# Image Processing for Physicists

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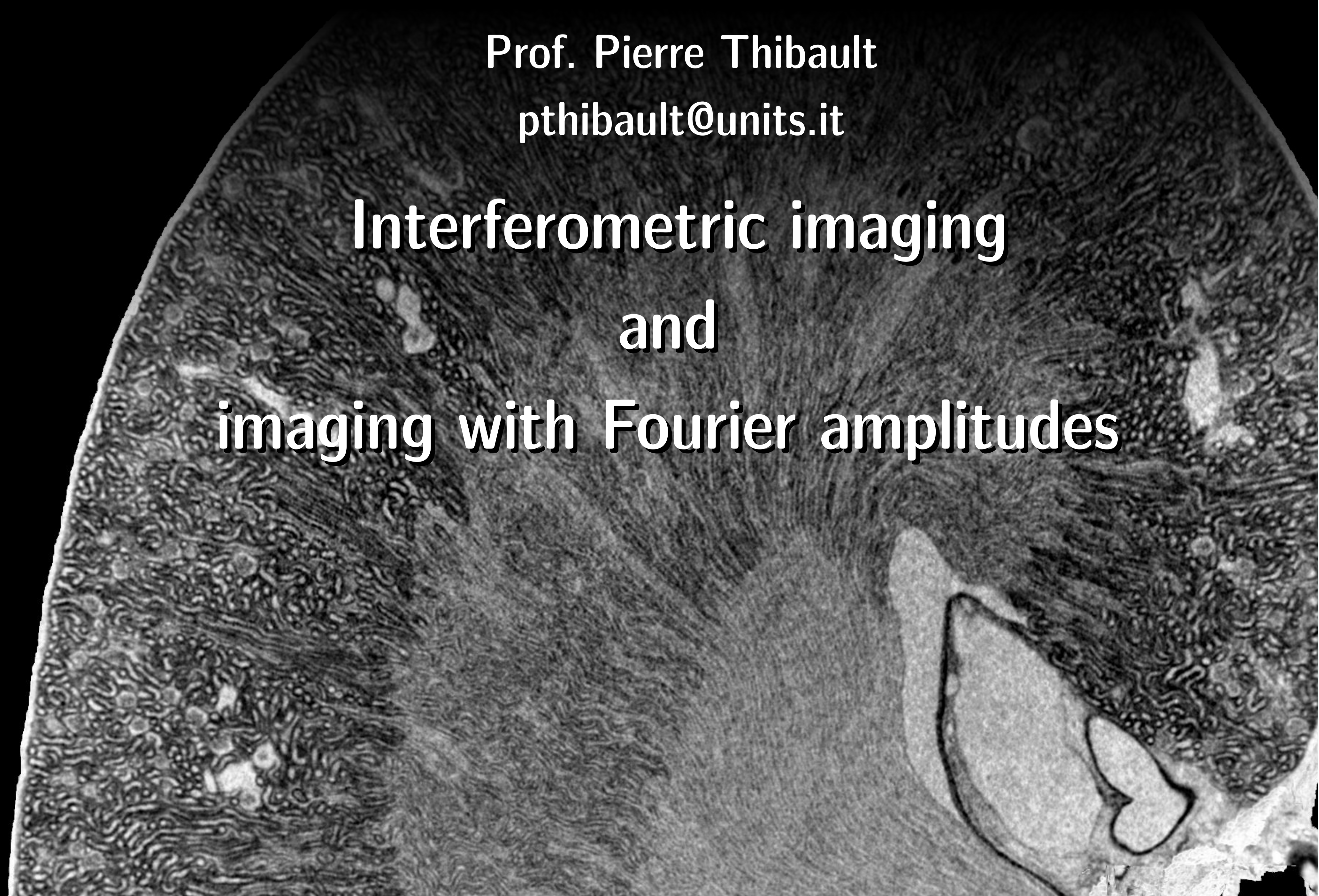
## Interferometric imaging

## and

## imaging with Fourier amplitudes



# Overview

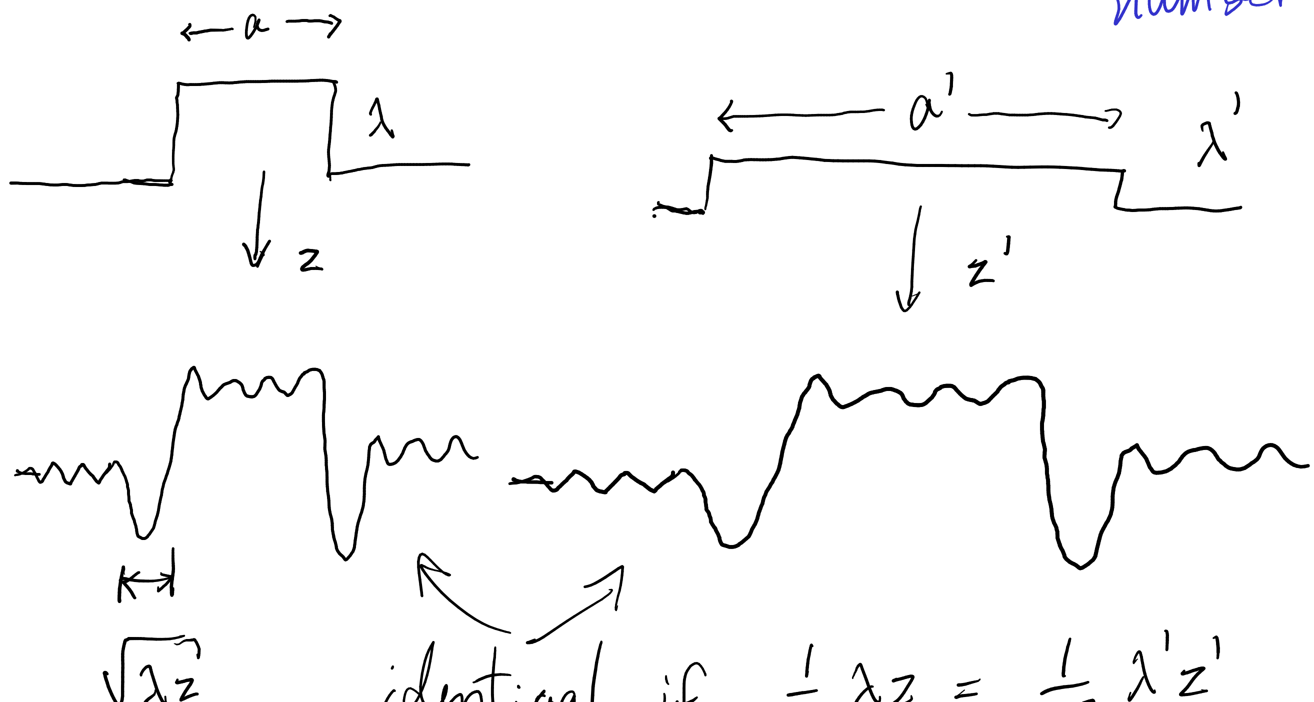
- The phase problem
- Holography: on/off-axis
- Grating interferometric imaging
- Imaging using far-field amplitude measurements
  - Fourier transform holography
  - Coherent diffraction imaging
  - Ptychography

# Wave propagation

Reminder: angular spectrum formulation

$$\psi(r;z) = \mathcal{F}^{-1}\left\{\mathcal{F}\{\psi(r;z=0)\}\exp\left(i\pi \underline{u^2\lambda z}\right)\right\}$$

unitless number




$$\frac{a^2}{\lambda z} = f$$

Fresnel number

$f \ll 1$ : far-field

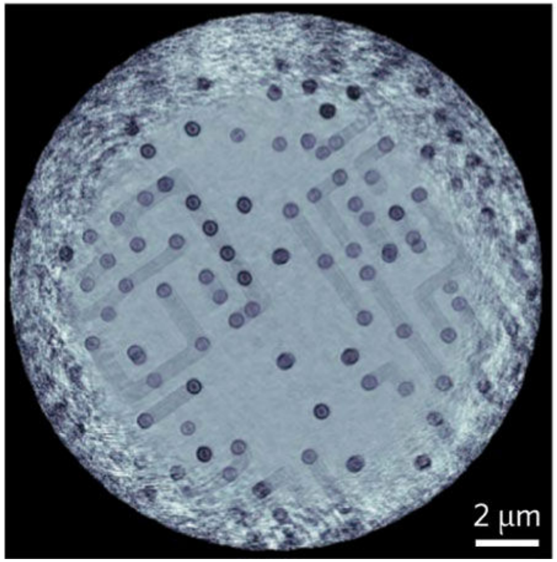
$f \gg 1$ : near-field

identical if $\frac{1}{a^2}\lambda z = \frac{1}{a'^2}\lambda' z'$

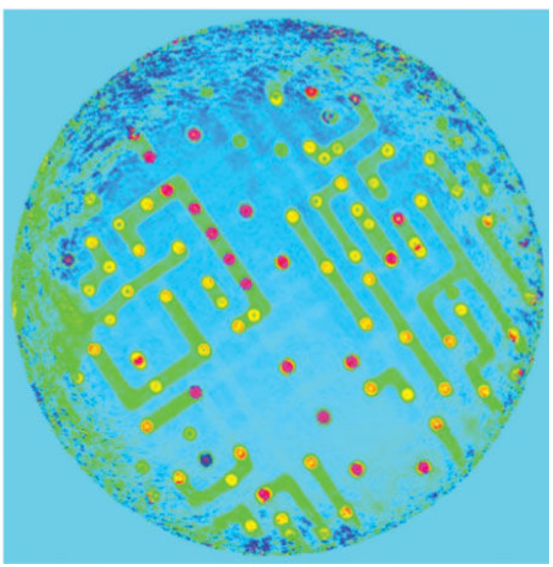
$\sqrt{\lambda z}$ = characteristic length

# Complex-valued images

X-ray transmission image




Amplitude
attenuation
of the wave



phase
delay in
the wavefield
(refraction)

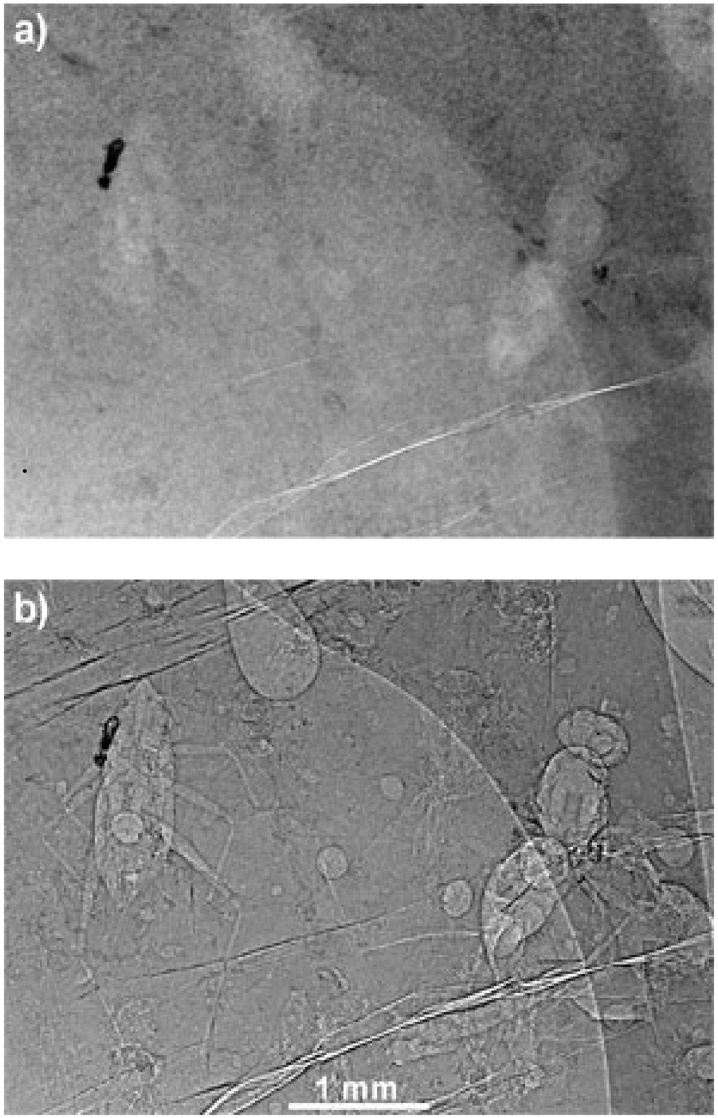
synthetic aperture radar

phase
unwrapping

phase
image

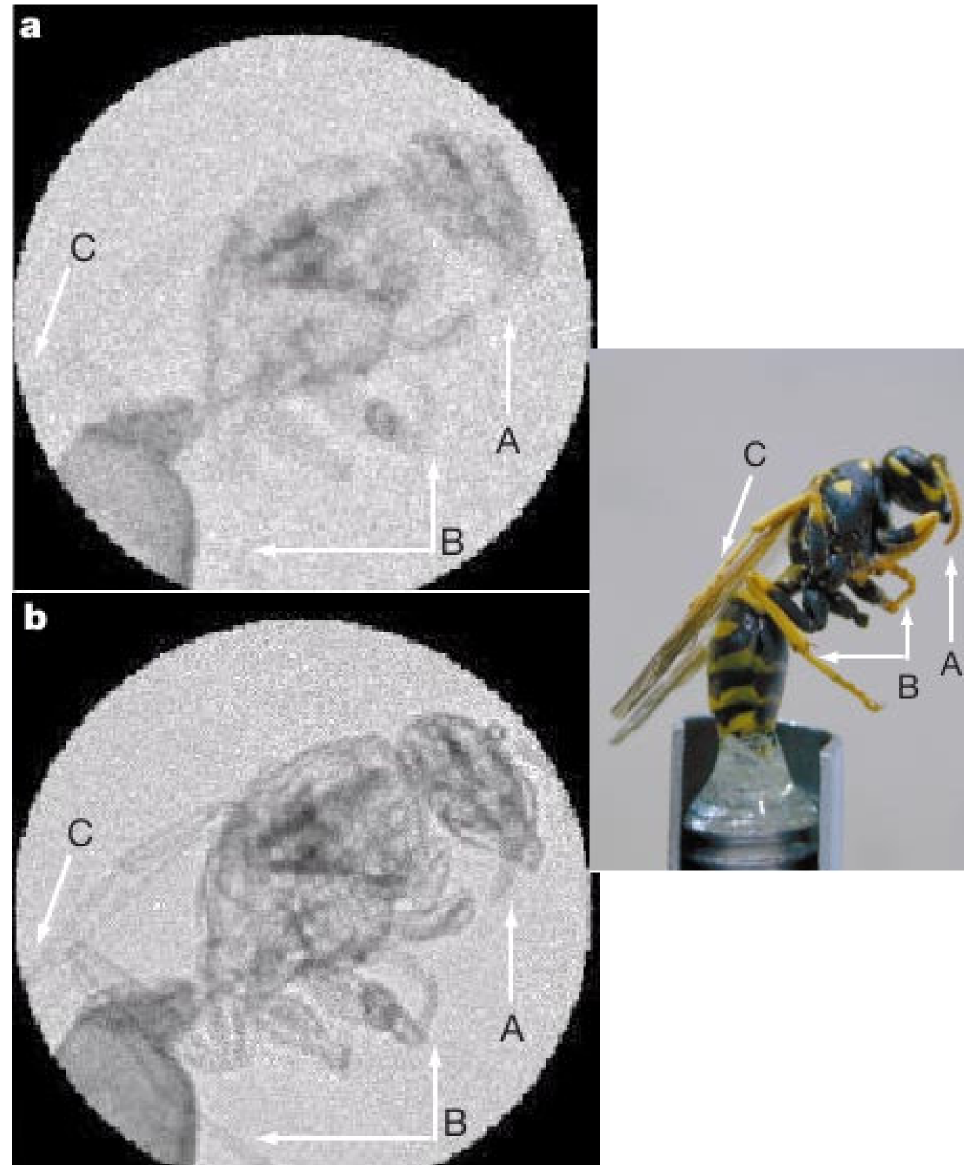
# Phase-contrast

## Hard X-ray propagation-based phase contrast



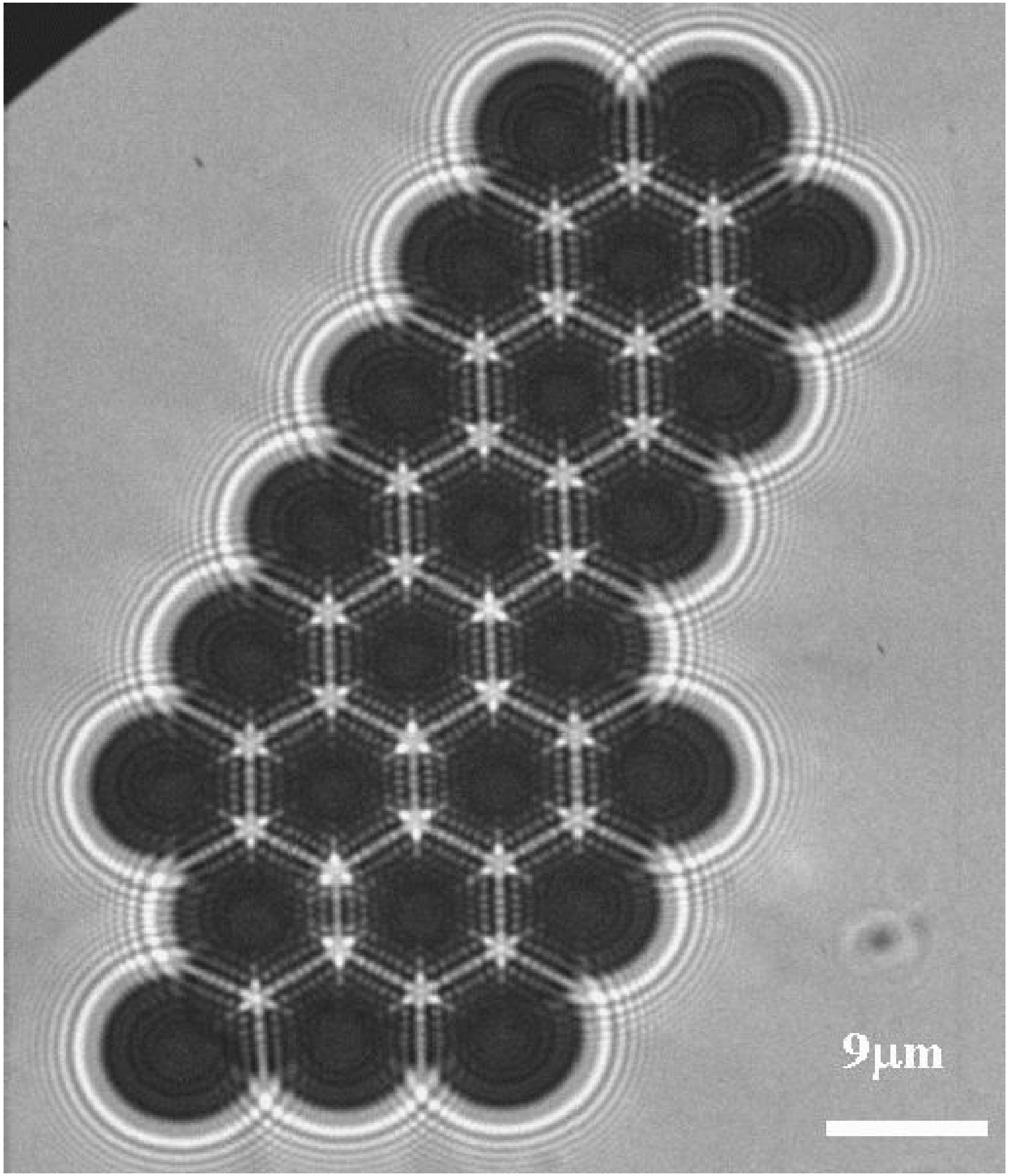

Source:
www.esrf.eu/news/general/amber/amber/

## Neutron phase contrast




Source: Allman et al. Nature **408** (2000).

# Inline holography




Source: Mayo et al. Opt Express **11** (2003).

