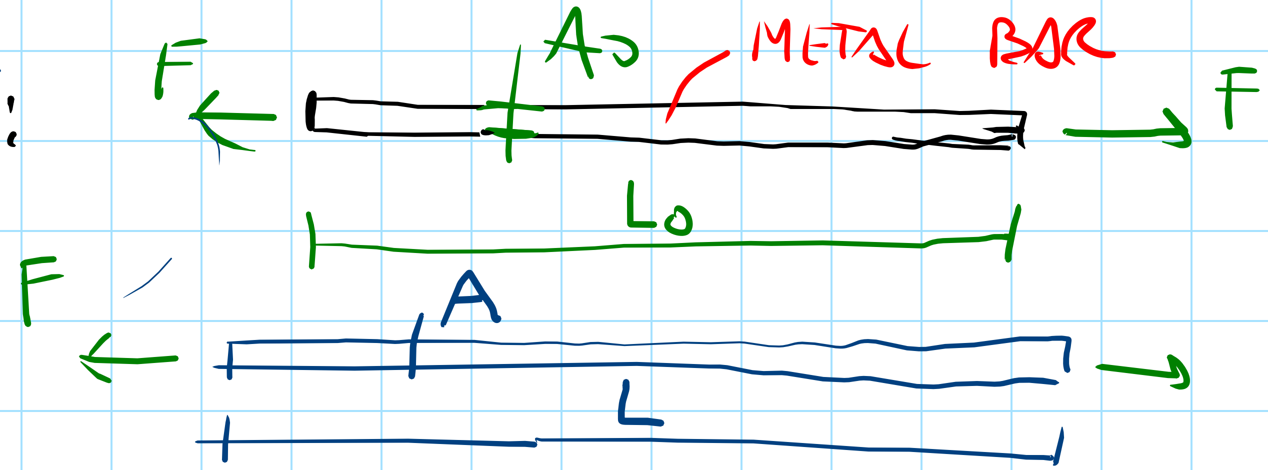
COMPUTATIONAL SOLID MECHANICS

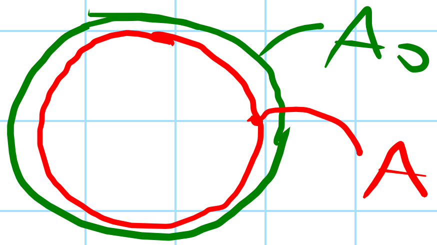
CSM, 11/3/26

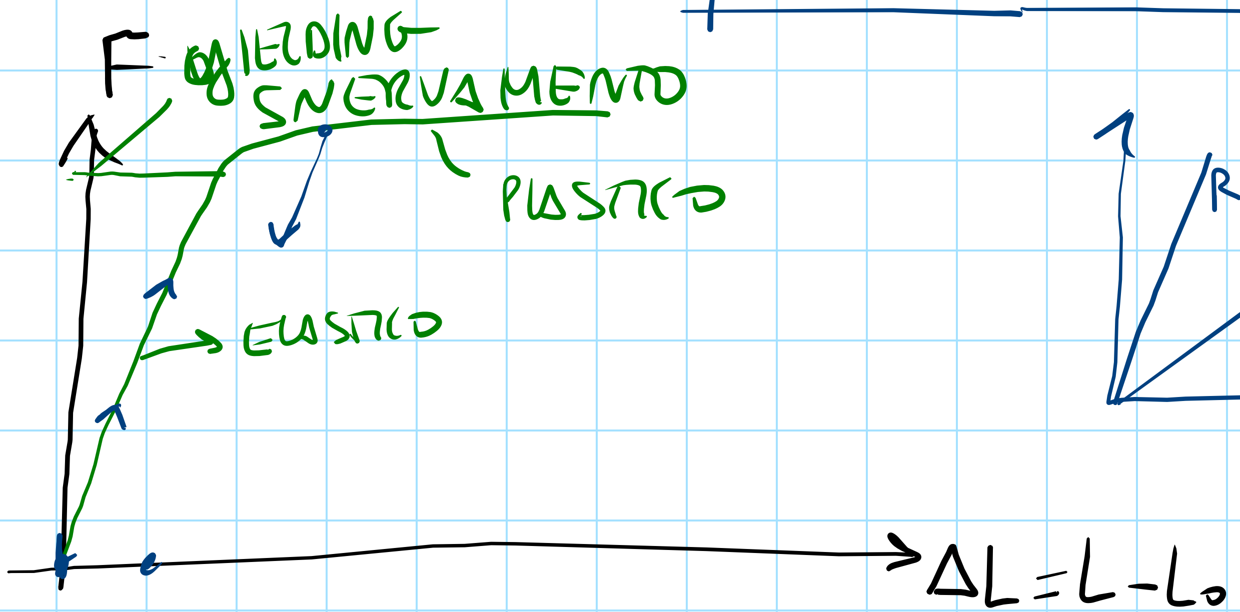
SOLID MECHANICS: EQUILIBRIUM, STRAIN/DEFORMATION, CONSTITUTIVE BEHAVIOUR OF SOLIDS SUBJECTED TO "EXTERNAL" ACTIONS

