose
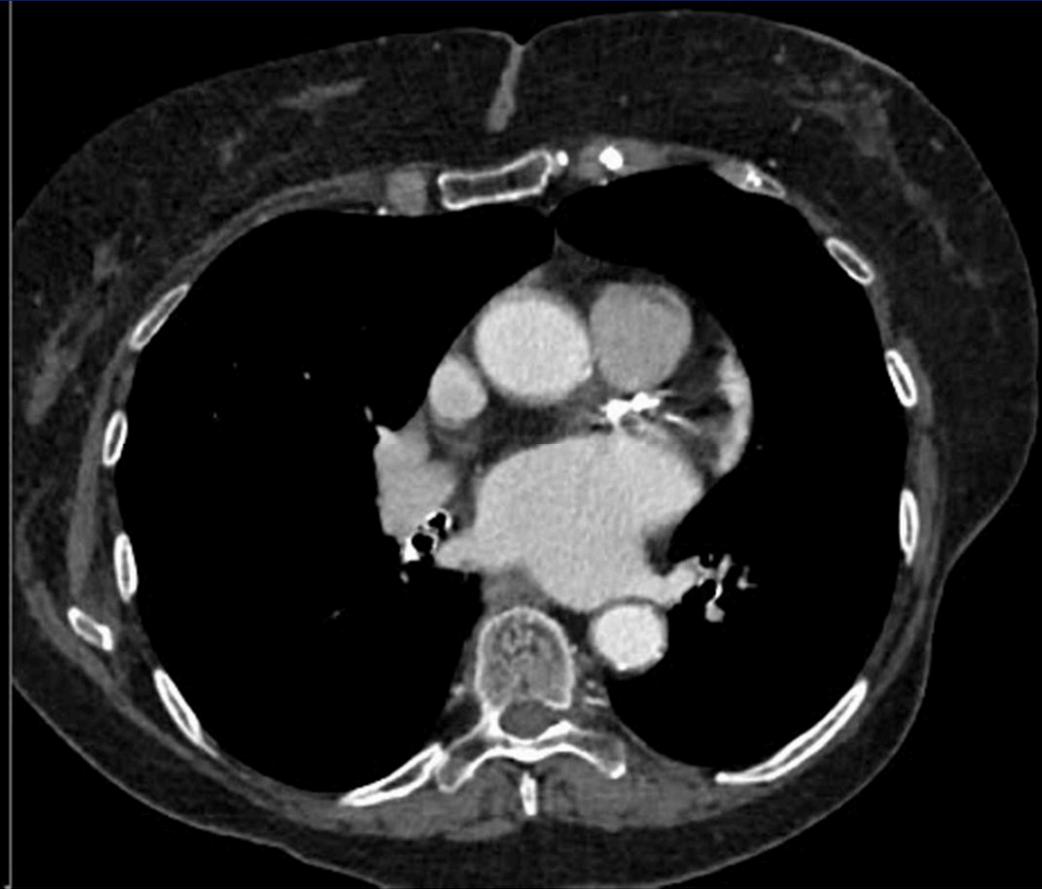
Tomography

Iterative methods

Medical CT

Iterative methods

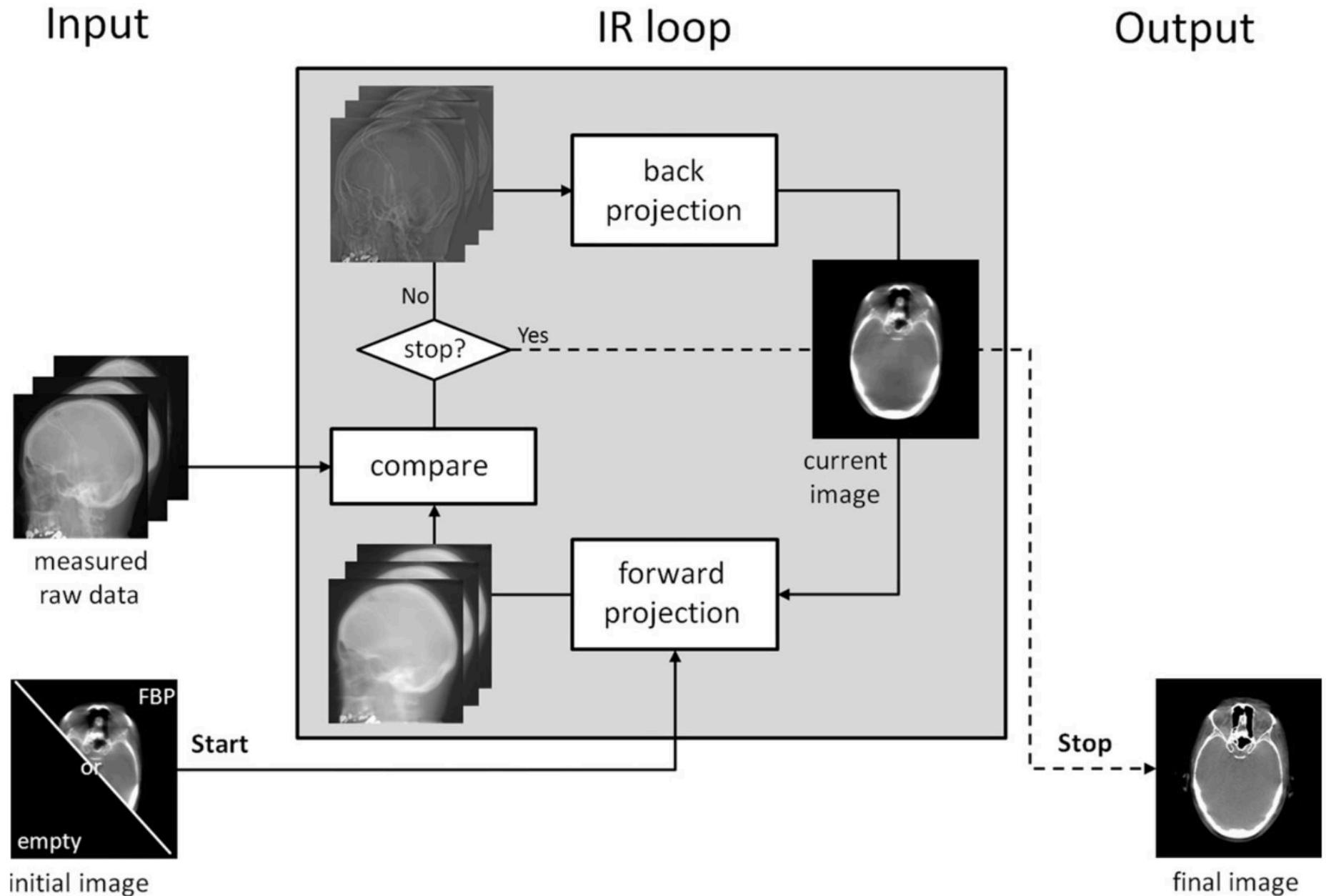| Abbreviation	Meaning	
ART	Algebraic reconstruction technique	Gordon et al. 1970
SART	Simultaneous ART	Anderson & Kak, 1984
SIRT	Simultaneous iterative reconstruction technique	Gilbert 1972
OS-SIRT	Ordered subset SIRT	Gordon et al. 1970
MART	Multiplicative algebraic reconstruction technique	
ML-EM	Maximum likelihood expectation-maximization	Lange & Carson 1984
OS-EM	Ordered subset expectation-maximization	Manglos et al 1995
OSC	Ordered subset convex algorithm	Kamphuis & Beekman 1998 Erdogan & Fessler 1999
ICD	Iterative coordinate descent	...
OS-ICD	Ordered subset ICD	
MBIR	Model-based iterative reconstruction	

