25 Novembre

Proposizione (Integrazione per parti)
Sia I un intervallo e $f, g \in C^1(I)$ vale la seguente formula

$$\int f'(x)\, g(x)\, dx = f(x)\, g(x) - \int f(x)\, g'(x)\, dx \qquad (1)$$

Dato un intervallo $[a,b]$ ~~ed~~ ed $f, g \in C^1([a,b])$ vale

$$\int_a^b f'(x)\, g(x)\, dx = f(x)\, g(x) \Big]_a^b - \int_a^b f(x)\, g'(x)\, dx \qquad (2)$$

<u>Dim</u> La (1) si ricava da

$$(f\, g)' = f'\, g + f\, g'$$

$$f'\, g = (f\, g)' - f\, g'$$

$$\int f'(x)\, g(x)\, dx = \int \left[(f(x) g(x))' - f(x) g'(x) \right] dx$$

$$= \int (f(x)\, g(x))'\, dx - \int f(x)\, g'(x)\, dx$$

$$= f(x)\, g(x) - \int f(x)\, g'(x)\, dx$$

$$\int_a^b f'(x)\, g(x)\, dx = \int f'(x)\, g(x)\, dx \Big]_a^b \overset{(1)}{=}$$

$$= \left[f(x)\, g(x) - \int f(x)\, g'(x)\, dx \right] \Big]_a^b$$

$$= f(x)\, g(x) \Big]_a^b - \int f(x)\, g'(x)\, dx \Big]_a^b$$

$$\left((\lambda F(x) + \mu G(x)) \right]_a^b = \lambda F(x) \Big]_a^b + \mu G(x) \Big]_a^b$$

$$= f(x)\, g(x) \Big]_a^b - \int_a^b f(x)\, g'(x)\, dx .$$

$$\int f'(x)\, g(x)\, dx = f(x)\, g(x) - \int f(x)\, g'(x)\, dx \qquad (1)$$

Dato un intervallo $[a,b]$ ed $f, g \in C^1([a,b])$ vale

$$\int_a^b f'(x)\, g(x)\, dx = f(x)\, g(x) \Big]_a^b - \int_a^b f(x)\, g'(x)\, dx \qquad (2)$$

$$\int x\, e^x\, dx = \int x\, (e^x)' = x\, e^x - \int (x)'\, e^x$$

$$= x\, e^x - \int e^x = x\, e^x - e^x + C$$

$$\int x^{100}\, e^x\, dx = \int x^{100} (e^x)'\, dx =$$

$$= x^{100}\, e^x - \int (x^{100})'\, e^x\, dx =$$

$$= x^{100}\, e^x - 100 \int x^{99}\, e^x\, dx$$

$$\int x\, e^{2x}\, dx = \int x \left(\frac{e^{2x}}{2}\right)' dx =$$

$$= x\, \frac{e^{2x}}{2} - \frac{1}{2} \int e^x\, dx$$

$$\int x \sin x \, dx = \qquad \int f' g = fg - \int f g'$$

$$= \int x \, (-\cos x)' \, dx$$

$$= -x \cos x + \int (x)' \cos x \, dx$$

$$= -x \cos x + \int \cos x \, dx = -x \cos x + \sin x + C$$

$$\int e^{x} \sin x \, dx$$

$$\int e^{x} e^{3x} \, dx = \int e^{4x} \, dx$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \qquad \operatorname{sh} x = \frac{e^{x} - e^{-x}}{2}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \operatorname{ch} x = \frac{e^{x} + e^{-x}}{2}$$

$$\int f'g = fg - \int f g'$$

$$\boxed{\int e^x \sin x \, dx}$$

$$= \int (e^x)' \sin x \, dx =$$

$$= e^x \sin x - \int e^x \cos x \, dx \qquad \int f'g = fg - \int fg'$$

$$= e^x \sin x - \left[\int (e^x)' \cos x \, dx\right]$$

$$= e^x \sin x - \left[e^x \cos x - \int e^x \underbrace{(\cos x)'}_{-\sin x}\right]$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x .$$