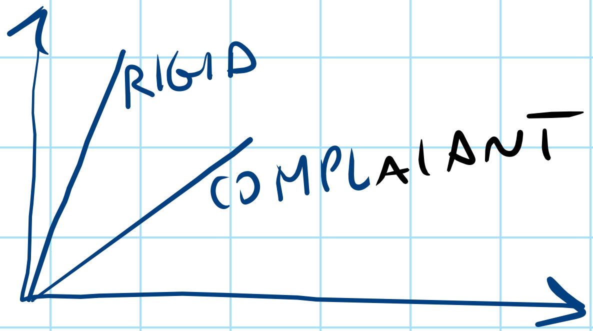
• MAIN TEST:



UNIAXIAL TENSION TEST





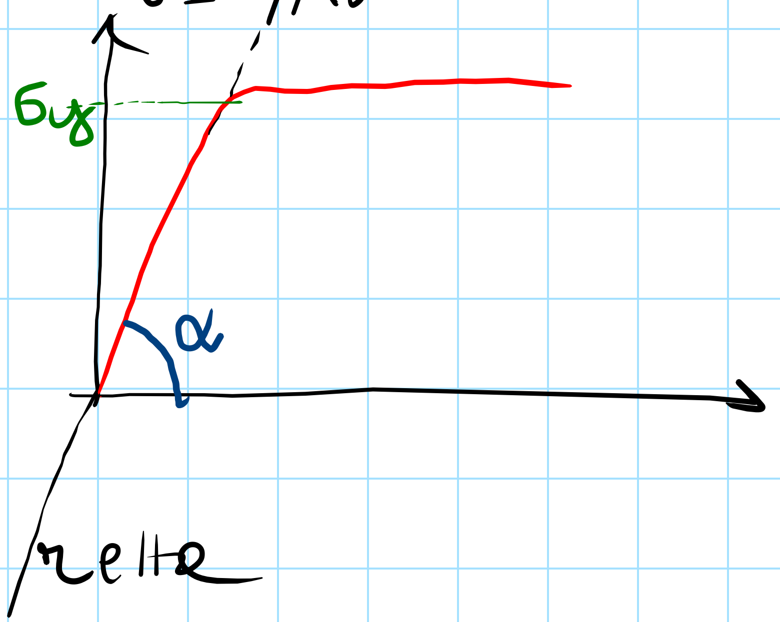


$A, L = f(F)$

$L > L_0$

$A < A_0$

$\sigma = F/A_0$ = MEAN STRESS $[F L^{-2}]$



$\frac{\Delta L}{L_0}$: STRAIN [-]

ε

$E = tg\,\alpha$: YOUNG MODULUS

ELASTIC MODULUS

$[E] = \left[\frac{F}{L^2}\right]$

$E \sim$ MPa, GPa

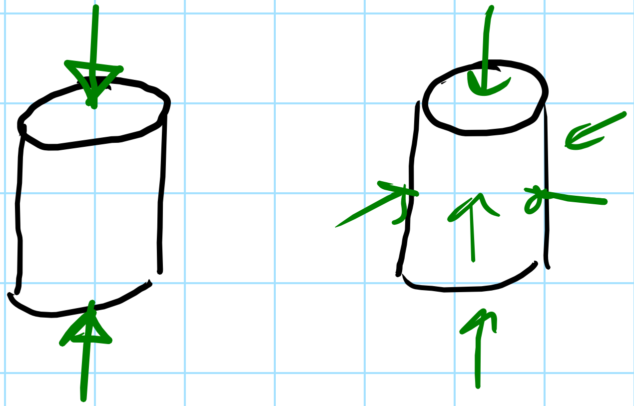
eq. retta : $\sigma = E\varepsilon$: LEGGE DI HOOKE (UNIAXIAL TENSION)

