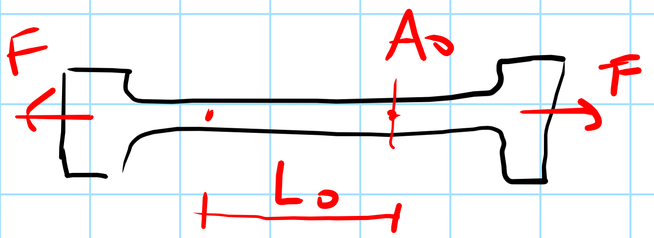
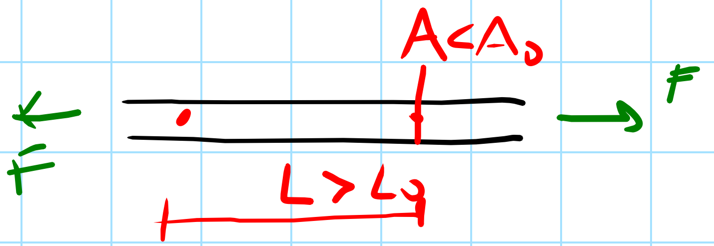
KINEMATICS: STRAIN, DEFORMATION

DEFORMAZIONE



UNIAXIAL STRESS (STEEL)

THE SPECIMEN ELONGATES



$$\sigma = \frac{F}{A_0} \quad ; \quad \varepsilon = \frac{\Delta L}{L_0} = \frac{L-L_0}{L_0}$$: LONGITUDINAL STRAIN [-]