Medical CT

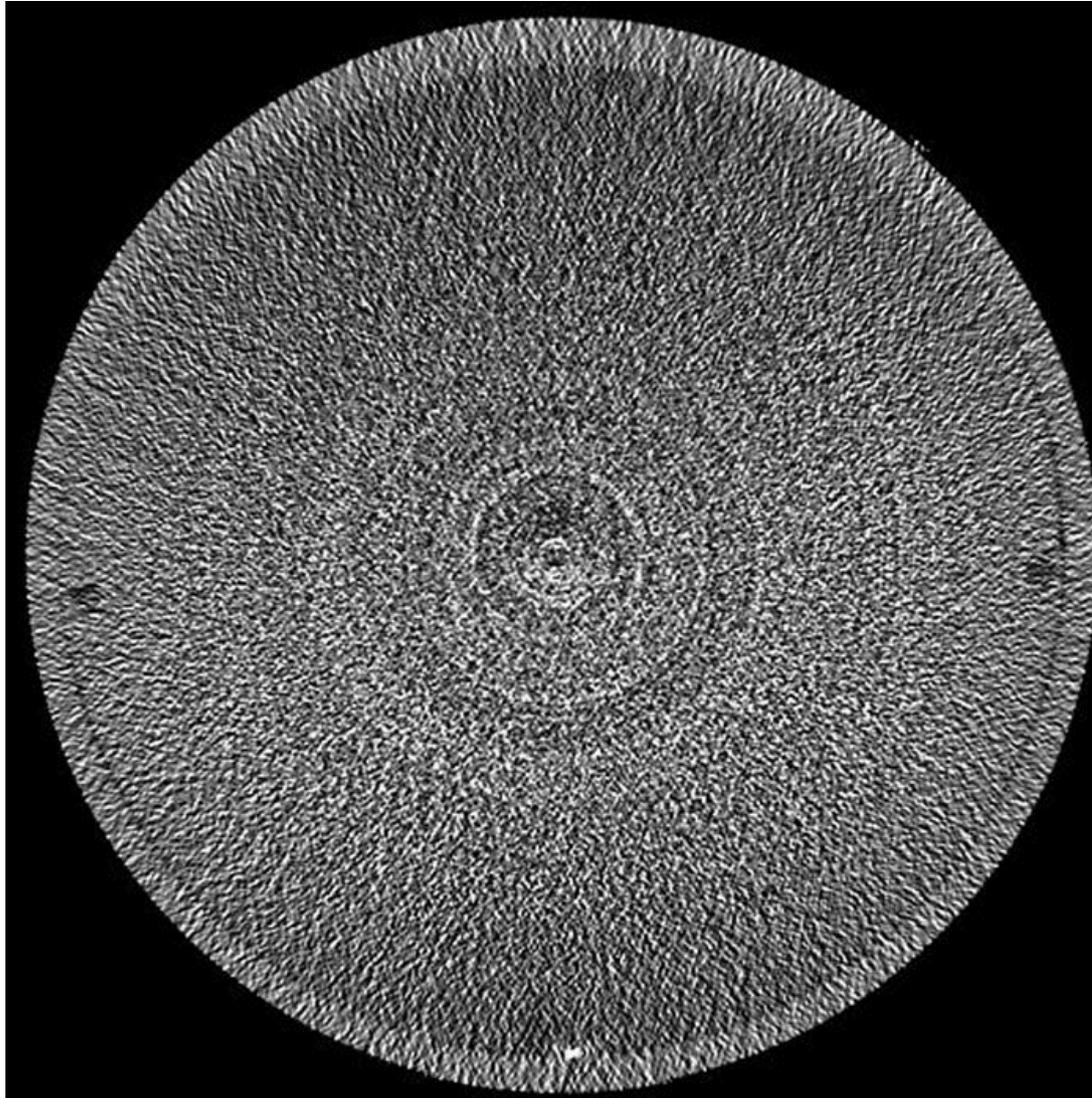