STEEL : $E = 210$ GPa

ALUMINUM : $= 70$ GPa

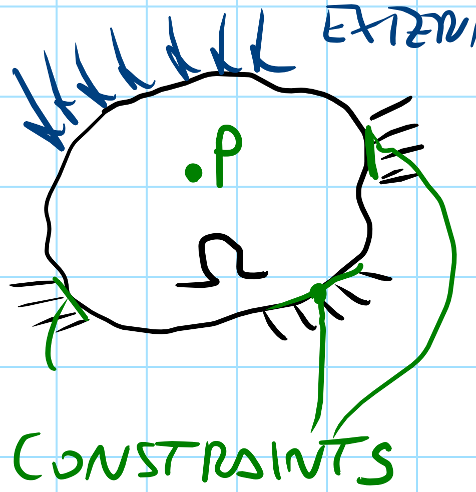
POLYURETHANE : $E = 10$ MPa

NOTE THAT FOR OTHER TYPES OF MATERIALS (I.E. BRITTLE MATERIALS SUCH AS CONCRETE, BRICKS, SOIL) THE UNIAXIAL TEST IS NOT USEFUL BECAUSE THOSE MATERIALS RESIST MAINLY UNDER COMPRESSION. THEN OTHER TEST METHODS ARE REQUIRED:



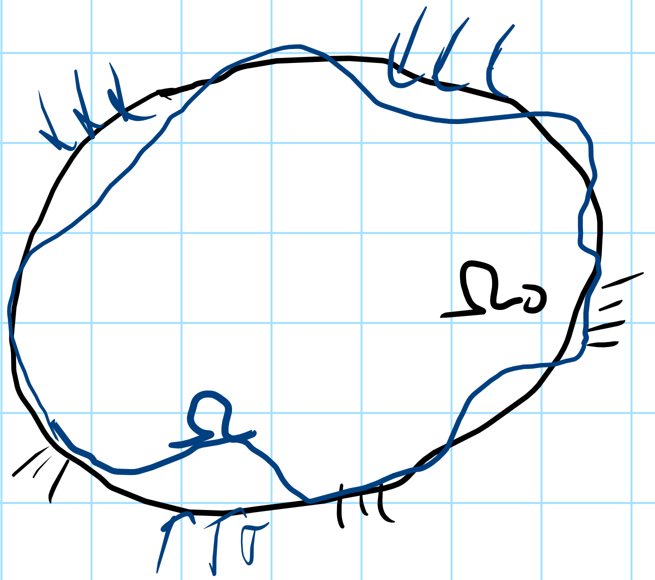
$\longrightarrow$ EXPERIMENTAL METHODS $\longrightarrow$ VALUE OF E.

IN A COMPLEX SIMULATION OF A 3D ELASTIC BODY:



GOAL: TO OBTAIN THE STATE OF STRESS AND STRAIN IN A GENERIC POINT $P \in \Omega$

HYPOTHESIS: WHEN CONSIDERING EQUILIBRIUM, WE ASSUME THE CURRENT CONFIGURATION COINCIDENT WITH THE INITIAL CONFIGURATION (WITH A SMALL ERROR).



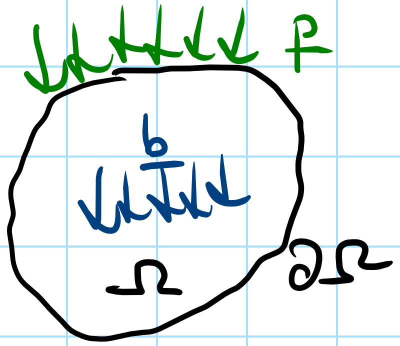
Ω_0: INITIAL CONF. (BODY AT REST, NO EXT. FORCES)

Ω: CURRENT CONF. (EFFECT OF EXT. FORCES)