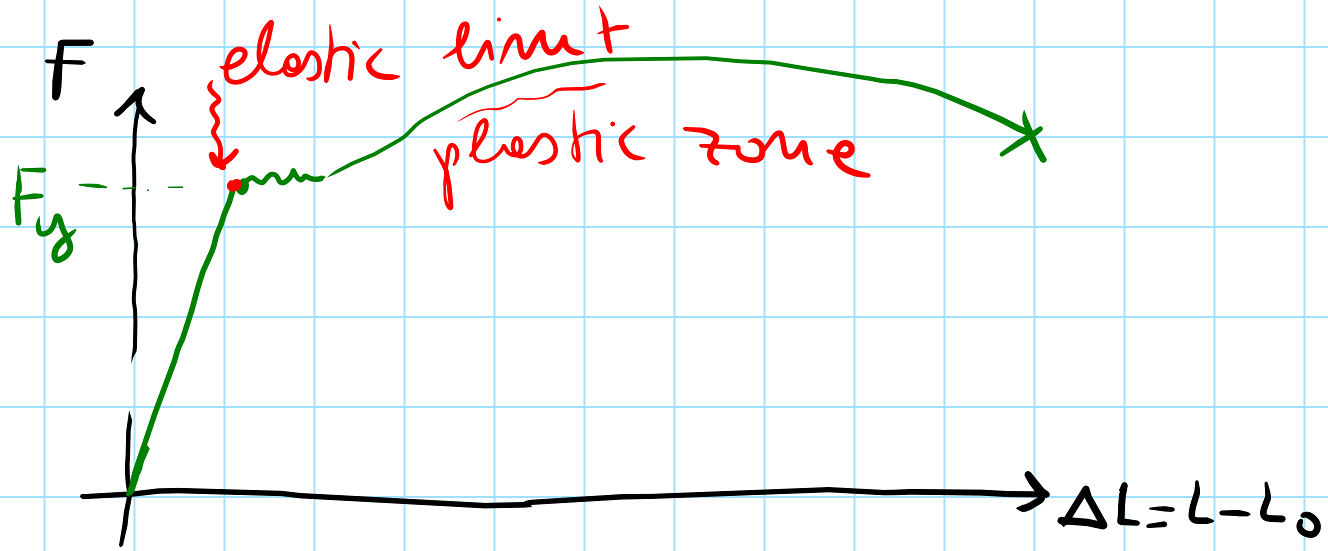
UNIAXIAL STRESS

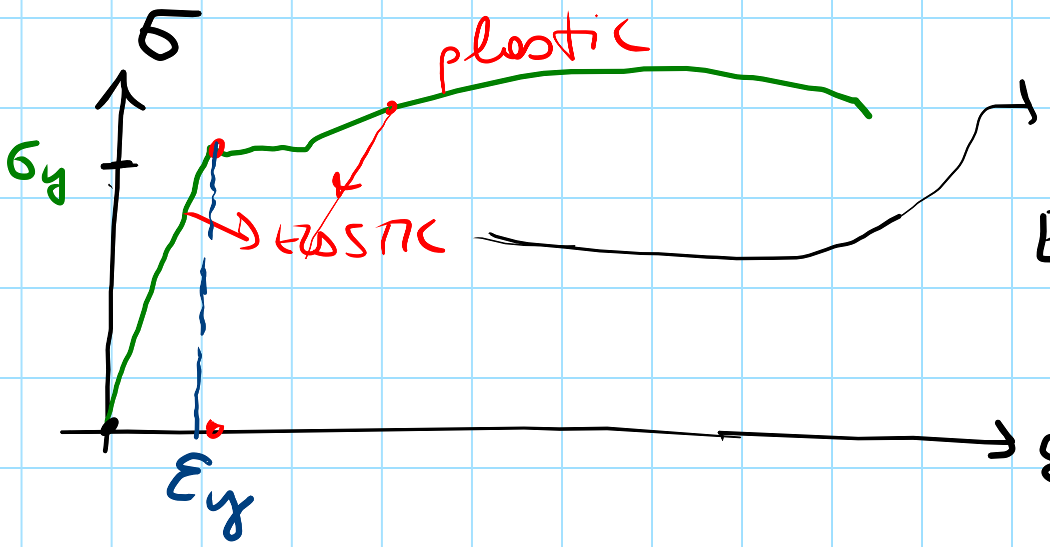
STEEL: $\sigma_y = 300$ MPa

$E \simeq 200$ GPa

ε_y ?

$$\varepsilon_y \simeq \frac{300}{200 \cdot 10^3} = 1.5 \cdot 10^{-3} = 1.5 ‰$$





$\sigma = E\,\varepsilon$

E: YOUNG MODULUS

ELASTIC MODULUS

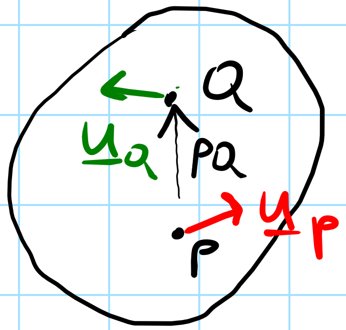
E: STEEL: 210 GPa , CONCRETE: 20 GPa

ALUMINIUM: 70 GPa

MORE ABOUT STRAIN (3D ANALYSIS)

IMPORTANT: RIGID-BODY MOTION OF A BODY

$\Rightarrow$ ABSENSE OF STRAIN

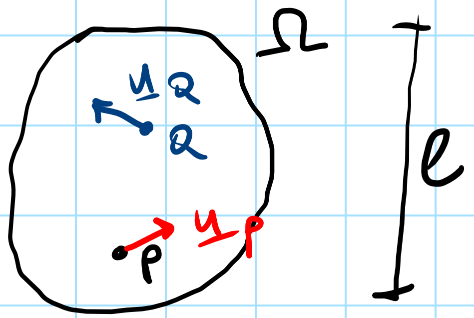


$$\underline{u}_Q = \underline{u}_P + \underset{\sim}{W}\, PQ$$

SKEW-SYMMETRIC

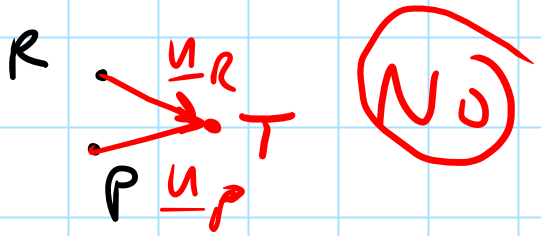
$$\underset{\sim}{W} = -\underset{\sim}{W}^T$$