# Inline holography

Measure $I(\vec{r}) = |\psi(r;z)|^2$

* plane monochromatic wave
* weak transmission of imaged object

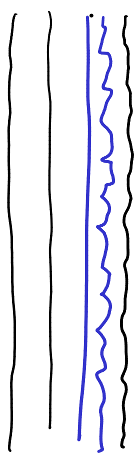
small perturbation

$$\psi(r; z=0) = A\left(1+\varepsilon(\vec{r}, z=0)\right)$$

object plane

constant (plane wave)

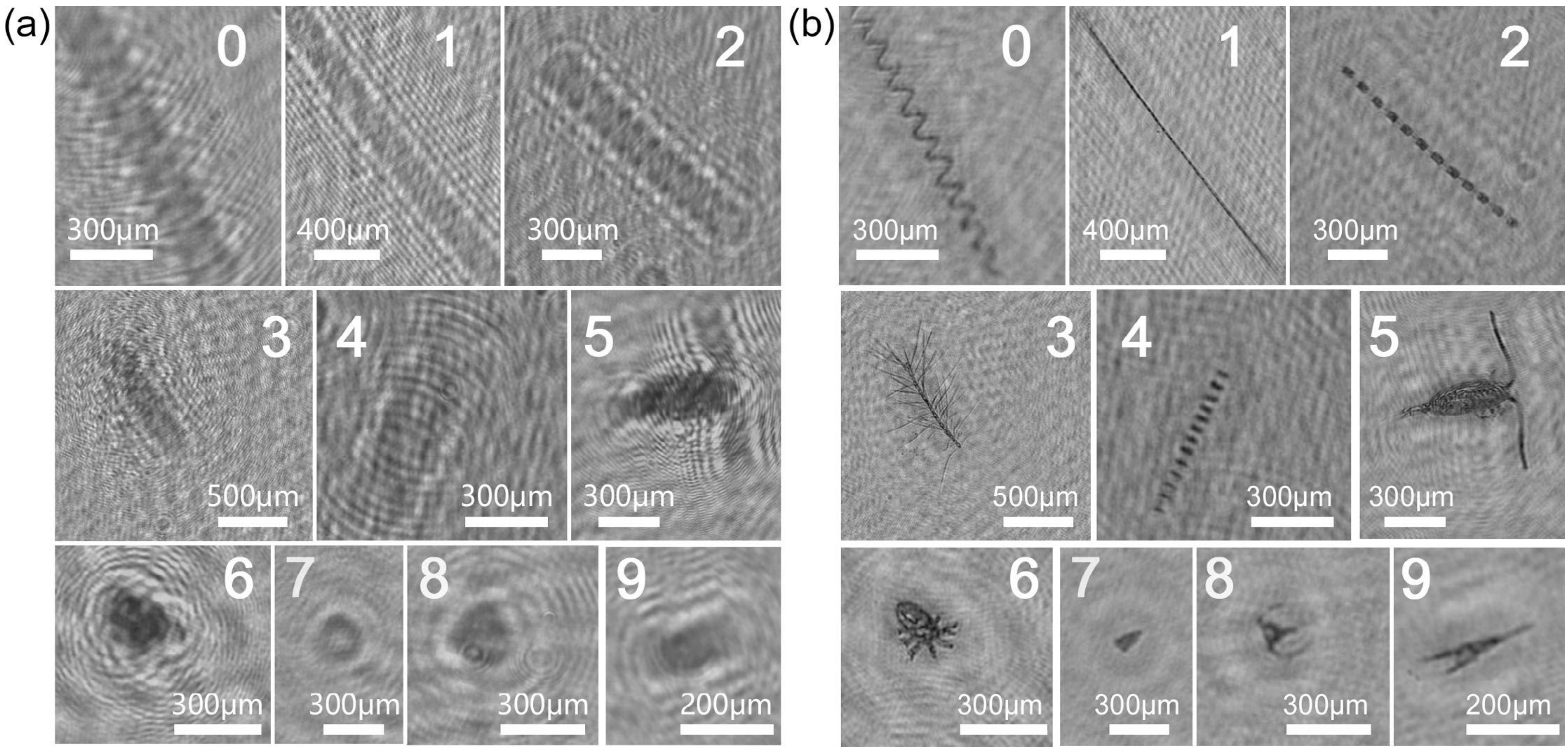
$$I(\vec{r}, z) = \left|A\left(1+\varepsilon(r;z)\right)\right|^2 = |A|^2\left(1+\varepsilon(r;z)+\varepsilon^*(r,z)+\cancel{|\varepsilon(r;z)|^2}\right)$$



$$\simeq |A|^2\Big(1+\underbrace{\varepsilon(\vec{r};z)}_{\text{propagated by } z}+\underbrace{\varepsilon^*(r;z)}_{\text{equivalent to propagation by } -z}\Big)$$

"twin image" problem

# Digital inline holography




trick: apply angular spectrum propagator

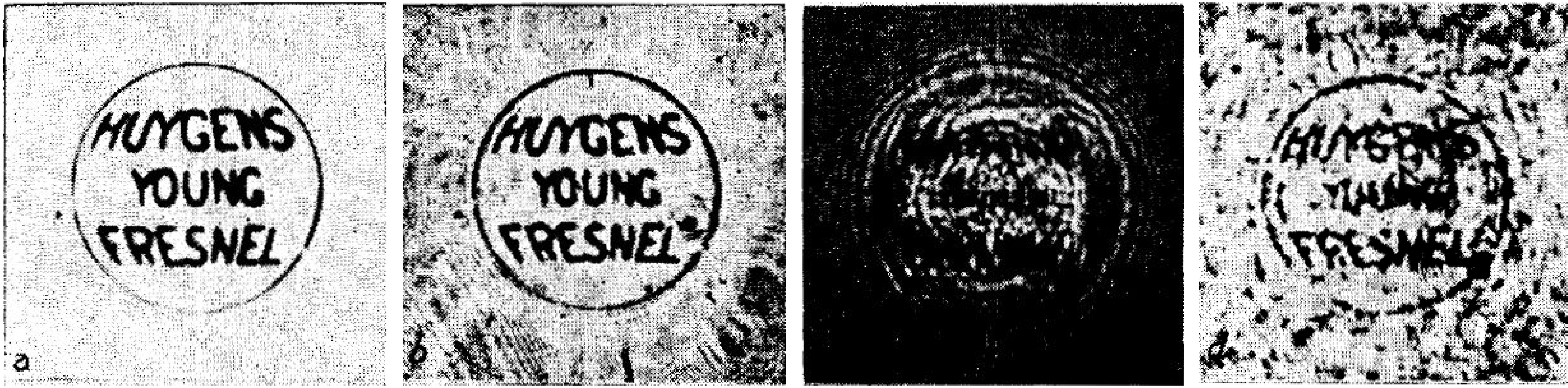
# The phase problem

We always measure $|\psi|^2$. phases are lost.

We need to recover the phase part of the wavefield

- sometimes the phase is interesting
- most often: the phase is an auxiliary quantity for proper interpretation of the wavefield.

# In-line holography




D. Gabor, Nature **161**, 777-778 (1948).

mask

"in focus"

after propagation

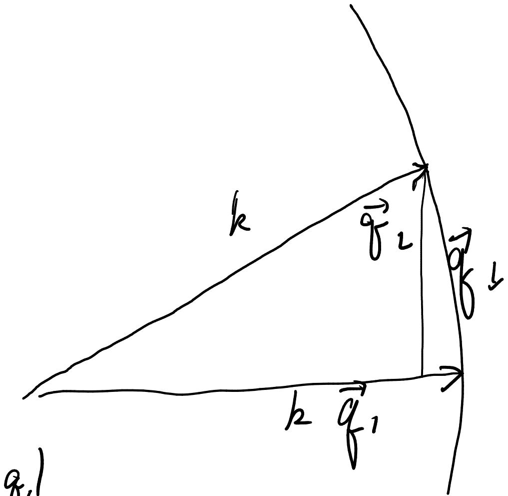
propagated

twin image
degrades quality

# Tilted reference

two tilted plane waves:

$$e^{i\vec{q}_1\cdot\vec{r}} + e^{i\vec{q}_2\cdot\vec{r}} = e^{ikz}\Big(1 + \underbrace{e^{i\vec{q}_\perp\cdot\vec{r}_\perp}}_{\text{propagates at angle } \sin\theta = \frac{|q_\perp|}{k}}\Big)$$



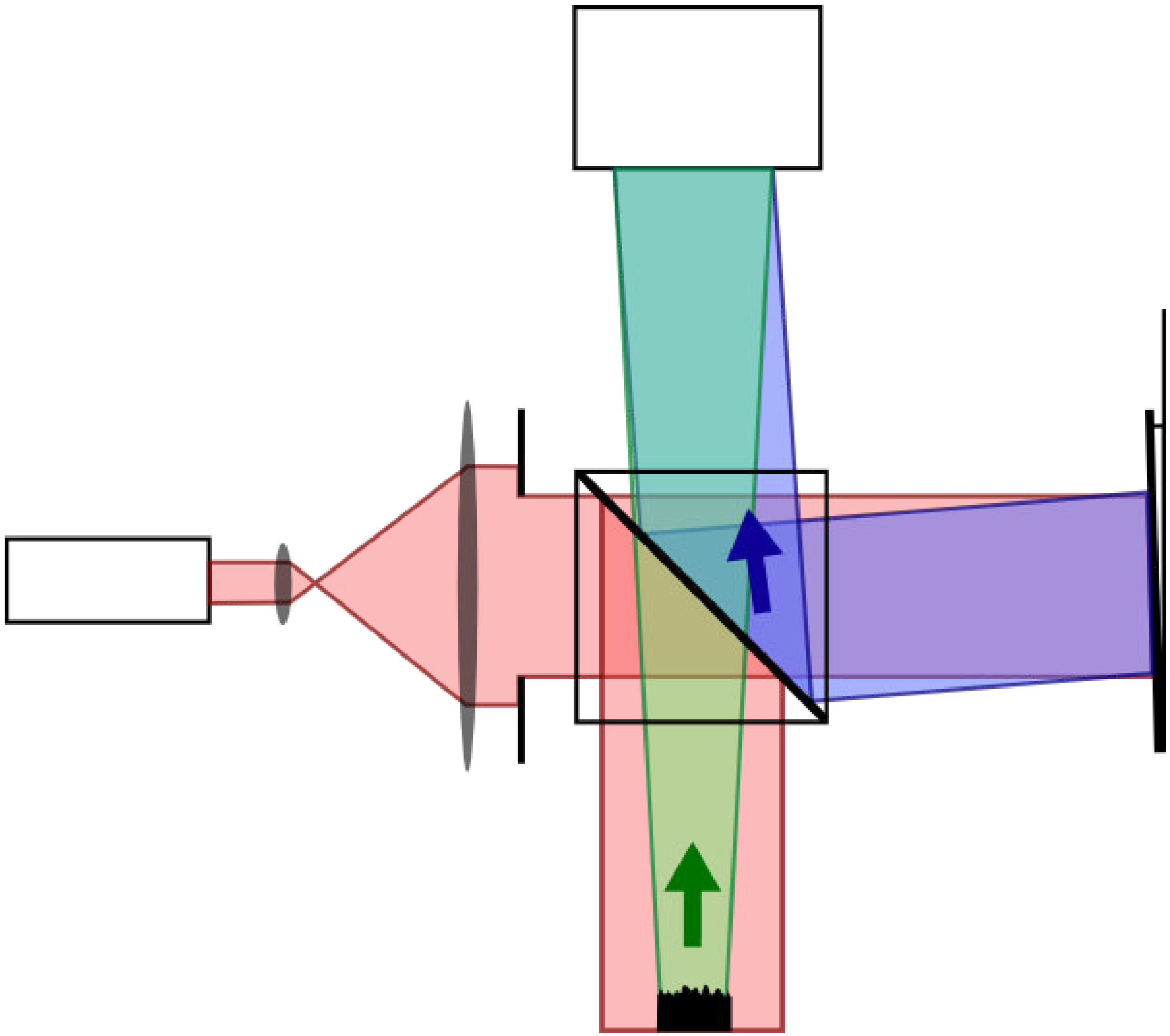

$$I = \left|1 + e^{iq_\perp\cdot r}\right|^2 = 1 + e^{iq\cdot r} + e^{-iq\cdot r} + 1 = 2 + 2\cos(\vec{q}\cdot\vec{r})$$

oscillations ("fringes") with spatial frequency

$$\vec{u} = \frac{\vec{q}_\perp}{2\pi}$$

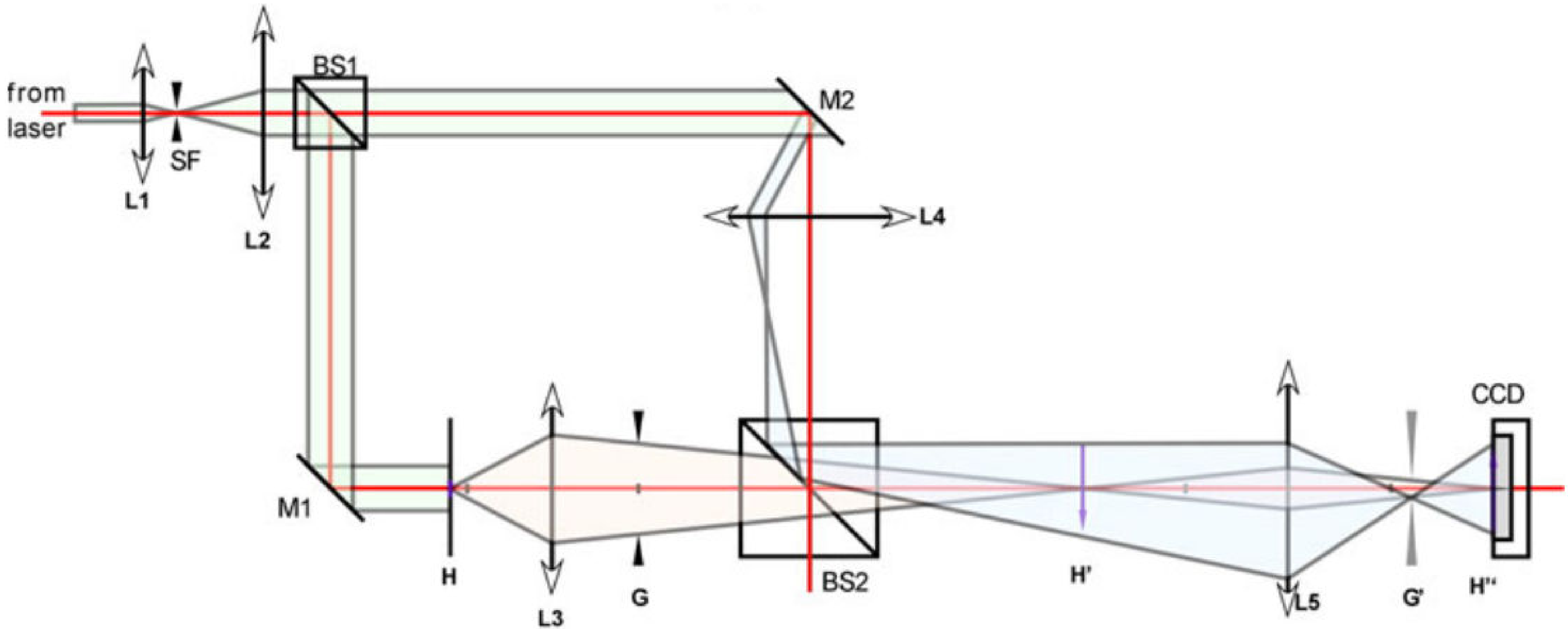
with a sample: $\left|a(\vec{r}) + e^{i\vec{q}'\cdot\vec{r}}\right|^2$

# Fringe interferometry



Twyman-Green interferometer

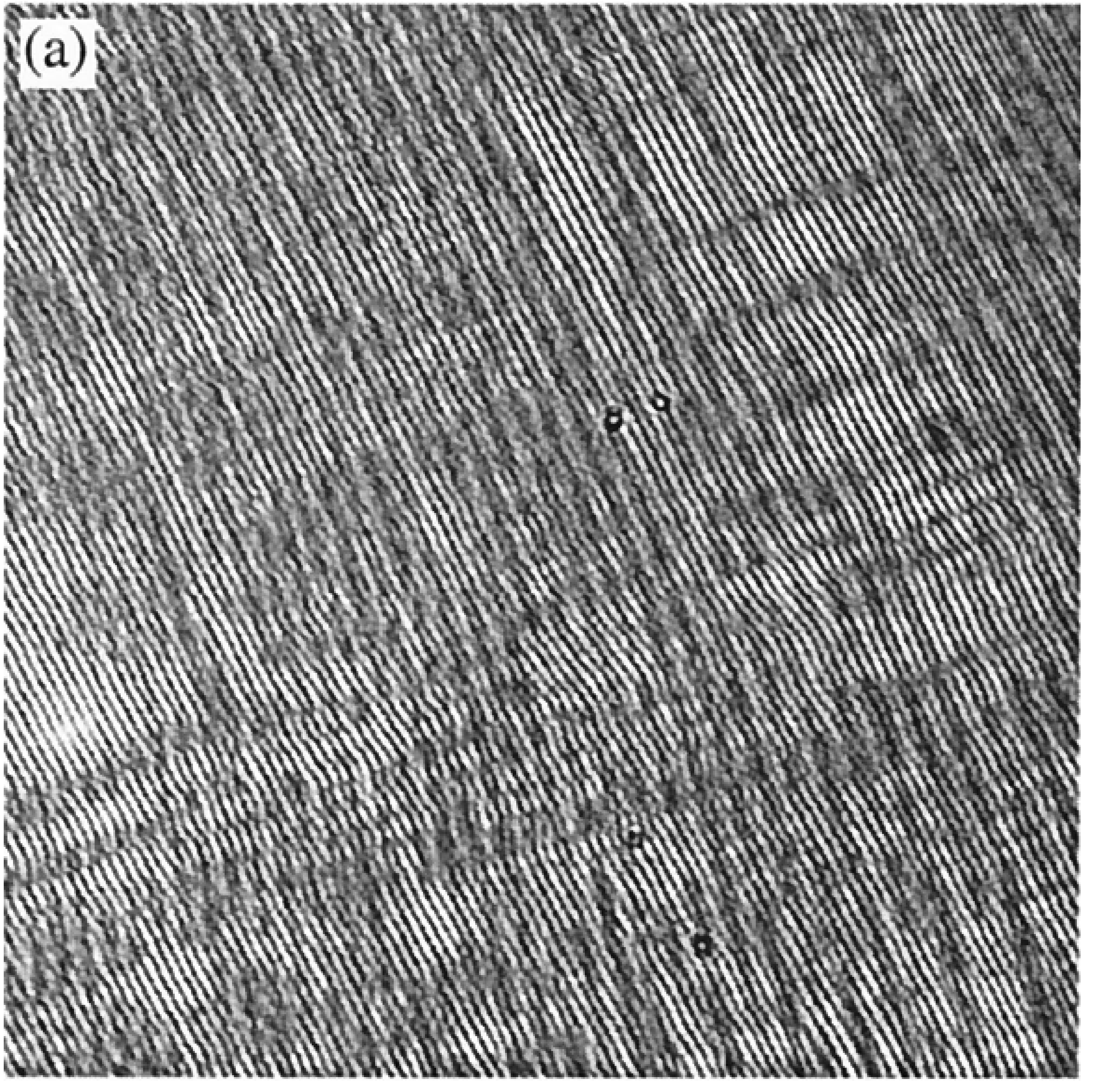
# Visible light interferometer




Mach-Zehnder interferometer

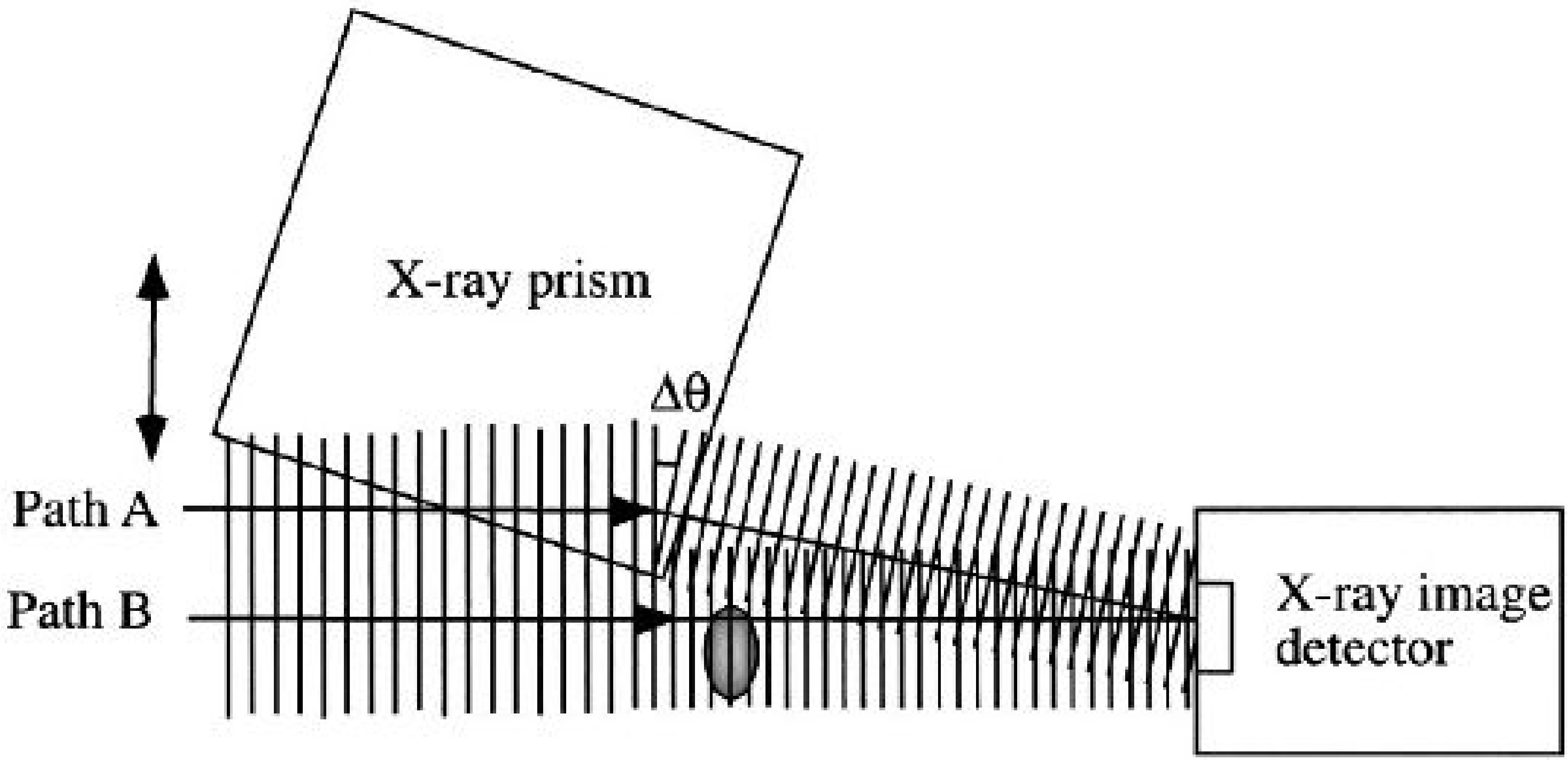
Source: M. K. Kim, SPIE Rev. **1**, 018005 (2010).