ASSUME: $\Omega_0 \simeq \Omega$

FUNDAMENTALS OF STRESS THEORY (TEORIA DELLA TENSIONE)

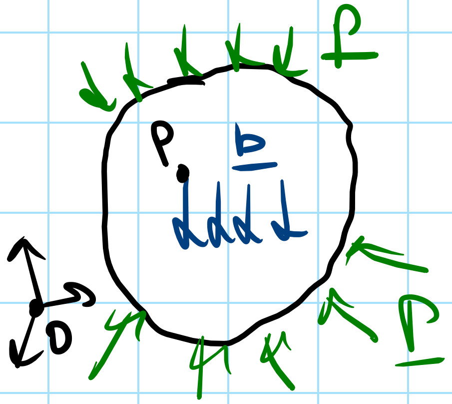
- TYPES OF EXTERNAL FORCES



$\underline{b}$; BODY FORCES (FORCE / UNIT VOLUME), $P \in \Omega$

$\underline{p}$: SURFACE FORCES (FORCE/UNIT SURFACE), $P \in \partial\Omega$
CONTACT FORCES

- EQUILIBRIUM OF A BODY Ω



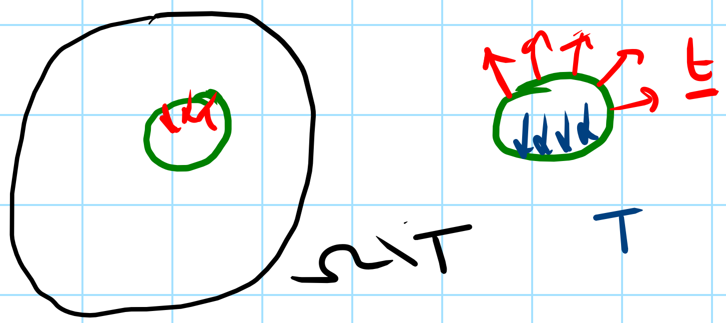
Ω IN EQUILIBRIUM

$$\int_{\Omega} \underline{b}\, dV + \int_{\partial\Omega} \underline{p}\, dS = \underline{0}$$ FORCE BALANCE

$$\int_{\Omega} OP \times \underline{b}\, dV + \int_{\partial\Omega} OP \times \underline{p}\, dS = \underline{0}$$ MOMENTUM BALANCE

TO BUILD A STRESS THEORY WE HAVE ALSO TO POSTULATE THE FOLLOWING:

EQUILIBRIUM MUST BE SATISFIED FOR ALL PARTS OF BODY Ω.



$\underline{t}$: CONTACT FORCES (INTERNAL)

$$\int_T \underline{b}\, dV + \int_{\partial T} \underline{t}\, dS = \underline{0} \qquad (\forall T)$$

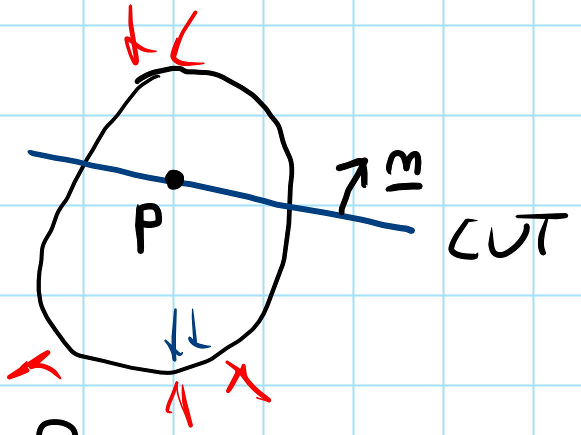
(... MOMENTA)

BUT ALSO EQUIL. IS STILL VALID ($\Omega \setminus T$)