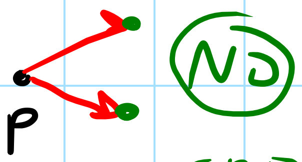
HYPOTHESES OF ANALYSIS OF STRAIN



- SMALL DISPLACEMENTS, $|\underline{u}_P| \ll \ell \;, \forall P \in \Omega$
- " DISPLACEMENT GRADIENT, $|\nabla \underline{u}_P| \ll 1 \;, \forall P \in \Omega$
- FUNCTION $\underline{u}_P\ (\forall P)$ IS 1 TO 1:

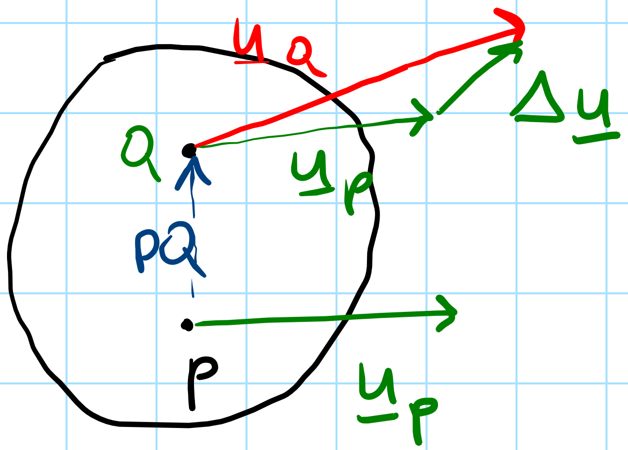
NEIGHBOURHOOD OF P





CREATION OF A CRACK

ANALYSIS OF STRAIN AT A POINT $P \in \Omega$



$$\underline{u}_Q = \underline{u}_P + \Delta \underline{u} \longrightarrow \text{TAYLOR EXPANS.}$$

$$\underline{u}_Q = \underline{u}_P + \underbrace{\nabla \underline{u}(P)}_{\substack{\text{TENSOR} \\ \text{(DISPL. GRADIENT)}}} \; \underbrace{PQ}_{\text{VECTOR}} + o(|PQ|)$$

$$\underline{u}_Q \simeq \underline{u}_P + \nabla \underline{u}(P)\, PQ = \underline{u}_P + (\overset{\text{SYMM.}}{\underset{\sim}{\varepsilon}} + \underset{\text{SKEW-SYMM}}{\underset{\sim}{W}})_P\, PQ = \underline{u}_P + \underbrace{\underset{\sim}{\varepsilon}\, PQ}_{\substack{\text{STRAIN} \\ \text{MOTION}}} + \underbrace{\underset{\sim}{W}\, PQ}_{\text{RIGID-BODY MOTION}}$$

$$\underset{\sim}{\varepsilon} = \frac{1}{2}\left(\nabla \underline{u} + \nabla \underline{u}^T\right) \qquad (\underset{\sim}{\varepsilon} = \underset{\sim}{\varepsilon}^T)$$

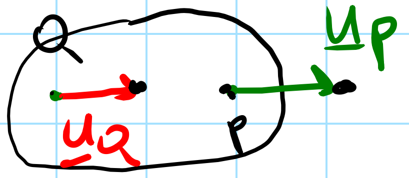
$$\underset{\sim}{W} = \frac{1}{2}\left(\nabla \underline{u} - \nabla \underline{u}^T\right) \qquad (\underset{\sim}{W} = -\underset{\sim}{W}^T)$$

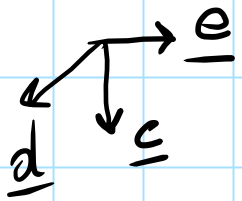
$\underset{\sim}{\varepsilon}$: STRAIN TENSOR ($= \underset{\sim}{0}$ FOR RIGID-BODY MOTION!)