# Fringe interferometry




Source: Cuche et al. Appl. Opt. **39**, 4070 (2000)

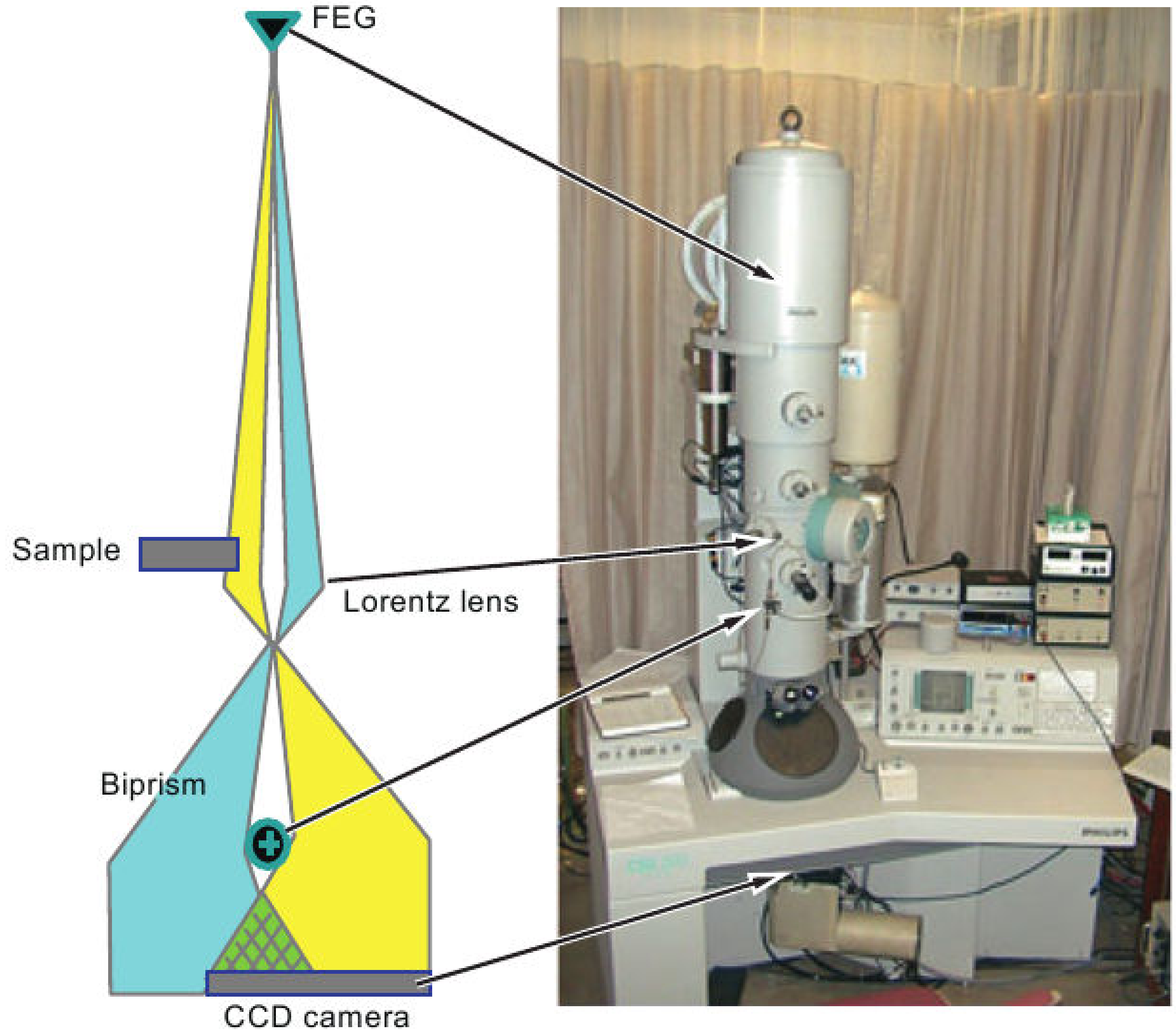
# Off-axis X-ray holography




Source: Y. Kohmura, J. Appl. Phys. **96**, 1781-1784 (2004)

# Off-axis electron holography

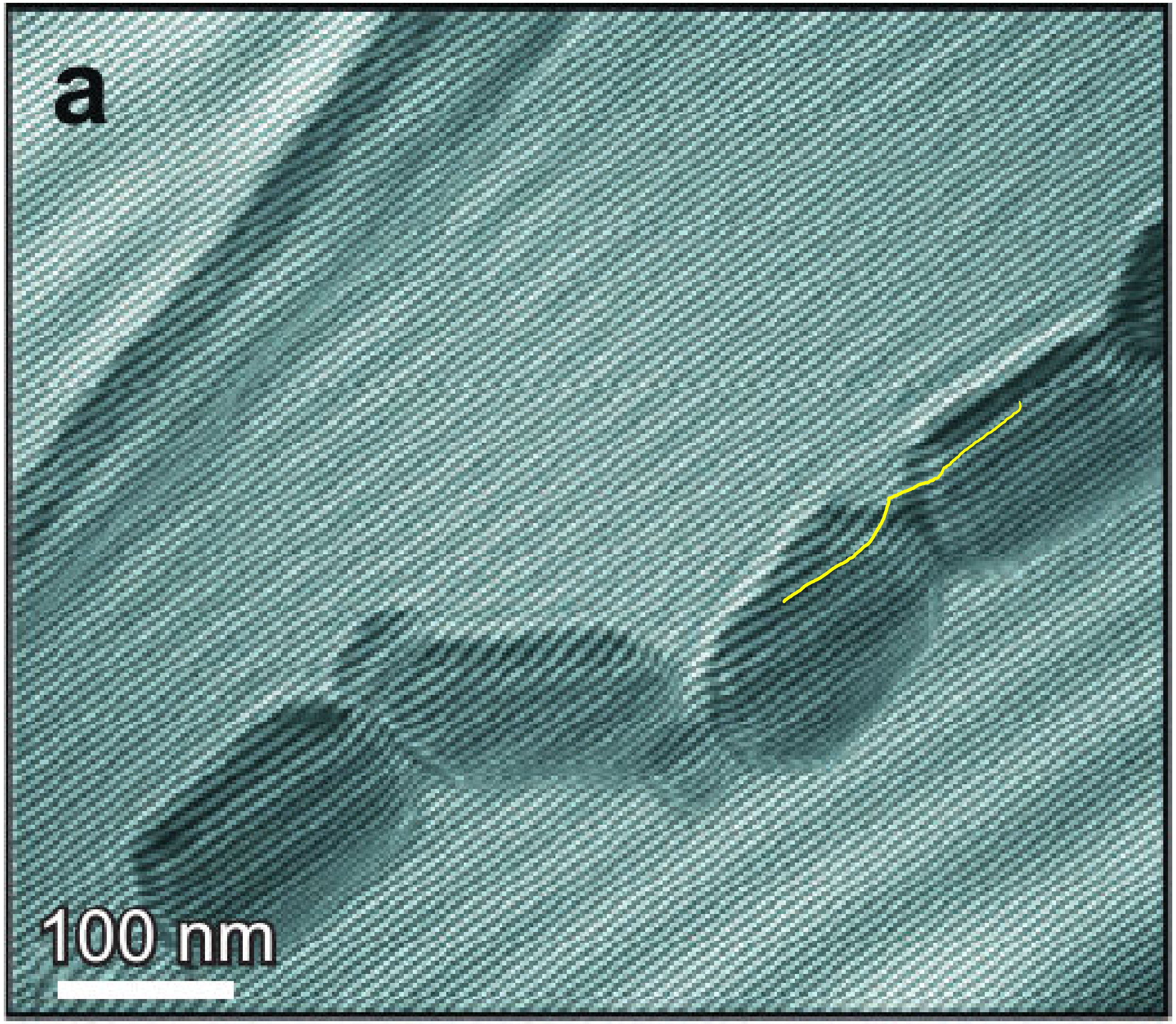
## Electron microscopy




**Hologram** **Philips CM200 FEG**

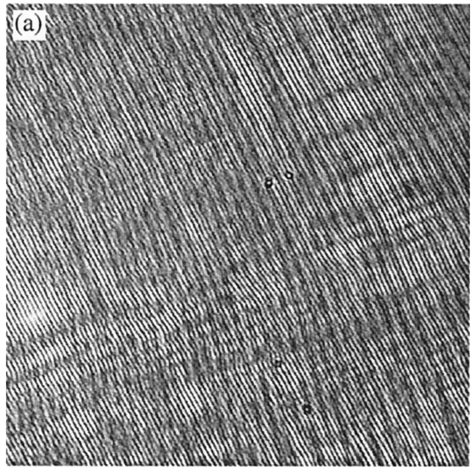
Source: M. R. McCartney, Ann. Rev. Mat. Sci. **37** 729-767 (2007)

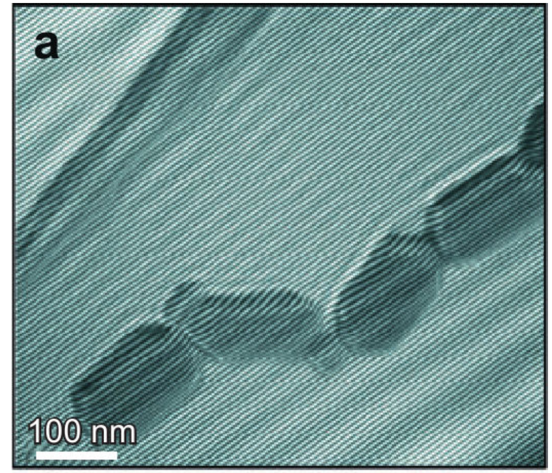
# Off-axis electron holography




Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

# Fringe interferometry




$a \to$ attenuation

$\varphi \to$ phase shift (refraction)

$$\underset{\text{object}}{\psi = \psi_o} + \underset{\text{reference}}{\psi_r}$$

$$\psi_r(\vec{r}) = A\, e^{i\vec{q}_\perp \cdot \vec{r}}$$

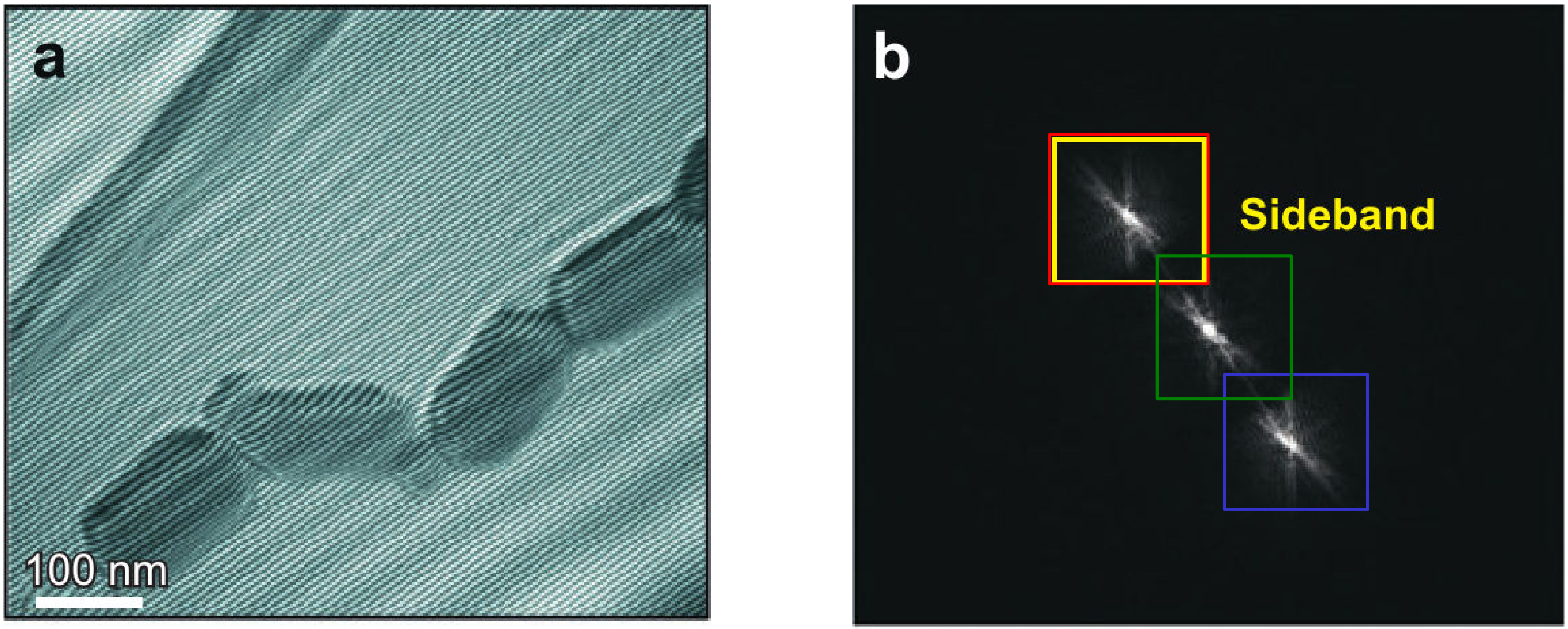
$$\psi_o(\vec{r}) = A\, a(\vec{r})\, e^{i\varphi(\vec{r})}$$

Measurement:

$$|\psi|^2 = (\psi_o + \psi_r)(\psi_o + \psi_r)^*$$

$$= |A|^2 \Big(1 + a^2(\vec{r}) + \overbrace{a(\vec{r})\, e^{i(\varphi - q_\perp \cdot r)} + a(\vec{r})\, e^{-i(\varphi - q_\perp \cdot r)}}^{2a(\vec{r}) \cos(\vec{q}_\perp \cdot \vec{r} - \varphi)}\Big)$$

# Off-axis holography

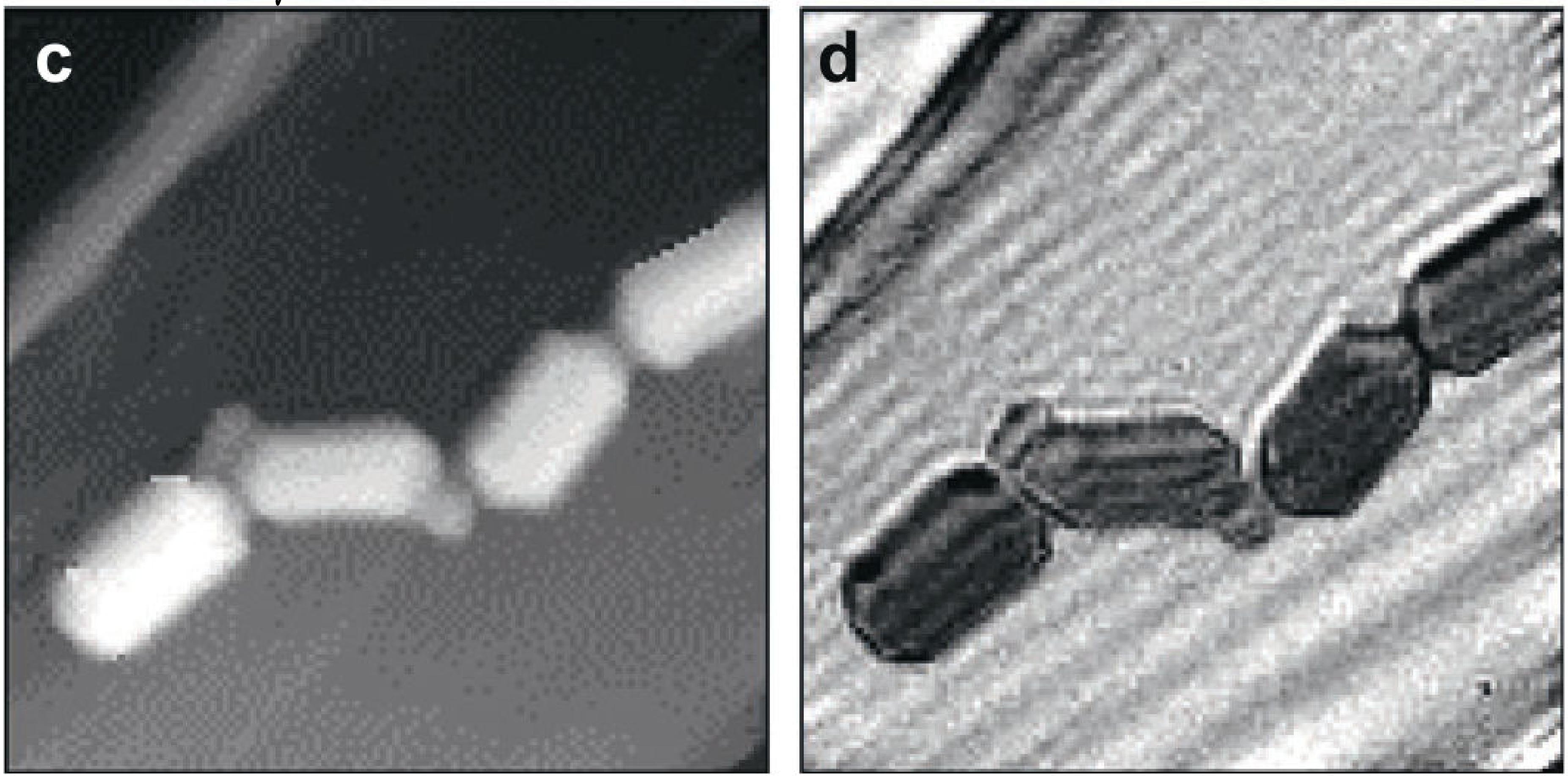



$$\mathcal{F}\{|\psi|^2\} = |A^2| \left[ \boxed{\mathcal{F}\{a^2(r)+1\}} + \boxed{\mathcal{F}\{\psi_0\}\left(\vec{u} + \frac{\vec{q}_\perp}{2\pi}\right)} \boxed{+ \mathcal{F}\{\psi_0^*\}\left(\vec{u} - \frac{\vec{q}_\perp}{2\pi}\right)} \right]$$

Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

# Off-axis holography

phase $\varphi$ attenuation (a)




Price paid: loss in resolution

Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

# Phase stepping

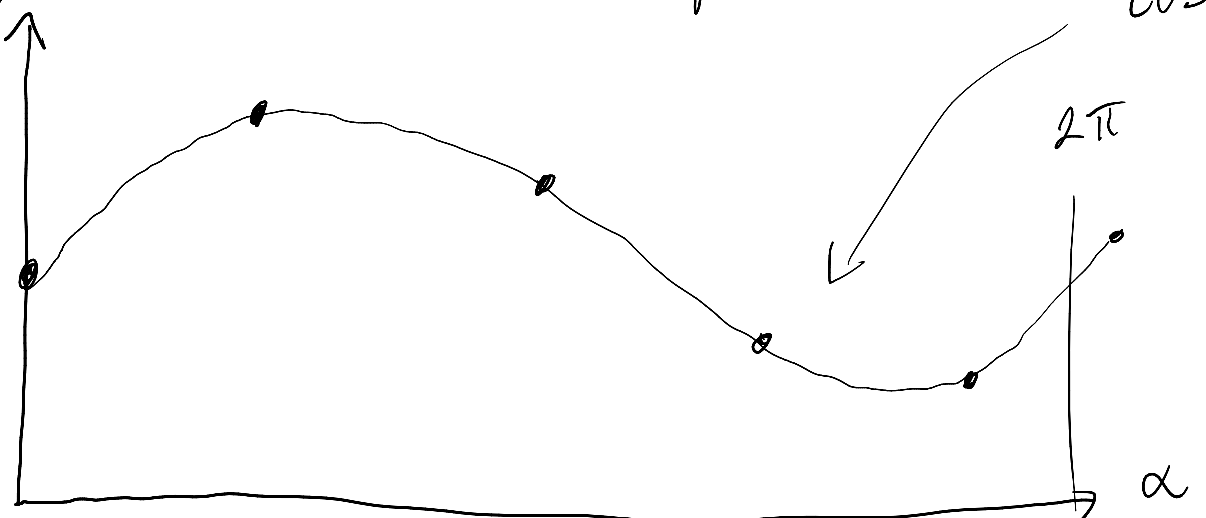
- Encoding phase **and** amplitude in a single image has a price: resolution

  $\rightarrow$ Take more than one image, changing the reference in each.

varies

$I(\vec{r})$ intensity of pixel $\vec{r}$

$\cos(\underbrace{\vec{q}_\perp \cdot \vec{r} - \varphi}_{\text{fixed for a given } \vec{r}} + \alpha)$

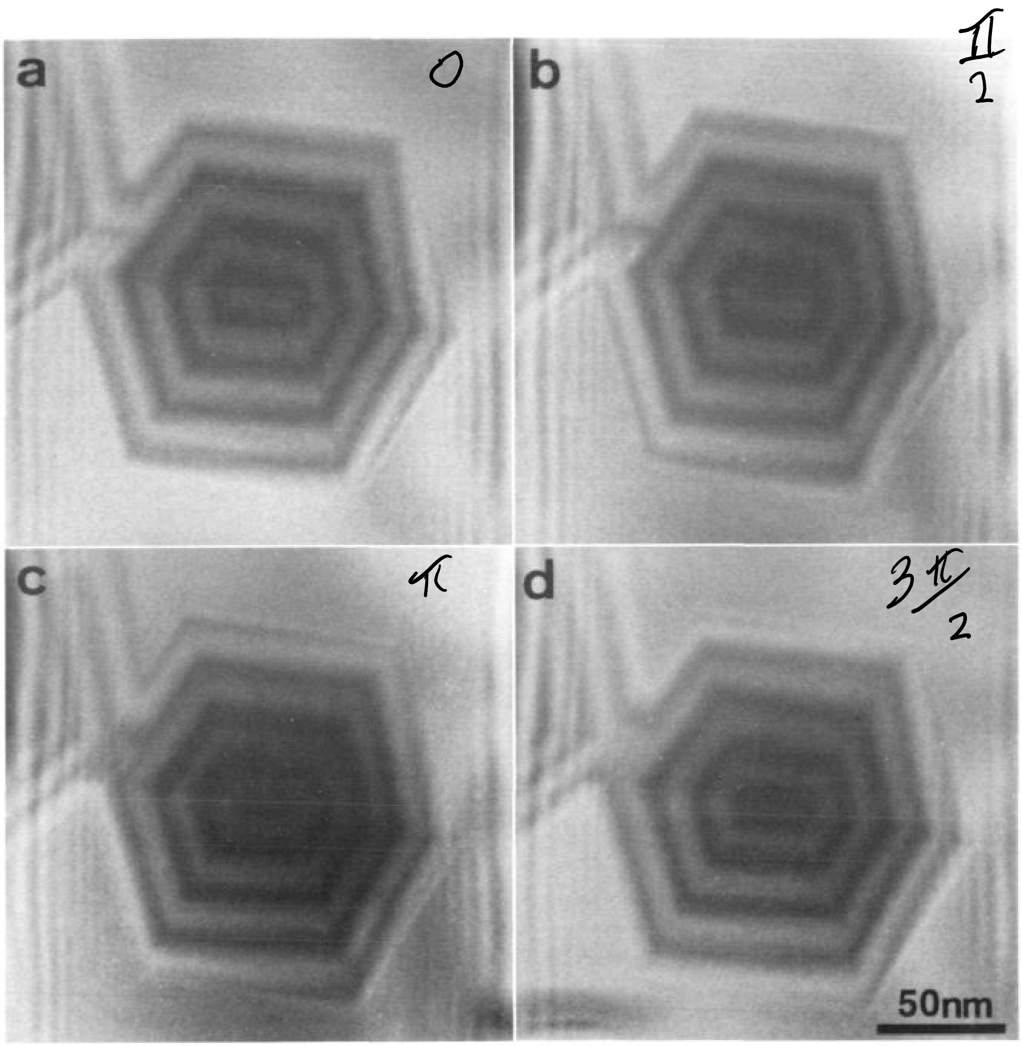



physical shift

$$\psi_r = A e^{i(q_\perp \cdot \vec{r} + \alpha)} = A e^{i(q_\perp \cdot (\vec{r} + \vec{\xi}))}$$