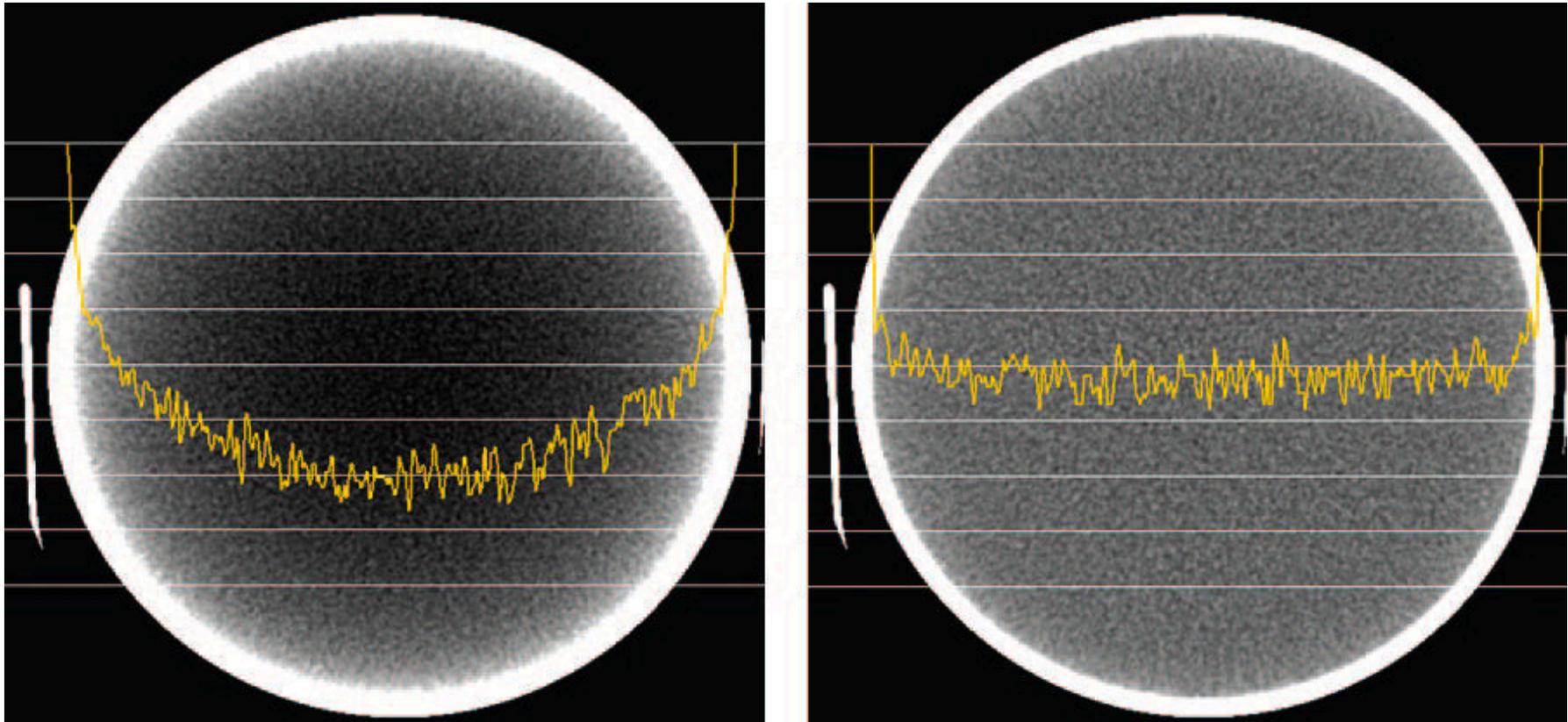# Image quality