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \longrightarrow \begin{matrix} \varepsilon_{11} = u_{1,1} \\ \varepsilon_{22} = u_{2,2} \end{matrix}, \quad \varepsilon_{12} = \frac{1}{2}\left(u_{1,2} + u_{2,1}\right)$$

MEANING OF THE COMPONENTS OF $\underset{\sim}{\varepsilon}$

– DIAGONAL OF $\underset{\sim}{\varepsilon}$





$\underline{u}_P = 0.5\,\underline{e}$

$\underline{u}_Q = 0.2\,\underline{e}$

$PQ = -20\,\underline{e}$

$$\underset{\sim}{\varepsilon} = \varepsilon_{11}\,\underline{e}\otimes\underline{e} + \varepsilon_{12}\,\underline{e}\otimes\underline{c} + \varepsilon_{13}\,\underline{e}\otimes\underline{d} + \dots$$

$$(*)\quad \underline{u}_Q - \underline{u}_P = \underset{\sim}{\varepsilon}\, PQ$$

$$\underline{v} = v_1\,\underline{e} + v_2\,\underline{c} + v_3\,\underline{d}$$

$\{\underline{e},\underline{c},\underline{d}\}$ ORTHONORMAL BASIS

$$(*):\quad 0.2\,\underline{e} - 0.5\,\underline{e} = \underset{\sim}{\varepsilon}\,\underline{e}\,(-20)$$

$$\underset{\sim}{\varepsilon}\,\underline{e} = \varepsilon_{11}\,\underline{e}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ & & \end{bmatrix}$$

$$\underbrace{(0.2 - 0.5)}\,\underline{e} = \underbrace{\varepsilon_{11}\,(-20)}\,\underline{e}$$

ALTERNATIVE

$$\varepsilon_e = \frac{\Delta L}{L_0} = \frac{20.3 - 20}{20} = \frac{0.3}{20} = 0.015 \quad \text{(ELONGATION)}$$

$$\varepsilon_{11} = \frac{-0.3}{-20} = +0.015$$

ELEMENTS ON THE DIAGONAL OF $\underset{\sim}{\varepsilon}$:

LONGITUDINAL STRAINS

$\varepsilon_{11} > 0$: ELONGATION

$\varepsilon_{11} < 0$: CONTRACTION

— OFF-DIAGONAL TERMS OF $\underset{\sim}{\varepsilon}$

EX: (*)

$$\begin{cases} u_1(x_1,x_2,x_3) = \dfrac{\gamma_{12}}{2} x_2 \\ u_2(\qquad) = \dfrac{\gamma_{12}}{2} x_1 \\ u_3(\qquad) = 0 \end{cases} \qquad (\gamma_{12} << 1,\ \gamma_{12} > 0)$$