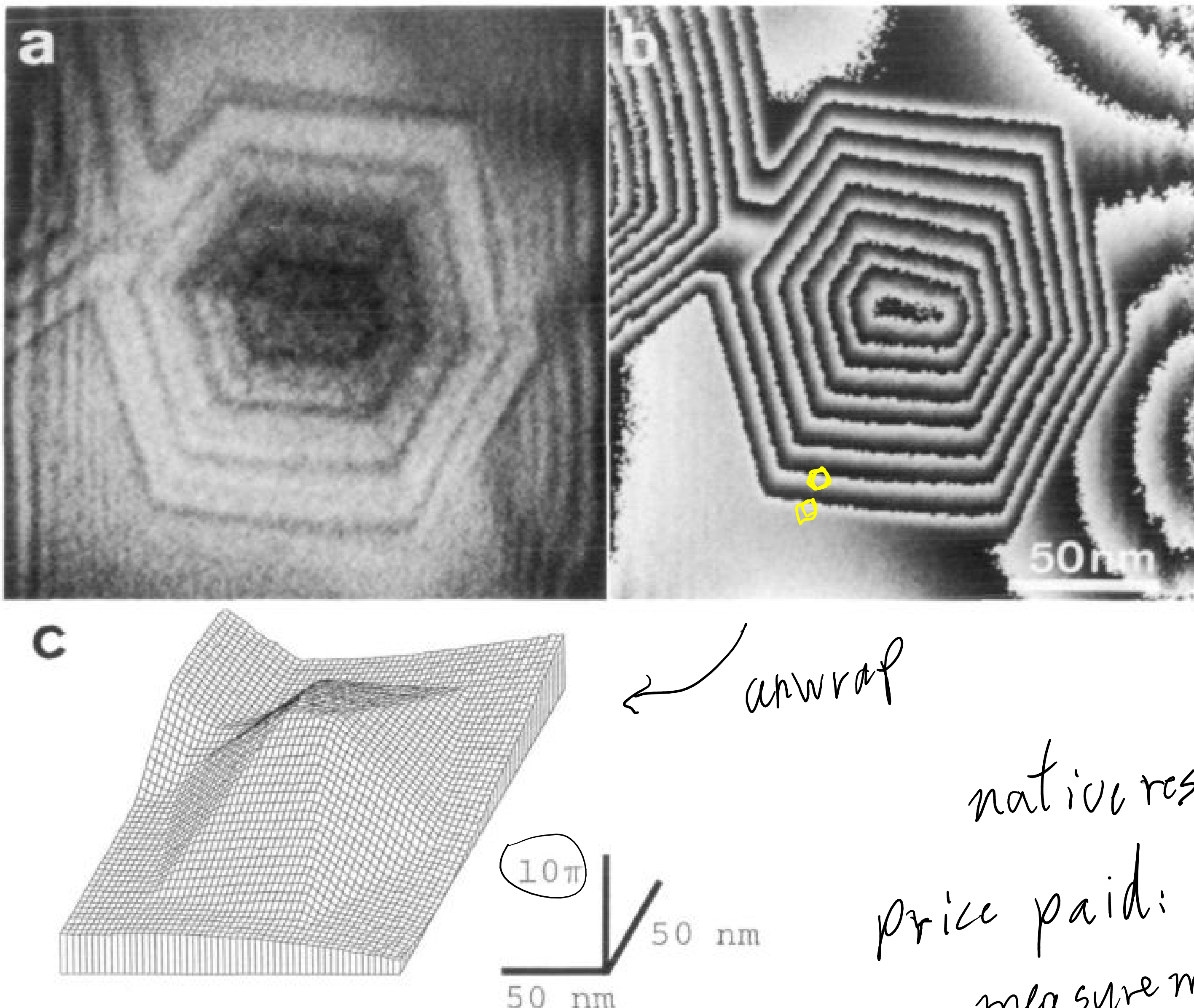
# Fringe scanning

Transmission
Electron
Microscopy




Source: K. Harada, J. Electron Microsc. **39** 470-476 (1990)

# Fringe scanning




unwrap

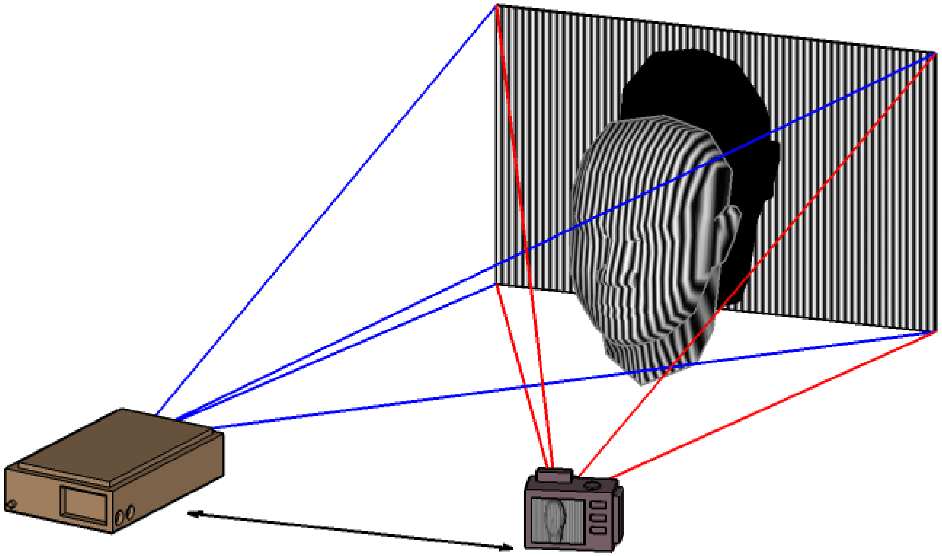
native resolution ✓

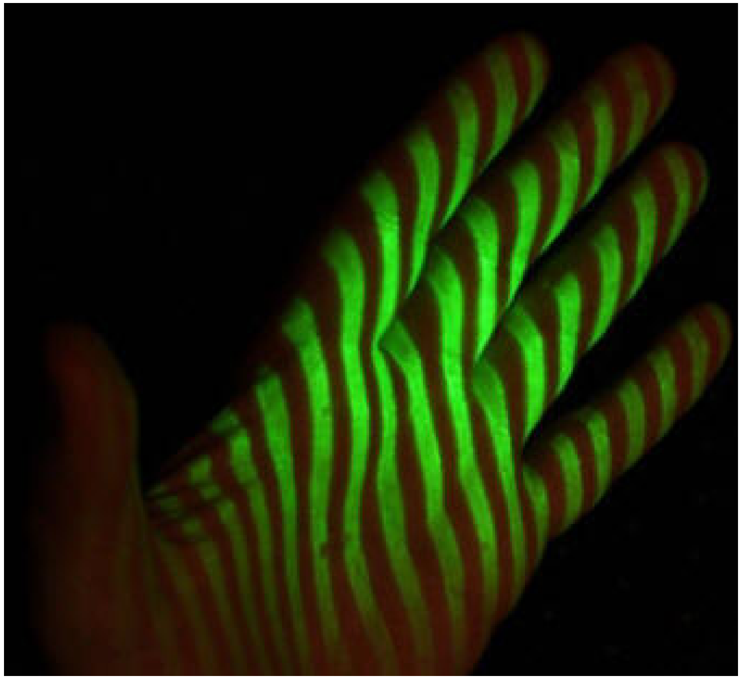
price paid: more measurements required

Source: K. Harada, J. Electron Microsc. **39** 470-476 (1990)

# Structured light sensing

- Project a structured light pattern onto sample
- Distortions of light pattern allow reconstruction of sample shape





# Phase unwrapping

- Phase is measured only in the interval [0, 2π)
- Physical phase shifts (which can be larger) are wrapped on this interval

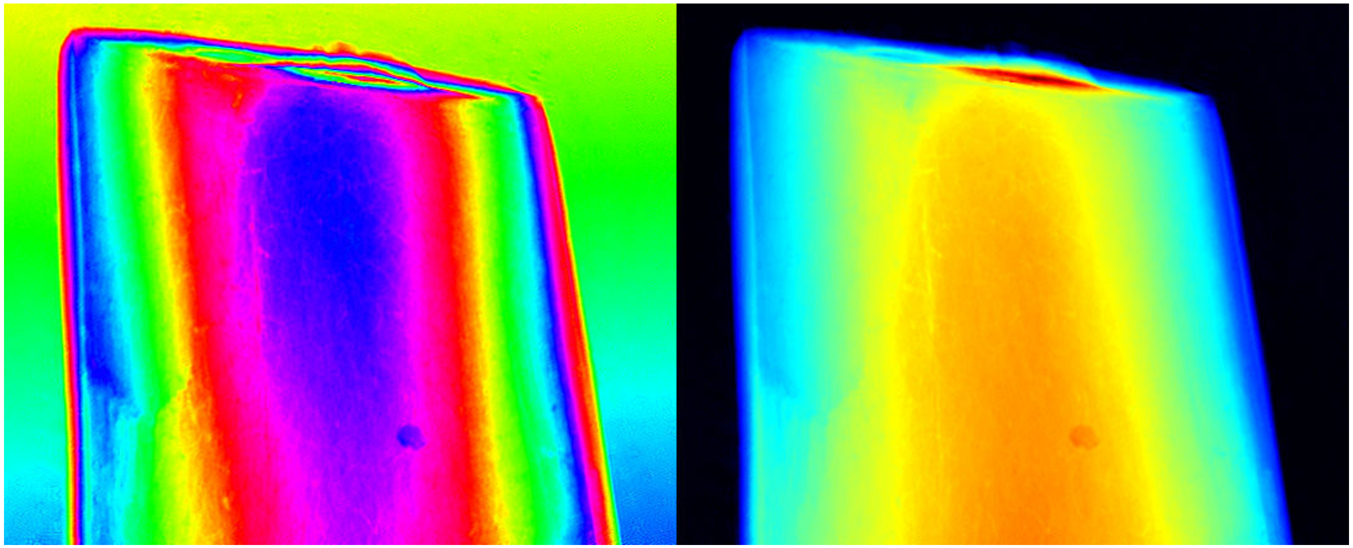
  $\rightarrow$ Any multiple of 2π is possible

- Unwrapping: use correlations in the image to guess the total phase shift.
- Main difficulties:
  - aliasing: phase shifts are too rapid for the image sampling
  - noise: produces local singularities (vortices)
- Many strategies exist

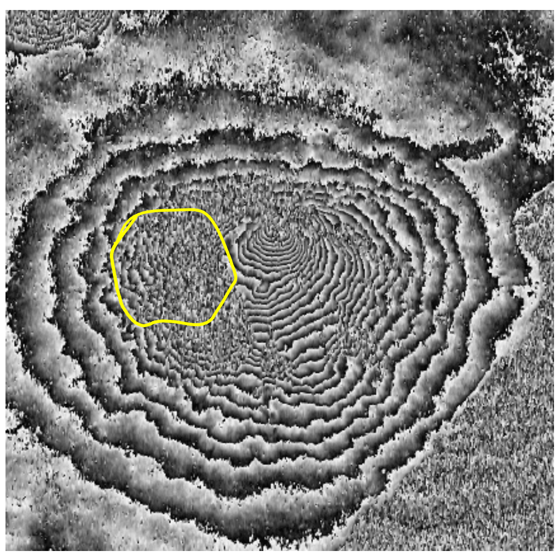
# Complex-valued images

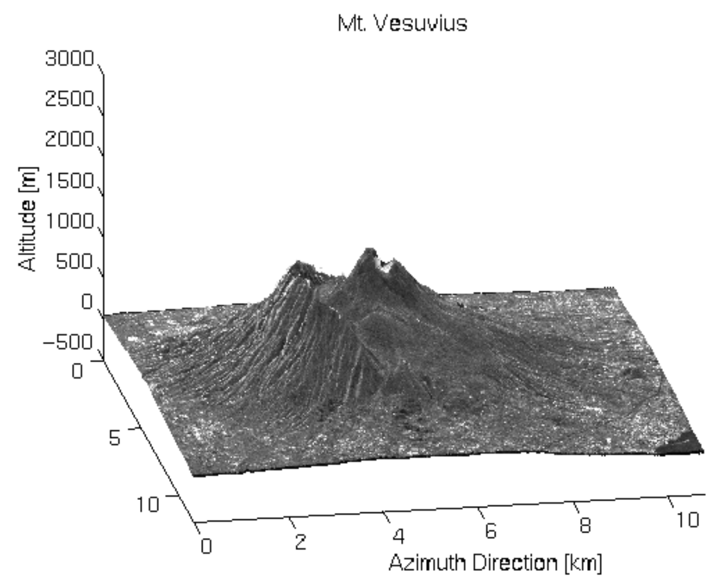
## Phase unwrapping

"easy"



aliasing



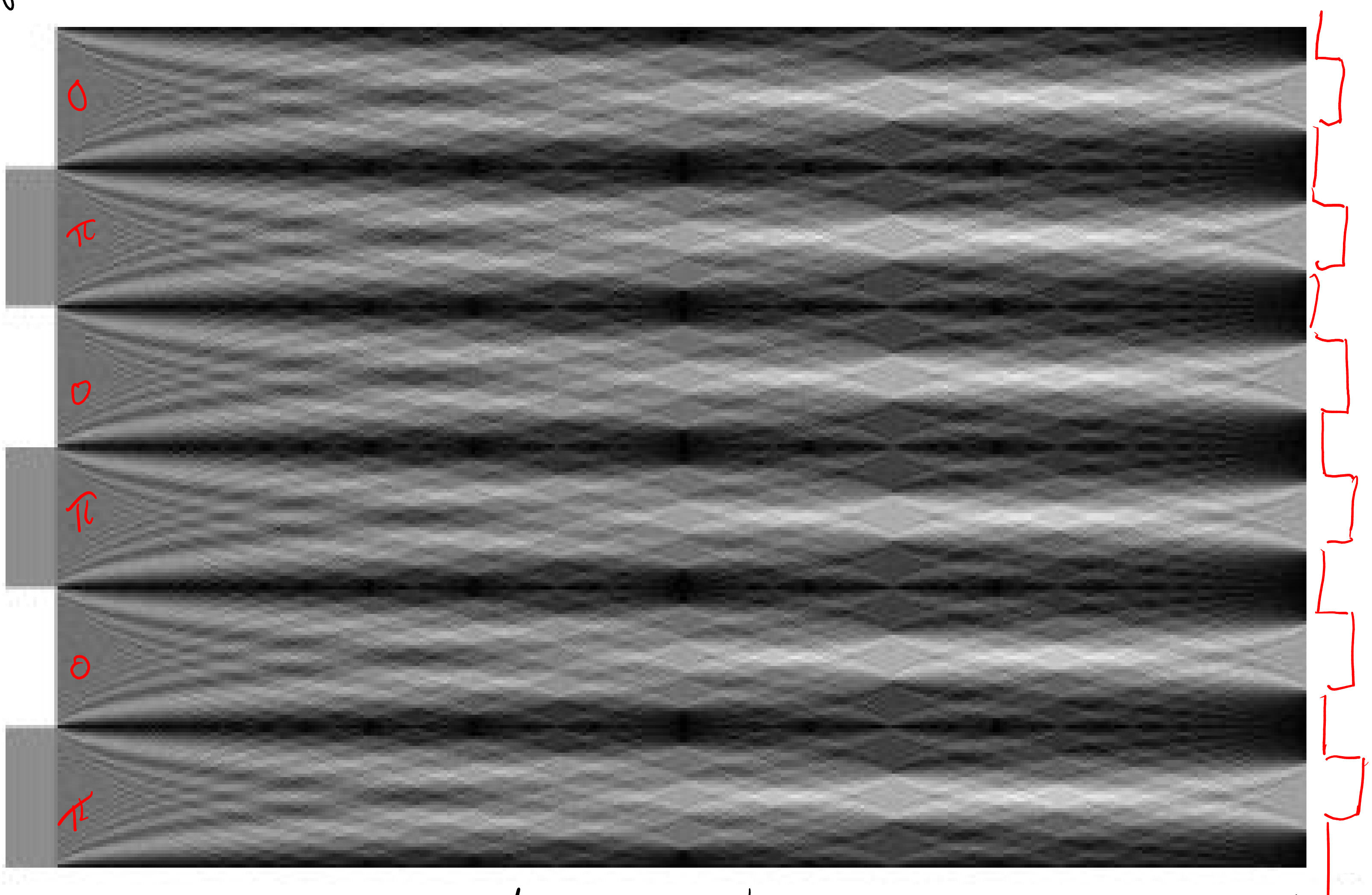



Source: http://earth.esa.int/workshops/ers97/program-details/speeches/rocca-et-al/

# Grating interferometry

phase grating

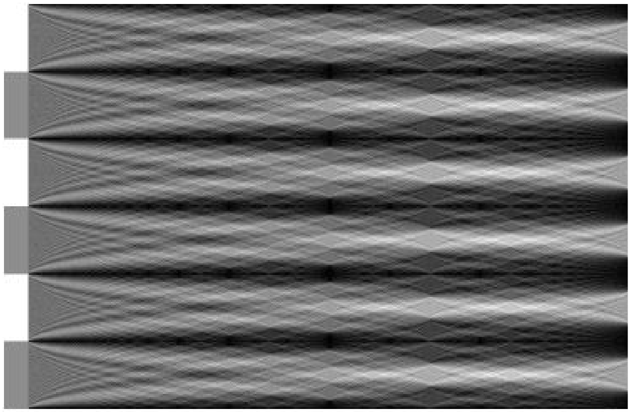
## Diffraction from a grating

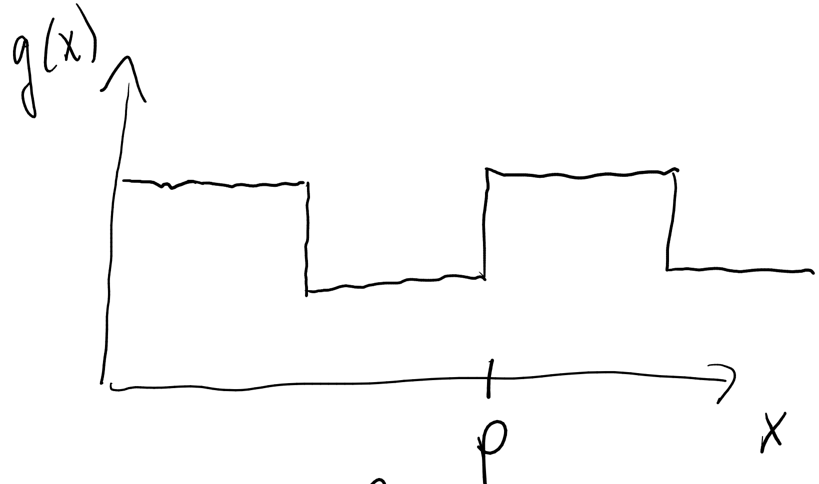



Talbot effect: periodic structure reforms periodically through propagation

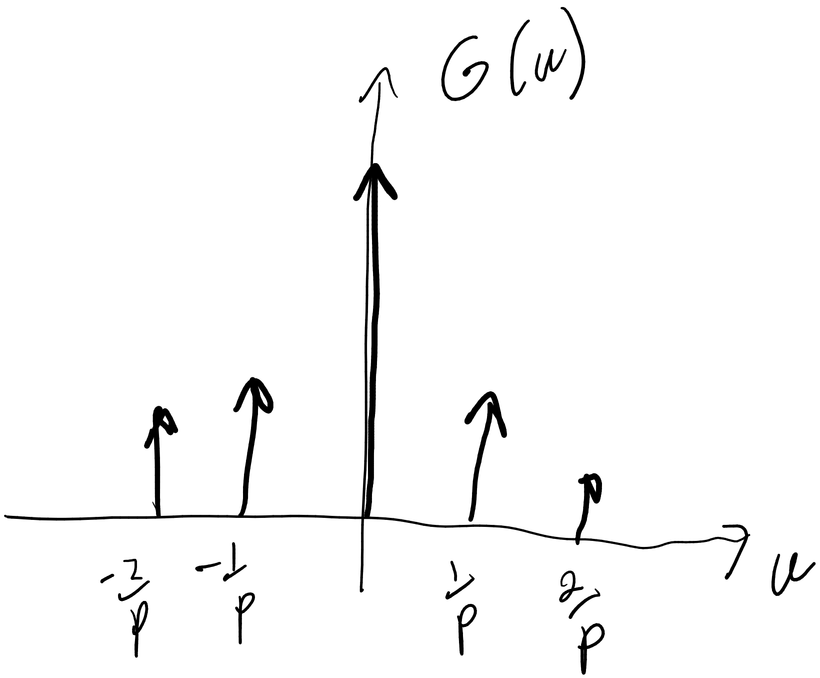
# Grating interferometry

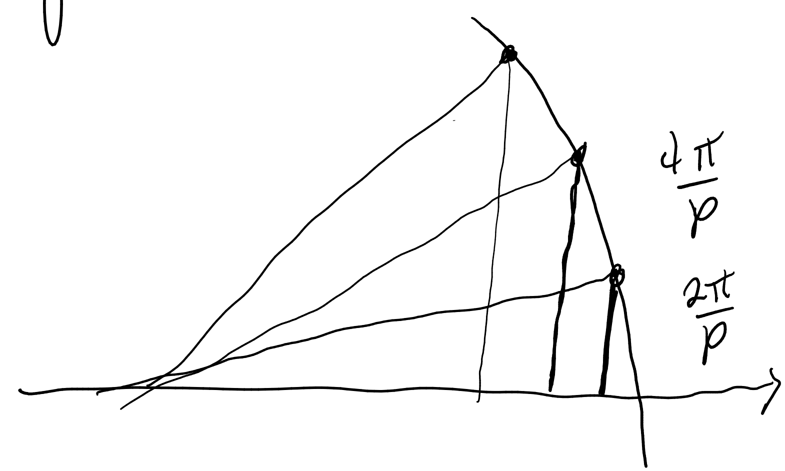
## Diffraction from a grating





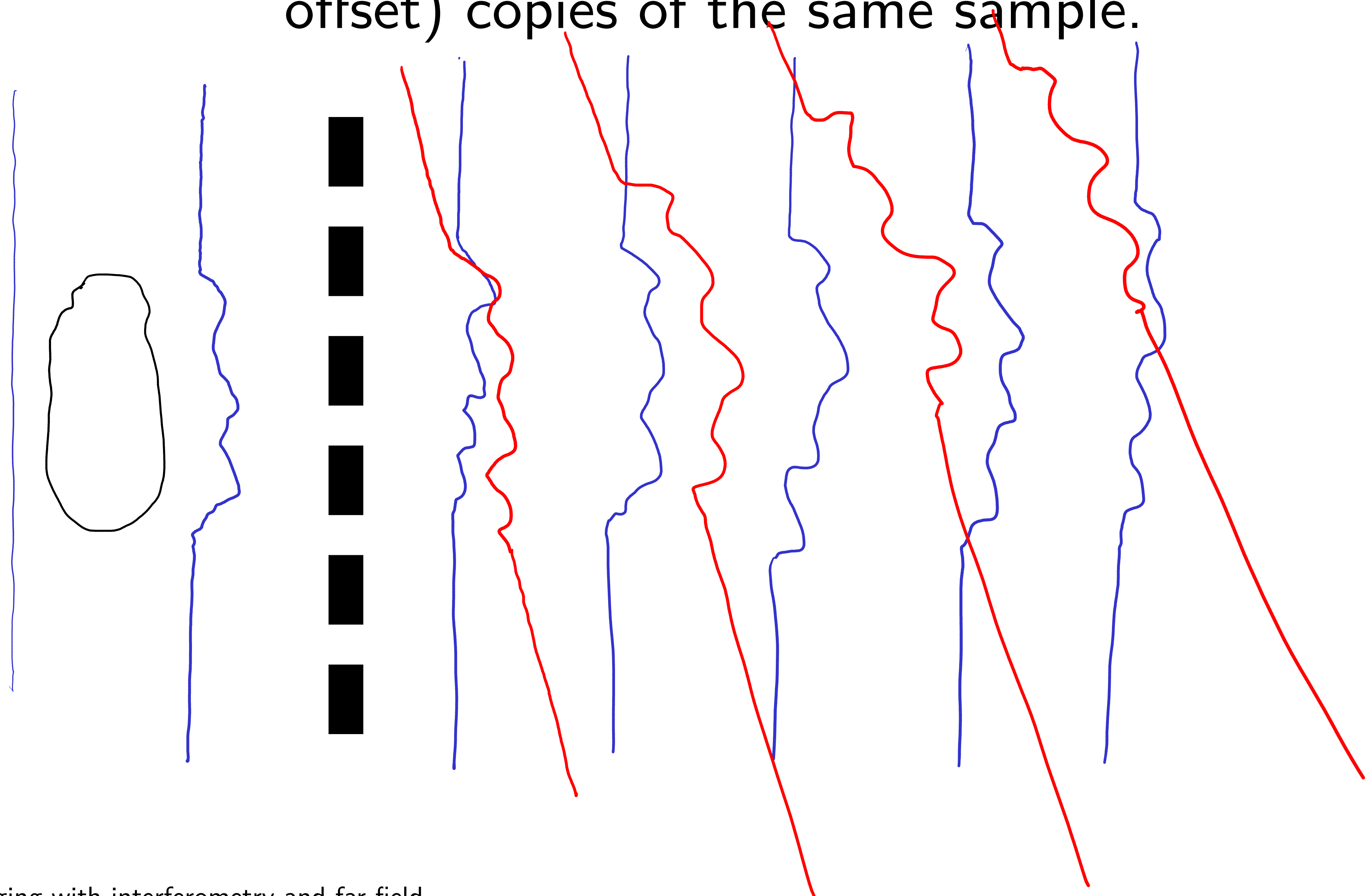

$$g(x) = \sum_{-\infty}^{\infty} g_n e^{2\pi i x n/p}$$



# Grating interferometry

Observing the interference between two (slightly offset) copies of the same sample.