Signal to noise
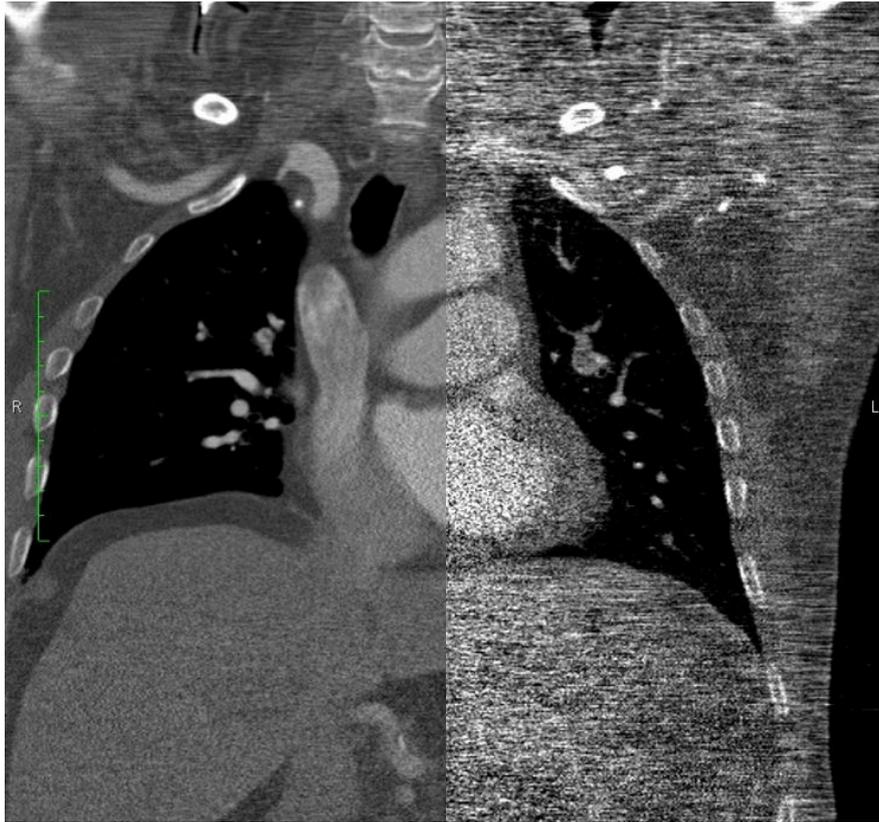
Artifacts

Artifacts

Ring artifacts: caused by damaged or
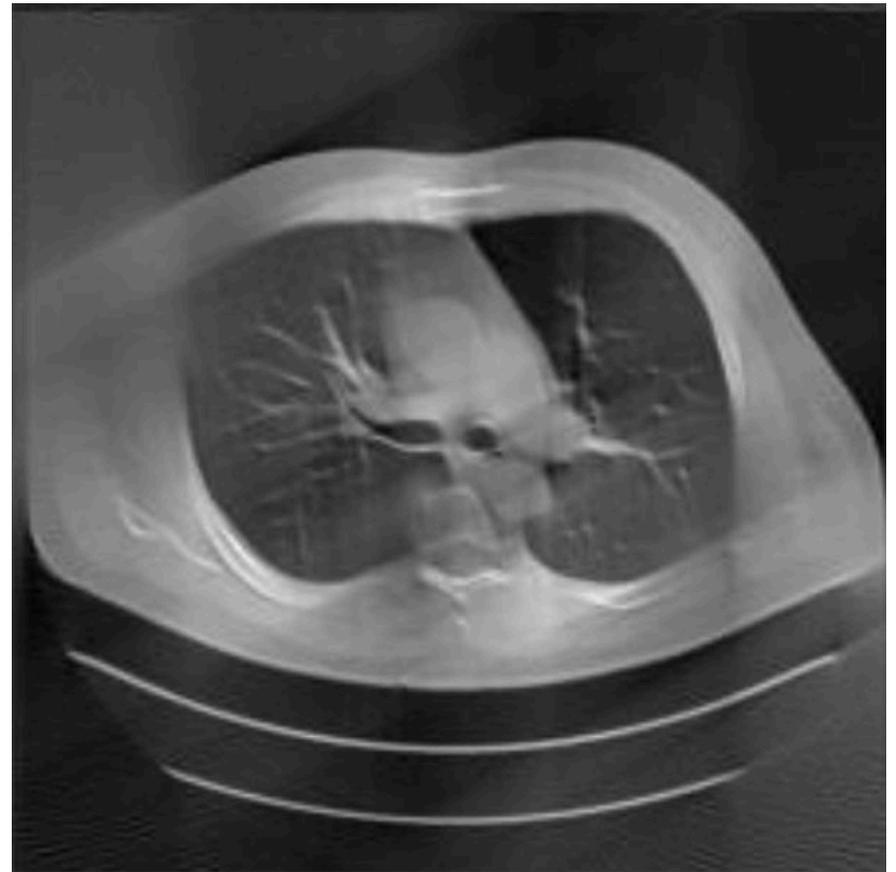miscalibrated detector pixels.