$$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x$$

$$2 \int e^x \sin x \, dx = e^x \sin x - e^x \cos x + C$$

$$\int e^x \sin x = \frac{e^x \sin x - e^x \cos x}{2} + C$$

$$\int e^{2x} \cos(3x) \, dx$$

$$\int f'g = fg - \int f g'$$

$$\int \lg x \, dx =$$

$$= \int 1 \lg x = \int (x)' \lg x =$$

$$= x \lg x - \int \cancel{x} \frac{1}{\cancel{x}} = x \lg x - \int 1 =$$

$$= x \lg x - x + c$$

$$\int f' g = f g - \int f g'$$

$$\int \arctan x \, dx =$$

$$\int (x)' \arctan x \, dx =$$

$$= x \arctan x - \int x \arctan' x \, dx$$

$$= x \arctan x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx$$

$$= x \arctan x - \frac{1}{2} \log(1+x^2) + C$$

Lemma (Cambio di variabile)

Siano I e J due intervalli,

$u(x): I \longrightarrow J$ e $f(u): J \to \mathbb{R}$

$u \in C^1(I)$, $f \in C^0(J)$. Denotiamo con

$\int f(u)\, du$ le primitive di f. Vale

$$\int f(u(x))\, u'(x)\, dx = \left(\int f(u)\, du\right)(u(x)) \quad (1)$$

[Lo si trova scritto $\int f(u(x))\, u'(x)\, dx = \int f(u)\, du$]

Dim. di (1).

$$\frac{d}{dx}\int f(u(x))\, u'(x)\, dx = f(u(x))\, u'(x)$$

$$\frac{d}{dx}\left(\int f(u)\, du\right)(u(x)) = \left(\frac{d}{du}\int f(u)\, du\right)(u(x))\; u'(x)$$

$$= f(u(x))\, u'(x)$$

$$\int f(u(x))\, u'(x)\, dx = \int f(u)\, du$$

$$\int f(u)\, du = \int f(u) \frac{du}{dx}\, dx$$

$$\int \tan x\, dx = \int \frac{\sin x}{\cos x}\, dx$$

$= -du$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\, dx$

per $u =$

$$= -\int \frac{1}{u}\, du = -\log|u| + c$$

$$= -\log|\cos x| + c$$

$$x = u + 1$$
$$u = x - 1$$
$$du = dx$$

$$\int \frac{x+1}{x-1}\,dx$$

$$= \int \frac{u+2}{u}\,du = \int \left(1 + \frac{2}{u}\right) du$$

$$= \int 1\,du + 2\int u^{-1}\,du$$

$$= u + 2\lg|u| + C = x - 1 + 2\lg|x-1| + C$$

$\sin^{2n+1}(x) \cos^m(x)\, dx$

$\mathbb{R}(\cos x, \sin x)$

$\int \cos^{2n+1}(x) \sin^m(x)\, dx$

$\boxed{u = \sin x}$

$3x^2y^7 + 2xy^4 + 5xy$

$= \int \cos^{2n}(x) \sin^m(x) \cos x\, dx$

$= \int (1-\sin^2 x)^n \sin^m(x) \boxed{\cos x\, dx}$

$u = \sin x$

$du = \boxed{\cos x\, dx}$

$= \int (1-u^2)^n u^m\, du$

$$\int \cos^2(x)\ \sin^2(x)\ dx =$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\sin^2(x) = \frac{1-\cancel{\sin}\cos(2x)}{2}$$

$$= \frac{1}{4}\int \left(1+\cos(2x)\right)\left(1-\cos(2x)\right) dx =$$

$$= \frac{1}{4}\int \left(1-\cos^2(2x)\right) dx$$

$$= \frac{x}{4} - \frac{1}{4}\int \cos^2(2x)\ dx \qquad \cos^2(2x) = \frac{1+\cos(4x)}{2}$$

$$= \frac{x}{4} - \frac{1}{8}\int \left(1+\cos(4x)\right) dx =$$

$$= \frac{x}{4} - \frac{x}{8} - \frac{1}{8}\underbrace{\int \cos(4x)\ dx}_{\frac{\sin(4x)}{4} + c}$$

$$\int \cos(4x)\, dx = \qquad u = 4x$$

$$du = 4\, dx$$

$$= \frac{1}{4}\int \cos u\ dx$$

$$= \frac{1}{4}\sin(u) + c = \frac{1}{4}\sin(4x) + C.$$