# Grating interferometry

Observing the interference between two (slightly offset) copies of the same sample.

phase grating, as an example, has dominant orders $\pm 1$

$$\psi(\vec{r}; z=0) = \psi_0(\vec{r}) \cdot g(\vec{r})$$

$$\psi(\vec{r}; z) = \mathcal{F}^{-1}\left\{ \mathcal{F}\left\{ \psi_0(\vec{r})\, g(\vec{r}) \right\} e^{-i\pi u^2 \lambda z} \right\}$$

$$= \mathcal{F}\left\{ \tilde{\psi}_0(\vec{u}) * G(\vec{u})\, e^{-i\pi u^2 \lambda z} \right\}$$

$$= \mathcal{F}^{-1}\left\{ \tilde{\psi}_0(\vec{u}) * \left( g_1\, \delta\left(\vec{u} - \frac{\hat{x}}{p}\right) + g_{-1}\, \delta\left(\vec{u} + \frac{\hat{x}}{p}\right) \right) e^{\cdots} \right\}$$

$$= \mathcal{F}^{-1}\left\{ g_1\, \tilde{\psi}_0\left(\vec{u} - \frac{\hat{x}}{p}\right) + g_{-1}\, \tilde{\psi}_0\left(\vec{u} + \frac{\hat{x}}{p}\right) e^{\cdots} \right\}$$

$$\mathcal{F}^{-1}\left\{\tilde{\psi}_0\left(u-\frac{\hat{x}}{p}\right)e^{-\pi i u^2 \lambda z}\right\} = \mathcal{F}^{-1}\left\{\tilde{\psi}_0(\vec{u}')e^{-i\pi\lambda z\left(u'^2+\frac{1}{p^2}+\frac{2u'_x}{p}\right)}\right\}$$

$$= \int_{-\infty}^{\infty} \tilde{\psi}\left(\vec{u}-\frac{\hat{x}}{p}\right) e^{-\pi i u^2 \lambda z}\, e^{2\pi i \vec{u}\cdot\vec{r}}\, du$$

$$\boxed{\begin{aligned} &\vec{u}' = \vec{u} - \frac{\hat{x}}{p} \qquad u^2 = \left(\vec{u}' + \frac{\hat{x}}{p}\right)^2 \\ &\qquad\qquad\qquad\quad = u'^2 + \frac{2u_x}{p} + \frac{1}{p^2} \end{aligned}}$$

$$= e^{-\frac{i\pi\lambda z}{p^2}} \int_{-\infty}^{\infty} d^2\vec{u}'\, \underbrace{\tilde{\psi}_0(\vec{u}')\, e^{-i\pi\lambda z u'^2}}_{\text{usual argument for propagation}}\, e^{-\frac{2\pi i \lambda z}{p}\vec{u}'\cdot\hat{x}}\, \underbrace{e^{2\pi i\left(\vec{u}'+\frac{\hat{x}}{p}\right)\cdot\vec{r}}}_{}$$

$$e^{2\pi i \vec{u}\cdot\left(\vec{r}-\frac{\lambda z \hat{x}}{p}\right)} \cdot e^{\frac{2\pi i x}{p}}$$

$$= e^{-\frac{i\pi\lambda z}{p^2}}\, \psi_0\left(\vec{r}-\frac{\lambda z}{p}\hat{x}\,;\, z\right) e^{\frac{2\pi i x}{p}}$$

⋮

$$\psi(\vec{r};z) = e^{-\frac{i\pi\lambda z}{p^2}}\left(g_1 e^{\frac{2\pi i x}{p}}\psi_0\left(\vec{r}-\frac{\lambda z}{p}\hat{x};z\right)\right.$$

$$\left.\left.+\, g_{-1}\, e^{-\frac{2\pi i x}{p}}\psi_0\left(\vec{r}+\frac{\lambda z}{p}\hat{x};z\right)\right)\right)$$

$$I(\vec{r};z) = |\psi(\vec{r};z)|^2$$

$$= \psi(\vec{r};z)\,\psi^*(\vec{r};z)$$

$$= |g_1|^2\left|\psi_0\left(\vec{r}-\frac{\lambda z}{p}\hat{x};z\right)\right|^2 + |g_{-1}|^2\left|\psi_0\left(\vec{r}+\frac{\lambda z}{p}\hat{x};z\right)\right|^2$$

$$+\, g_1 g_{-1}^* e^{\frac{4\pi i x}{p}}\psi_0\left(\vec{r}-\frac{\lambda z}{p}\hat{x};z\right)\psi_0^*\left(\vec{r}+\frac{\lambda z}{p}\hat{x};z\right) + c.c.$$

$$\psi_0 = a(r)e^{i\varphi(r)} \qquad\qquad g_1 = g_{-1}^*$$

$$I(\vec{r};z) = |g_1|^2 \left( \left| a\left(\vec{r} - \frac{\lambda z}{p}\right) \right|^2 + \left| a\left(r + \frac{\lambda z}{p}\hat{x}\right) \right|^2 \right)$$

$$+ |g|^2 e^{\frac{4\pi i x}{p}} a\left(\vec{r} - \frac{\lambda z}{p}\right) a\left(\vec{r} + \frac{\lambda z}{p}\right) e^{i\varphi\left(\vec{r} - \frac{\lambda z}{p}\hat{x}\right) - i\varphi\left(\vec{r} + \frac{\lambda z}{p}\hat{x}\right)}$$

$$+ \text{c.c.}$$

$\frac{\lambda z}{p}$ assumed to be small compared to resolution of the system

$$I(\vec{r};z) \simeq 2|g_1|^2 |a(\vec{r})|^2$$

$$+ 2|g_1|^2 |a(r)|^2 \cos\left( \frac{4\pi x}{p} - 2\frac{\lambda z}{p}\frac{\partial \varphi}{\partial x} \right)$$

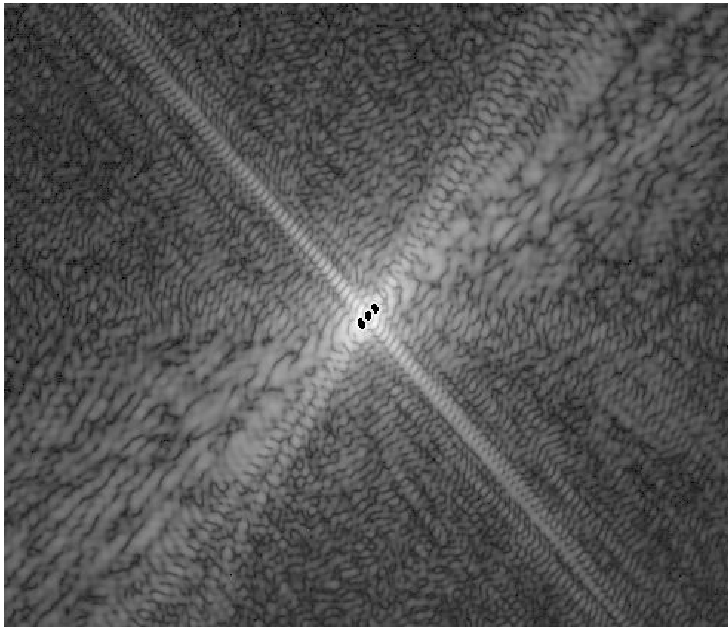
$$\varphi\left(\vec{r} - \frac{\lambda z}{p}\hat{x}\right) - \varphi\left(\vec{r} + \frac{\lambda z}{p}\hat{x}\right) \simeq -2\frac{\lambda z}{p}\frac{\partial \varphi}{\partial x}$$

# Far-field diffraction

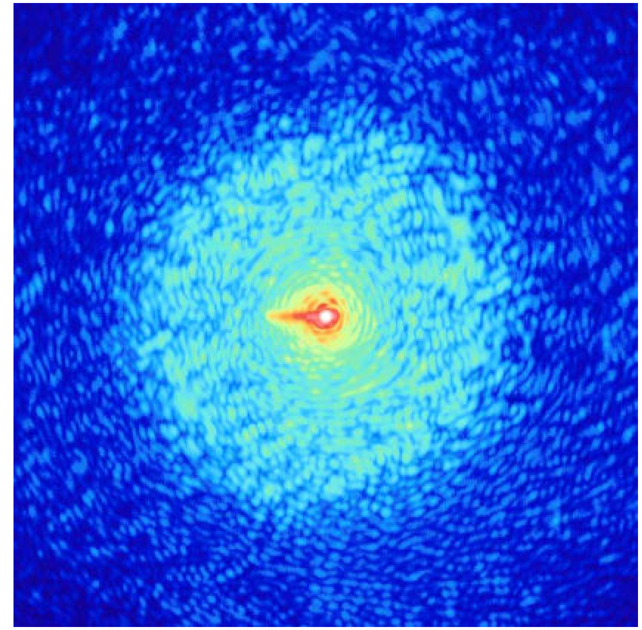
## The Fraunhofer regime

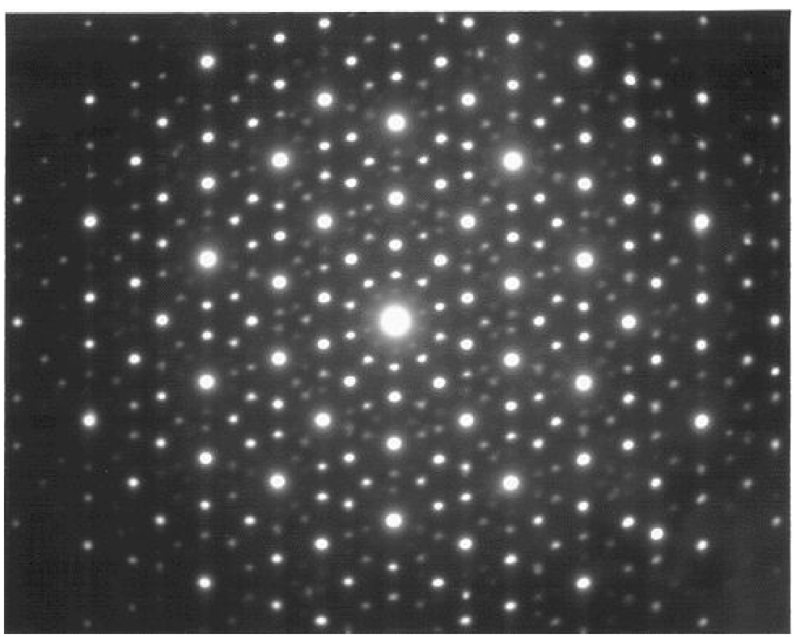

$$|\psi(\vec{r}; z \to \infty)|^2 = |\mathcal{F}\psi|^2\left(\vec{u} = \frac{\vec{r}}{\lambda z}\right) = I(\vec{u})$$

# Diffraction patterns

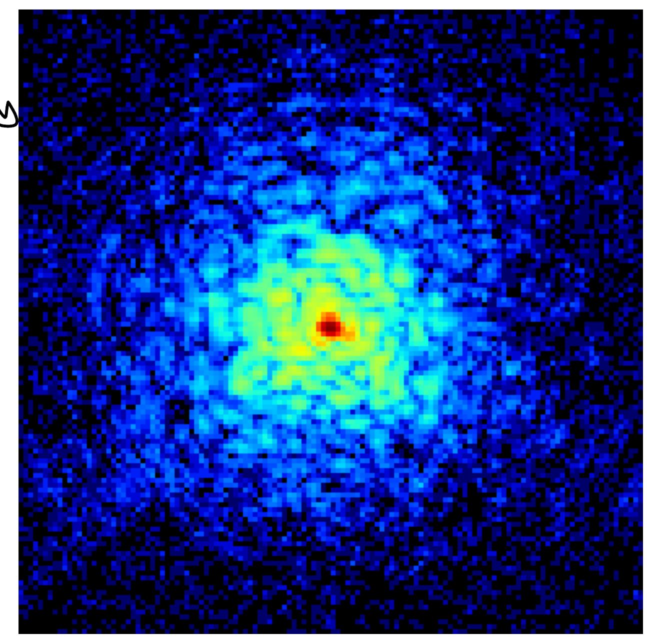


speckles



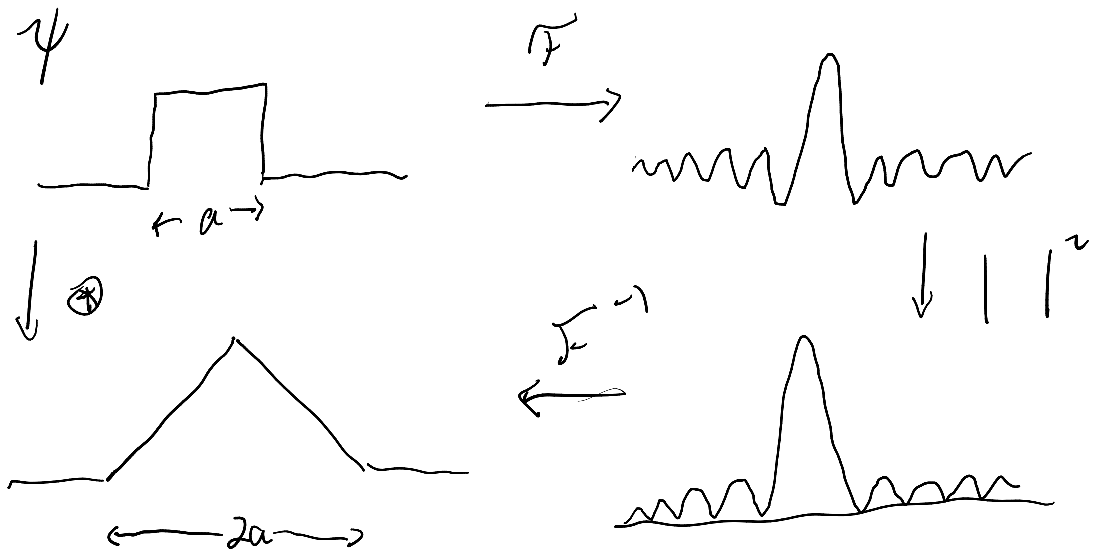


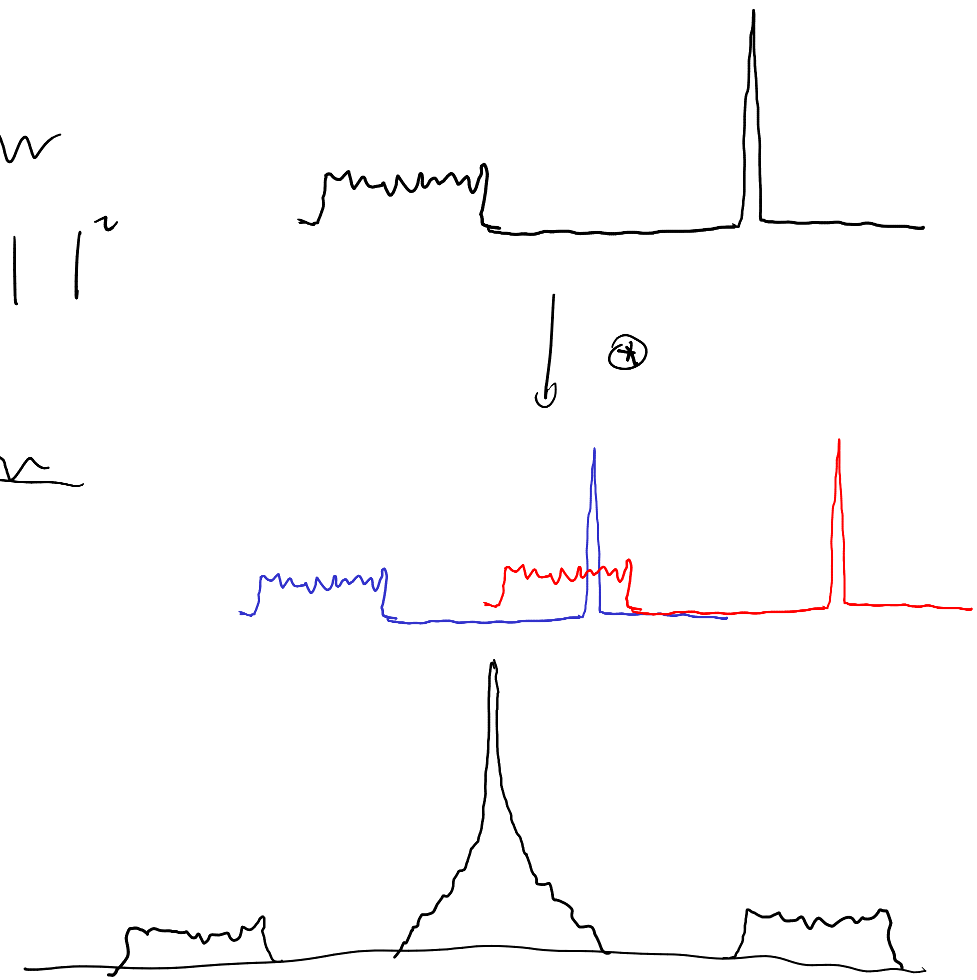
Bragg peaks



# Diffraction and autocorrelation

$$\mathcal{F}^{-1}\{I(\vec{u})\} = \mathcal{F}^{-1}\{\psi(u)\,\psi^*(u)\} = \underbrace{\psi(\vec{r}) \circledast \psi(\vec{r})}_{\text{autocorrelation}}$$



