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Corso di Termofluidodinamica Computazionale

Homework No. 3
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Problem 3

Problem description

Consider the simple 2D grid constituted by triangular equilateral cells illustrated in figure 1, taken directly from Chapt. 9 in [1]. The coordinates of the mesh nodes (vertices) are provided in the associated file `grid.nod` and are also given, for convenience, in table 1.

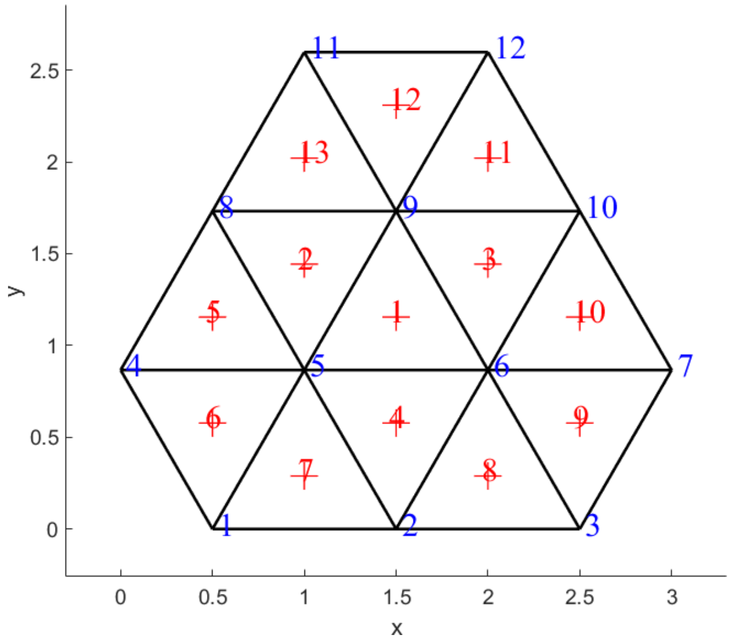

Figure 1: Grid for problem 3.

Node	x	y
1	0.5	0.0
2	1.5	0.0
3	2.5	0.0
4	0.0	0.8660254
5	1.0	0.8660254
6	2.0	0.8660254
7	3.0	0.8660254
8	0.5	1.7320508
9	1.5	1.7320508
10	2.5	1.7320508
11	1.0	2.5980762
12	2.0	2.5980762

Table 1: Coordinates of the nodes (vertices) of the grid of figure 1.

The cell definitions are stored in the cell connectivity matrix. This is a matrix of node (vertex) numbers where each row of the matrix contains the connectivity - the nodes - of the corresponding cell. It is given in the file `grid.ele` and also in table 2.

Cell	Nodes		
1	5	6	9
2	5	9	8
3	6	10	9
4	2	6	5
5	4	5	8
6	1	5	4
7	1	2	5
8	2	3	6
9	3	7	6
10	6	7	10
11	9	10	12
12	9	12	11
13	8	9	11

Table 2: Cell connectivity for the grid of figure 1.

Question

Consider the three following cases of temperature distribution, and corresponding (exact) analytical expression of the gradient:

Linear

$$\begin{aligned} T(x,y) &= 100x + 30y + 5 \\ \nabla T(x,y) &= 100\,\mathbf{i} + 30\,\mathbf{j} \end{aligned}$$

Quadratic

$$\begin{aligned} T(x,y) &= 100\left(x^2 + y^2 + 1\right) \\ \nabla T(x,y) &= 200x\,\mathbf{i} + 200y\,\mathbf{j} \end{aligned}$$

Cubic

$$\begin{aligned} T(x,y) &= 100\left(x^3 + y^2 + xy + 5\right) \\ \nabla T(x,y) &= 100\left(3x^2 + y\right)\mathbf{i} + 100\,(2y + x)\,\mathbf{j} \end{aligned}$$

Compute numerically, for the three temperature distributions, the temperature gradient at the centroid of cell *1* using the cell-based Gauss-Green method [1] and compare the results with the exact gradient.

For the calculations develop a code/script (MATLAB, OCTAVE, Scilab or Python recommended) accordingly to the requirements described in problem 4 of Homework 2015-16.

[1] For this problem, since the cell faces are midway between the centroids of the two attached cells, using weighted interpolation has no effect.

References

[1] F. Moukalled, L. Mangani, M. Darwish, *The Finite Volume Method in Computational Fluid Dynamics: An Advanced Introduction With OpenFOAM and Matlab*, Springer-Verlag, 2015.