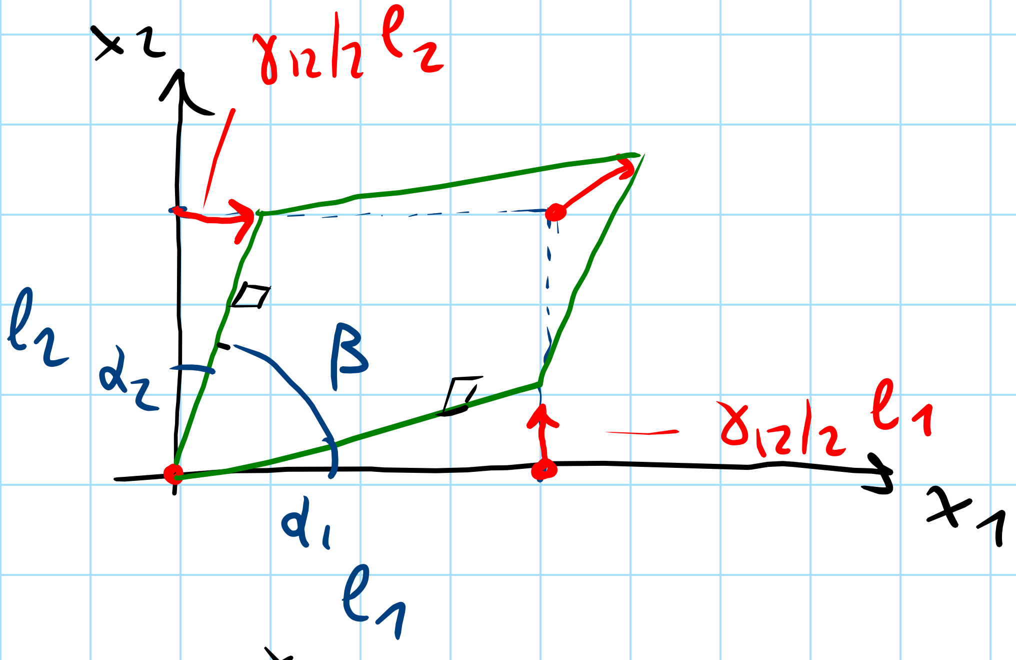
PURE SHEAR DEFORMATION

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} 0 & \gamma_{12}/2 & 0 \\ \dfrac{\gamma_{12}}{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \neq f(x_1 x_2 x_3)$$

HOMOGENEOUS STRAIN

$$[\underset{\sim}{W}] = [\underset{\sim}{0}]$$



$$\alpha_1 = \alpha_2 = \frac{\gamma_{12}}{2}$$

γ_{12} GOVERNS THE CHANGE IN ANGLES OF AN INITIAL PARALLELEPIPED SHAPE IN THE NEIGHBOURHOOD OF THE POINT P.

$$\frac{\pi}{2} = \beta + \alpha_1 + \alpha_2 \rightarrow \frac{\pi}{2} = \beta + 2\alpha \rightarrow \frac{\pi}{2} = \beta + \gamma_{12}$$

$$\gamma_{12} = \frac{\pi}{2} - \beta$$

γ_{12}: CHANGE IN SHAPE (SHEAR STRAIN) (SCORRIMENTO)

EX $\quad u_1 = 0.005\, x_2$, $\quad u_2 = 0.005\, x_1$ $\rightarrow \gamma_{12} = 0.01$, $\beta = \frac{\pi}{2} - 0.01 \Rightarrow \beta = 89°.43$

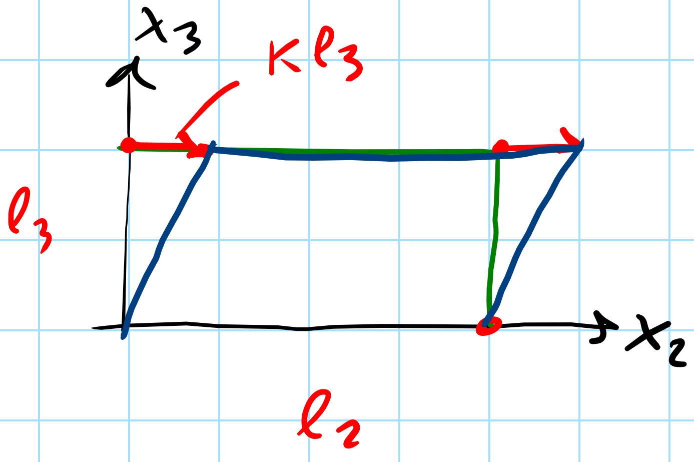
EX: SIMPLE SHEAR

$$\begin{cases} u_1 = 0 \\ u_2 = K x_3 \\ u_3 = 0 \end{cases} \qquad (0 < K \ll 1)$$

$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}K \\ 0 & \frac{1}{2}K & 0 \end{bmatrix} \; ; \; [\underset{\sim}{W}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}K \\ 0 & -\frac{1}{2}K & 0 \end{bmatrix}$$

RIGID BODY!

$$\underline{u}_Q - \underline{u}_P = \underbrace{\underset{\sim}{\varepsilon}\, \underline{PQ}}_{①} + \underbrace{\underset{\sim}{W}\, \underline{PQ}}_{②}$$