# Fourier transform holography

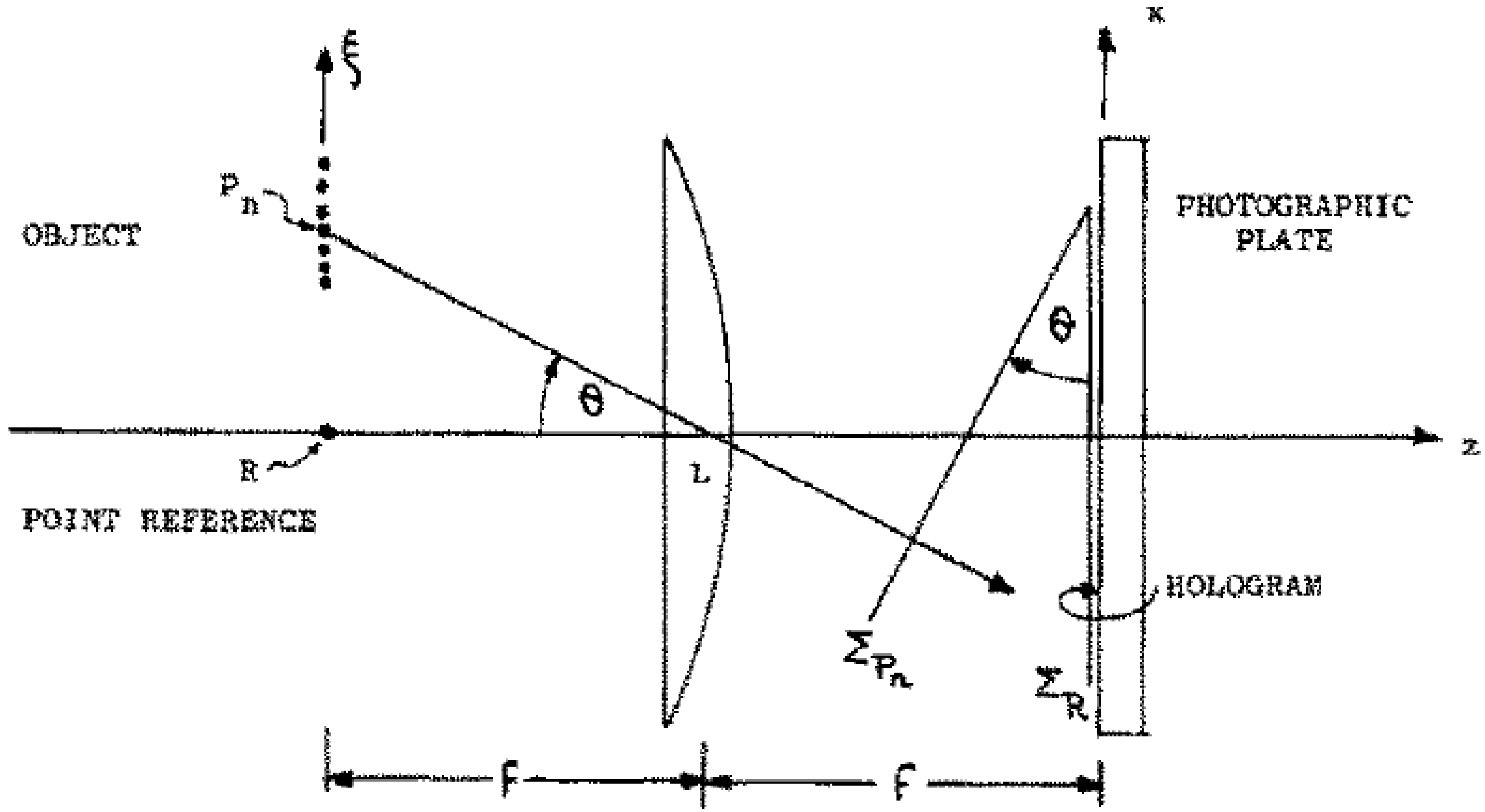
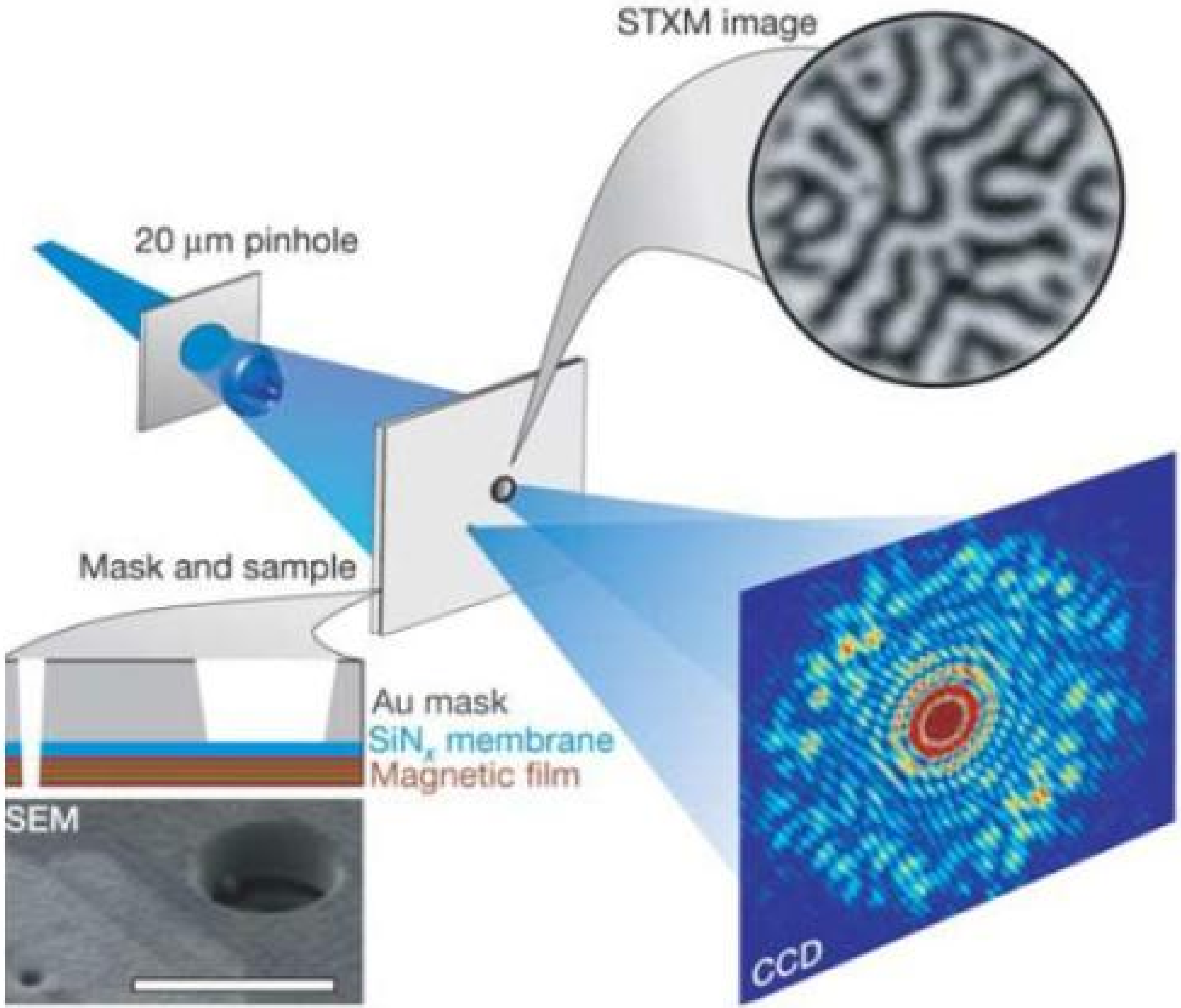



**Fig. 1. Recording of a Fourier-transform hologram with a lens $L$. $\Sigma_R$ = reference wavefront.**

Source: G. Stroke, Appl. Phys. Lett. **6**, 201-203 (1965).

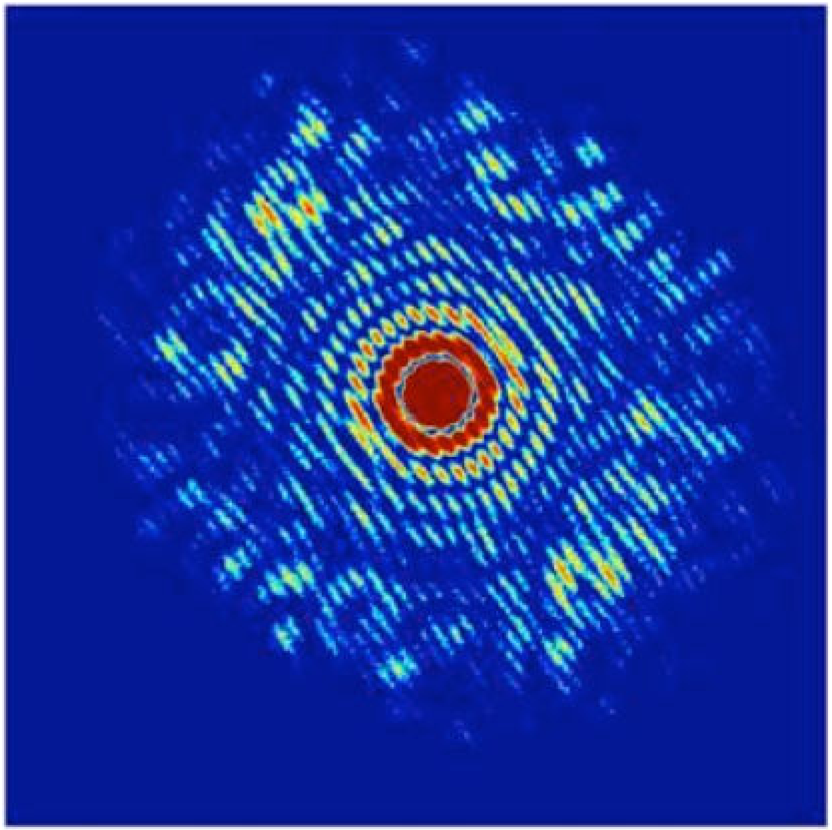
# Fourier transform holography

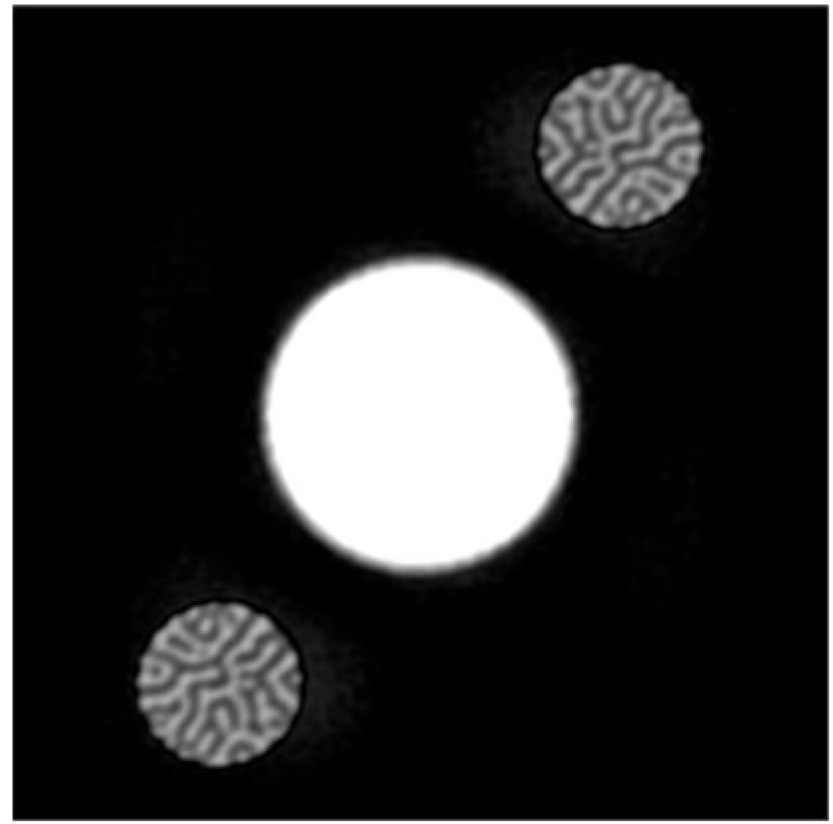



Source: S. Eisebitt et al., Nature **432**, 885-888 (2004).

# Fourier transform holography

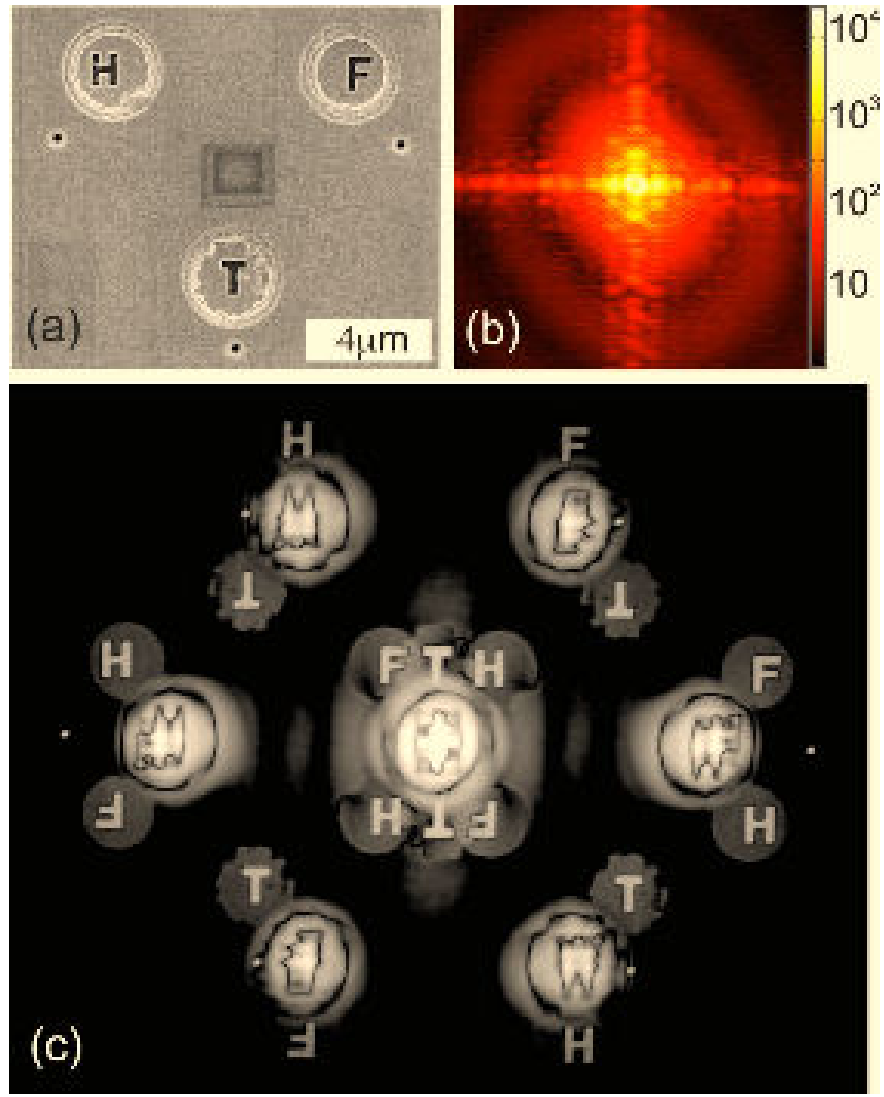
# Fourier transform holography





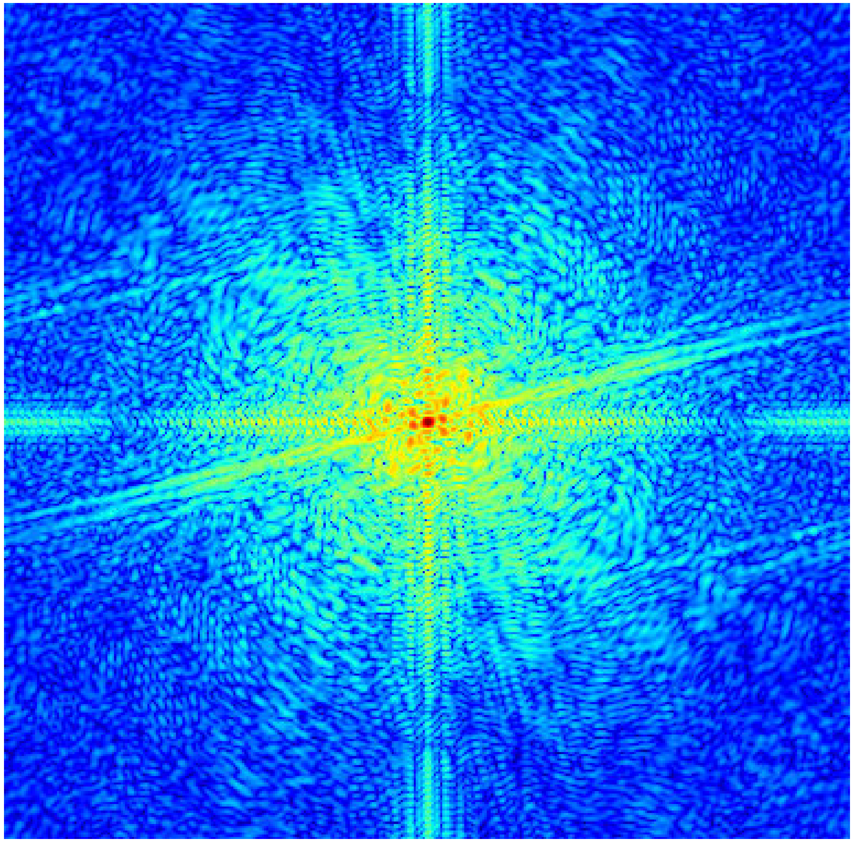
# Fourier transform holography

## Multiple references




Source: W. Schlotter et al., Opt.. Lett. **21**, 3110-3112 (2006).