Artifacts

Beam hardening: deviation from the exponential law caused by the attenuation of a broad spectrum (thicker objects seem more transmissive than they should)
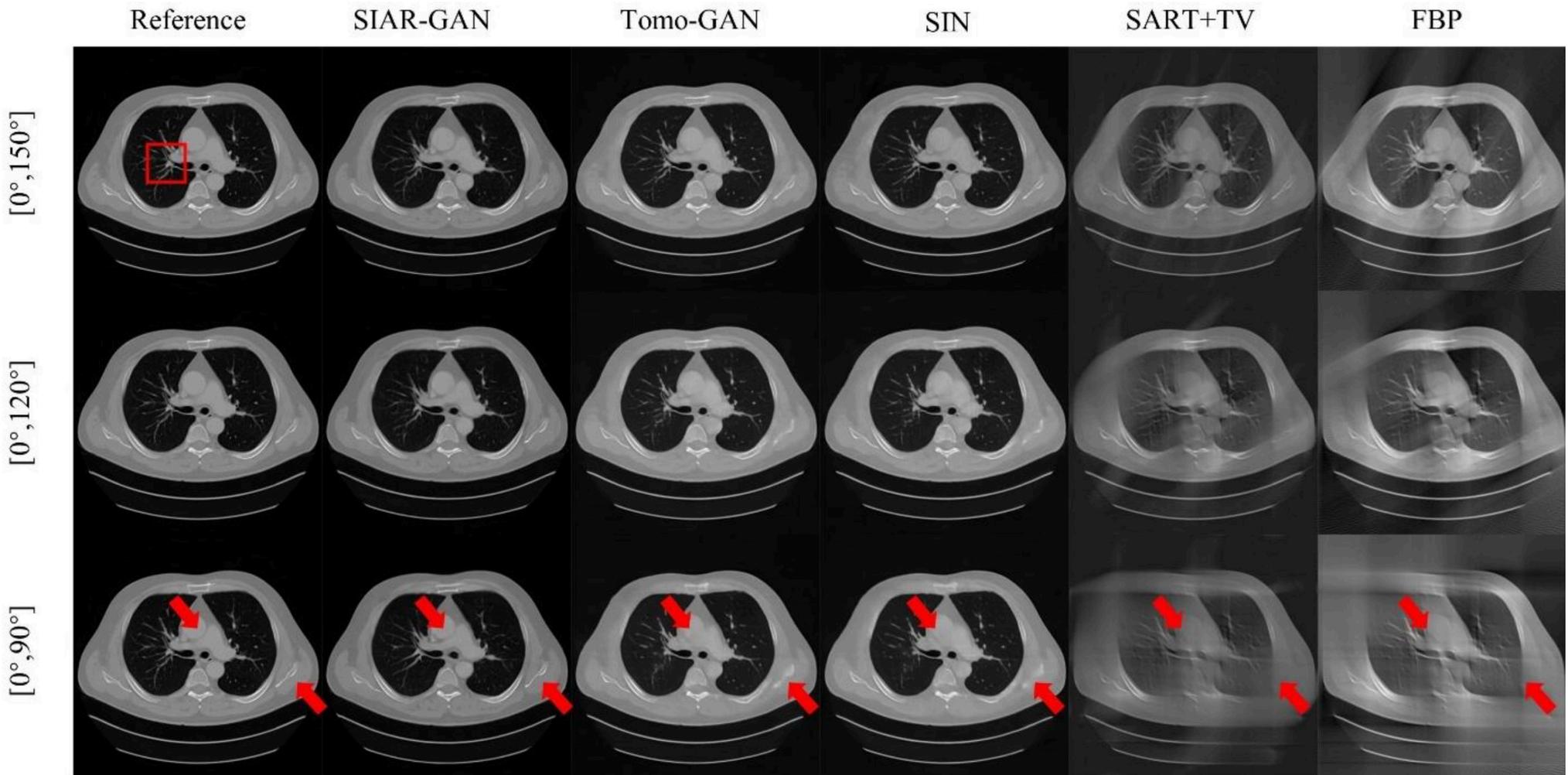
Artifacts

"Missing wedge": caused by an incomplete sinogram
(also called limited angle tomography)