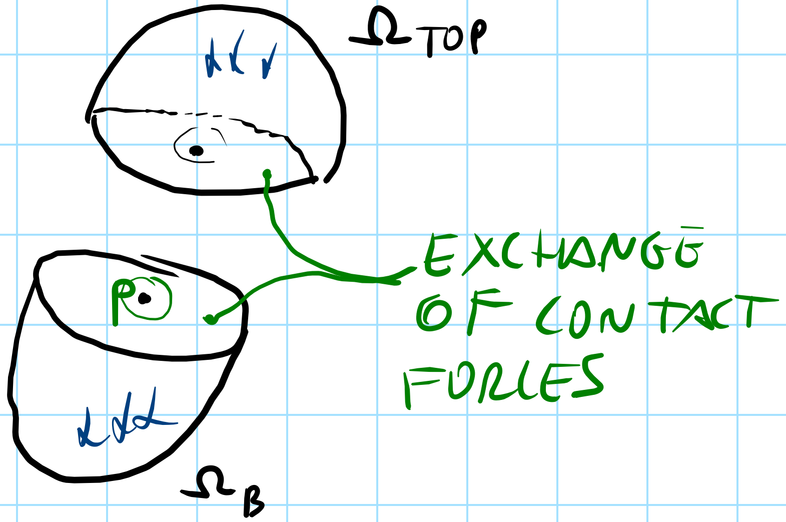
NOTE $\underline{t}(P)$ ARE FORCE/UNIT SURFACE [$\underline{t} \sim$ MPa], THEY ARE ALSO CALLED "TRACTIONS" (TENSION)

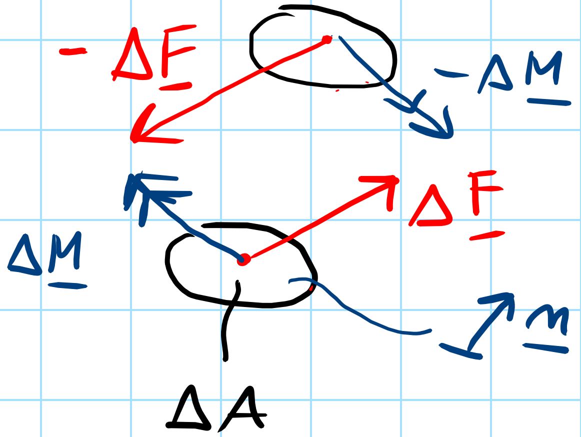
AND ARE AN UNKNOWN OF THE MODELING PROBLEM!

LOCAL ANALYSIS OF STRESS



Ω IN EQUIL





NEIGHBOURHOOD OF P

AREA ΔA

OUTWARD UNIT NORMAL

BOTH Ω_{TOP}, Ω_B IN EQUILIBRIUM FOR THE POSTULATE

$$\underline{t}(P,\underline{m}) = \lim_{\Delta A \to 0} \frac{\Delta \underline{F}}{\Delta A}$$ TRACTION VECTOR

$$\lim_{\Delta A \to 0} \frac{\Delta \underline{M}}{\Delta A} = \underline{0}$$ MOMENT-TRACTION VECTOR NULL

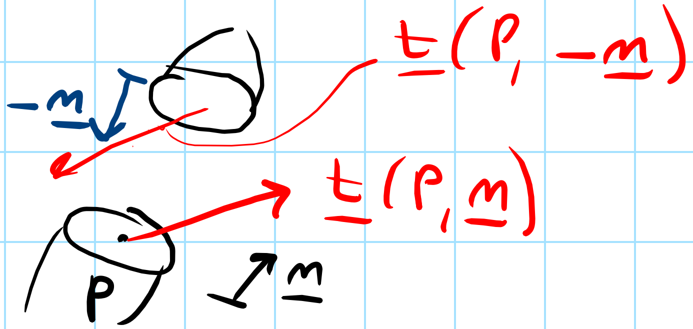
CAUCHY'S SIMPLE MATERIALS (1827)

A THEORY WHERE

$$\lim_{\Delta A \to 0} \frac{\Delta \underline{M}}{\Delta A} = \underline{m}(P,m)$$ IS DUE TO COSSERAT BROTHERS (1909)

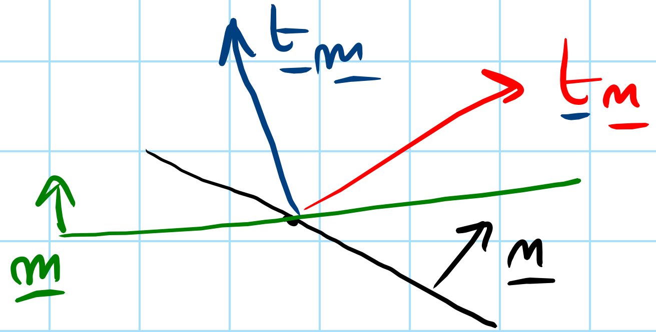
(MICROSTRUCTURED MATERIALS)