# Coherent diffractive imaging

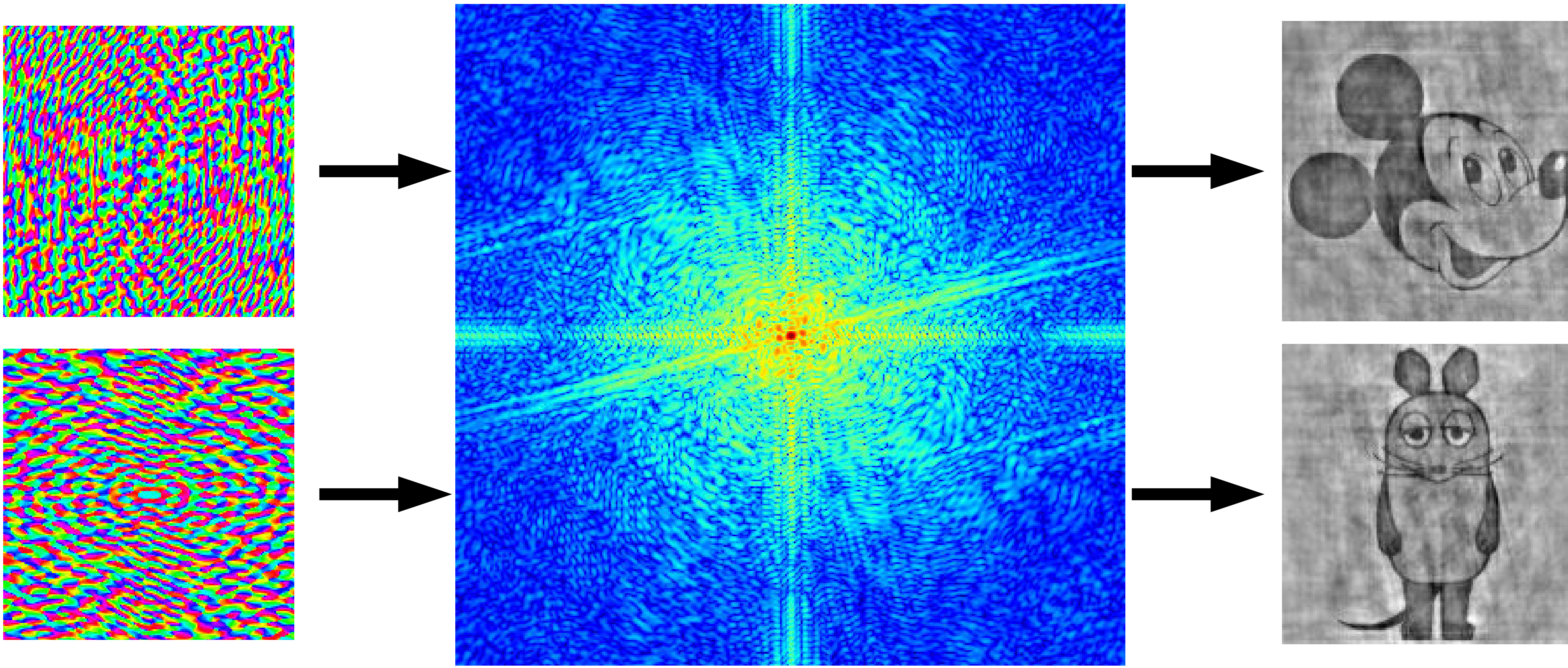


# The phase problem

$e^{i\varphi(u)}$

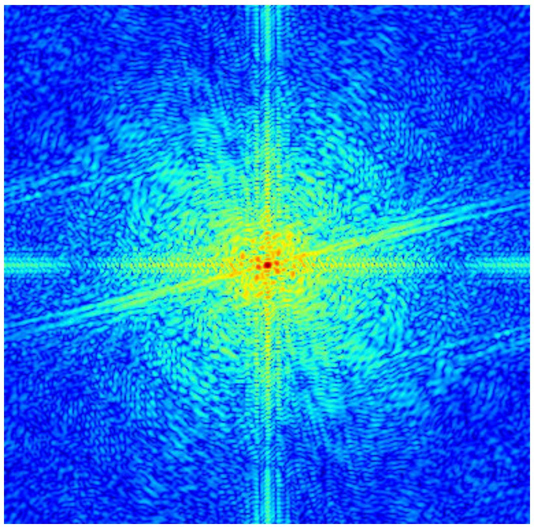
$|\mathcal{F}\{\psi\}|^2(\vec{u})$

phases carry very important information

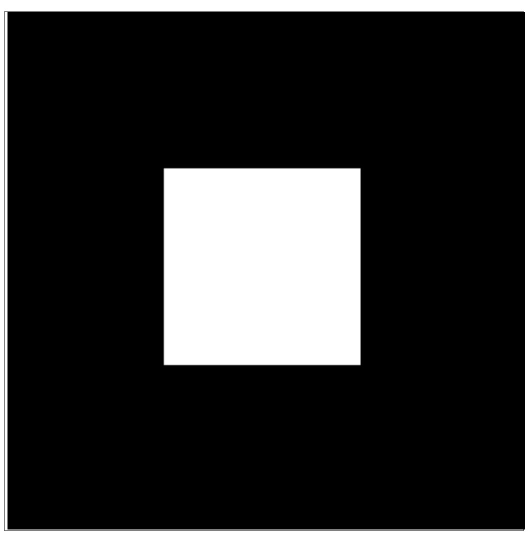


# Coherent diffractive imaging

Two constraints

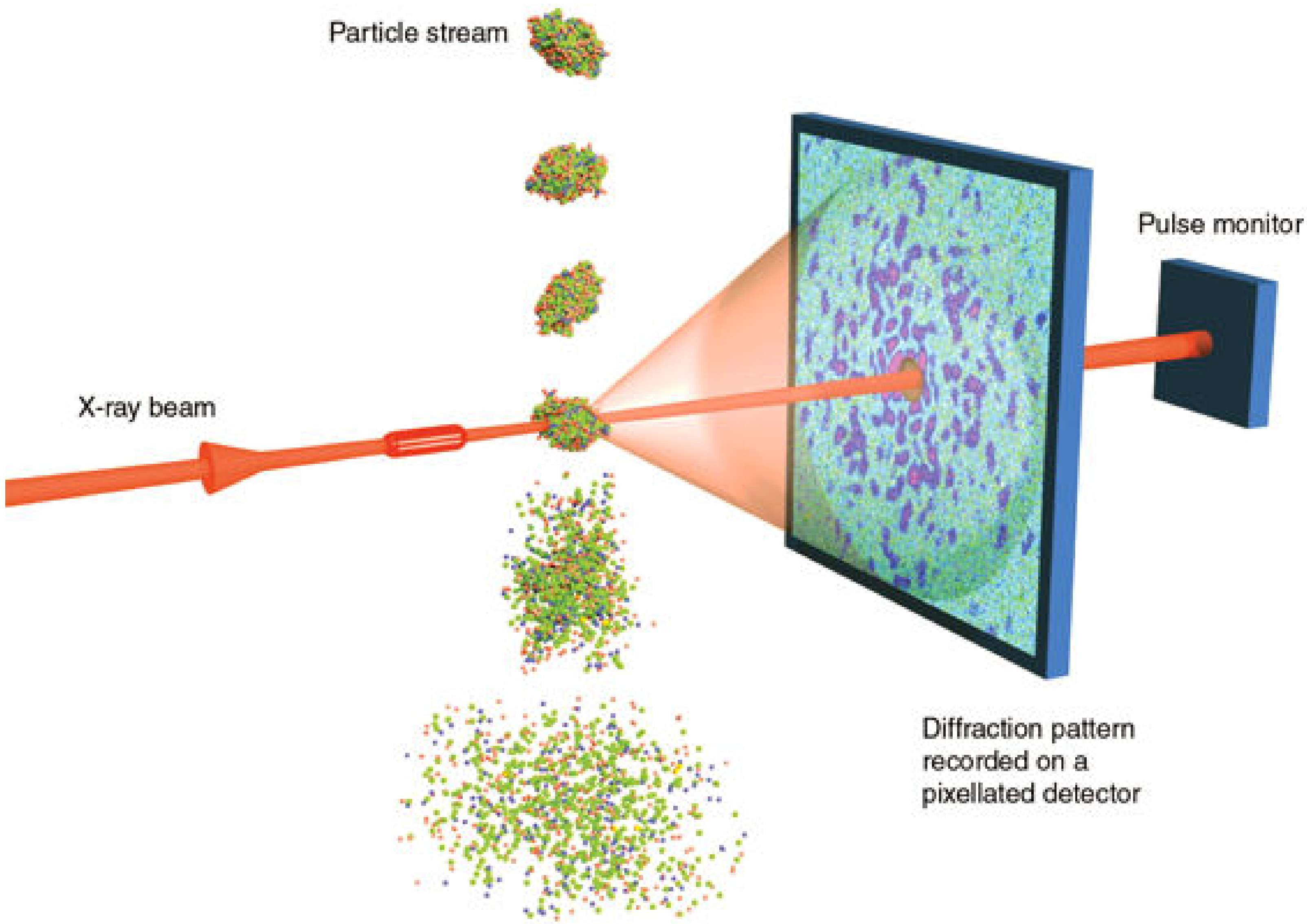


1. Solution is consistent with measured Fourier amplitude



2. Solution is isolated in space

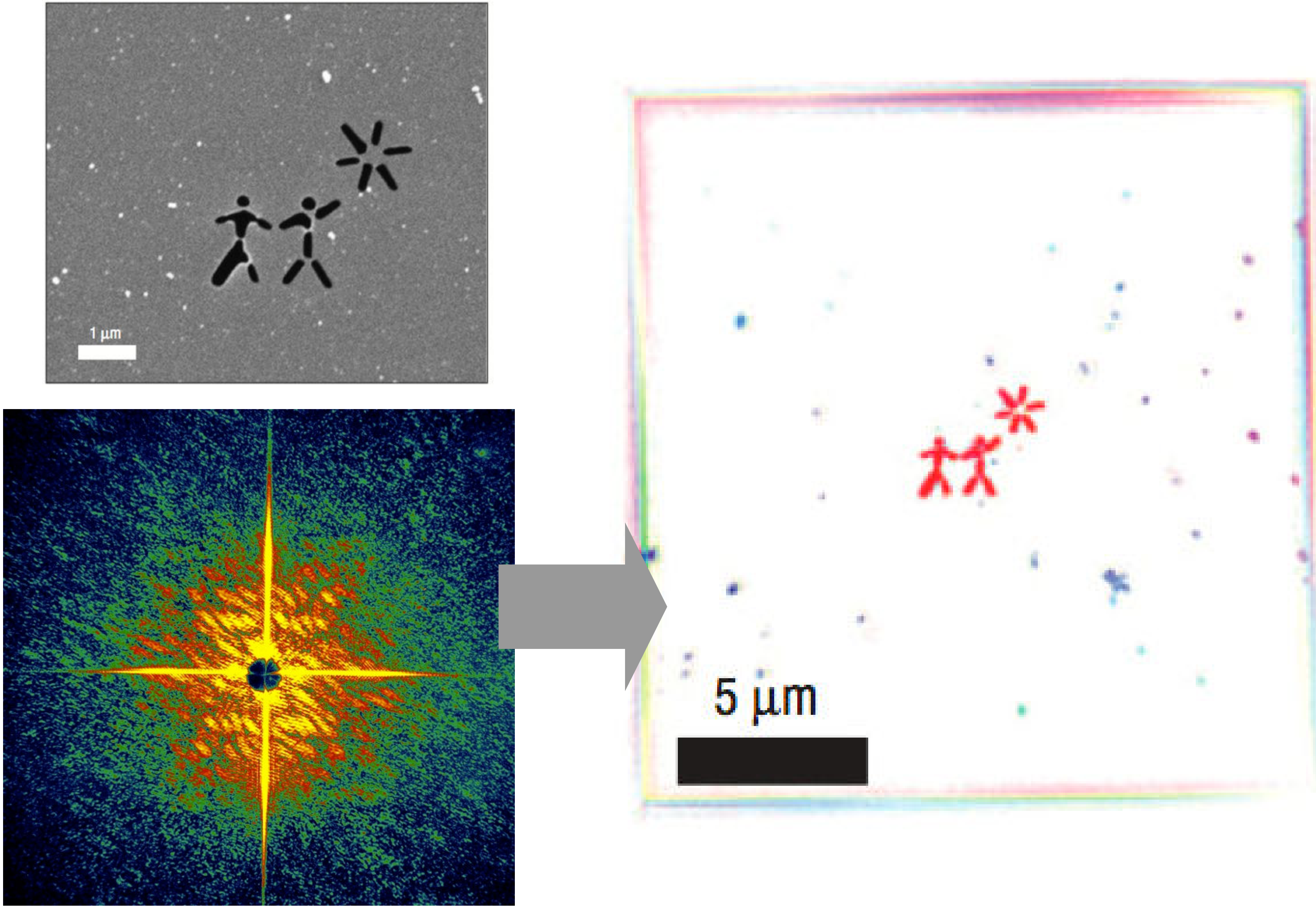
# Radiation damage limits on radiation




R. Neutze *et al*, Nature **406**, 752 (2000)

K. J. Gaffney *et al*, Science **316**, 1444 (2007)

# “Diffraction before destruction”



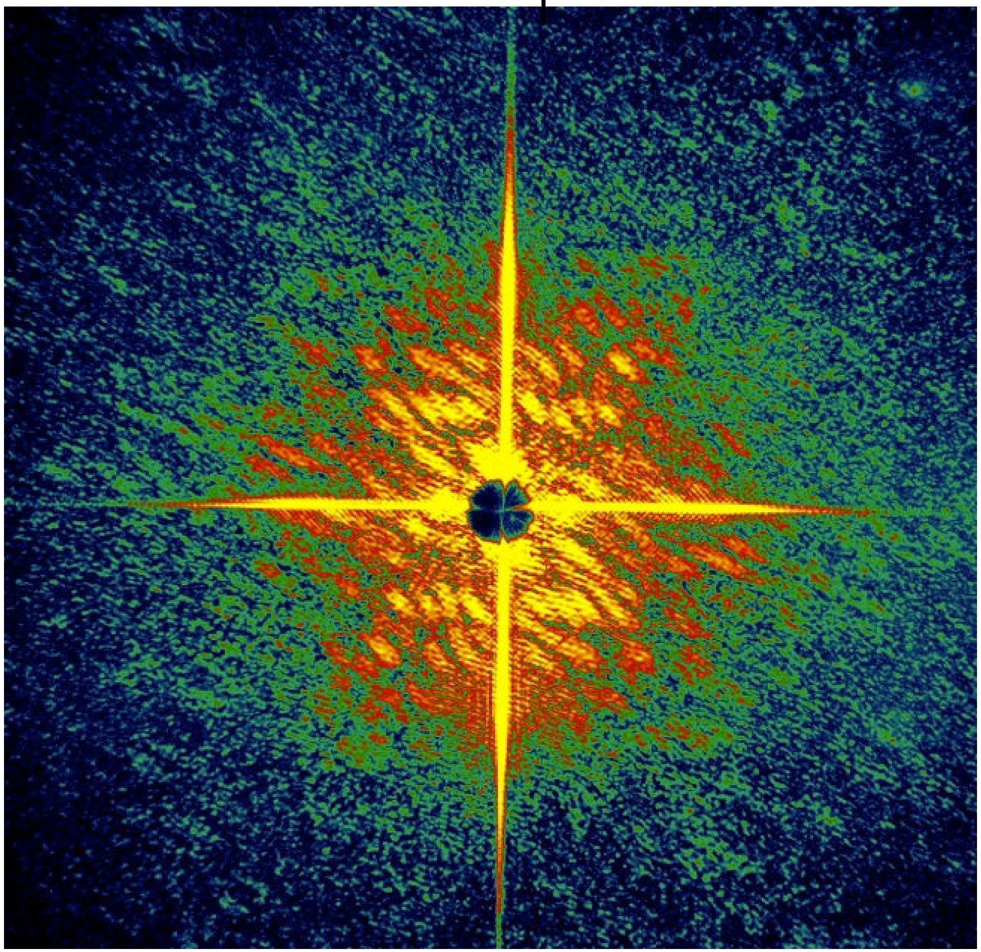

H. N. Chapman *et al*, Nat. Phys. **2**, 839 (2006)

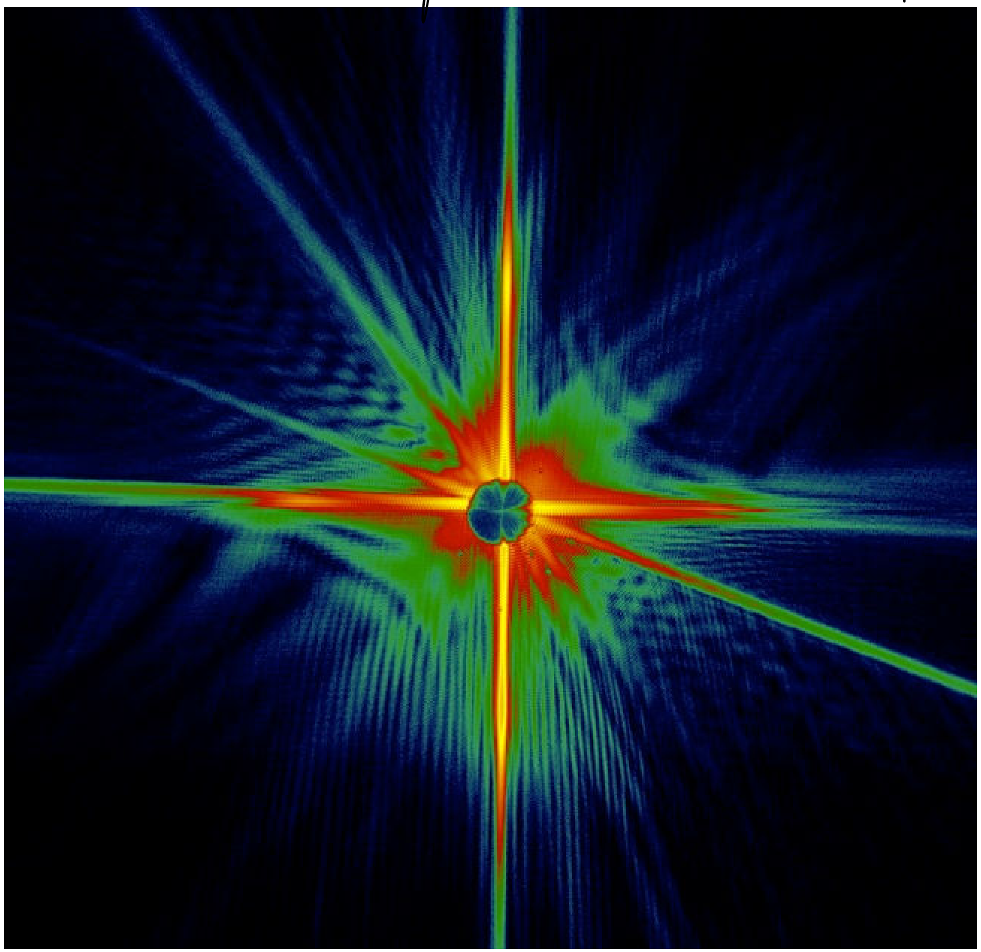
# “Diffraction before destruction”

## The imaging pulse vaporized the sample

1st pulse

2nd pulse (no more sample)





H. N. Chapman *et al*, Nat. Phys. **2**, 839 (2006)

# Ptychography

- Scanning an isolated illumination on an extended specimen
- Measure full coherent diffraction pattern at each scan point
- Combine everything to get a reconstruction

**Dynamische Theorie der Kristallstrukturanalyse durch Elektronenbeugung im inhomogenen Primärstrahlwellenfeld**

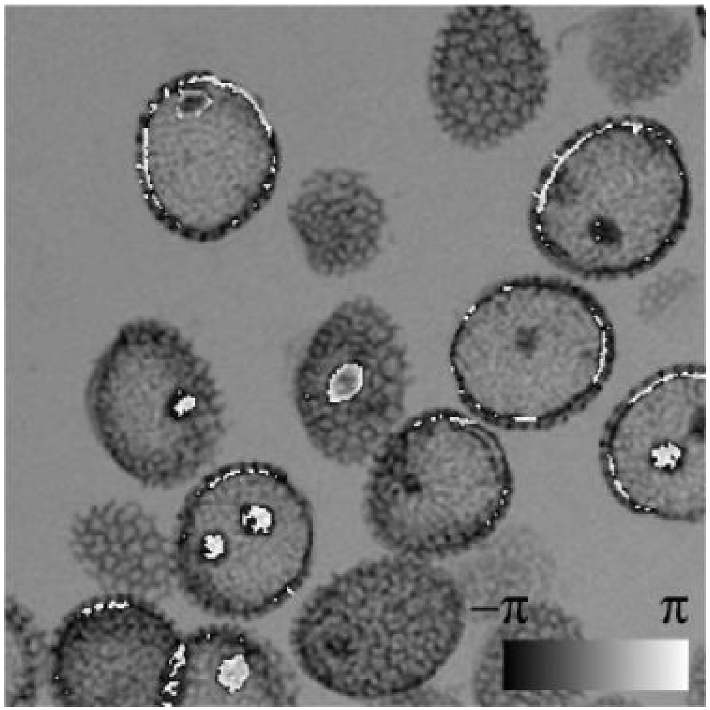
**Von R. Hegerl und W. Hoppe**

Some time ago a new principle was proposed for the registration of the complete information (amplitudes and phases) in a diffraction diagram, which does not – as does Holography – require the interference of the scattered waves with a single reference wave. The basis of the principle lies in the interference of neighbouring scattered waves which result when the object function $\varrho(x, y)$ is multiplied by a generalized primary wave function $p(x, y)$ in Fourier space (diffraction diagram) this is a convolution of the Fourier transforms of these functions. The above mentioned interferences necessary for the phase determination can be obtained by suitable choice of the shape of $p(x, y)$. To distinguish it from holography this procedure is designated "ptychography" ($\pi\tau\upsilon\xi$ = fold). The procedure is applicable to periodic and aperiodic structures. The relationships are simplest for plane lattices. In this paper the theory is extended to space lattices both with and without consideration of the dynamic theory. The resulting effects are demonstrated using a practical example.

# Ptychography

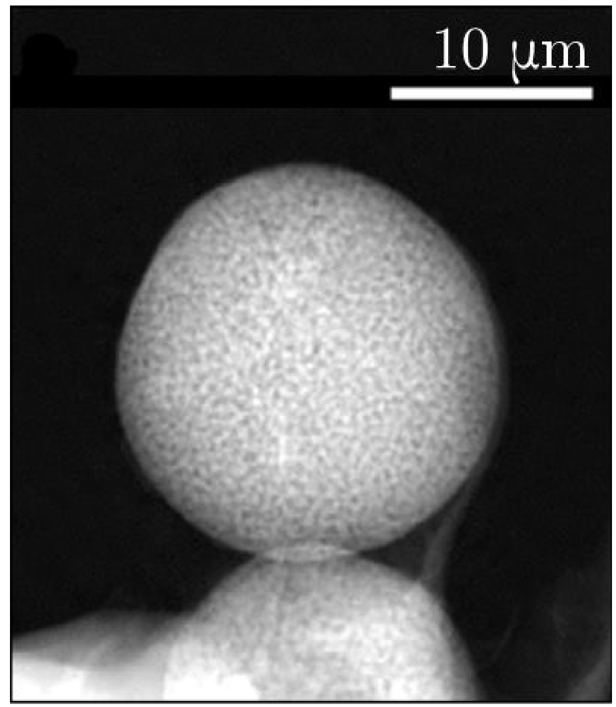
## A few examples

Visible light



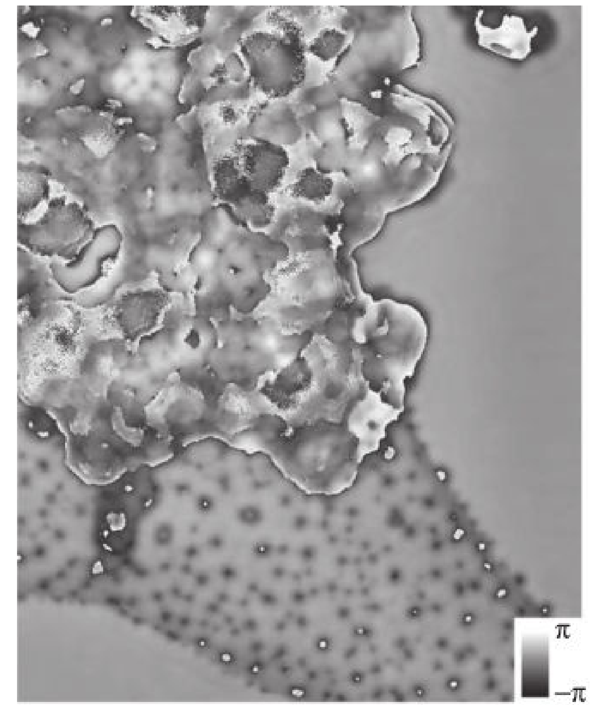

A. Maiden *et al.*, Opt. Lett. **35**, 2585-2587 (2010).

X-rays



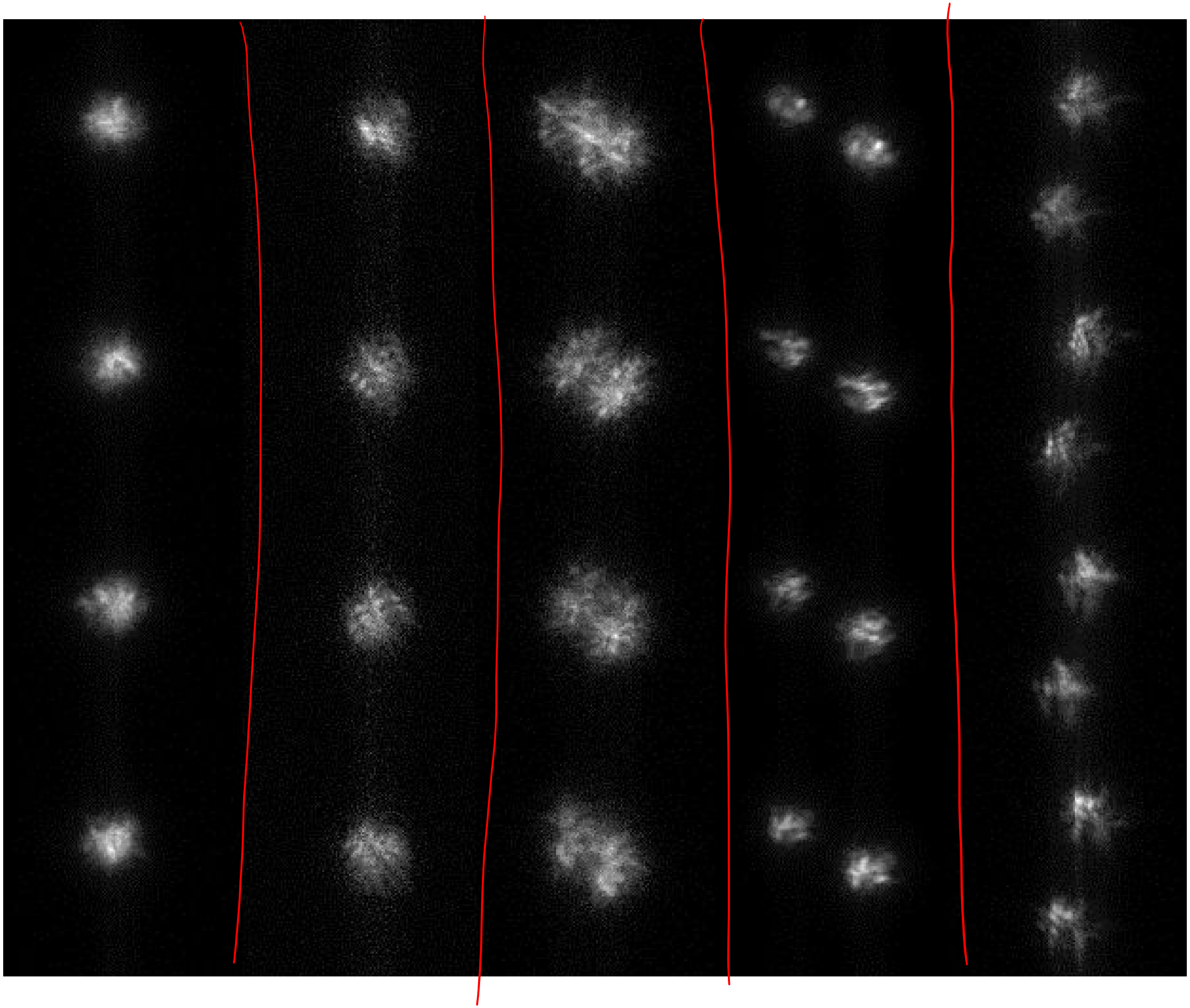

P. Thibault *et al.*, New J. Phys **14**, 063004 (2012).

electrons




M. Humphry *et al.*, Nat. Comm. **3**, 730 (2012).

# Speckle imaging in astronomy



different measurements of the same objects.