Artifacts

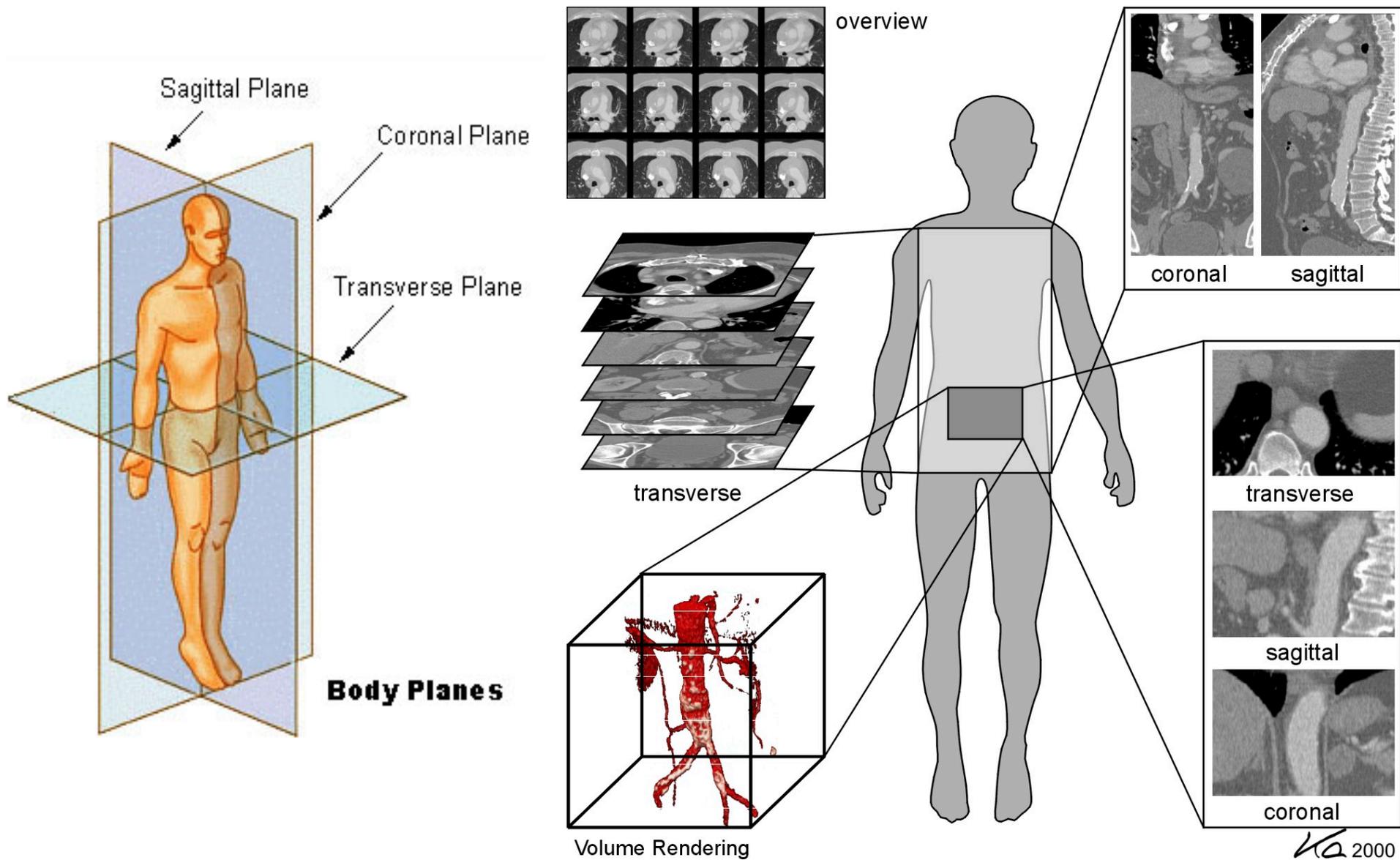## "Missing wedge": promising results using machine learning

Medical CT

Artifacts

Photon starvation: strongly absorbing features discard useful signal for the reconstruction of nearby areas