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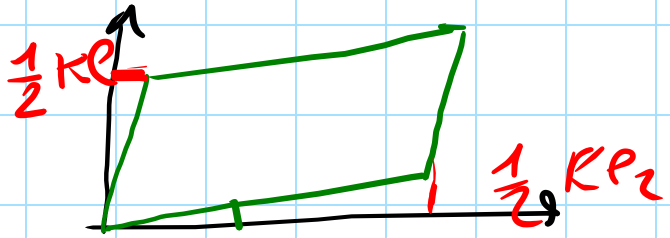
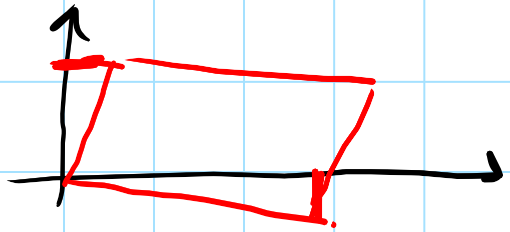


① $\underset{\sim}{\varepsilon}\, PQ$

+



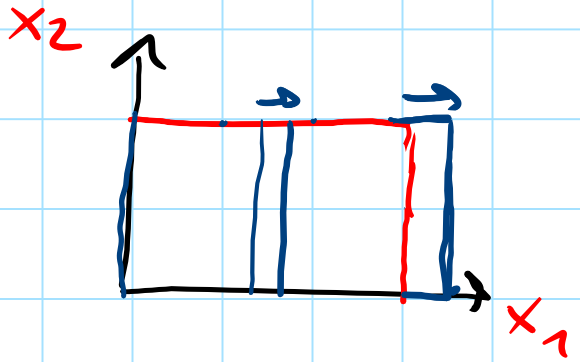
② $\underset{\sim}{W}\, PQ$

THE SIMPLE SHEAR DEFORMATION IS COMPOSED OF A "STRAIN" PART AND A "RIGID-BODY" PART.

IT REPRESENTS A GENERIC DEFORMATION MAP.

OTHER RELEVANT DEFORMATION / STRAIN STATES

- SIMPLE EXTENSION / ELONGATION



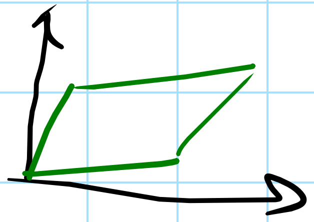
$$[\underset{\sim}{\varepsilon}] = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

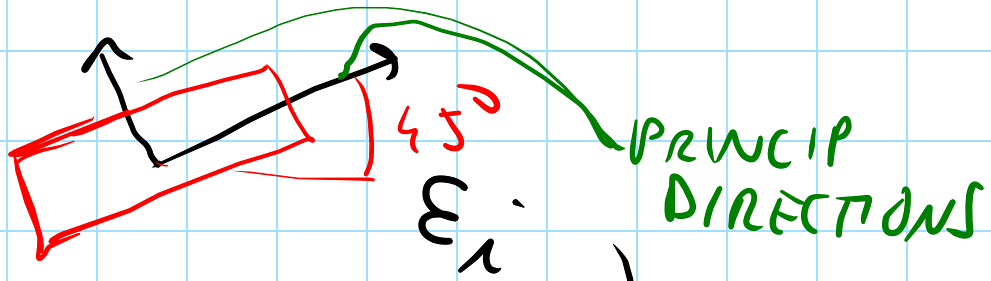
- PURE SHEAR $[\underset{\sim}{\varepsilon}] = \begin{bmatrix} 0 & * & 0 \\ * & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$*: \neq 0$

- PLANE STRAIN: $[\underset{\sim}{\varepsilon}] = \begin{bmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 0 \end{bmatrix}$
(IN x_1-x_2)
(DEFORM. PIANA)

LEX: PURE SHEAR





WE DO NOT TALK ABOUT "PRINCIPAL STRAINS" (EIGENVALUES OF $\underset{\sim}{\varepsilon}$)
AND "PRINCIPAL DIRECTIONS OF STRAIN" (EIGENVECTORS OF $\underset{\sim}{\varepsilon}$) $\underline{u}_i$

HOWEVER, DUE TO $\underset{\sim}{\varepsilon} = \underset{\sim}{\varepsilon}^T \Rightarrow \varepsilon_i$ and $\underline{u}_i$ ARE MUTUALLY ORTHOGONAL
$\in \mathbb{R}$