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**Homework No. 2  
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## 1 Problems

- Following *Homework 1 - AA, 2019-20*, second case, consider a longitudinal, plain fin of rectangular profile, as shown in figure 1, which is cooled by both convection and radiation. It has a thermal conductivity value of  $k = 50 \text{ W/(m K)}$ , a thickness  $t = 1 \text{ mm}$ , a length  $L = 20 \text{ mm}$  and it extends indefinitely along the third direction. The convective heat transfer coefficient is  $h = 50 \text{ W/(m}^2 \text{ K)}$  and the temperature of the surrounding fluid is  $T_\infty = 25 \text{ }^\circ\text{C}$ . Also in this case the radiation heat transfer can be accounted for assuming that the fin surface is gray-diffuse, with a global emissivity  $\epsilon = 0.7$ , that the fluid is not participating nor emitting, and that the temperature of the surroundings is  $T_s = 15^\circ\text{C}$ . The temperature of the base of the fin is maintained at a temperature  $T_b = 400 \text{ }^\circ\text{C}$ , while the tip of the fin can be assumed perfectly insulated.

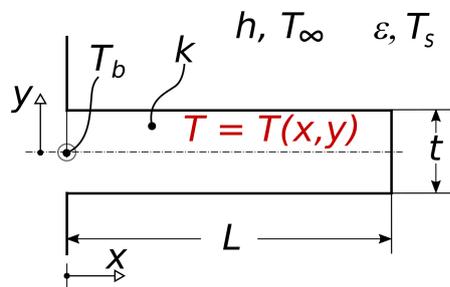


Figure 1: Two-dimensional longitudinal fin of rectangular profile.

In this case, disregard the usual assumption of 1D temperature distribution (see [1, 2]), i.e.

$$T \approx T(x)$$

and consider a full 2D temperature distribution [3]:

$$T = T(x, y)$$

Using the Finite Volume method, develop a 2D steady numerical model for the fin and, using an *adequate* number of cells, compute the heat transfer for unit width of the fin  $q'_{num2D}$  [W/m]. Compare the result with that obtained with the 1D model of *Homework 1*. What is the % error using the 1D assumption ?

- Repeat the same analysis of problem 1, but considering the lower aspect-ratio  $L/t$  fin of *Homework 1*, e.g. thickness  $t = 10 \text{ mm}$  and length  $L = 20 \text{ mm}$  and again compare the results with the 1D model. What is the % error using the 1D assumption in this second case? It is lower or higher ? Why ?

Plot a contour map of the temperature field.

**TIP**

Also in this case, it is suggested to express the heat transfer by radiation in a form similar to that of convection heat transfer:

$$\begin{aligned} q'_{rad} &= \epsilon \sigma A_f (T_f^4 - T_s^4) \\ &= A_f h_r (T_f - T_s) \end{aligned}$$

where  $T_f$  is the (local) temperature of the fin surface,  $\sigma = 5.670367 \times 10^{-8} \text{ W}/(\text{m}^2 \text{ K}^4)$  is the Stefan-Boltzmann constant and

$$h_r = \epsilon \sigma (T_f^2 + T_s^2) (T_f + T_s)$$

is the *radiation heat transfer coefficient*. Since  $h_r$  depends on  $T_f$ , an iterative procedure, with a check of convergence, should be applied.

In this way, the total heat transfer from an elementary surface of the fin can be expressed as

$$q'_{tot} = q'_{conv} + q'_{rad} = A_f [h (T_f - T_\infty) + h_r (T_f - T_s)]$$

Again, proper attention must be paid to the unit of measure of temperature.

**References**

- [1] G. Comini, G. Cortella, *Fondamenti di trasmissione del calore*, 4a Ed., S.G.E. Editore, (2013).
- [2] F. P. Incropera, D. P. Dewitt, T. L. Bergman, A. S. Lavine, *Fundamentals of Heat and Mass Transfer*, 6th Ed., Wiley, (2007).
- [3] A. D. Kraus, A. Aziz, J. Welty, *Extended Surface Heat Transfer*, Wiley, (2001).