Medical CT

Tomographic Display

Body Planes

Sagittal Plane

Coronal Plane

Transverse Plane

overview

transverse

Volume Rendering

coronal   sagittal

transverse

sagittal

coronal

2000

Tomography

Volume rendering display

Tomography

source: Kalender, Publicis, 3rd ed. 2011
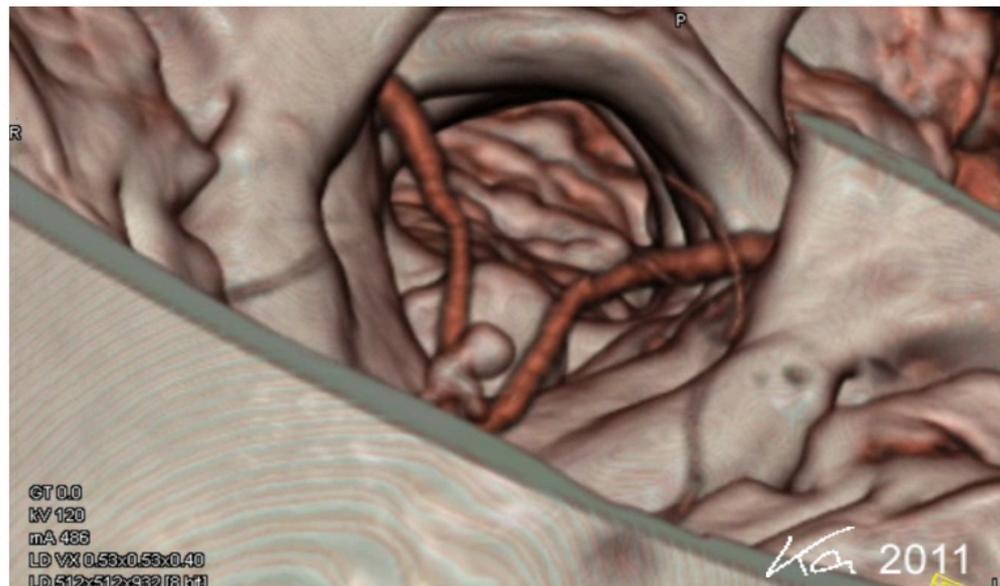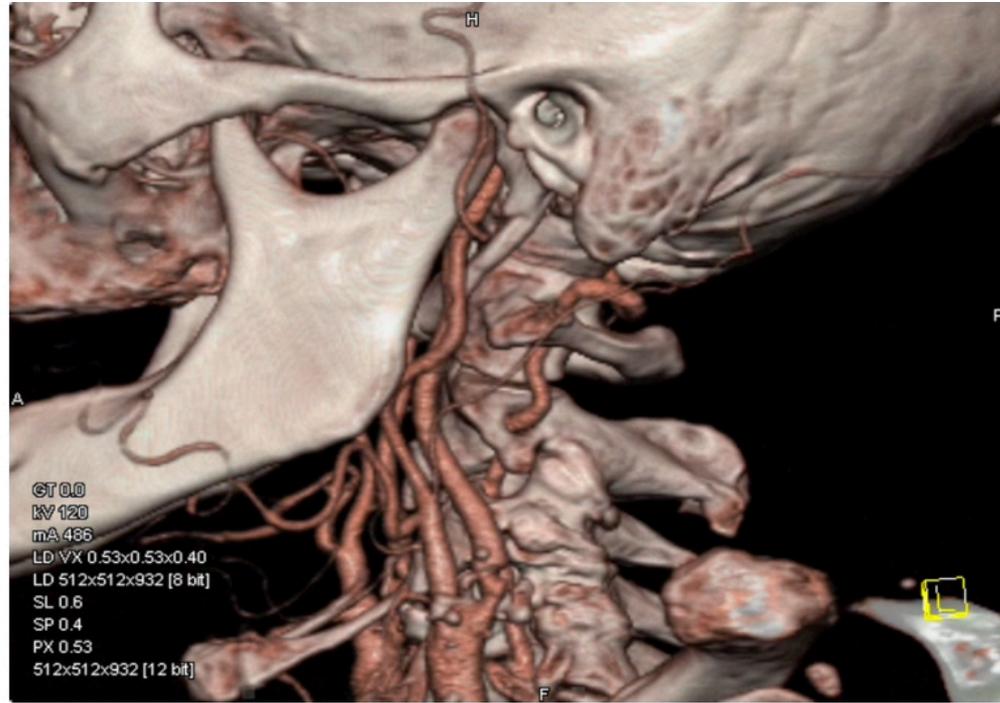
Summary

- Computed tomography: reconstruction from projections

- Analytic approach:

    - Projections and tomographic slices are related by the Fourier slice theorem

    - Standard algorithm uses filtered back-projection

- Algebraic approach:

    - Tomography as a system of linear equations

    - Iterative methods are used for large matrix inversions

    - More powerful but computationally more costly

- Imperfect data leads to artifacts