OBS. THAT $\underline{t}(P,-\underline{n}) = -\underline{t}(P,\underline{n})$

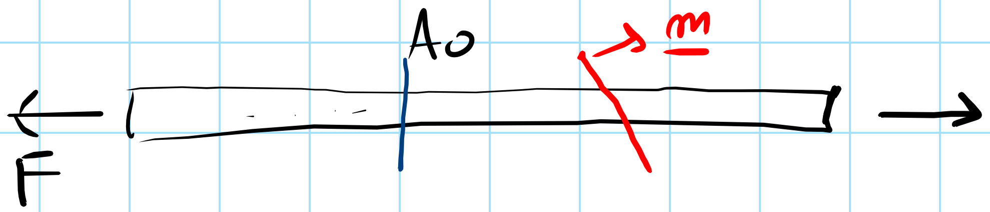


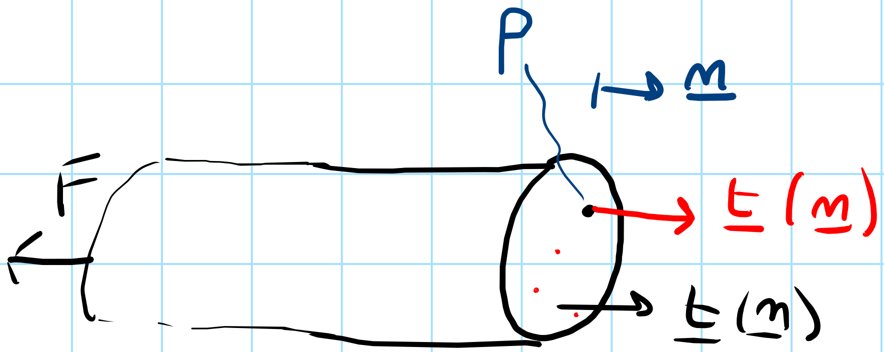
VERY IMPORTANT:

$\underline{t}(P,\underline{n}) \neq \underline{t}(P,\underline{m})$



EX



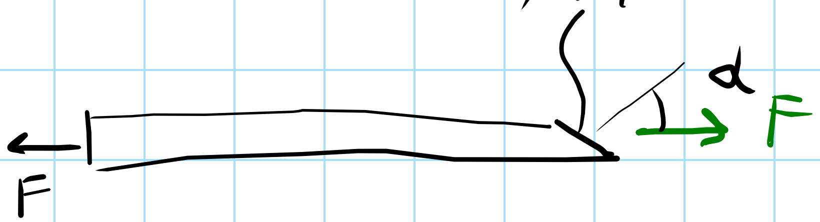


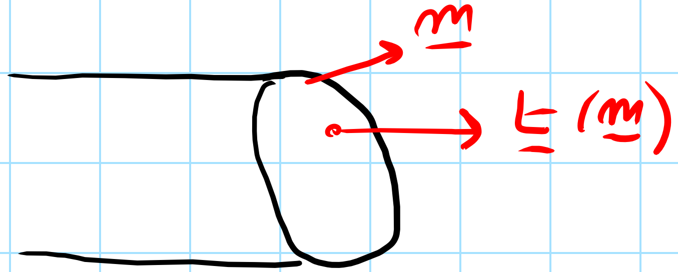
$\underline{t}(\underline{n}) = \frac{F}{A_0}\underline{n}$

BALANCE ALONG $\underline{n}$:

$$-F\underline{n} + \int_{A_0} \frac{F}{A_0}\underline{n}\, dS = \left(-F + \frac{F}{A_0}\int_{A_0} dS\right)\underline{n} = \underline{0}$$

$\tilde{A} \neq A_0$





$|\underline{t}(\underline{m})| = \frac{F}{\tilde{A}}\underline{m}$

$\tilde{A} \neq A$ $\quad |\underline{t}(\underline{n})| \neq |\underline{t}(\underline{m})|$ $\quad F = \int_{\tilde{A}} \frac{F}{\tilde{A}}\, dS$ $\quad A = \tilde{A}\cos\alpha$