Difference is caused by turbulence

Source:http://www.cis.rit.edu/research/thesis/bs/2000/hoffmann/thesis.html

# Speckle imaging in astronomy

## Model

One measurement

$$I(\vec{r}) = O * P$$

instantaneous point-spread function

$$\tilde{I}(\vec{u}) = \tilde{O}(\vec{u}) \cdot M \leftarrow \text{MTF}$$

$$|\tilde{I}(\tilde{u})|^2 = |\tilde{O}(\tilde{u})|^2 |M|^2$$

many measurements

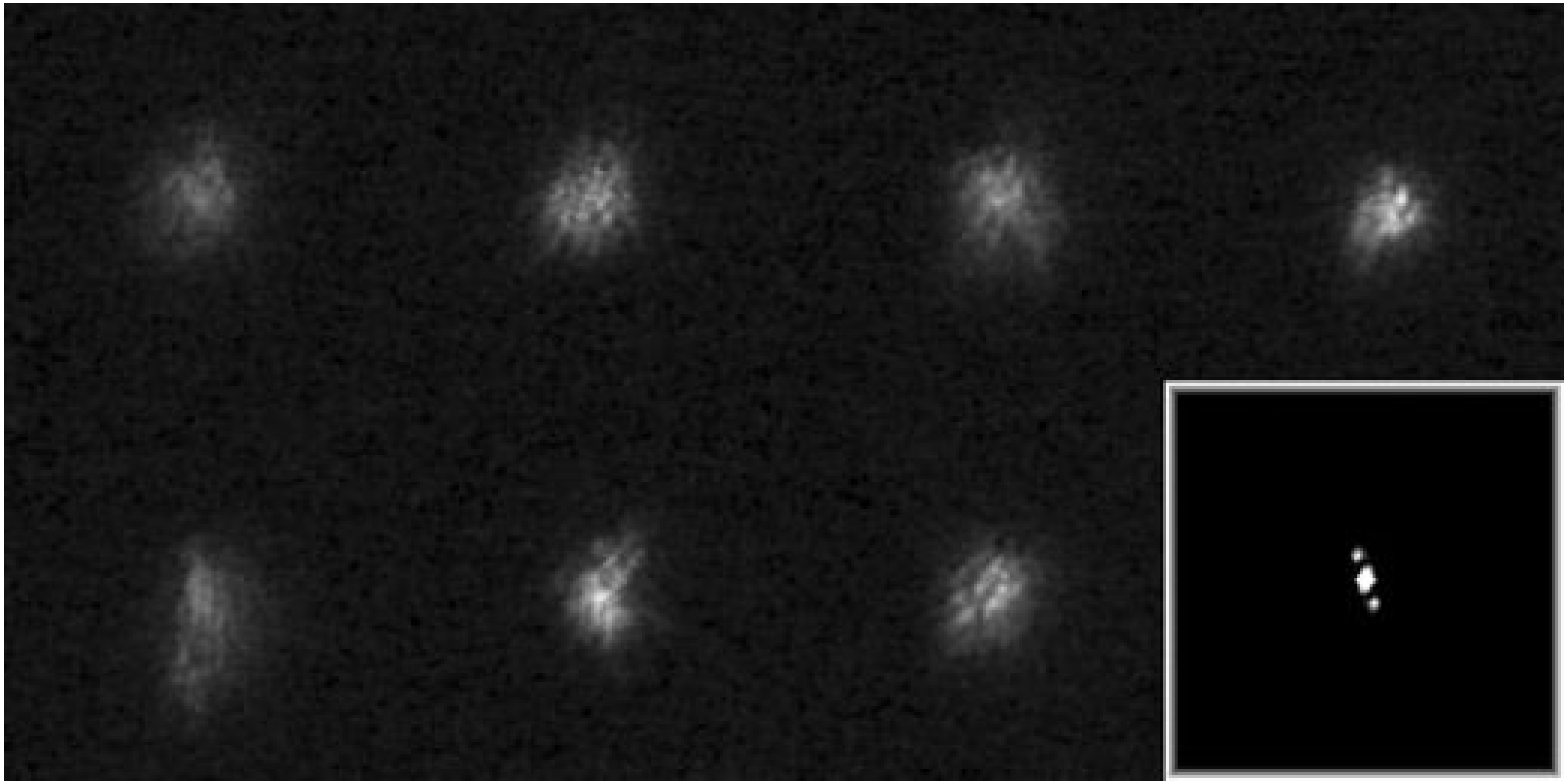
$$\langle |\tilde{I}(\tilde{u})|^2 \rangle = |\tilde{O}|^2 \underbrace{\langle |M|^2 \rangle}$$

can be modeled from fluid dynamics

$$|\tilde{O}|^2 \simeq \frac{\langle |\tilde{I}|^2 \rangle}{\langle |M_{model}|^2 \rangle}$$

recovering $O$ from $|\tilde{O}|^2$ same as Coherent diffractive imaging!

# Speckle imaging in astronomy
## Retrieval of the autocorrelation

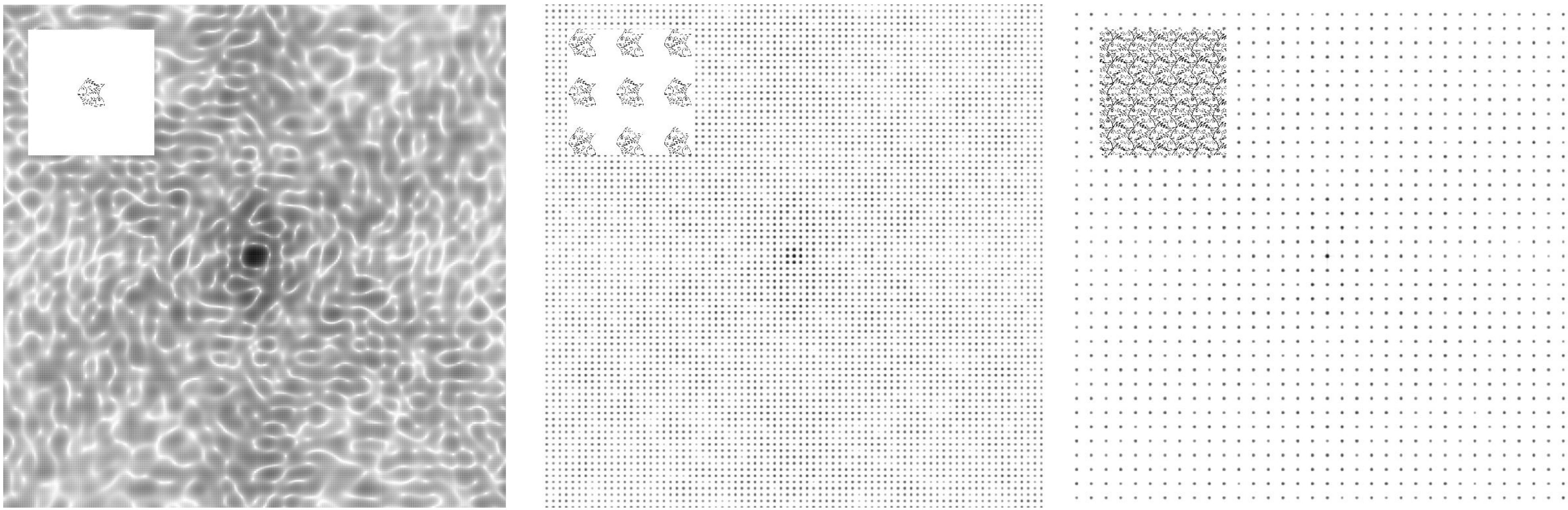


autocorrelation

Source: http://www.astrosurf.com/hfosaf/uk/speckle10.htm

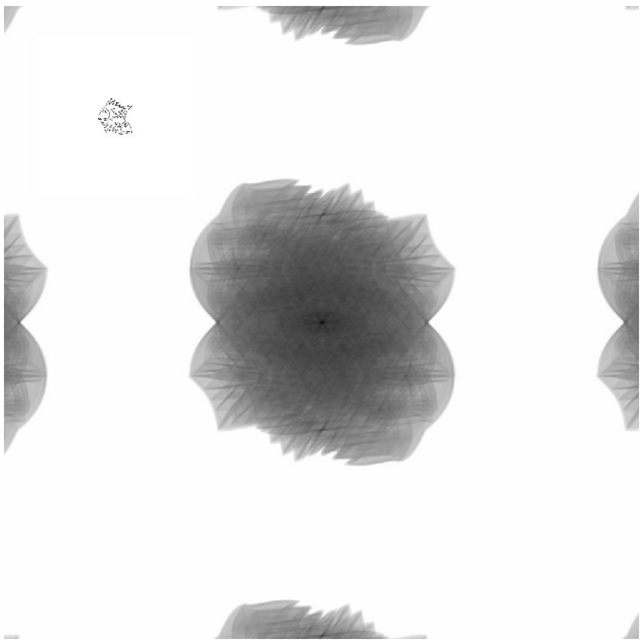
# Crystallography

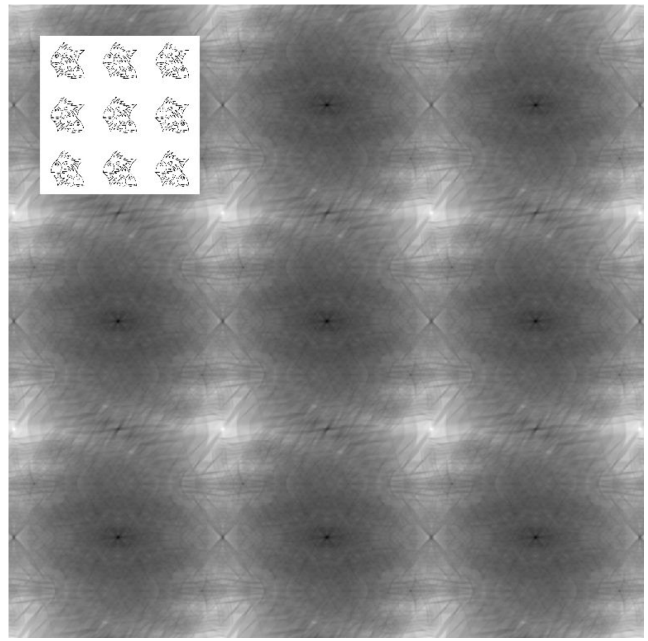
## Diffraction by a crystal: Bragg peaks

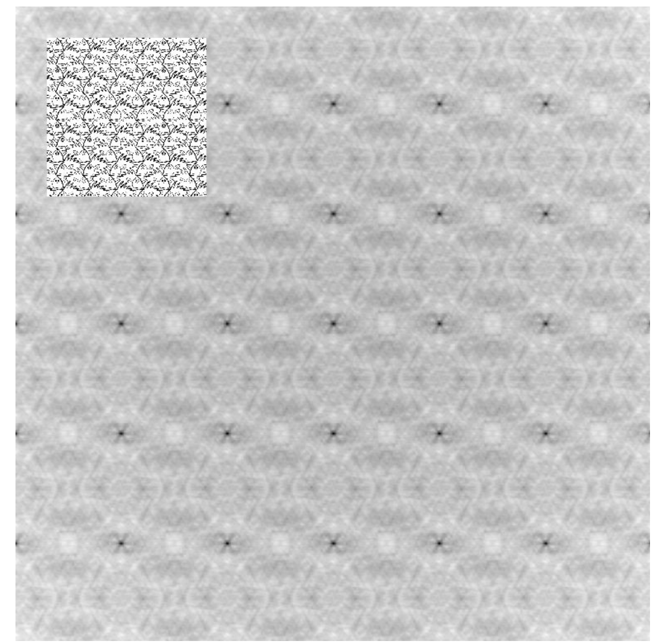


# Crystallography

## Fourier transform of intensity: autocorrelation





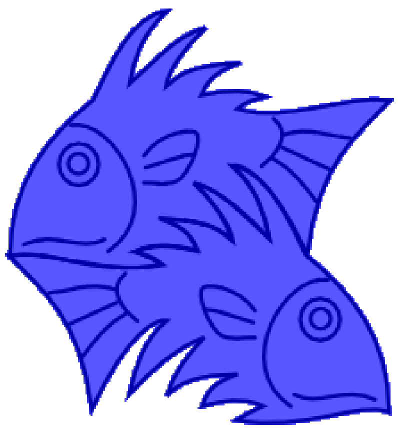


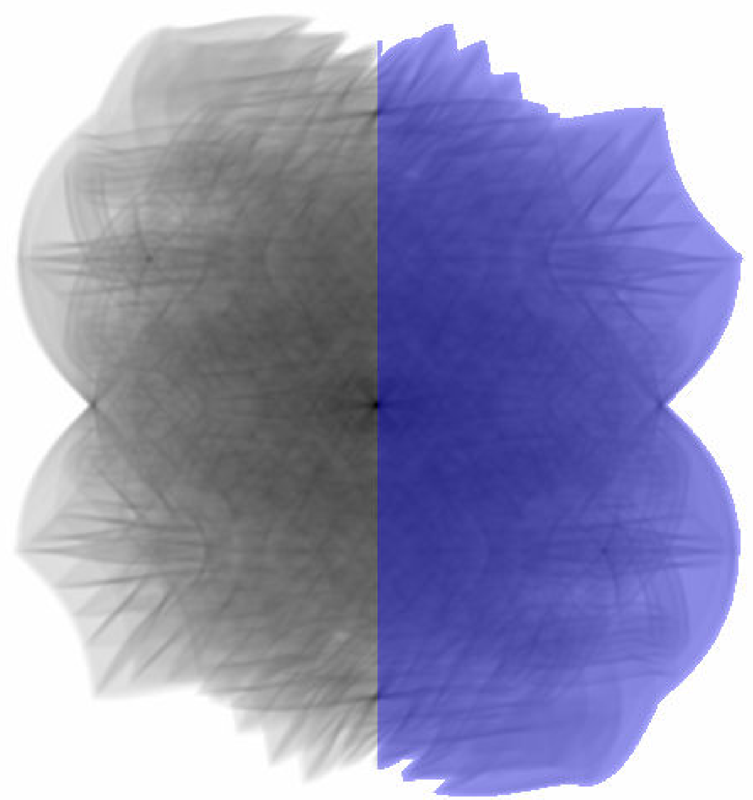
F.T. of crystal diffraction

called Patterson map

# Crystallography

## Problem is overconstrained with an isolated sample

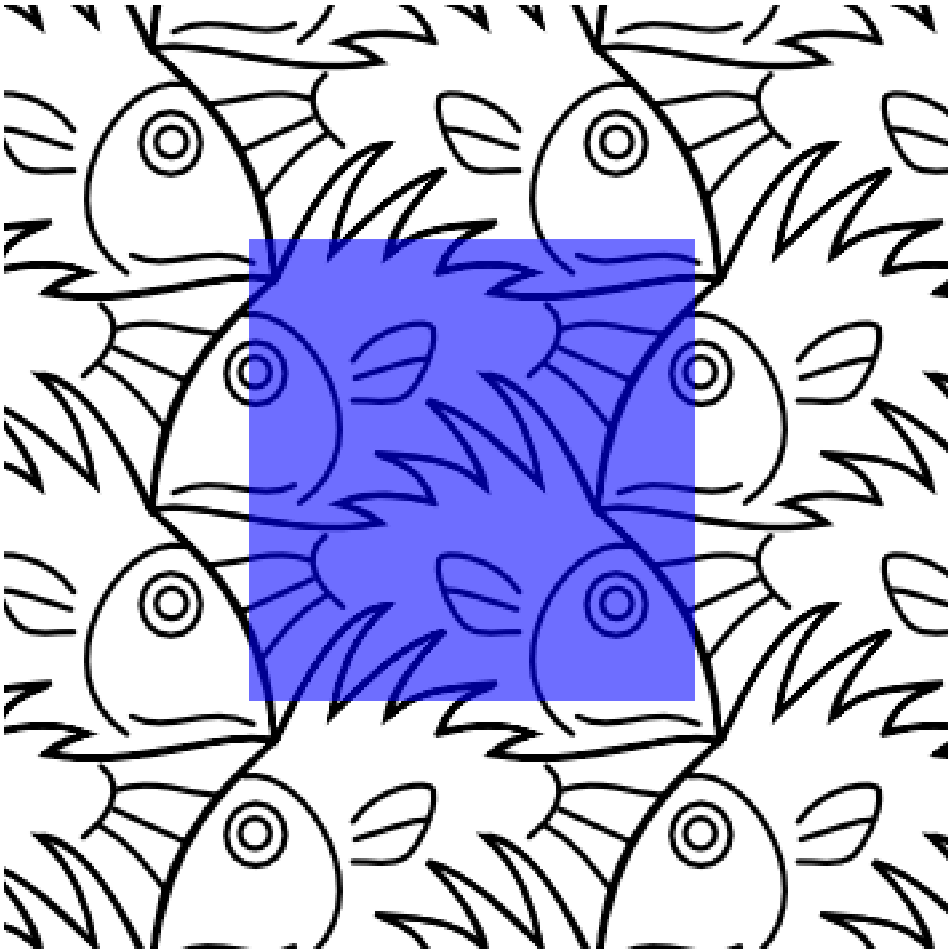


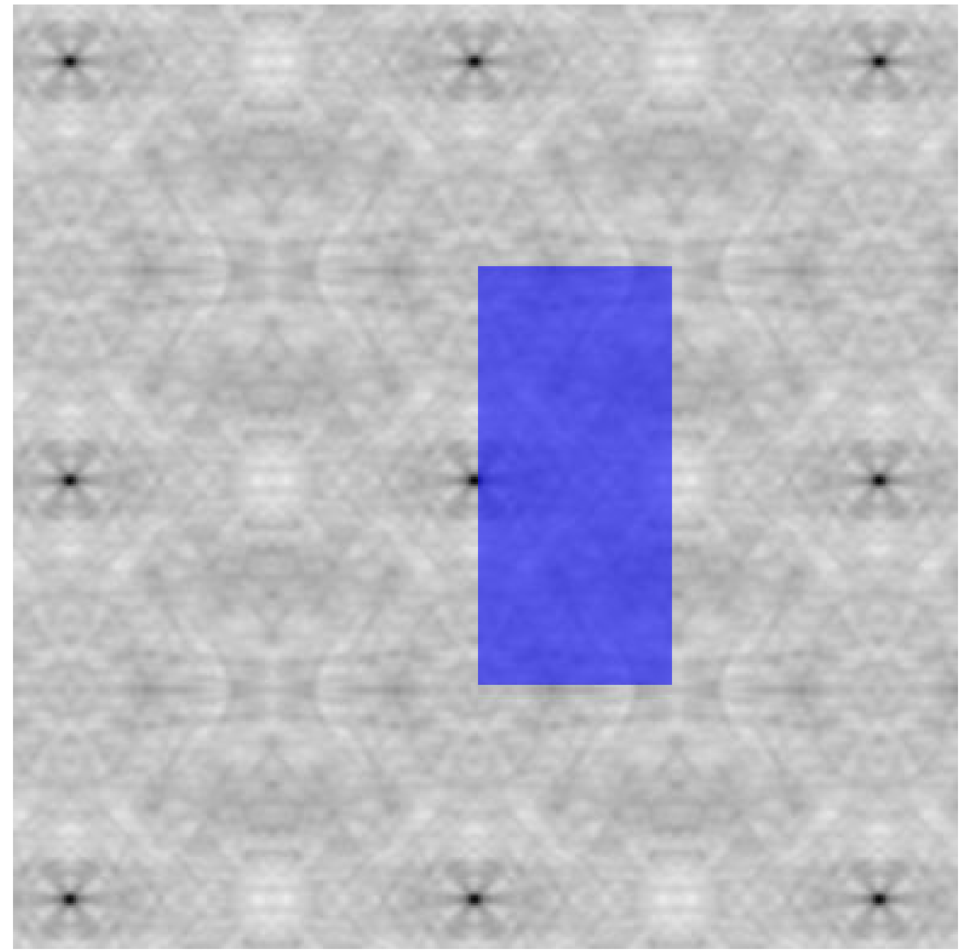


**unknowns = N** **constraints ≥ 2N**

# Crystallography

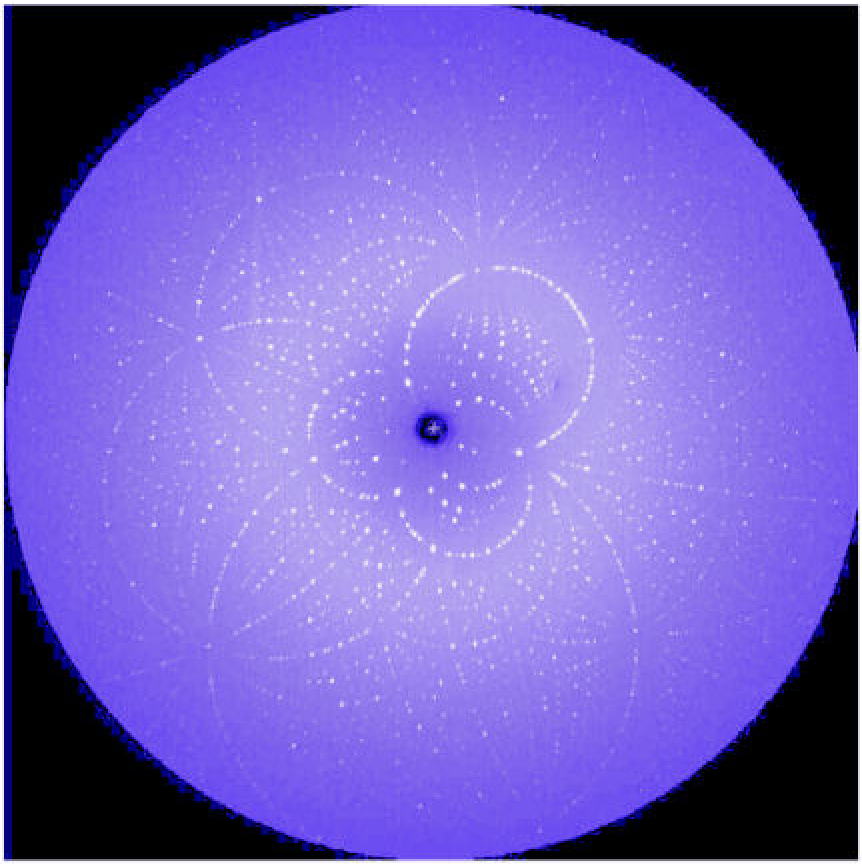
## Problem is **under**constrained with a crystal





**unknowns = N** **constraints = N/2**

# Crystallography
## Structure determination



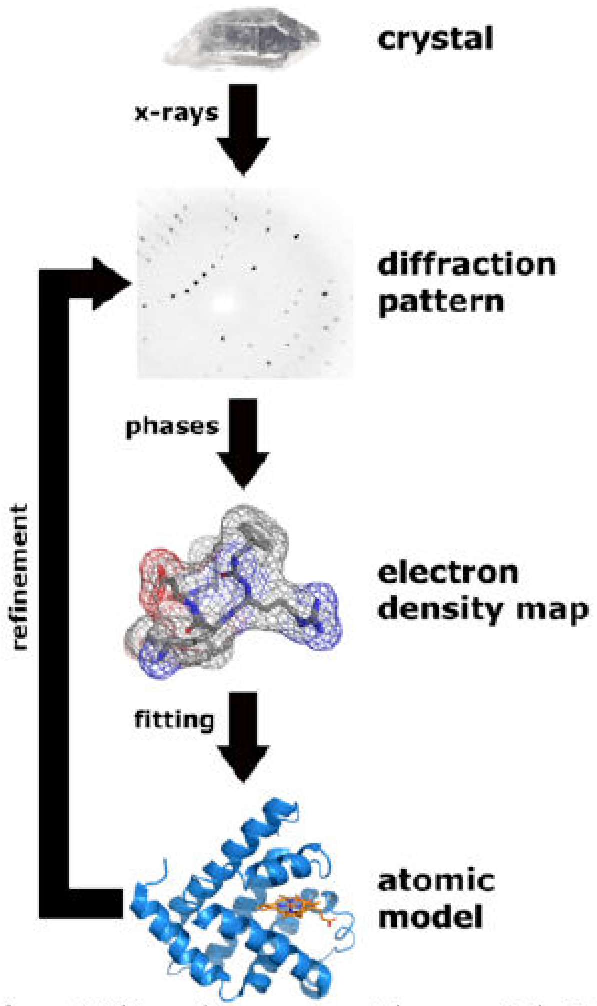



Image from Wikimedia courtesy Thomas Splettstoesser

# Crystallography

## Structure determination

- Hard problem: few measurements for the number of unknowns
- Luckily: crystals are made of atoms $\rightarrow$ strong constraint
- Also common: combining additional measurements (SAD, MAD, isomorphous replacement, ...)

# Summary

$\approx$ lensless imaging

## Imaging from far-field amplitudes

- Used when image-forming lenses are unavailable (or unreliable) or to obtain more quantitative images.
- In general difficult because of the phase problem
- Solved with the help of additional information:
  - Strong *a priori* knowledge (e.g. CDI: support)
  - Multiple measurements (e.g. ptychography)